Strings and Branes

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Plan of the Lectures

(1) Target–Space Supersymmetry

(2) The M2–Brane

(3) Other M–Branes

(4) Branes in Ten Dimensions

(5) Dualities

Recommended Literature

[1] J. Polchinski, *String Theory*, Volumes I and II, Cambridge University Press, 1998.

[2] D. Lüst and S. Theisen, *Lectures on String Theory*, Springer Verlag (1989).

[3] E. Kiritsis, *Introduction to Superstring Theory*, Leuven University Press (1997).

[4] A. Dabholkar, *Lectures on Orientifolds and Duality*, `hep-th/9804208`.

[5] P.K. Townsend, *M–theory from its Superalgebra*, Cargèse Lectures 1997, `hep-th/9712004`.

[6] B. de Wit and J. Louis, *Supersymmetry and Dualities in various dimensions*, Cargèse Lectures 1997, `hep-th/9801132`.

Recommended Literature

[7] K. Stelle, *BPS Branes in Supergravity*, ICTP Lectures 1997, `hep-th/9803116`.

[8] A. Van Proeyen, *Tools for supersymmetry*, Călimăneşti lectures 1998, `hep-th/9910030`.

[9] A. Salam and E. Sezgin, editors, *Supergravity in Diverse Dimensions*, North Holland/World Scientific, 1989.

[10] R. Argurio, *Brane Physics in M-theory*, Ph.D. thesis Université Libre de Bruxelles, `hep-th/9807171`.

Credits

The material presented in these lectures is not original. For further references, see the recommended literature. The algebraic approach of these lectures is taken from [5].

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(1) Target-Space Supersymmetry

Bosonic Spacetime Symmetries

Poincaré or Conformal

$\Downarrow$

Fermionic Spacetime Symmetries

Super-Poincaré or Super-Conformal

$\Downarrow$

Supersymmetry (SUSY)

Bosons B $\overset{\text{SUSY}}{\Longleftrightarrow}$ Fermions F

The Supersymmetry algebra

$$\delta \mathrm{B} = \bar{\epsilon}\mathrm{F} \quad \Rightarrow \quad \delta \mathrm{F} = \gamma^{\mu}\left(\partial_{\mu}\mathrm{B}\right)\epsilon$$

$$\Downarrow$$

$$\{Q_{\alpha}, Q_{\beta}\} = \left(\gamma^{\mu}\mathcal{C}^{-1}\right)_{\alpha\beta}P_{\mu}$$

Local supersymmetry leads to gravity:

Supergravity (SUGRA)

$$\begin{aligned} \text{Bosons} \quad &\xrightarrow{Q} \quad \text{Fermions} \\ &\xrightarrow{Q} \quad \text{Bosons translated by } \mathrm{P}_{\mu} \end{aligned}$$

$$\sharp \;\; \text{Bosons} \quad = \quad \sharp \;\; \text{Fermions}$$

Exceptions: D = 1, D = (0, s), D = (1,1)

Exercise I

Spinors

in

Arbitrary Dimensions

Super-Poincaré Algebras

$$
\begin{aligned}
[M_{\mu\nu}, M^{\rho\sigma}] &= -2\delta^{[\rho}_{[\mu} M_{\nu]}{}^{\sigma]} \\
[P_\mu, M_{\nu\rho}] &= \eta_{\mu[\nu} P_{\rho]} \\
[P_\mu, P_\nu] &= 0
\end{aligned}
$$

$$
\begin{aligned}
\{Q^i_\alpha, Q_{\beta j}\} &= (\gamma^\mu \mathcal{C}^{-1})_{\alpha\beta} P_\mu \delta^i_j \\
[M_{\mu\nu}, Q^i_\alpha] &= -\tfrac{1}{4}(\gamma_{\mu\nu} Q)^i_\alpha \\
[P_\mu, Q^i_\alpha] &= 0
\end{aligned}
$$

Extended SUSY $\rightarrow$

$$[T_A, Q^i_\alpha] = (U_A)^i{}_j Q_\alpha{}^j$$

$$\Downarrow$$

R–symmetry

Various Dimensions

D	Type	Q	R	rep.
1	M	1	$SO(N)$	–
2	MW	1	$SO(N_L) \times SO(N_R)$	–
3	M	2	$SO(N)$	1+1
4	M^+	4	$U(N)$	2+2
5	SM	8	$USp(N)$	4+4
6	SMW	8	$USp(N_L) \times USp(N_R)$	4+4
7	SM	16	$USp(N)$	8+8
8	M^-	16	$U(N)$	8+8
9	M	16	$SO(N)$	8+8
10	MW	16	$SO(N_L) \times SO(N_R)$	8+8
11	M	32	$SO(N)$	128+128
12	M^+	64	$U(N)$	–

Massless Representations

$$P^2 = 0 \quad \Rightarrow \quad P_\mu = (P, \pm P, 0, \cdots, 0)$$

$$\mathcal{C} = \gamma^0 \;\Rightarrow\; \{Q_\alpha, Q_\beta\} = P(1 \mp \gamma_{01})_{\alpha\beta}$$

$$\Downarrow$$

Q/2 charges, transforming as a spinor of SO(D-2), generate a Clifford algebra

Creation operators in fundamental of

U(Q/4)

$$\Downarrow$$

2 Chiral Spinor representations of

SO(Q/2) $\supset$ SO(D-2)

Example I

D=10 Q=16

8 super charges transforming as a spinor $\mathbf{8_s}$ of SO(8)

$$\begin{aligned} SU(4) &\rightarrow 1 + 4 + 6 + \bar{4} + 1 \\ &= \mathbf{8_v} + \mathbf{8_c} \quad \text{of} \quad SO(8) \end{aligned}$$

supermultiplet	bosons	fermions
vector multiplet	$\mathbf{8_v}$	$\mathbf{8_c}$
graviton multiplet	$\mathbf{35_v} + \mathbf{28} + \mathbf{1}$	$\mathbf{56_c} + \mathbf{8_s}$
gravitino multiplet	$\mathbf{35_c} + \mathbf{28} + \mathbf{1}$	$\mathbf{56_c} + \mathbf{8_s}$
gravitino multiplet	$\mathbf{56_v} + \mathbf{8_v}$	$\mathbf{56_s} + \mathbf{8_c}$

Example II

D=11 Q=32

16 super charges transforming as a spinor of SO(9)

$$
\begin{aligned}
SU(8) &\rightarrow 1+8+28+56+70+56+28+8+1 \\
&= 128+128 \quad \text{of} \quad SO(16)
\end{aligned}
$$

$$
\begin{aligned}
128 &\rightarrow 44+84 : \quad g_{\mu\nu}+C_{\mu\nu\rho} \\
128 &\rightarrow 128 \quad : \quad \psi_\mu
\end{aligned}
$$

Example III

D=6 $\qquad$ $\mathrm{N} = (\mathrm{N_L}, \mathrm{N_R})$

N=(1,0) or Q=8

4 super charges transforming as the helicities $(\pm 1/2; \pm 1/2)$ of $\mathrm{SU_+(2)} \times \mathrm{USp(1)_L}$

$$|0,-1/2> \; + \; |\pm 1/2, 0> \; + \; |0, 1/2> \; \rightarrow$$
$$(1,1;2,1)_C \; + \; (2,1;1,1)_C$$

$(\mathrm{N_L}, \mathrm{N_R})$	supermultiplet	#	bosons
(1,0)	hyper multiplet	4 + 4	ϕ^i
(1,0)	tensor multiplet	4 + 4	$B^+_{\mu\nu}\,,\phi$
(1,0)	vector multiplet	4 + 4	V_μ
(2,0)	tensor multiplet	8 + 8	$B^+_{\mu\nu}\,,\phi^{[IJ]}$
(1,1)	vector multiplet	8 + 8	$V_\mu\,,\phi^{i\bar{j}}$

Various Dimensions

D\Q	32	16	8	4
11	•			
10	IIA $(1,1)$ IIB $(2,0)$	Type I $(1,0)$		
9	•	•		
8	•	•		
7	•	•		
6	(2,2)	(1,1) (2,0)	(1,0)	
5	•	•	•	
4	•	•	•	•
	N=8	N=4	N=2	N=1

Super-AdS Algebras

Poincaré is Inönü-Wigner contract. of AdS

$$[P_\mu, P_\nu] = \frac{1}{2R^2} M_{\mu\nu}$$

$\mathrm{AdS}_D = \mathrm{SO(D\text{-}1,2)}$ $\quad$ $\mathrm{Conf}_D = \mathrm{SO(D,2)}$

$$\Downarrow$$

$$\mathrm{AdS}_{D+1} = \mathrm{Conf}_D$$

Jacobi's $\rightarrow$

$[P_\mu, Q] = x\gamma_\mu Q$ with $x^2 = \frac{1}{16R^2}$

AdS in $D + 1 > 7$ requires "central charges"

D=11 $\rightarrow$ OSp(1|32)

Classification

D	superalgebra	R	$\sharp$ fermionic
1	OSp(N|2)	O(N)	2N
	SU(N|1, 1)	SU(N) × U(1)	4N
	SU(2|1, 1)	SU(2)	8
	$\mathrm{OSp}(4^\star\|2\mathrm{N})$	SU(2)×USp(2N)	8N
	G(3)	G_2	14
	$F^0(4)$	SO(7)	16
	$\mathrm{D}^0(2,1\|\alpha)$	SU(2) × SU(2)	8
3	OSp(N|4)	SO(N)	4N
4	SU(2, 2|N)	SU(N) × U(1)	8N
	SU(2, 2|4)	SU(4)	32
5	$\mathrm{F}^2(4)$	SU(2)	16
6	$\mathrm{OSp}(8^\star\|\mathrm{N})$	USp(N) N even	8N