

The

Strong

Equivalence

Principle

in

Higher Order Gravities

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Do HOGs of the form

$$S[g_{\mu}] = \frac{1}{g^2} \int d^d x \sqrt{|g|} \left\{ R + F(g, R, \nabla R) \right\}$$

satisfy the SEP?

Common lore : **NO!**

i) Extra d.o.f.

ii) Spacetime-dependent coupling constants from cosmological curvature.

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i) Extra d.o.f. ← (Not always : Cubic)
(Einstein Gravity)
(Bueno & Cano 2016)

ii) Spacetime-dependent coupling constants from cosmological curvature. ← (Maybe not so easy to test)

A more basic/fundamental question:

Does gravity couple to gravitational energy in the same way it couples to other kinds of energy? (Through $T^{\mu\nu}$)

We are going to give a general answer in the "post-Newtonian" approx. $g_{\mu\nu} = \eta_{\mu\nu} + \gamma h_{\mu\nu}$

$$g_{\mu\nu} = \eta_{\mu\nu} + \gamma h_{\mu\nu};$$

$$S[g] = S^{(0)}[h] + \gamma S^{(1)}[h] + \gamma^2 S^{(2)}[h] + \dots;$$

$$-2 \frac{\delta S}{\delta h_{\mu\nu}} = D^{(0)\mu\nu} - \gamma t^{(0)\mu\nu} + \dots$$

wave operator

energy-momentum tensor
of the gravitational field
derived from $S^{(0)}$?



Our goal is to answer this question and determine the general properties of $t^{(0)\mu\nu}$ for HOGs.

The energy-momentum tensor

$$S[\phi] = \int d^d x \mathcal{L}(\partial^\mu \phi)$$

1st Noether th.: invariance under constant δx^μ

$$\partial_\mu t_{\text{can}}^\mu{}_\nu = \frac{\delta S}{\delta \phi} \partial_\nu \phi ; (= 0 \text{ on-shell})$$

$$t_{\text{can}}^\mu{}_\nu = \eta^\mu{}_\nu \mathcal{L} - \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} \partial_\nu \phi - \frac{\partial \mathcal{L}}{\partial \partial_\mu \partial_\alpha \phi} \partial_\nu \partial_\alpha \phi$$

$$+ \partial_\alpha \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \partial_\alpha \phi} \right) \partial_\nu \phi + \dots$$

We can add to t_{can}^μ terms of the form

$$\partial_S \psi^{\mu\nu} ; \quad \text{with } \psi^{\mu\nu} = \psi^{[\mu\nu]} ;$$

For instance $t_{can}^\mu + \partial_S \psi^{\mu\nu} = t_{Belinfante}^\mu$

$$t_{Belinfante}^\mu = t_{Belinfante}^\mu$$

We can also add terms proportional to the e.o.m.

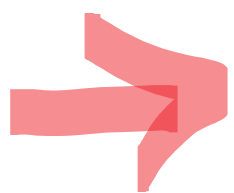
$$A^\mu \frac{\delta S}{\delta \phi}$$

We want to show that

$$t^{(0)\mu\nu} \equiv 2 \frac{\delta \mathcal{L}^{(1)}}{\delta h_{\mu\nu}} = t_{\text{can}}^{(0)\mu\nu} + \partial_\alpha \psi_{\text{Some}}^{\alpha\mu\nu} + A_{\text{Some}}^{\mu\nu} g^\sigma \frac{\delta \mathcal{L}^{(0)}}{\delta h_{\sigma\alpha}};$$

with

$$t_{\text{can}}^{(0)\mu\nu} \equiv \eta^{\mu\nu} \mathcal{L}^{(0)} - \frac{\partial \mathcal{L}^{(0)}}{\partial h_{\alpha\rho}} \partial^\nu h^{\alpha\rho} \dots$$


$$\partial_\mu t_{\text{can}}^{(0)\mu\nu} = \frac{\delta \mathcal{L}^{(0)}}{\delta h_{\sigma\alpha}} \partial^\nu h_{\sigma\alpha};$$

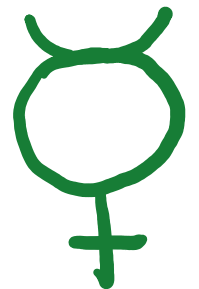
In GR it can be checked that

$$t_{GR}^{(0)\mu\nu} \equiv 2 \frac{\delta S^{(1)}}{\delta h_{\mu\nu}} = t_{can}^{(0)\mu\nu} + \partial_\sigma \psi_{GR}^{\mu\nu};$$

but $\psi_{GR}^{\mu\nu} \neq \psi_{Belinfante}^{\mu\nu}$;

$t_{GR}^{(0)\mu\nu}$ is unambiguously defined by $S_{E-H}^{(1)}$

and changes are physically relevant!



In GR $t^{(\circ)\mu\nu}$ is completely determined by the gauge invariance associated to g.c.t.s because $S^{(1)}$ is.

What happens in HOGs?

Under infinitesimal g.c.t.s $\delta_{\xi} x^{\mu} = \xi^{\mu}(x)$

$$\delta_{\xi} \underset{\text{HOG}}{S[g]} = 0;$$

In the post-Dirac approximation.

$$\delta_{\xi} = \delta_{\xi}^{(0)} + \gamma \delta_{\xi}^{(1)} + \gamma^2 \delta_{\xi}^{(2)} + \dots$$

For $h_{\mu\nu}$ $\delta_{\xi} h_{\mu\nu} = \left(\delta_{\xi}^{(0)} + \gamma \delta_{\xi}^{(1)} \right) h_{\mu\nu}; \quad \left(\begin{array}{c} \text{no} \\ \delta(\gamma^2) \end{array} \right)$

$$\delta_{\xi}^{(0)} h_{\mu\nu} = 2 \partial_{(\mu} \xi_{\nu)};$$

$$\begin{aligned} \delta_{\xi}^{(1)} h_{\mu\nu} &= \mathcal{L}_{\xi} h_{\mu\nu} = \\ &= \xi^{\sigma} \partial_{\sigma} h_{\mu\nu} + 2 \partial_{(\mu} \xi^{\sigma} h_{\nu)\sigma}; \end{aligned}$$

$$\delta_{\xi} \mathcal{S}[g] = 0; \quad \text{HOG}$$

(Order by order in ξ)

$$\left\{ \begin{array}{l} \delta_{\xi}^{(0)} \mathcal{S}^{(0)}[h] = 0; \\ \delta_{\xi}^{(0)} \mathcal{S}^{(n)}[h] + \delta_{\xi}^{(1)} \mathcal{S}^{(n-1)}[h] = 0; \\ n \geq 1 \end{array} \right.$$

2nd Noether th:

$$\left\{ \begin{array}{l} \partial_{\mu} \frac{\delta \mathcal{S}^{(0)}}{\delta h_{\mu\nu}} = 0; \\ \partial_{\mu} \frac{\delta \mathcal{S}^{(1)}}{\delta h_{\mu\nu}} = -\partial_{\mu} \left(\frac{\delta \mathcal{S}^{(0)}}{\delta h_{\mu\sigma}} h_{\sigma}^{\nu} \right) + \frac{1}{2} \frac{\delta \mathcal{S}^{(0)}}{\delta h_{\sigma\sigma}} \partial^{\nu} h_{\sigma\sigma}; \end{array} \right.$$

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1st Noether th:

$$\partial_{\mu} t_{\text{can}}^{\mu\nu} = \frac{\delta S^{(0)}}{\delta h_{\sigma\sigma}} \partial^{\nu} h_{\mu\sigma};$$

$$\delta_{\xi} \mathcal{S}[g] = 0; \quad \text{HOG}$$

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x2

$$\partial_{\mu} \left\{ 2 \frac{\delta S^{(1)}}{\delta h_{\mu\nu}} - t_{\text{can}}^{(0)\mu\nu} + 2 \frac{\delta S^{(0)}}{\delta h_{\mu\sigma}} h_{\sigma}^{\nu} \right\} = 0$$

$$\delta_{\xi} S[g] = 0; \quad \text{HOG}$$

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$$\begin{cases} \delta_{\xi}^{(0)} S^{(0)}[h] = 0; \\ \delta_{\xi}^{(0)} S^{(n)}[h] + \delta_{\xi}^{(1)} S^{(n-1)}[h] = 0; \\ n \geq 1 \end{cases}$$

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$$2 \frac{\delta S^{(1)}}{\delta h_{\mu\nu}} = t_{\text{con}}^{(0)\mu\nu} - 2 \frac{\delta S^{(0)}}{\delta h_{\mu\sigma}} h_{\sigma}^{\nu} + \partial_{\sigma} \psi^{\mu\nu};$$

$$\begin{aligned}
 & t^{(0)\mu\nu} \\
 & \parallel \\
 & 2 \frac{\delta \mathcal{L}^{(1)}}{\delta h_{\mu\nu}} = t^{(0)\mu\nu} - 2 \frac{\delta \mathcal{L}^{(0)}}{\delta h_{\mu\sigma}} h_{\sigma}^{\nu} + \partial_{\sigma} \psi^{\mu\nu};
 \end{aligned}$$

Q.E.D.

This result can be extended to higher orders in \mathcal{X} :

n^{th} Noether identity:

$$\partial_\mu \frac{\delta S^{(n)}}{\delta h_{\mu\nu}} = -\partial_\mu \left(\frac{\delta S^{(n-1)}}{\delta h_{\mu\sigma}} h_\sigma^\nu \right) + \frac{1}{2} \frac{\delta S^{(n-1)}}{\delta h_{\sigma\sigma}} \partial^\nu h_{\sigma\sigma};$$

This result can be extended to higher orders in α :

n^{th} Another identity:

$$\partial_\mu \frac{\delta S^{(n)}}{\delta h_{\mu\nu}} = -\partial_\mu \left(\frac{\delta S^{(n-1)}}{\delta h_{\mu\sigma}} h_{\sigma}^{\nu} \right) + \frac{1}{2} \frac{\delta S^{(n-1)}}{\delta h_{\sigma\tau}} \partial^\nu h_{\sigma\tau};$$

\downarrow

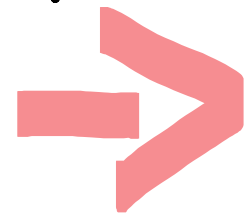
$$\partial_\mu t_{\text{can}}^{(n-1)\mu\nu}$$

This result can be extended to higher orders in χ :

n^{th} Noether identity:

$$\partial_\mu \frac{\delta \mathcal{S}^{(n)}}{\delta h_{\mu\nu}} = -\partial_\mu \left(\frac{\delta \mathcal{S}^{(n-1)}}{\delta h_{\mu\sigma}} h_\sigma^\nu \right) + \frac{1}{2} \frac{\delta \mathcal{S}^{(n-1)}}{\delta h_{\sigma\tau}} \partial^\nu h_\sigma^\tau;$$

$\times 2$



$$\partial_\mu \left(2 \frac{\delta \mathcal{S}^{(n)}}{\delta h_{\mu\nu}} - t_{\text{can}}^{(n-1)\mu\nu} + 2 \frac{\delta \mathcal{S}^{(n-1)}}{\delta h_{\mu\sigma}} h_\sigma^\nu \right) = 0$$

This result can be extended to higher orders in χ :

n^{th} Noether identity:

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$\times 2$

$$\partial_\mu \left(2 \frac{\delta \mathcal{S}^{(n)}}{\delta h_{\mu\nu}} - t_{\text{can}}^{(n-1)\mu\nu} + 2 \frac{\delta \mathcal{S}^{(n-1)}}{\delta h_{\mu\sigma}} h_\sigma^\nu \right) = 0$$

$$2 \frac{\delta \mathcal{S}^{(n)}}{\delta h_{\mu\nu}} = t_{\text{can}}^{(n-1)\mu\nu} - 2 \frac{\delta \mathcal{S}^{(n-1)}}{\delta h_{\mu\sigma}} h_\sigma^\nu + \partial_\sigma \psi^{(n-1)\sigma\mu\nu}$$

$$\begin{aligned}
& \chi \left\{ \begin{aligned} 2 \frac{\delta \mathcal{S}^{(1)}}{\delta h_{\mu\nu}} &= t_{\text{can}}^{(0)\mu\nu} - 2 \frac{\delta \mathcal{S}^{(0)}}{\delta h_{\mu\sigma}} h_{\sigma}^{\nu} + \partial_{\sigma} \mathcal{F}^{(0)\sigma\mu\nu}; \end{aligned} \right\} \\
& + \chi^2 \left\{ \begin{aligned} 2 \frac{\delta \mathcal{S}^{(2)}}{\delta h_{\mu\nu}} &= t_{\text{can}}^{(1)\mu\nu} - 2 \frac{\delta \mathcal{S}^{(1)}}{\delta h_{\mu\sigma}} h_{\sigma}^{\nu} + \partial_{\sigma} \mathcal{F}^{(1)\sigma\mu\nu}; \end{aligned} \right\} \\
& + \chi^3 \left\{ \begin{aligned} 2 \frac{\delta \mathcal{S}^{(3)}}{\delta h_{\mu\nu}} &= t_{\text{can}}^{(2)\mu\nu} - 2 \frac{\delta \mathcal{S}^{(2)}}{\delta h_{\mu\sigma}} h_{\sigma}^{\nu} + \partial_{\sigma} \mathcal{F}^{(2)\sigma\mu\nu}; \end{aligned} \right\} \dots
\end{aligned}$$

$$\begin{aligned}
& \gamma \left\{ 2 \frac{\delta \mathcal{S}^{(0)}}{\delta h_{\mu\nu}} = t_{\text{can}}^{(0)\mu\nu} - 2 \frac{\delta \mathcal{S}^{(0)}}{\delta h_{\mu\sigma}} h_{\sigma}^{\nu} + \partial_{\sigma} \psi^{(0)\sigma\mu\nu}; \right\} \\
& + \gamma^2 \left\{ 2 \frac{\delta \mathcal{S}^{(2)}}{\delta h_{\mu\nu}} = t_{\text{can}}^{(1)\mu\nu} - 2 \frac{\delta \mathcal{S}^{(1)}}{\delta h_{\mu\sigma}} h_{\sigma}^{\nu} + \partial_{\sigma} \psi^{(1)\sigma\mu\nu}; \right\} \\
& + \gamma^3 \left\{ 2 \frac{\delta \mathcal{S}^{(3)}}{\delta h_{\mu\nu}} = t_{\text{can}}^{(2)\mu\nu} - 2 \frac{\delta \mathcal{S}^{(2)}}{\delta h_{\mu\sigma}} h_{\sigma}^{\nu} + \partial_{\sigma} \psi^{(2)\sigma\mu\nu}; \right\} \dots
\end{aligned}$$

$$\begin{aligned}
& 2 \left(\gamma \frac{\delta \mathcal{S}^{(0)}}{\delta h_{\mu\nu}} + \gamma^2 \frac{\delta \mathcal{S}^{(2)}}{\delta h_{\mu\nu}} + \dots + \gamma^m \frac{\delta \mathcal{S}^{(m)}}{\delta h_{\mu\nu}} \right) = \\
& = \gamma t_{\text{can}}^{(0)\mu\nu} + \gamma^2 t_{\text{can}}^{(1)\mu\nu} + \dots + \gamma^m t_{\text{can}}^{(m)\mu\nu} \\
& - 2 \gamma \left(\frac{\delta \mathcal{S}^{(0)}}{\delta h_{\mu\nu}} + \gamma \frac{\delta \mathcal{S}^{(1)}}{\delta h_{\mu\nu}} + \dots + \gamma^{m-1} \frac{\delta \mathcal{S}^{(m-1)}}{\delta h_{\mu\nu}} \right) h_{\sigma}^{\nu} \\
& + \partial_{\sigma} \left(\gamma \psi^{(0)\sigma\mu\nu} + \gamma^2 \psi^{(1)\sigma\mu\nu} + \dots + \gamma^m \psi^{(m-1)\sigma\mu\nu} \right)
\end{aligned}$$

So this order, the e.o.m. are

$$\frac{\delta S^{(0)}}{\delta h_{\mu\nu}} + \cancel{\eta} \frac{\delta S^{(1)}}{\delta h_{\mu\nu}} + \dots + \cancel{\eta^{m-1}} \frac{\delta S^{(m-1)}}{\delta h_{\mu\nu}} = -\cancel{\eta^m} \frac{\delta S^{(m)}}{\delta h_{\mu\nu}}$$

and this term vanishes on-shell to $\mathcal{O}(\eta^m)$

$$t^{\mu\nu}(h) = t_{\text{can}}^{\mu\nu} + \partial_\alpha \psi^{\alpha\mu\nu} + \mathcal{O}(\eta^{m+1})$$



$$D^{(0)\mu\nu} = \cancel{\eta} t^{\mu\nu}; \quad t^{\mu\nu} = t^{(0)\mu\nu} + \cancel{\eta} t^{(1)\mu\nu} + \dots$$

$$\begin{array}{c} \text{"} \\ -2 \frac{\delta S^{(0)}}{\delta h_{\mu\nu}} \\ \text{"} \end{array} \quad \begin{array}{c} \text{"} \\ 2 \frac{\delta S^{(1)}}{\delta h_{\mu\nu}} \\ \text{"} \end{array} \quad \begin{array}{c} \text{"} \\ 2 \frac{\delta S^{(2)}}{\delta h_{\mu\nu}} \\ \text{"} \end{array} \quad \dots$$

$$t_{\text{can}}^{\mu\nu} = t_{\text{can}}^{(0)\mu\nu} + \cancel{\eta} t_{\text{can}}^{(1)\mu\nu} + \dots \text{ from } S^{(0)} + \cancel{\eta} S^{(1)} + \dots$$

Finally, using similar variational methods we can determine the gauge transformation properties of $t^{\mu\nu}(h)$ to any order in φ :

$$\delta_{\xi}^{(0)} \frac{\delta \mathcal{S}^{(n)}}{\delta h_{\alpha\beta}} = -\delta_{\xi}^{(1)} \frac{\delta \mathcal{S}^{(n-1)}}{\delta h_{\alpha\beta}} + \partial_{\xi} \left(\xi^{\sigma} \frac{\delta \mathcal{S}^{(n-1)}}{\delta h_{\alpha\beta}} \right) - 2 \partial^{(\alpha} \xi^{\beta)} \frac{\delta \mathcal{S}^{(n-1)}}{\delta h_{\rho\sigma}};$$

At lowest order (checked for GR)

$$\begin{cases} \delta_{\xi}^{(0)} D^{(0)\mu\nu} = 0; \\ \delta_{\xi}^{(0)} t^{(0)\mu\nu} = \delta_{\xi}^{(1)} D^{(0)\mu\nu} + \partial_{\xi} \left(\xi^{(0)} D^{(0)\mu\nu} \right) - 2 \partial^{(\mu} \xi^{\nu)} D^{(0)\mu\nu} \end{cases}$$