Strings and Branes

Madrid lectures, March 12-16, 2001

Exercise I : Spinors in Arbitrary Dimensions

(1) Consider $A \equiv \Gamma_1 \cdots \Gamma_t$. Show that

$$A^{-1} = (-)^t \Gamma_t \cdots \Gamma_1 \qquad \text{and} \qquad A^\dagger A = 1\,. \tag{1}$$

(2) We define

$$\tilde{F}_{a_1 \ldots a_n} = (\mathrm{i})^{D/2+t} \frac{1}{n!} \varepsilon_{a_1 \ldots a_D} F^{a_D \ldots a_{n+1}}\,, \qquad D = 2n. \tag{2}$$

Show that

$$\tilde{\tilde{F}}_{a_1 \ldots a_n} = F_{a_1 \ldots a_n} \qquad \text{and} \qquad \tilde{F}^2 = (-)^{D/2} F^2\,. \tag{3}$$

(3) Show that

$$\left(\mathcal{C}\Gamma^{(n)}\right)^T = -\epsilon (-)^{n(n-1)/2} (-\eta)^n \mathcal{C}\Gamma^{(n)}\,. \tag{4}$$

(4) Show that in D=8 dimensions we can only have

$$(\epsilon,\, \eta) = (-1, +1) \qquad \text{or} \qquad (\epsilon,\, \eta) = (-1, -1) \tag{5}$$

(5) Show that

$$B^* B = -\epsilon \eta^t (-)^{t(t+1)/2}\,. \tag{6}$$