How to measure the distance to the Sun with a yardstick

ΣΑΒΒΑΣ ΝΕΣΣΕΡΗΣ - SAVVAS NESSERIS

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2nd Discovery Youngster Symposium

How to measure the distance to the Sun with a yardstick

1) How big is the Earth?

Ἐρατοσθένης (Eratosthenes) c. 276 BC – c. 195 BC

2) How far away is the Moon?

Ἀρίσταρχος (Aristarchos): 310 BC – ca. 230 BC
Ἵππαρχος (Hipparchos) c. 190 BC – c. 120 BC

3) What is the distance to the Sun?

Ἀρίσταρχος (Aristarchos): 310 BC – ca. 230 BC

Ἵππαρχος (Hipparchos) c. 190 BC – c. 120 BC

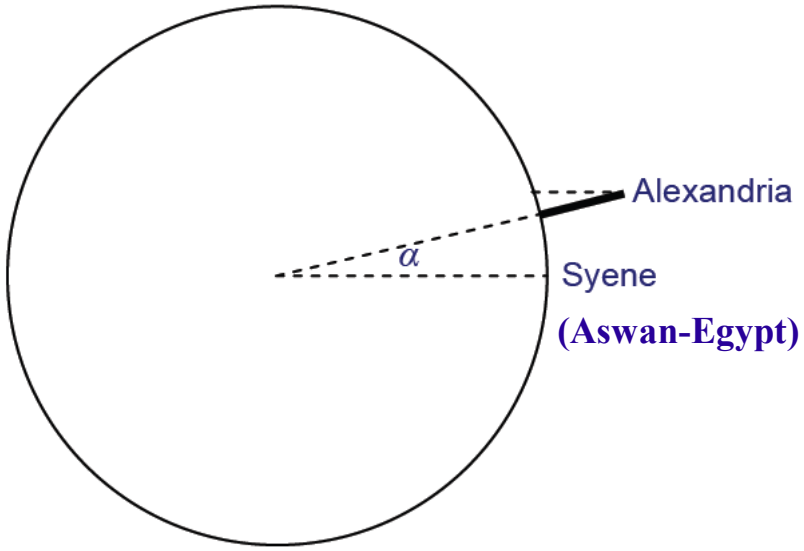
1) Measuring the Earth's diameter

Ἐρατοσθένης (Eratosthenes) ο Β

c. 276 BC – c. 195 BC

He was a mathematician, poet, philologist, competitive athlete, geographer, astronomer and music theorist. He founded the discipline of geography, of scientific chronology and he invented the leap day.

During the summer solstice:



$$\alpha \simeq \frac{2\pi}{50} \sim 7^o 12'$$

generic **Olympic**

$$AS = 5000 \text{ stadia}$$

$$\text{where stadion} = [157m, 176m, 185m]$$

Athenian

$$R_E = \frac{AS}{\alpha} = \frac{250000}{2\pi} st. =$$

$$= [6.25 \cdot 10^3 km, 7.36 \cdot 10^3 km]$$

2% **16%**

Real value: $R_E = 6.37 \cdot 10^3 km$

Get Directions | My Maps

Distance Measurement Tool

Click on the map to trace a path you want to measure.

Units:

(•) Metric ( ) English I'm feeling geeky

Total distance:
850.512 km

Delete last point | Reset

$$\text{stadion} = [157m, 176m, 185m]$$

10%

10% x 850km=85km!

Best guess:
Olympic stadium

Get Directions | My Maps

Distance Measurement Tool

Click on the map to trace a path you want to measure.

Units:

Metric English I'm feeling geeky

Total distance:
850.512 km

Delete last point | Reset

Units:
Ångström

Total distance:
8.50512e+15 Å

Units:
Light-year

Total distance:
8.98992e-11 ly

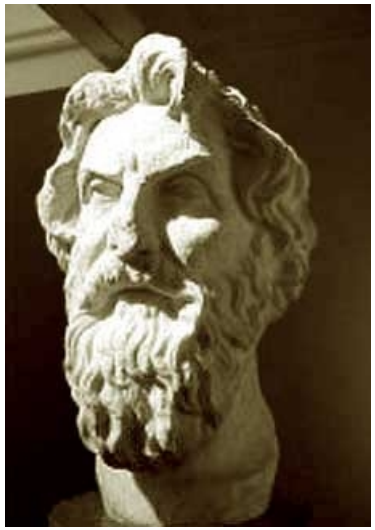
Units:
TeX point

Total distance:
2.41994e+9 pt

Units:
Olympic swimming pool

Total distance:
17010.2 pools

2) Measuring the distance to the Moon

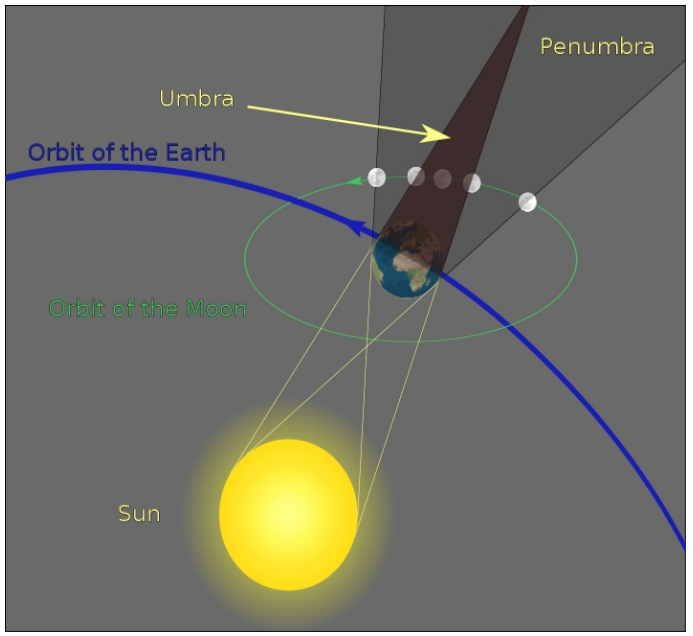


Ἀρίσταρχος (Aristarchos):

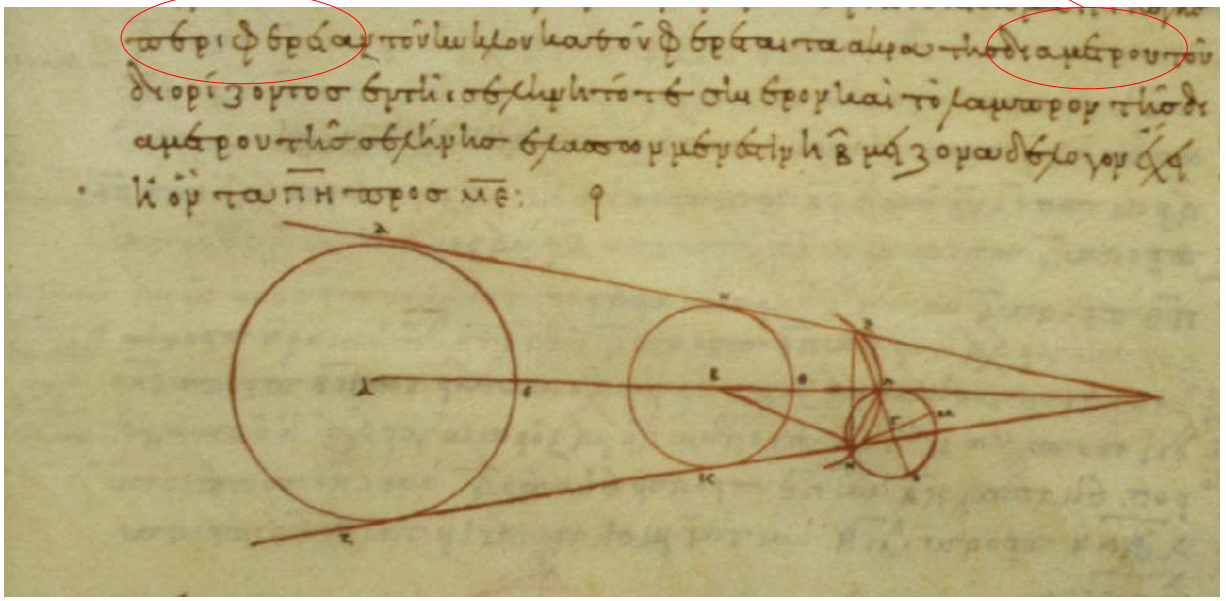
310 BC – ca. 230 BC

He was an astronomer and mathematician. He was the first to present the heliocentric model, which was "re-invented" 1800 years later by Copernicus.

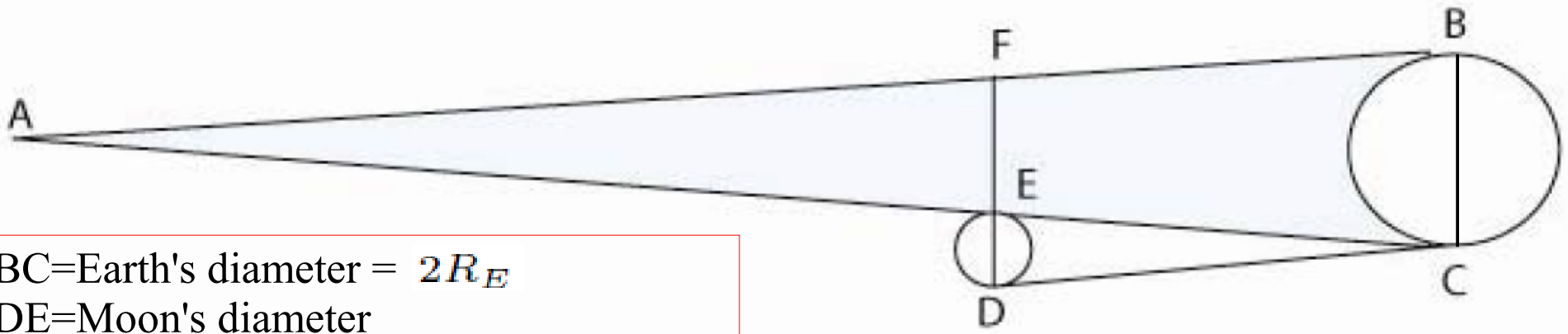
During a Lunar eclipse:



periphery diameter



2) Measuring the distance to the Moon



BC=Earth's diameter = $2R_E$
DE=Moon's diameter
θ=angular diameter of the Sun = $[0.5^o, 2^o]$
CE=Distance to the Moon

$$F\hat{A}E = E\hat{C}D$$
$$A\hat{E}F = D\hat{E}C$$
$$A\hat{F}E = E\hat{D}C$$

→ Triangles AFE and EDC are similar

From observations: $\frac{FE}{ED} = 2.5 \rightarrow \frac{AE}{EC} = 2.5 \rightarrow \frac{AC}{EC} = 3.5$

$$\theta \approx \frac{BC}{AC} = \frac{BC}{3.5EC} = \frac{2R_E}{3.5d_M} \rightarrow \frac{d_M}{R_E} = \frac{2}{3.5\theta}$$

2^o → 16.4 (Aris.)

0.5^o → 65.5 (Hip.)

Real value ~ 60

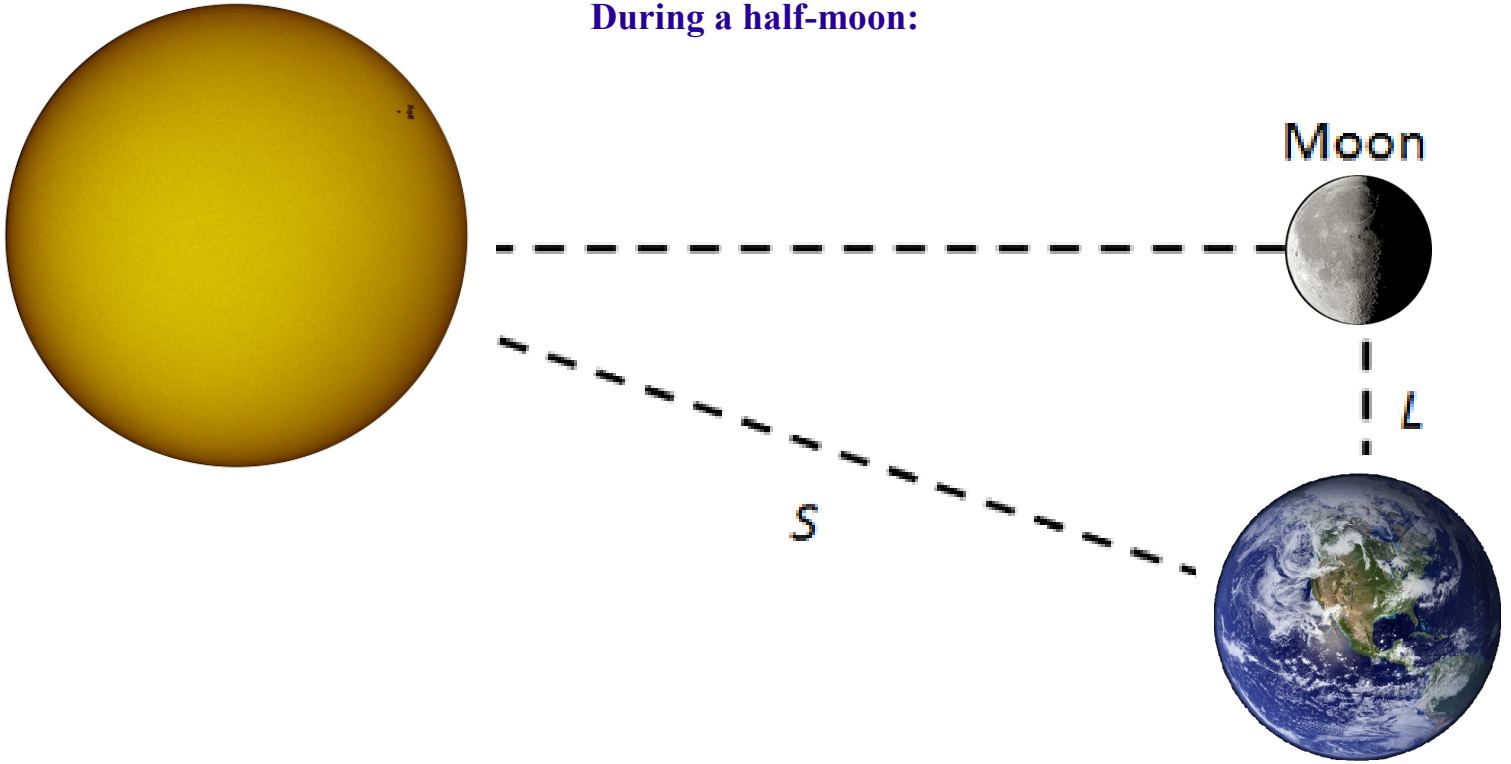
3) Measuring the distance to the Sun

Ἵππαρχος (Hipparchos)

c. 190 BC – c. 120 BC

He was an astronomer, geographer and mathematician and he invented trigonometry. He also developed a reliable method to predict solar eclipses, discovered the Earth's precession and compiled the first star catalog.

During a half-moon:



3) Measuring the distance to the Sun

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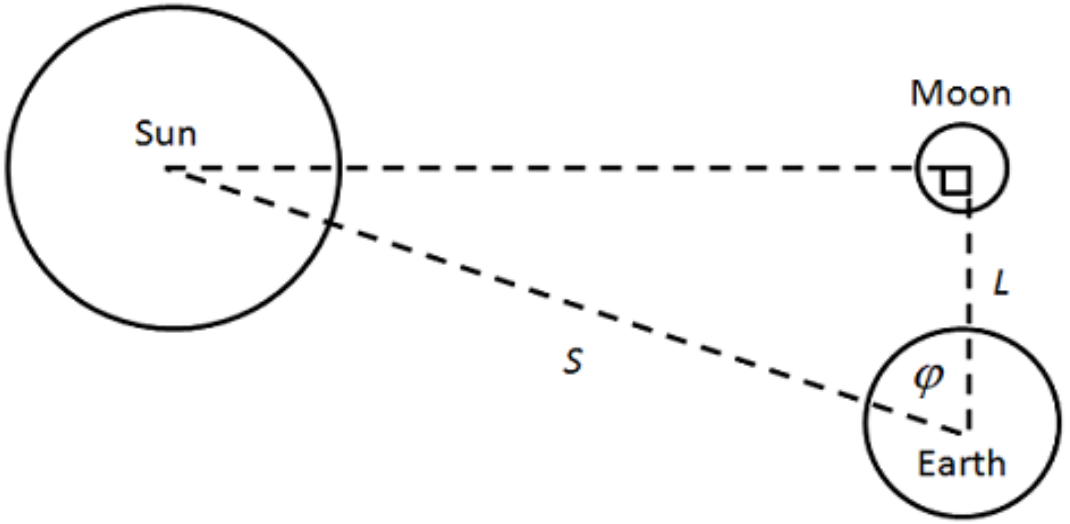


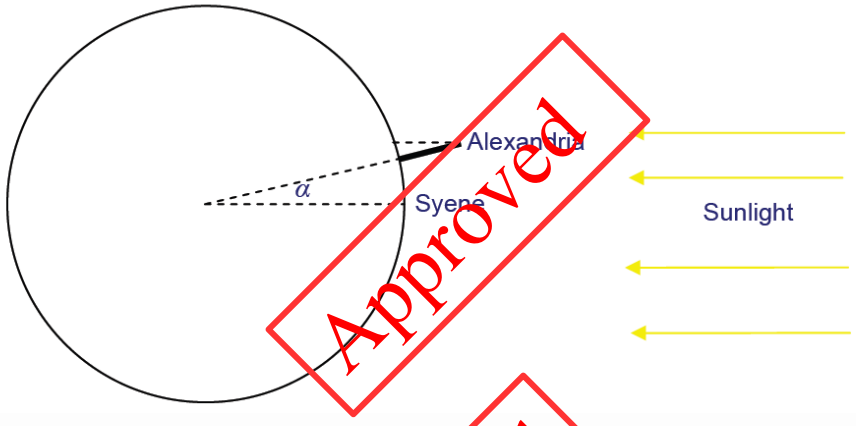
$$\frac{\pi}{2} - \phi \approx \frac{2\pi}{120} \simeq 3^o \Rightarrow 18 < \frac{S}{L} < 20$$

$$\frac{S}{L} = \frac{1}{cos(\phi)} \simeq 19 \quad \text{(Aris.)}$$

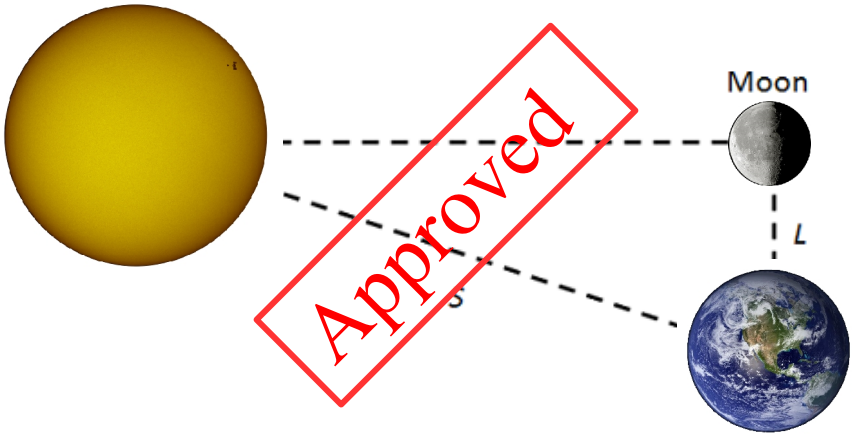
$$\text{(Hip.)} \quad \frac{\pi}{2} - \phi \approx 1^o \Rightarrow \frac{S}{L} = 57$$

$$\text{Real value: } \frac{S}{L} = [370 - 410]$$

4) Is this method "planet-independent"?



Approved



Approved

Failed

Final results:

1) How big is the Earth?

$$R_E = [6.25 \cdot 10^3 km, 7.36 \cdot 10^3 km]$$

2) How far away is the Moon?

$$\frac{d_M}{R_E} = [16.4, 65.5]$$

3) What is the distance to the Sun?

$$\frac{d_S}{d_M} = [19, 57]$$

Final results:

1) How big is the Earth?

$$R_E = [6.25 \cdot 10^3 km, 7.36 \cdot 10^3 km]$$

2) How far away is the Moon?

$$\frac{d_M}{R_E} = [16.4, 65.5]$$

3) What is the distance to the Sun?

$$\frac{d_S}{d_M} = [19, 57]$$

$$d_S = [2 \cdot 10^6 km, 2 \cdot 10^7 km]$$

O(100) O(10)

Real value: $d_S = 1.5 \cdot 10^8 km$