# Lecture 3: Other M-Branes

$$C^{(p+1)} \quad \overset{\text{Poincare duality}}{\Leftrightarrow} \quad C^{(D-p-3)}$$

$$p - \text{brane} \quad \Leftrightarrow \quad (D-p-4) - \text{brane}$$

$$S_{\mathrm{D=11\ SUGRA}} \sim \int_{M^{11}} C^{(3)} dC^{(3)} dC^{(3)}$$

$$dC^{(6)} - C^{(3)} dC^{(3)} = {}^{*}dC^{(3)}$$

$$S_{11} = \frac{1}{16\pi G_N} \int d^{11}x \sqrt{-g}\{-R - \frac{1}{2 \times 7!} F^2_{(7)} + \cdots\}$$

$$F^{(7)} = dC^{(6)}$$

# The M5–Brane

$$
\begin{aligned}
ds^2 &= H^{-1/3} dx^2_{(6)} + H^{2/3} dx^2_{(5)} \,, \\
F^{(7)}_{012345m} &= \pm \partial_m H^{-1} \,, \qquad m = 6, 7, \cdots, 10
\end{aligned}
$$

$$
H = 1 + \left(\frac{r_0}{r}\right)^3
$$

$$
(1 \pm \gamma_{012345})\, \epsilon_\infty = 0 \,, \qquad \epsilon = H^{-1/12} \epsilon_\infty
$$

$$
\mathrm{M5} \;:\; \times | \times \times \times \times \times - - - - -
$$

## Mass

$\tau_5$ is Mass per unit 5–volume

$$\tau_5 = \frac{1}{2\kappa^2}\int_{\partial M^5} d^4\Sigma^m \left(\partial^n h_{mn} - \partial_m h^b{}_b\right)$$

$$m = 6, 7, \cdots, 10$$
$$a = 1, 2, \cdots, 10$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,, \qquad \mu = 0, 1, \cdots, 10$$

$$h_{mn} = \tfrac{2}{3}\left(\frac{r_0}{r}\right)^3 \delta_{mn} \qquad h^b{}_b = \tfrac{5}{3}\left(\frac{r_0}{r}\right)^3$$

$$d^4\Sigma^m = r^3 y^m d\Omega^{(4)}$$

$$\Downarrow$$

$$\tau_5 = \frac{3}{2\kappa^2} r_0^3 \Omega_4$$

## Charge

$\mu_5$ is Charge per unit 5–volume

$$\mu_5 = \frac{1}{2\kappa^2}\int_{\partial M^5} d^4\Sigma^{m_1\cdots m_4}\,{}^*F^{(7)}_{m_1\cdots m_4}$$

$$L^{-6} \qquad L^{-9} \qquad L^{4} \qquad L^{-1}$$

$${}^*F^{(7)}_{m_1\cdots m_4} = \pm 3 r_0^3 \epsilon_{m_1\cdots m_4 m_5}\frac{y^{m_5}}{r^5}$$

$$\Downarrow$$

$$\mu_5 = \pm\frac{3}{2\kappa^2} r_0^3 \Omega_4 \quad \Rightarrow \quad \tau_5 = |\mu_5|$$

Dirac Quantization Condition

$$\mu_2\mu_5 = \frac{2\pi n}{\kappa^2}$$

# Worldvolume Action

- scalar multiplets: D = T + p+1 $\leq$ 11
  D=11: 8 scalars in 3 dimensions

- (2,0) tensor multiplet: $\Rightarrow$ M5–brane:
  (5 scalars, 1 SD tensor) in 6 dimensions

$$
\begin{aligned}
S_{\mathrm{M5}} &= \tau_5 \int d^6\xi \sqrt{|\det g|}\,\left\{1 + \tfrac{1}{6}\mathcal{H}^2 + \cdots\right\} \\
&+ \mu_5 \int d^6\xi\ \left\{C^{(6)} + C^{(3)}\mathcal{H}\right\}
\end{aligned}
$$

$$
\mathcal{H} = db^{(2)} + C^{(3)}\,, \qquad \mathcal{H} = {}^\star\mathcal{H} + \cdots
$$

## Self-duality

$$
\begin{aligned}
{}^{\star\star}H_{(\mathrm{D}/2)} &= (-)^{D/2+t}H_{(D/2)} \\
({}^{\star}H_{(D/2)})^2 &= (-)^t (H_{(D/2)})^2
\end{aligned}
$$

## Examples

$$
\begin{aligned}
\mathrm{D}=(0,4) \quad &: \quad {}^{\star\star}H_{(2)} = H_{(2)} \text{ and } (H_{(2)})^2 \neq 0 \\
\mathrm{D}=(1,5) \quad &: \quad {}^{\star\star}H_{(3)} = H_{(3)} \text{ and } (H_{(3)})^2 = 0
\end{aligned}
$$

$$
S = \int d^6x \left[-\tfrac{1}{4}\left(H_{\mu\nu\rho} - H^*_{\mu\nu\rho}\right) H^{*\mu\nu\sigma} v^\rho v_\sigma\right]
$$

$$
v_\mu = \partial_\mu a/\sqrt{(\partial a)^2}
$$

$$
\delta B_{\mu\nu} = \psi_{[\mu} v_{\nu]}\,, \qquad \delta a = 0
$$

## Exercise III

Show that from the E.O.M. of $b^{(2)}$

$$\partial^\mu \left( v^\sigma v_{[\mu} H^-_{\nu\rho\sigma]} \right) = 0$$

it follows that

$$H^-_{\mu\nu\rho} = 0$$

## The M–theory Monopole

$$
\begin{aligned}
ds^2 &= dx_{(7)}^2 + H^{-1}\left(dz + A_i dx^i\right)^2 + H dx_i^2 \,, \\
F_{ij} &= \epsilon_{ijk}\partial_k H \,, \qquad\qquad i = 1,2,3
\end{aligned}
$$

$$
H = 1 + \frac{r_0}{r} \qquad\qquad r_0 = \tfrac{1}{2}R
$$

$$
\text{KK} \quad : \quad \times | \times \times \times \times \times \times \; z \; - - -
$$

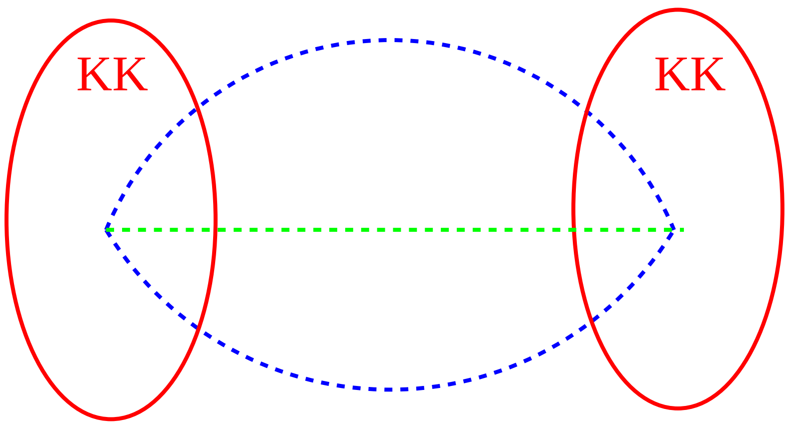
z is a U(1)–isometry direction $\Rightarrow$ Spatial Killing vector $k = \partial/\partial z$

$$
k^2 = k^\mu k^\nu g_{\mu\nu} \equiv (R/\ell_p)^2
$$

$$
\left(1 - \Gamma_{0123456}\right)\epsilon \;=\; 0 \;\Rightarrow\; 6 - \text{Brane?}
$$

# Two–center Solution

$$H = 1 + \frac{r_0}{|\mathbf{X} + \mathbf{X}_0|} + \frac{r_0'}{|\mathbf{X} - \mathbf{X}_0|}$$




$\mathbf{X} = -\mathbf{X}_0$ $\mathbf{X} = +\mathbf{X}_0$

# Worldvolume Action

6-Brane $\Rightarrow$ D=7 vector multiplet:

3 scalars + 1 vector $\Rightarrow$

## Gauged Sigma Model

$$
\begin{aligned}
S_{\mathrm{KK-M}} &= \tau \int d^7\xi\, k^2 \sqrt{|\det\left(\Pi_{ij} + k^{-1}\mathcal{F}_{ij}\right)|} \\
&+ \mu \int d^7\xi\, \left\{ (i_k N)^{(7)} + \cdots \right\}
\end{aligned}
$$

with

$$
\Pi_{ij} = \partial_i X^\mu \partial_j X^\nu \left( g_{\mu\nu} + k_\mu k_\nu / k^2 \right)
$$

# The M9(k)–Brane

$$(1-\Gamma_{0123456789})\,\epsilon = 0 \ \text{or} \ (1-\Gamma_{11})\,\epsilon = 0$$

Assume U(1) isometry in worldvolume direction $\Rightarrow$ 8-Brane

$$\text{M9(k)} \quad : \quad \times| \times \times \times \times \times \times \times \times z \; -$$

$$\begin{aligned} S_{\text{M9(k)}} &= \tau \int d^9\xi\, k^3 \sqrt{|\det\left(\Pi_{ij} + k^{-1}\mathcal{F}_{ij}\right)|} \\ &+ \mu \int d^9\xi\, \left\{ i_k C^{(10)}(k) + \cdots \right\} \end{aligned}$$

$$S_{\text{D=11 SUGRA}} \sim \frac{1}{2} \int d^{11}x\; m^2 k^4 \sqrt{-g}$$

M9(k)–Brane $\overset{R\to\infty}{\Rightarrow}$ End of the world Brane

# Effective Tensions

All WV–actions (except the wave) are of the form

$$S = \tau \int d^{p+1}\xi \; k^{\beta} \sqrt{|\det g|} + \cdots$$

$$\tau_{\text{eff}} = \frac{R^{\beta}}{(\ell_p)^{p+1+\beta}} \; : \; \text{Effective Tension}$$

| | | |
|---|---|---|
| W–M | : | – – |
| M2 | : | $p = 2\,,\ \beta = 0$ |
| M5 | : | $p = 5\,,\ \beta = 0$ |
| KK–M | : | $p = 6\,,\ \beta = 2$ |
| M9(k) | : | $p = 8\,,\ \beta = 3$ |

# Summary

## Central Charges

$$
\begin{aligned}
P_\mu &\Rightarrow \mathrm{W{-}M} \\
Z_{\mu\nu} &\Rightarrow \mathrm{M2\ and\ M9(k)} \\
Z_{\mu_1\cdots\mu_5} &\Rightarrow \mathrm{M5\ and\ KK{-}M}
\end{aligned}
$$

## Solutions

$$
\begin{aligned}
\mathrm{M2, M5} &\quad : \quad \mathrm{2 - Block\ Solutions} \\
\mathrm{W{-}M, KK{-}M} &\quad : \quad \mathrm{off - diagonal\ solutions} \\
\mathrm{M9(k)} &\quad : \quad \mathrm{Killing\ vector}
\end{aligned}
$$

## WV–Actions

$$
\begin{aligned}
\mathrm{W{-}M} &\quad : \quad \mathrm{massless\ particle} \\
\mathrm{M2} &\quad : \quad \mathrm{scalar\ multiplet} \\
\mathrm{M5} &\quad : \quad \mathrm{tensor\ multiplet} \\
\mathrm{KK{-}M, M9(k)} &\quad : \quad \mathrm{D = 7, D = 9\ vector\ multiplet}
\end{aligned}
$$