Lecture 4: Branes in Ten Dimensions

- Type IIA Branes

- Type IIB Branes

Massless N=IIA Representation

$$\{Q^{\pm}_{\alpha}, Q^{\pm}_{\beta}\} \quad = \quad \left(P^{\pm}\gamma^{\mu}\mathcal{C}^{-1}\right)_{\alpha\beta} P_{\mu}$$

$$\begin{aligned}
(\mathbf{8_v} + \mathbf{8_s})_{\mathrm{L}} \quad &\otimes \quad (\mathbf{8_v} + \mathbf{8_c})_{\mathrm{R}} \\
&= \quad \underbrace{\mathbf{35_v} + \mathbf{28} + \mathbf{1}}_{\mathrm{NS-NS}} + \underbrace{\mathbf{8_v} + \mathbf{56_v}}_{\mathrm{R-R}} \\
&\quad + \underbrace{\mathbf{56_s} + \mathbf{8_c}}_{\mathrm{NS-R}} + \underbrace{\mathbf{56_c} + \mathbf{8_s}}_{\mathrm{R-NS}} \\
&= \quad \mathbf{128} + \mathbf{128}
\end{aligned}$$

$$\begin{aligned}
\mathbf{44} \quad &\Longrightarrow \quad \mathbf{35_v} + \mathbf{8_v} + \mathbf{1} \\
\mathbf{84} \quad &\Longrightarrow \quad \mathbf{28} + \mathbf{56_v} \\
\mathbf{128} \quad &\Longrightarrow \quad \mathbf{56_s} + \mathbf{56_c} + \mathbf{8_s} + \mathbf{8_c}
\end{aligned}$$

Type IIA central charges

$$
\begin{aligned}
\{Q_\alpha, Q_\beta\} \quad = \quad & \left(\gamma^\mu \mathcal{C}^{-1}\right)_{\alpha\beta} P_\mu + \left(\gamma_{11}\mathcal{C}^{-1}\right)_{\alpha\beta} Z \\
& + \left(\gamma^\mu \gamma_{11}\mathcal{C}^{-1}\right)_{\alpha\beta} Z_\mu \\
& + \tfrac{1}{2!}\left(\gamma^{\mu\nu}\mathcal{C}^{-1}\right)_{\alpha\beta} Z_{\mu\nu} \\
& + \tfrac{1}{4!}\left(\gamma^{\mu_1\cdots\mu_4}\gamma_{11}\mathcal{C}^{-1}\right)_{\alpha\beta} Z_{\mu_1\cdots\mu_4} \\
& + \tfrac{1}{5!}\left(\gamma^{\mu_1\cdots\mu_5}\gamma_{11}\mathcal{C}^{-1}\right)_{\alpha\beta} Z_{\mu_1\cdots\mu_5}
\end{aligned}
$$

$$
\begin{aligned}
\tfrac{1}{2} \times 32 \times 33 \quad = \quad & 1 + 10 + 10 + 45 + 210 \\
& + 126^+ + 126^-
\end{aligned}
$$

Heterotic Automorphism

$$\{Q^+_\alpha\,,Q^+_\beta\} = \left(P^+\gamma^\mu\mathcal{C}^{-1}\right)_{\alpha\beta}(P+Z)_\mu + \tfrac{1}{5!}\left(P^+\gamma^{\mu_1\cdots\mu_5}\mathcal{C}^{-1}\right)_{\alpha\beta} Z^+_{\mu_1\cdots\mu_5}$$

$$\{Q^-_\alpha\,,Q^-_\beta\} = \left(P^-\gamma^\mu\mathcal{C}^{-1}\right)_{\alpha\beta}(P-Z)_\mu + \tfrac{1}{5!}\left(P^-\gamma^{\mu_1\cdots\mu_5}\mathcal{C}^{-1}\right)_{\alpha\beta} Z^-_{\mu_1\cdots\mu_5}$$

$$\{Q^+_\alpha\,,Q^-_\beta\} = \left(P^+\mathcal{C}^{-1}\right)_{\alpha\beta} Z + \tfrac{1}{2!}\left(P^+\gamma^{\mu\nu}\mathcal{C}^{-1}\right)_{\alpha\beta} Z_{\mu\nu} + \tfrac{1}{4!}\left(P^+\gamma^{\mu_1\cdots\mu_4}\mathcal{C}^{-1}\right)_{\alpha\beta} Z_{\mu_1\cdots\mu_4}$$

$$Q^\pm_\alpha \xrightarrow{(-)^{F^c_L}} \pm Q^\pm_\alpha$$

$$\{\mathrm{Z}\,,\mathrm{Z}_{\mu\nu}\,,\mathrm{Z}_{\mu_1\cdots\mu_4}\} \xrightarrow{(-)^{F^c_L}} -\{\mathrm{Z}\,,\mathrm{Z}_{\mu\nu}\,,\mathrm{Z}_{\mu_1\cdots\mu_4}\}$$

$$\boxed{(-)^{F^c_L}}$$

Green-Schwarz Lightcone Action

$$S_{\rm IIA} \ \sim \ \int d^2\xi \left\{ \partial_+ X^i \partial_- X^i - i S^{\dot a} \partial_- S^{\dot a} - i \tilde S^a \partial_+ \tilde S^a \right\}$$

with

$$X^i \ : \ \mathbf{8_v} \,, \qquad S^{\dot a} \ : \ \mathbf{8_c} \,, \qquad S^a \ : \ \mathbf{8_s}$$

$$(-)^{F^c_L} \ : \quad S^{\dot a} \to -S^{\dot a} \qquad \Rightarrow$$

$$S_0^{\dot a} \to -S_0^{\dot a} \quad \text{with} \quad \{S_0^{\dot a} \,, S_0^{\dot b}\} = \delta^{\dot a \dot b}$$

$$\mathbf{8_c} \quad \to \quad \mathbf{4}(-\tfrac{1}{2}) + \bar{\mathbf{4}}(\tfrac{1}{2}) \quad \to \quad \mathbf{8_v} \ (\text{even}) + \mathbf{8_s} \ (\text{odd})$$

$$\mathbf{8_c} \to -\mathbf{8_c} \qquad \Rightarrow \qquad \mathbf{8_s} \to -\mathbf{8_s}$$

$$(\mathbf{8_v} + \mathbf{8_s})_{\mathbf{L}} \quad \otimes \quad (\mathbf{8_v} + \mathbf{8_c})_{\mathbf{R}}$$

$$\Longrightarrow \quad \mathbf{35_v} + \mathbf{28} + \mathbf{1} + \mathbf{56_s} + \mathbf{8_c}$$

Type IIA Branes

$$
\begin{aligned}
P_\mu &\rightarrow \text{W}-\text{A}\,,\\
Z &\rightarrow \text{D0}\,,\\
Z_\mu &\rightarrow \text{NS}-\text{1A and NS}-\text{9A(k)}\,,\\
Z_{\mu\nu} &\rightarrow \text{D2 and D8}\,,\\
Z_{\mu_1\cdots\mu_4} &\rightarrow \text{D4 and D6}\,,\\
Z_{\mu_1\cdots\mu_5} &\rightarrow \text{NS}-\text{5A and KK}-\text{A}
\end{aligned}
$$

We find a gravitational wave (W–A), a fundamental string (NS–1A), Dp-branes (p = 0,2,4,6,8), a solitonic five-brane (NS–5A), a Kaluza-Klein monopole (KK–A) and a nine-brane (NS–9A(k))

M–Reduction

$$P_\mu \;\to\; P_\mu\,,\; Z$$

$$\text{W–M} \;\to\; \text{W–A}\,,\; \text{D0}$$

$$Z_{\mu\nu} \;\to\; Z_{\mu\nu}\,,\; Z_\mu$$

$$\text{M2} \;\to\; \text{D2}\,,\; \text{NS–1A}$$

$$\text{M9(k)} \;\to\; \text{D8}\,,\; \text{NS–9A(k)}$$

$$Z_{\mu_1\cdots\mu_5} \;\to\; Z_{\mu_1\cdots\mu_5}\,,\; Z_{\mu_1\cdots\mu_4}$$

$$\text{M5} \;\to\; \text{NS–5A}\,,\; \text{D4}$$

$$\text{KK–M} \;\to\; \text{KK–A}\,,\; \text{D6}$$

IIA–SUGRA

$$\mathbf{35_v} + \mathbf{1} + \mathbf{28} + \mathbf{8_v} + \mathbf{56_v} + \mathbf{56_s} + \mathbf{56_c} + \mathbf{8_s} + \mathbf{8_c}$$

$$\Big\{ g\,,\ \phi\,,\ B^{(2)}\,,\ C^{(1)}\,,\ C^{(3)}\,,\ \underline{C^{(9)}}\,, \underline{B^{(10)}}\,;\ \psi_\mu^{\pm}\,,\ \lambda^{\pm} \Big\}$$

$C^{(9)}$: D=10 cosmological constant

$B^{(10)}$: Charge of NS–9A(k) brane

Discrete Symmetries

$$f^{\pm} \quad \overset{(-)^{F_L^c}}{\rightarrow} \quad \pm f^{\pm}\,,$$

$$\{C^{(1)}\,,\ C^{(3)}\,,\ C^{(9)}\} \quad \overset{(-)^{F_L^c}}{\rightarrow} \quad -\{C^{(1)}\,,\ C^{(3)}\,,\ C^{(9)}\}$$

$$x \xrightarrow{I_{10}\Omega} -x \quad \text{and} \quad \{B^{(2)}\,, C^{(3)}\} \quad \xrightarrow{I_{10}\Omega} \quad -\{B^{(2)}\,, C^{(3)}\}$$

Gravitational Solutions

$$S_{\mathrm{SUGRA}} = \int d^{\mathrm{D}}x\sqrt{-g}\{-R+\cdots\}$$

Gravitational Wave

$$\begin{aligned} ds^2 &= -(2-H)\,dt^2 + H dz^2 - 2(1-H)\,dtdz \\ &\quad + \left(dx_2^2 + \cdots dx_{(\mathrm{D}-1)}^2\right) \end{aligned}$$

KK–Monopole (D $\geq$ 5)

$$\begin{aligned} ds^2 &= dx_{(\mathrm{D}-4)}^2 + H^{-1}\left(dz + A_i dx^i\right)^2 + H dx_i^2\,, \\ F_{ij} &= \epsilon_{ijk}\partial_k H\,, \qquad\qquad i = 1,2,3 \end{aligned}$$

Two–Block Solutions

$$\mathcal{L} = \frac{\sqrt{-g}}{2\kappa_D^2}\left[-R - \tfrac{4}{D-2}(\partial\phi)^2 - \frac{1}{2(p+2)!}e^{a\phi}F_{(p+2)}^2\right]$$

$$\begin{aligned} ds^2 &= H^{\alpha}\ dx_{(p+1)}^2 + H^{\beta}\ dx_{(D-p-1)}^2 \\ e^{\phi} &= H^{\frac{(D-2)a}{4\Delta}}\,, \qquad\qquad g_s = 1 \\ F_{0\cdots pm} &= \pm\sqrt{\frac{4}{\Delta}}\partial_m H^{-1}\,, \qquad\qquad \left(p = \frac{D}{2}\ :\ \text{selfdual}\right) \end{aligned}$$

with

$$\begin{aligned} \alpha &= \frac{-4(D-p-3)}{(D-2)\Delta}\,, \qquad \beta = \frac{4(p+1)}{(D-2)\Delta} \\ \Delta &= \frac{(D-2)a^2}{8} + 2\frac{(p+1)(D-p-3)}{(D-2)} \end{aligned}$$

$$H = 1 + \left(\frac{r_0}{r}\right)^{D-p-3}\,, \qquad\qquad D-p-3 \neq 0, -2$$

Two–Block Solutions Cont'd

Brane	a	p	Δ
M2	0	2	4
M5	0	5	4
Dp	$-\frac{1}{2}(p-3)$	p even	4
NS–1A	–1	1	4
NS–5A	1	5	4

$\Delta < 4$: intersecting branes

$\Delta = \dfrac{4}{n}$: susy intersections

Δ is invariant under reductions

Diagonal Dimensional reduction

z belongs to Worldvolume Direction

$$\begin{array}{ccc} \mathrm{M2} & \rightarrow & \mathrm{NS{-}1A} \\ \mathrm{M5} & \rightarrow & \mathrm{D4} \end{array}$$

$$p\text{--Brane} \quad \longrightarrow \quad (p-1)\text{--Brane}$$

with <u>same</u> value of Δ

Also:

$$\begin{array}{ccc} \mathrm{W{-}M} & \rightarrow & \mathrm{D0} \\ \mathrm{KK{-}M} & \rightarrow & \mathrm{KK{-}A} \\ \mathrm{M9(k)} & \rightarrow & \mathrm{D8} \end{array}$$

Vertical Dimensional reduction

z belongs to Transverse Direction

$$\Downarrow$$

Multi-Center Solutions

$$H = 1 + \sum_{\alpha} \left(\frac{r_{0,\alpha}}{|\vec{y} - \vec{y}_{\alpha}|} \right)^{D-p-3} \qquad r_{0,\alpha} > 0$$

$$\Downarrow$$

$$H = 1 + \int_{\infty}^{\infty} dz \left(\frac{r_0^2}{(\tilde{r}^2 + z^2)} \right)^{\frac{D-p-3}{2}} = 1 + \left(\frac{\tilde{r}_0}{\tilde{r}} \right)^{D-p-4}$$

$$D - p - 4 \geq 1$$

OR

H is harmonic on $\mathbb{E}^{D-p-1}$ $\Rightarrow$ harmonic on $\mathbb{E}^{D-p-2}$

Vertical Dimensional reduction

$$\mathrm{M2} \rightarrow \mathrm{D2}$$
$$\mathrm{M5} \rightarrow \mathrm{NS5{-}A}$$

$$p\mathrm{-Brane} \longrightarrow p\mathrm{-Brane}$$

with <u>same</u> value of Δ

Also:

$$\mathrm{W{-}M} \rightarrow \mathrm{W{-}M}$$
$$\mathrm{KK{-}M} \xrightarrow{z} \mathrm{D6}$$
$$\mathrm{M9(k)} \rightarrow \mathrm{NS9{-}A(k)}$$

Solutions

$$g_S = e^{4/(D-2)\phi} g_E \quad \Rightarrow \quad \mathcal{L} \sim \frac{\sqrt{-g}}{2\kappa_D^2} e^{-2\phi} R + \cdots$$

$$\begin{aligned} ds^2 &= H^{-1/2} dx^2_{(p+1)} + H^{1/2} dx^2_{(9-p)} \\ e^{2\phi} &= H^{-1/2(p-3)} \qquad\qquad \text{(Dp)} \\ F_{0\cdots pm} &= \partial_m H^{-1} \end{aligned}$$

$$\begin{aligned} ds^2 &= H^{-1} dx^2_{(2)} + dx^2_{(8)} \\ e^{2\phi} &= H^{-1} \qquad\qquad \text{(NS-1A)} \\ F_{01m} &= \partial_m H^{-1} \end{aligned}$$

$$\begin{aligned} ds^2 &= dx^2_{(6)} + H dx^2_{(4)} \\ e^{2\phi} &= H \qquad\qquad \text{(NS-5A)} \\ F_{012345m} &= \partial_m H^{-1} \end{aligned}$$

Worldvolume Multiplets

Scalar Multiplets $\rightarrow$ NS–1A, M2, NS–5

p+1	T	T	T	T
1	1	2	4	8
2	1	2	4	8
3	1	2	4	8
4		2	4	
5			4	
6			4	

$$\underbrace{D-(p+1)}_{\text{scalars}} = \frac{Q}{4}$$

Vector Multiplets → Dp

p+1	T	T	T	T
1	2	3	5	9
2	1	2	4	8
3	0	3	1	7
4		0	2	6
5			1	5
6			0	4
7				3
8				2
9				1
10				0

$$\underbrace{D-(p+1)}_{\text{scalars}}+\underbrace{p-1}_{\text{vector}}=D-2=\frac{Q}{4}$$

Tensor Multiplets $\Rightarrow$ M5, NS–5A

p+1	T	T
6	1	5

DBI Actions

$$S^{(\mathrm{Dp})} = -\tau_p \int_{M^{p+1}} d^{p+1}\xi e^{-\phi} \sqrt{|g_{ij} + \mathcal{F}_{ij}|} + C\ e^{\mathcal{F}}$$

where

$$\mathcal{F}_{ij} = F_{ij} - \mathcal{B}_{ij}\,, \qquad F_{ij} = 2\partial_{[i}V_{j]}$$

$$\delta_\kappa Z^M E_M{}^\alpha = [\bar{\kappa}\,(1+\Gamma)]^\alpha$$

where Γ is defined by

$$\Gamma = \frac{\sqrt{|g|}}{\sqrt{|g+\mathcal{F}|}} \sum_{n=0}^{\infty} \frac{1}{2^n n!} \gamma^{j_1 k_1 \cdots j_n k_n} \mathcal{F}_{j_1 k_1} \ldots \mathcal{F}_{j_n k_n} \times$$
$$\times (\Gamma_{11})^{n+\frac{p-2}{2}} \Gamma_{(0)}$$

Exercise IV

Show that

$$\Gamma^2 = \mathbb{1}_{32}$$

Effective Tensions

After M–reduction

$$\ell_p = \ell_s g_s^{1/3}\,,$$

$$R = \begin{cases} l_s g_s & \text{reduction in the R direction} \\ R & \text{reduction in other direction} \end{cases}$$

we find

$$\tau_{\rm eff} = \frac{(g_s)^\alpha R^\beta}{(\ell_s)^{p+1+\beta}}$$

$$\begin{aligned}
\text{NS}-1\text{A} &: \quad \alpha = 0\,, \quad \beta = 0 \\
\text{Dp} &: \quad \alpha = -1\,, \quad \beta = 0 \\
\text{NS}-5\text{A} &: \quad \alpha = -2\,, \quad \beta = 0 \\
\text{KK}-\text{A} &: \quad \alpha = -2\,, \quad \beta = 2 \\
\text{NS}-9\text{A(k)} &: \quad \alpha = -4\,, \quad \beta = 3
\end{aligned}$$

M-theory origin

$$\ell_s = \frac{\ell_p^{3/2}}{R^{1/2}}\,, \qquad g_s = \left(\frac{R}{\ell_p}\right)^{3/2}$$

$$\Downarrow$$

$$\tau_{\rm eff}(R) = R^{\frac{3}{2}\alpha+\frac{1}{2}(p+1)}$$

brane	$\tau_{\rm eff}(R)$	M-Theory
NS–1A	R	Diag. Red. of M2
NS–5A	–	Direct Red. of M5
D0	1/R	KK modes
D2	–	Direct Red. of M2
D4	R	Diag. Red. of M5
D6	R^2	KK–monopole
D8	R^3	M9–brane ?

Massless N=IIB Representation

$$\{Q^i_\alpha\,, Q^j_\beta\} \;=\; \delta^{ij}\left(P^+\gamma^\mu \mathcal{C}^{-1}\right)_{\alpha\beta} P_\mu$$

$$\begin{aligned}
(\mathbf{8_v}+\mathbf{8_c}) \;&\otimes\; (\mathbf{8_v}+\mathbf{8_c}) \\
&= \underbrace{\mathbf{35_v}+\mathbf{28}+\mathbf{1}}_{\text{NS}-\text{NS}}+\underbrace{\mathbf{1}+\mathbf{28}+\mathbf{35_c}}_{\text{R}-\text{R}} \\
&\quad + \underbrace{\mathbf{56_s}+\mathbf{8_c}}_{\text{NS}-\text{R}}+\underbrace{\mathbf{56_s}+\mathbf{8_c}}_{\text{R}-\text{NS}} \\
&= 128 + 128
\end{aligned}$$

$\mathbf{1}, \mathbf{28}, \mathbf{8_c}, \mathbf{56_s}$ are doublets of SO(2)

Type IIB central charges

$$
\begin{aligned}
\{Q^i_\alpha, Q^j_\beta\} \quad = \quad & \delta^{ij}\left(P^+\gamma^\mu\mathcal{C}^{-1}\right)_{\alpha\beta} P_\mu \\
& + \left(P^+\gamma^\mu\mathcal{C}^{-1}\right)_{\alpha\beta} Z^{(ij)}_\mu \\
& + \tfrac{1}{3!}\epsilon^{ij}\left(P^+\gamma^{\mu\nu\rho}\mathcal{C}^{-1}\right)_{\alpha\beta} Z_{\mu\nu\rho} \\
& + \tfrac{1}{5!}\delta^{ij}\left(P^+\gamma^{\mu_1\cdots\mu_5}\mathcal{C}^{-1}\right)_{\alpha\beta} Z^+_{\mu_1\cdots\mu_5} \\
& + \tfrac{1}{5!}\left(P^+\gamma^{\mu_1\cdots\mu_5}\mathcal{C}^{-1}\right)_{\alpha\beta} Z^{+,(ij)}_{\mu_1\cdots\mu_5}
\end{aligned}
$$

$Z^{(ij)}_\mu$, $Z^{+,(ij)}_{\mu_1\cdots\mu_5}$ are doublets of SO(2) (symmetric traceless representations)

$$
\begin{aligned}
\tfrac{1}{2}\times 32\times 33 = {} & 10+10+10+120 \\
& + 3\ \times 126^+
\end{aligned}
$$

Heterotic/Type I Automorphism

$$
\begin{aligned}
\{Q^i_\alpha, Q^j_\beta\} \quad = \quad & \left(P^+\gamma^\mu\mathcal{C}^{-1}\right)_{\alpha\beta}\left(\delta^{ij}P_\mu + (\tau_1)^{ij}Z^{(1)}_\mu + (\tau_3)^{ij}Z^{(3)}_\mu\right) \\
& + \tfrac{1}{3!}(i\tau_2)^{ij}\left(P^+\gamma^{\mu\nu\rho}\mathcal{C}^{-1}\right)_{\alpha\beta}Z_{\mu\nu\rho} \\
& \tfrac{1}{5!}\left(P^+\gamma^{\mu_1\cdots\mu_5}\mathcal{C}^{-1}\right)_{\alpha\beta}\left\{\delta^{ij}Z^+_{\mu_1\cdots\mu_5}\right. \\
& \left. +(\tau_1)^{ij}Z^{(1),+}_{\mu_1\cdots\mu_5} + (\tau_3)^{ij}Z^{(3),+}_{\mu_1\cdots\mu_5}\right\}
\end{aligned}
$$

Heterotic

$$
\begin{aligned}
Q^i_\alpha \quad & \xrightarrow{(-)^{F_L}} \quad (\tau_3)^{ij}Q^j_\alpha \\
\left\{Z^{(1)}_\mu, Z_{\mu\nu\rho}, Z^{+,(1)}_{\mu_1\cdots\mu_5}\right\} \quad & \xrightarrow{(-)^{F_L}} \quad -\left\{Z^{(1)}_\mu, Z_{\mu\nu\rho}, Z^{+,(1)}_{\mu_1\cdots\mu_5}\right\}
\end{aligned}
$$

Type I

$$
\begin{aligned}
Q^i_\alpha \quad & \xrightarrow{\Omega} \quad (\tau_1)^{ij}Q^j_\alpha \\
\left\{Z^{(3)}_\mu, Z_{\mu\nu\rho}, Z^{+,(3)}_{\mu_1\cdots\mu_5}\right\} \quad & \xrightarrow{\Omega} \quad -\left\{Z^{(3)}_\mu, Z_{\mu\nu\rho}, Z^{+,(3)}_{\mu_1\cdots\mu_5}\right\}
\end{aligned}
$$

$$\boxed{(-)^{F_L}}$$

$$(-)^{F_L}: \quad S^a \to -S^a \qquad \Rightarrow$$

$$S_0^a \to -S_0^a \quad \text{with} \quad \{S_0^a\,,\,S_0^b\} = \delta^{ab}$$

$$\mathbf{8_s} \quad \to \quad \mathbf{4}(\tfrac{1}{2}) + \bar{\mathbf{4}}(-\tfrac{1}{2}) \quad \to \quad \mathbf{8_v}\ (\text{even}) + \mathbf{8_c}\ (\text{odd})$$

$$\mathbf{8_s} \to -\mathbf{8_s} \qquad \Rightarrow \qquad \mathbf{8_c} \to -\mathbf{8_c}$$

$$(\mathbf{8_v} + \mathbf{8_c})_{\mathbf{L}} \quad \otimes \quad (\mathbf{8_v} + \mathbf{8_c})_{\mathbf{R}}$$
$$\Longrightarrow\ \mathbf{35_v} + \mathbf{28} + \mathbf{1} + \mathbf{56_s} + \mathbf{8_s}$$

Worldsheet Parity Ω

Green-Schwarz Lightcone Action

$$S_{\mathrm{IIB}} \quad \sim \quad \int d^2\xi \left\{ \partial_+ X^i \partial_- X^i - i S^a \partial_- S^a - i \tilde{S}^a \partial_+ \tilde{S}^a \right\}$$

with

$$X^i \, : \, \mathbf{8_v} \, , \qquad S^a \, : \, \mathbf{8_s} \, , \qquad S^a \, : \, \mathbf{8_s}$$

$$\Omega : \qquad \sigma' = \ell - \sigma \, , \qquad S^a \leftrightarrow \tilde{S}^a$$

$$\tilde{\alpha}^i_n e^{-\frac{2\pi i n}{\ell}(\sigma+\tau)} + \alpha^i_n e^{\frac{2\pi i n}{\ell}(\sigma-\tau)} \rightarrow$$
$$\tilde{\alpha}^i_n e^{\frac{2\pi i n}{\ell}(\sigma-\tau)} + \alpha^i_n e^{-\frac{2\pi i n}{\ell}(\sigma+\tau)} \qquad \text{or} \;\; \alpha^i_n \leftrightarrow \tilde{\alpha}^i_n$$

$$\Omega | N, \tilde{N}, k > \; = \; | \tilde{N}, N, k >$$

$$\Downarrow$$

$$28 \;\; \rightarrow \;\; -28$$

$$(1, 35_c) \;\; \rightarrow \;\; -\,(1, 35_c)$$

Type IIB Branes

$$
\begin{array}{rcl}
P_\mu & \to & \mathrm{W{-}B}\,, \\
Z_\mu^{(ij)} & \to & \mathrm{D1\ and\ NS{-}1B}\,, \\
 & & \mathrm{D9\ and\ NS{-}9B}\,, \\
Z_{\mu\nu\rho} & \to & \mathrm{D3\ and\ D7}\,, \\
Z^{+,(ij)}_{\mu_1\cdots\mu_5} & \to & \mathrm{D5\ and\ NS{-}5B}\,, \\
Z^{+}_{\mu_1\cdots\mu_5} & \to & \mathrm{KK{-}B}
\end{array}
$$

We find a gravitational wave (W–B), a fundamental string (NS–1B), Dp-branes (p = 1,3,5,7,9), a solitonic five-brane (NS–5B), a Kaluza-Klein monopole (KK–B) and a further nine-brane (NS–9B)

Effective Tensions

$$\tau = \frac{(g_s)^\alpha R^\beta}{(\ell_s)^{p+1+\beta}}$$

$$\begin{aligned}
\text{NS}-1\text{B} &: \quad \alpha = 0\,, \quad \beta = 0 \\
\text{Dp} &: \quad \alpha = -1\,, \quad \beta = 0 \\
\text{NS}-5\text{B} &: \quad \alpha = -2\,, \quad \beta = 0 \\
\text{KK}-\text{B} &: \quad \alpha = -2\,, \quad \beta = 2 \\
\text{NS}-9\text{B} &: \quad \alpha = -4\,, \quad \beta = 0
\end{aligned}$$

Solutions/Worldvolume Actions

NS–5A solution = NS–5B solution

but

NS–5A action $\neq$ NS–5B action

$$\text{NS}-5\text{A} \quad : \quad \{X^\mu\,,\ \theta\,,\ b^{(2),+}\,,\ b^{(0)}\}\ \ 3+1$$

$$\text{NS}-5\text{B} \quad : \quad \{X^\mu\,,\ \theta\,,\ b^{(1)}\}\qquad\qquad 4$$

All Type IIA/IIB WV actions are related via M-reduction, T-duality and/or S-duality

Summary

Brane	Worldvolume	Multiplet
NS–1A	$1+1$	(8,8) scalar
NS–1B	$1+1$	(8,8) scalar
NS–5A	$5+1$	(2,0) tensor
NS–5B	$5+1$	(1,1) vector
Dp	$p+1$	vector
KK–A	$5+1$	(1,1) vector
KK–B	$5+1$	(2,0) tensor
NS–9A(k)	$8+1$	vector
NS–9B	$9+1$	vector