Progress on supersymmetric solutions of gauged SUGRAs

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In collaboration with

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Why gauged SUGRAs ?

- They describe many interesting physical situations

→ Charged fields → Abelian or → non-Abelian interactions

→ Scalar potentials → Vacuum selection → SSB → Inflation → Alternative asymptotics (AdS)

AdS/CFT ← Alternative asymptotics (AdS)

Focus on $\mathcal{N}=2$, $d=5$ SUGRAs

(8 supercharges)

coupled to vector multiplets

(~~hypers~~, ~~tensors~~)

- We can get solutions to cubic models of $\mathcal{N}=2$, $d=4$ SUGRA by dimensional reduction.
- More possibilities than $d=4$ (vacua, black rings...)
- Interesting geometrical problems.

· Which are the possible gaugings ?

Global symmetries : 1 R - symmetry (SU(2))
(on fermions only)

2 Isometries of scalar manifold
(compatible with Real Special Geometry)

2 Always non-Abelian (We'll just take SU(2))

1 a) Abelian
(F-I) b) non-Abelian $\Rightarrow$ 2 simultaneously

① - a $U(1)_R$ gaugings via Fayet-Iliopoulos terms

$$V(\phi) \leq 0 \Rightarrow \text{asymptotically } AdS_5$$

② non-Abelian gaugings of the Real Special manifold ($SU(2)$) SEYM THEORIES

$$V(\phi) = 0 \Rightarrow \text{asymptotically flat}$$

... → ① - b ⇒ ② Gauging of the $SU(2)_R$ R-symmetry group

$$V(\phi) \leq 0 \Rightarrow \text{asymptotically } AdS_5$$

work in progress ① - a ⊕ ② $U(1)_R \times SU(2)$ gauging

$$V(\phi) \leq 0 \Rightarrow \text{asymptotically } AdS_5$$

Plan of the talk

SU(2) SEYM

1.– Equations for timelike susy solutions.

2.– Examples of solutions.

3.– Embedding in String Theory.

U(1)

4.– Equations for timelike susy solutions

5.– New ansatz and new possibilities

Equations for timelike susy solutions

Hübscher, Meessen, O., Vaulà 2008
Bellorín, O., 2007, Bellorín 2008
Bueno, Meessen, O., Ramírez 2015
Meessen, O., Ramírez 2015

1 Timelike susy solutions of $\mathcal{N}=2, d=4$, SEYM

2 Timelike
&
3 Null

susy solutions of $\mathcal{N}=2, d=5$, SEYM
with one additional isometry

SAME EQUATIONS!

+ 3 sets of rules

.

Bogomol'nyi eqs: $$\tfrac{1}{2}\varepsilon_{\underline{rsw}}\breve{F}^{\Lambda}{}_{\underline{sw}} - \breve{\mathfrak{D}}_{\underline{r}}\Phi^{\Lambda} = 0\,,$$

Dyon eqs: $$\breve{\mathfrak{D}}_{\underline{r}}\breve{\mathfrak{D}}_{\underline{r}}\Phi_{\Lambda} - g^2 f_{\Lambda\Sigma}{}^{\Omega} f_{\Delta\Omega}{}^{\Gamma}\Phi^{\Sigma}\Phi^{\Delta}\Phi_{\Gamma} = 0\,,$$

Bubble eqs: $$\Phi_{\Lambda}\breve{\mathfrak{D}}_{\underline{r}}\breve{\mathfrak{D}}_{\underline{r}}\Phi^{\Lambda} - \Phi^{\Lambda}\breve{\mathfrak{D}}_{\underline{r}}\breve{\mathfrak{D}}_{\underline{r}}\Phi_{\Lambda} = 0\,,$$

Variables: Φ^{Λ}, Φ_{Λ}, $\breve{A}^{\Lambda}{}_{\underline{r}} \longrightarrow \breve{\mathfrak{D}}_{\underline{r}}$, $\breve{F}^{\Lambda}{}_{\underline{r}}$

$\Lambda = 0, 1, \ldots, n_{V_4}$

$n_{V_4} = n_{V_5} + 1$

$\underline{r}, \underline{s} = 1, 2, 3$

$\mathbb{E}^3$ indices

① Bogomol'nyi Equations

$$\frac{1}{2}\varepsilon_{\underline{rsw}}\breve{F}^{\Lambda}{}_{\underline{sw}} - \breve{\mathfrak{D}}_{\underline{r}}\Phi^{\Lambda} \;=\; 0\,,$$

Magnetic gauge field in Minkowski 1+3

Time-independent adjoint Higgs field

YMH action in 1+3: $\int d^4x \left\{ -\frac{1}{4}\breve{F}^{\Lambda}\breve{F}^{\Lambda} + \frac{1}{2}\breve{\mathfrak{D}}\Phi^{\Lambda}\breve{\mathfrak{D}}\Phi^{\Lambda} \right\}$

Time-independent magnetic configuration

$$\| \qquad -\frac{1}{2}\int d^4x \left[*_3\breve{F}^{\Lambda} \pm \breve{\mathfrak{D}}\Phi^{\Lambda} \right]^2$$

1^{st} order B. eqs $\Rightarrow$ 2^{nd} order YMH e.o.m.

The solutions are BPS magnetic monopoles.

Relation to $d=5$:

a) Kronheimer 1985: selfdual instantons in GH spaces

$$\| $$

BPS monopoles in $\mathbb{E}^3$

GH spaces: $\quad ds^2 = H^{-1}(dz+\alpha)^2 + H\, d\vec{x}^{\,3}$

$$\underset{\Phi^0}{\underset{\downarrow}{dH}} = *_3\, \underset{A^0}{\underset{\downarrow}{d\alpha}}$$

(Abelian B. eqs)

b) Gauntlett et al. 2002
Bellorín, O. 2007

$\mathcal{N}=2$, $d=5$, SEYM timelike susy solutions with one additional isometry have GH base spaces and selfdual YM fields

Solutions to the SU(2) Bogomol'nyi Eqs.

a) Spherically symmetric (Protogenov 1977)

General form
$$\begin{cases} \breve{A}^A = -h(r)\,\varepsilon^A{}_{rs}\,x^r\,dx^s\,; \\ \breve{\Phi}^A = -f(r)\,\delta^A{}_r\,x^r\,; \end{cases}$$

BPS 't Hooft-Polyakov magnetic monopole

$$f = -\frac{1}{g r^2}\Big[1 - \mu r \coth(\mu r + s)\Big]\,;$$

$$h = \frac{1}{g r^2}\Big[\frac{\mu r}{\sinh(\mu r + s)} - 1\Big]\,;$$

Protogenov's hair parameter

Coloured monopoles → BPST instantons $\left(H = \frac{1}{r}\right)$

$$f = -\frac{1}{g r^2 (1+\lambda^2 r)}\,; \quad h = -f\,;$$

f) Multicenter solutions

Ramírez's multimonopole solution 2015

$$\Phi^A = -\delta^{Ar}\frac{1}{gP}\partial_{\underline{r}}P\,; \qquad \check{A}^A{}_{\underline{r}} = -\varepsilon^A{}_{rs}\frac{1}{gP}\partial_{\underline{s}}P\,;$$

$$\partial_{\underline{r}}\partial_{\underline{r}}P = 0\,; \qquad P = \lambda^2 + \frac{1}{r}$$ → Coloured monopole

No more simple solutions known

② Dyon Equations

$$\breve{\mathfrak{D}}_{\underline{r}}\breve{\mathfrak{D}}_{\underline{r}}\Phi_\Lambda - g^2 f_{\Lambda\Sigma}{}^{\Omega} f_{\Delta\Omega}{}^{\Gamma}\Phi^\Sigma\Phi^\Delta\Phi_\Gamma = 0\,,$$

Determined by the B. Eqs.

a) Trivial solution: $\Phi_\Lambda = 0;$

b) Dyon solution: $\Phi_\Lambda = k\,\Phi^\Lambda;$ (compact groups)

c) Ramírez's dyon:

$$\left(\begin{aligned} \Phi^A &= -\delta^{Ar}\frac{1}{gP}\partial_{\underline r} P;\\ \breve{A}^A{}_{\underline r} &= -\varepsilon^A{}_{rs}\frac{1}{gP}\partial_{\underline s} P; \end{aligned}\right)$$

$$\boxed{\begin{aligned} \Phi_A &= -\frac{1}{gP}\delta_A{}^{\underline r}\partial_{\underline r} Q;\\ \frac{1}{P}\partial_{\underline r}\partial_{\underline r} Q &= 0; \end{aligned}}$$

③ Bubble Equations

$$\underbrace{\Phi_\Lambda \breve{\mathfrak{D}}_{\underline{r}} \breve{\mathfrak{D}}_{\underline{r}} \Phi^\Lambda}_{= 0} - \underbrace{\Phi^\Lambda \breve{\mathfrak{D}}_{\underline{r}} \breve{\mathfrak{D}}_{\underline{r}} \Phi_\Lambda}_{= 0} \;=\; 0\,,$$

$$\tfrac{1}{2}\varepsilon_{\underline{rsw}} \breve{F}^{\Lambda}{}_{\underline{sw}} - \breve{\mathfrak{D}}_{\underline{r}} \Phi^\Lambda \;=\; 0\,, \xrightarrow{\text{Bianchi YM}} \breve{\mathfrak{D}}_{\underline{r}} \breve{\mathfrak{D}}_{\underline{r}} \Phi^\Lambda = 0\,;$$

$$\breve{\mathfrak{D}}_{\underline{r}} \breve{\mathfrak{D}}_{\underline{r}} \Phi_\Lambda - g^2 f_{\Lambda\Sigma}{}^{\Omega} f_{\Delta\Omega}{}^{\Gamma} \Phi^\Sigma \Phi^\Delta \Phi_\Gamma \;=\; 0\,, \xrightarrow{f_{(\Lambda\Sigma)}{}^{\Gamma} = 0} \Phi^\Lambda \breve{\mathfrak{D}}_{\underline{r}} \breve{\mathfrak{D}}_{\underline{r}} \Phi_\Lambda = 0\,;$$

except at the singularities:

Fixed relative positions in Abelian multicenter (Denef, Bates)

For Ramirez's multicenter dyon

$$P = P_0 + \sum_\alpha \frac{P_\alpha}{|\vec{x} - \vec{x}_\alpha|}\,; \quad Q = Q_0 + \sum_\alpha \frac{Q_\alpha}{|\vec{x} - \vec{x}_\alpha|}$$

NO RESTRICTIONS

The bubble eqs are the integrability conditions of:

$$\partial_{[\underline{r}}\omega_{\underline{s}]} = 2\varepsilon_{rsw}\left(\Phi_\Lambda \breve{\mathfrak{D}}_{\underline{w}}\Phi^\Lambda - \Phi^\Lambda \breve{\mathfrak{D}}_{\underline{w}}\Phi_\Lambda\right)$$

For Ramirez's multicenter dyon $\omega^{NA}_{\underline{r}} = -4\varepsilon_{rsw}\frac{\partial_{\underline{s}}P}{P}\frac{\partial_{\underline{w}}Q}{P}$

and it does not contribute at the horizons $r_a \to 0$

Asymptotically $\omega^{NA}_{\underline{r}} \sim 1/r^5$

The solutions Φ^{Λ}, $\bar{\Phi}_{\Lambda}$, $\breve{A}^{\Lambda}{}_{\underline{r}}$ of these equations are the building blocks of the SEYM solutions

Let's build some!

N=2,d=4 SEYM solutions

Rules to construct solutions of the $\mathbb{CP}^3$ model:

Metric:
$$ds^2 = e^{2U}(dt+\omega)^2 - e^{-2U}dx^r dx^r\,,$$

where
$$\begin{cases} e^{-2U} = W(\mathcal{I})\,. \qquad \mathsf{W}(\mathcal{I}) = \tfrac{1}{2}\eta_{\Lambda\Sigma}\mathcal{I}^\Lambda\mathcal{I}^\Sigma + 2\eta^{\Lambda\Sigma}\mathcal{I}_\Lambda\mathcal{I}_\Sigma\,. \\ \partial_{[\underline{r}}\omega_{\underline{s}]} = 2\varepsilon_{rsw}\left(\Phi_\Lambda\breve{\mathfrak{D}}_{\underline{w}}\Phi^\Lambda - \Phi^\Lambda\breve{\mathfrak{D}}_{\underline{w}}\Phi_\Lambda\right) \end{cases}$$

$$(\overset{0}{+}\underbrace{\overset{A}{-\,-\,-}}_{SO(3)})$$

and
$$\mathcal{I}^\Lambda = -\sqrt{2}\Phi^\Lambda\,, \qquad \mathcal{I}_\Lambda = -\sqrt{2}\Phi_\Lambda\,,$$

(To simplify the presentation we focus on the metrics and ignore scalars and vector fields)

1) Spherically-symmetric solutions: monopole
Abelian BH + monopole
coloured BH

2) Multicenter coloured BHs

1.- Global monopole

Abelian sector $\Lambda = 0$ $\rightarrow$ $\Phi^0 =$ constant, $\Phi_0 = 0$

Non-Abelian sector $\Lambda = A$ $\rightarrow \begin{cases} \Phi^A \text{ Higgs field of BPS 't Hooft-Polyakov} \\ \Phi_A = 0 \end{cases}$

$$\Rightarrow \quad \omega = 0; \quad \boxed{e^{-2U} = 1 + (\mu/g)^2 - \frac{1}{g^2 r^2}\left[1 - \frac{\mu r}{\sinh \mu r}\cosh \mu r\right]^2}$$

$$e^{-2U} \in \Big[\underbrace{1 + \tfrac{1}{2}(\mu/g)^2}_{r=0},\ \underset{\substack{\uparrow \\ r\sim\infty}}{1}\Big), \quad M = \frac{\mu}{g^2 G_N^{(4)}};$$

Globally regular, horizonless solution.

2.- Global monopole + RN BH

$\Phi_0 = 0$

Abelian sector $\Lambda = 0$ $\rightarrow$ $\Phi^0 = \text{constant} + \frac{\overset{\circ}{p}{}^0/2}{r}$;

Non-Abelian sector $\Lambda = A$ $\rightarrow$ Φ^A Higgs field of BPS 't Hooft-Polyakov; $\Phi_A = 0$

$$\Rightarrow \omega = 0\,;\quad \boxed{e^{-2U} = \left[\sqrt{1+(\mu/g)^2} + \frac{p^0/2}{r}\right]^2 - \frac{1}{g^2 r^2}\left[1 - \frac{\mu r}{\sinh \mu r}\coth \mu r\right]^2}$$

$$e^{-2U} \in \Big(\underset{r\to 0}{\infty}\,,\ \underset{r\sim\infty}{1}\Big)\,,\quad G_N^{(4)} M = \sqrt{\frac{1+(\mu/g)^2}{4}}\,p^0 + \frac{\mu}{g^2}\,;$$

$$e^{-2U} \underset{r\sim 0}{\sim} \frac{(p^0)^2/4}{r^2}\,,\qquad S = \pi (p^0)^2/4\,;$$

No contribution to the entropy.

3.- Coloured Black Hole

Non-Abelian sector : $\begin{cases} \Phi^A = \dfrac{1}{g r^2(1+\lambda^2 r)} \delta^A{}_r x^r ; \\ \Phi_A = 0 ; \end{cases}$

Since $\Phi^A \Phi^A \underset{r\sim 0}{\sim} \dfrac{1}{g^2 r^2}$, we need a charge in the Abelian sector

Abelian sector : $\Phi^0 = 1 + \dfrac{p^0/2}{r}$; $\Phi_0 = 0$;

$$\Rightarrow \omega = 0 ; \quad \boxed{e^{-2U} = \left(1 + \frac{p^0/2}{r}\right)^2 - \frac{1}{g^2 r^2 (1+\lambda^2 r)^2} \quad ;}$$

$e^{-2U} \underset{r\to\infty}{\sim} 1 + \dfrac{p^0}{r}$; $M = \dfrac{p^0}{2 G_N^{(4)}}$; The non-Abelian field only at horizon?

$e^{2U} \sim \left[(p^0/2)^2 - \dfrac{1}{g^2}\right] \dfrac{1}{r^2}$; $S = \dfrac{\pi}{G_N^{(4)}} \left[(p^0/2)^2 - \dfrac{1}{g^2}\right]$;

4.- Dumbbell solution

Non-Abelian sector :

$$\begin{cases} \Phi^A = \dfrac{1}{g r^2 (1+\lambda^2 r)} \delta^A{}_r x^r ; \\ \Phi_A = 0 ; \end{cases}$$

Abelian sector :

$$\Phi^0 = \cancel{1} + \frac{p^0/2}{r} ; \quad \Phi_0 = 0 ;$$

Will not be asymptotically flat

$$\Rightarrow \omega = 0 ; \quad \boxed{e^{-2U} = \frac{(p^0/2)^2}{r^2} - \frac{1}{g^2 r^2 (1+\lambda^2 r)^2} ;}$$

$$e^{-2U} \underset{r \sim \infty}{\sim} \frac{(p^0/2)^2}{r^2} \Rightarrow AdS_2 \times S^2 \text{ at } r = \infty$$

Different radii

$$e^{-2U} \underset{r \sim 0}{\sim} \left[(p^0/2)^2 - \frac{1}{g^2} \right] \frac{1}{r^2} ; \Rightarrow AdS_2 \times S^2 \text{ at } r = 0$$

5.- Multicenter Coloured Black Hole

$$H = h + \sum_{\alpha=1}^{N} \frac{p_\alpha}{r_\alpha}\,, \qquad P = \lambda + \sum_{\alpha=1}^{N} \frac{s_\alpha}{r_\alpha}\,, \qquad Q = -\sum_{\alpha=1}^{N} \frac{\eta_\alpha s_\alpha/2}{r_\alpha}\,,$$

$$\Phi^0 = -H\,, \qquad \vec{\Phi} = -\frac{1}{gP}\vec{\nabla}P\,, \qquad \vec{\mathcal{J}} = \frac{2}{gP}\vec{\nabla}Q\,,$$

$\vec{\Phi} = (\Phi^A)$

$\vec{\mathcal{J}} = 2(\Phi_A)$

Non-Abelian sector: Ramírez's dyon

Abelian sector: Papapetrou-Majumdar (multi-Reissner-Nordström)

$$\boxed{\begin{aligned} e^{-2U} &= H^2 - \vec{\Phi}^2 - \vec{\mathcal{J}}^2\,, & \vec{Z} &= e^{-i\gamma}\frac{\vec{\Phi} + i\vec{\mathcal{J}}}{H}\,, \\ \vec{\omega} &= 2g^2\,\vec{\Phi}\times\vec{\mathcal{J}}\,, & V &= 2g^2 e^{4U}\,|\vec{\Phi}\times\vec{\mathcal{J}}|^2 \end{aligned}}$$

The metric function e^{-2U} can be written like this:

$$e^{-2U} = h + \sum_{\alpha=1}^{N} \frac{2M_\alpha}{r_\alpha} + \sum_{\alpha=1}^{N} \left[E_\alpha + (1+\eta_\alpha^2)\, R_\alpha\right] \frac{1}{r_\alpha^2}$$

$$+ \sum_{\alpha>\beta}^{N} \left[E_{\alpha\beta} - E_\alpha - E_\beta + 2(1+\eta_\alpha\eta_\beta)\, R_{\alpha\beta}\right] \frac{1}{r_\alpha r_\beta}$$

Manifestly positive functions

where

Mass of α^{th} BH

$$M_\alpha \equiv h p_\alpha \,, \qquad > 0$$

Entropy of α^{th} BH

$$E_\alpha \equiv p_\alpha^2 - (1+\eta_\alpha^2)/g^2 \,, \qquad > 0$$

$\Rightarrow e^{-2U} > 0$

$E(\alpha+\beta)$

$$E_{\alpha\beta} \equiv (p_\alpha + p_\beta)^2 - 4/g^2 - (\eta_\alpha + \eta_\beta)^2/g^2 \qquad > E_\alpha + E_\beta$$

$\vec{\omega}$ is regular at each r_α and there are no CTCs.

N=2,d=5 SEYM solutions

ST[2, n] model

$C_{0xy} = \frac{1}{6}\eta_{xy}$

$\left. \begin{array}{l} A^0_\mu, \; A^x_\mu \\ \phi^x \end{array} \right\} \; x = 1, 2 \dots n$

$\eta = (\underset{\substack{\downarrow \\ 1}}{+} \; \underset{\substack{\downarrow \\ 2}}{-} \; \underbrace{- \cdots}_{A})$ $\leftarrow su(2)$

$\underbrace{1 \;\; 2}_{k, \phi} \quad \underbrace{A}_{\ell^A}$

$$S = \int d^5x\sqrt{g}\left\{R + \partial_\mu\phi\partial^\mu\phi + \tfrac{4}{3}\partial_\mu \log k \partial^\mu \log k + 2e^{-\phi}k^{-2}\mathfrak{D}_\mu\ell^A\mathfrak{D}^\mu\ell^A\right.$$

$$-\tfrac{1}{12}e^{2\phi}k^{-4/3}F^0\cdot F^0 + \tfrac{1}{12}\left(\eta_{xy}e^{-\phi}k^{2/3} - 9h_xh_y\right)F^x\cdot F^y$$

$$\left.+\frac{1}{24\sqrt{3}}\frac{\varepsilon^{\mu\nu\rho\sigma\alpha}}{\sqrt{g}}A^0{}_\mu\eta_{xy}F^x{}_{\nu\rho}F^y{}_{\sigma\alpha}\right\},$$

These models can be obtained from Heterotic Supergravity compactified on T^5 and truncated to $\mathcal{N}=2$

Rules to construct timelike solutions of the ST[2,n] model:

Metric: (only)

$$ds^2 = \hat{f}^{\,2}(dt+\hat{\omega})^2 - \hat{f}^{\,-1}\left[H^{-1}(dz+\chi)^2 + H dx^r dx^r\right]$$

where

$$\left\{\begin{aligned}
\hat{f}^{\,-1} &= H^{-1}\left\{\tfrac{1}{4}\left(6HL_0 + 8\eta_{xy}K^xK^y\right)\left[9H^2\eta^{xy}L_xL_y + 48HK^0L_xK^x\right.\right.\\
&\qquad \left.\left. +64(K^0)^2\eta_{xy}K^xK^y\right]\right\}^{1/3}.\\
\hat{\omega} &= \omega_5(dz+\chi) + \omega\,,\\
\omega_5 &= M + 16\sqrt{2}H^{-2}C_{IJK}K^IK^JK^K + 3\sqrt{2}H^{-1}L_IK^I\,,\\
\partial_{[\underline{r}}\omega_{\underline{s}]} &= 2\varepsilon_{rsw}\left(\Phi_\Lambda\breve{\mathfrak{D}}_{\underline{w}}\Phi^\Lambda - \Phi^\Lambda\breve{\mathfrak{D}}_{\underline{w}}\Phi_\Lambda\right)
\end{aligned}\right.$$

and $K^I = \delta^I{}_\Lambda\Phi^{\Lambda+1}\,,\qquad L_I = -\frac{2\sqrt{2}}{3}\delta_I{}^\Lambda\Phi_{\Lambda+1}\,,\qquad H = -2\sqrt{2}\Phi^0\,,\qquad M = +\sqrt{2}\Phi_0\,,$

Simplest non-Abelian Black Hole

Abelian sector:
(3-charge BH)

$$\begin{cases} L_0 = -\frac{2\sqrt{2}}{3}\Phi_1 = B_0 + q_0/\rho^2 ; \\ L_\pm = L_1 \pm L_2 = -\frac{2\sqrt{2}}{3}\left(\Phi_2 \pm \Phi_3\right) = B_\pm + q_\pm/\rho^2 ; \end{cases}$$

Non-Abelian sector:
("Coloured monopole")

$$\Phi^A = \frac{1}{g\,r^2(1+\lambda^2 r)}\,\delta^A{}_r\, x^r$$

GH metric:
($\mathbb{R}^4 - \{0\}$)

$$H = 1/r \;; \qquad r = \rho^2/4 \;;$$

$\mathbb{R}^3$ $\mathbb{R}^4$

Kronheimer

BPST instanton in $\mathbb{R}^4$

$$\Phi^2 = \Phi^A\Phi^A = \frac{2\kappa^4}{3g^2\rho^4(\rho^2+\kappa^2)^2} \;;$$

$$\hat{\omega} = 0 \; ; \quad \hat{f}^{-3} = \underbrace{\left(L_0 - \frac{g^2}{3}\Phi^2\right)}_{\tilde{L}_0} L_+ L_- \; ;$$

$$\tilde{L}_0 B_0 + \frac{q_0}{\rho^2} - \frac{2}{9g^2}\frac{\kappa^2}{\rho^2(\rho^2+\kappa^2)^2} \; ;$$

$O\left(\frac{1}{\rho^2}\right)$ on the horizon $\rho \to 0$

$O\left(\frac{1}{\rho^2}\right)$ at $\rho \to \infty$

$O\left(\frac{1}{\rho^6}\right)$ at $\rho \to \infty$

→ Same puzzle as in $d=4$.

BUT

$$\tilde{L}_0 = B_0 + \left(q_0 - \frac{2}{9g^2}\right)\frac{1}{\rho^2} + \frac{2}{9g^2}\frac{\rho^2+2\kappa^2}{(\rho^2+\kappa^2)^2},$$

!!

It is remarkable that we can rewrite $\tilde{L}_0$ like this:

$$\tilde{L}_0 = B_0 + (q_0 - \frac{2}{9g^2})\frac{1}{\rho^2} + \frac{2}{9g^2}\frac{\rho^2 + 2\kappa^2}{(\rho^2+\kappa^2)^2},$$

Suggests: $q_0 - \frac{2}{9g^2}$ some standard brane charge $\frac{1}{\rho^2}$

What is $\frac{2}{9g^2}$?

Let's switch off everything else: $\begin{cases} q_0 - \frac{2}{9g^2} = 0 \\ q_\pm = 0 \end{cases}$

What do we get?

The full solution has this form:

$$
\begin{aligned}
ds^2 &= \hat{f}^2 dt^2 - \hat{f}^{-1}(d\rho^2 + \rho^2 d\Omega^2_{(3)}) \,, \\
\hat{f}^{-3} &= 1 + \frac{2e^{-\phi_\infty} k_\infty^{2/3}}{3g^2} \frac{\rho^2 + 2\kappa^2}{(\rho^2+\kappa^2)^2} \,, \\
A^0 &= -\frac{1}{\sqrt{3}} \hat{f}^3 dt \,, \qquad A^A = \frac{\kappa^2}{g(\rho^2+\kappa^2)} v_L^A \,, \\
e^{2\phi} &= e^{2\phi_\infty} \hat{f}^{-3} \,, \qquad k = k_\infty \hat{f}^{3/4} \,,
\end{aligned}
$$

Spherically symmetric, globally regular, horizon-less, asymptotically flat

"GLOBAL INSTANTON"

What is a "global instanton"?

let's uplift the solution to $d=10$ Heterotic Supergravity (other uplifts more difficult or impossible)

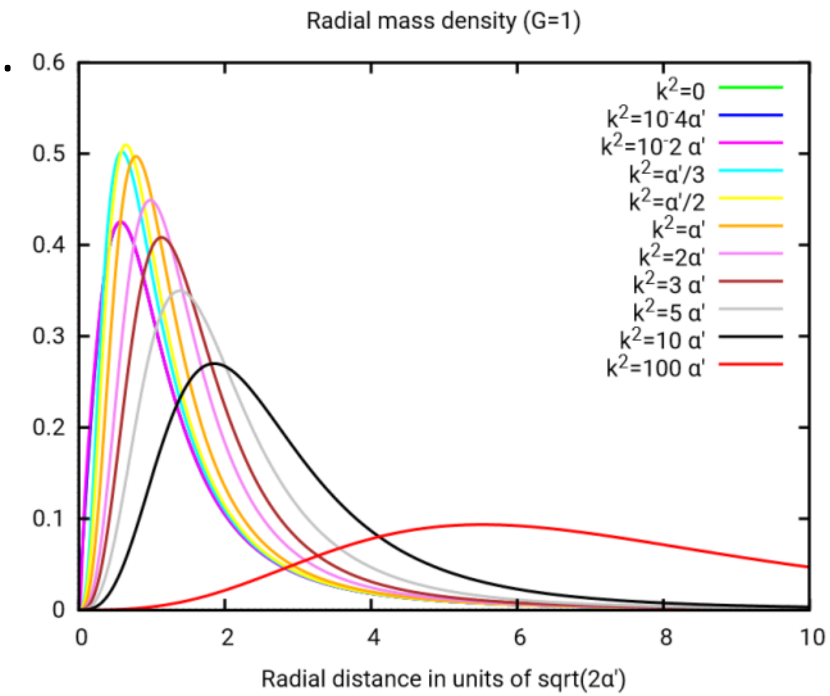
$$g_5 = k_\infty^{1/3}\, e^{-\phi_\infty/2} / \sqrt{12\alpha'} \;;$$

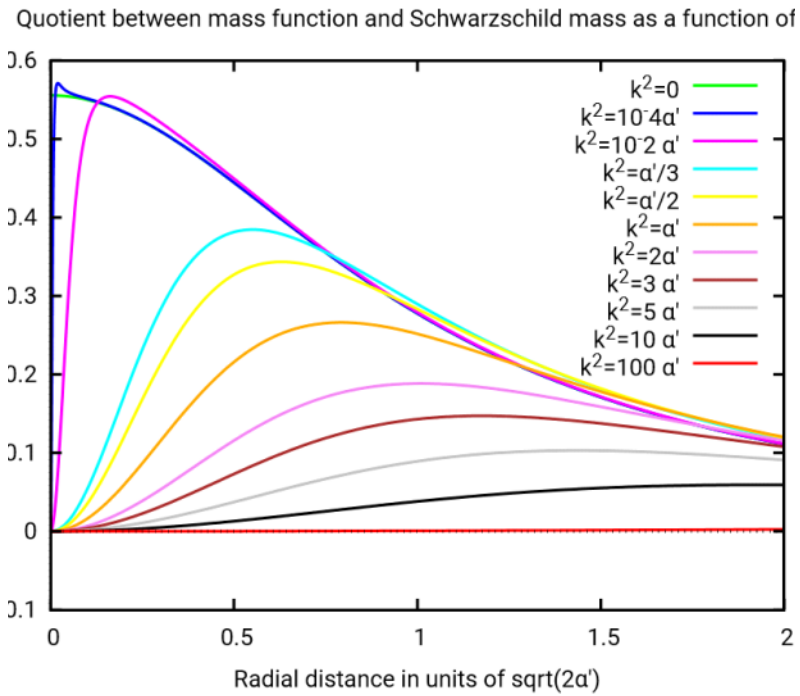
$$g_s = e^{\phi_\infty} \;;$$

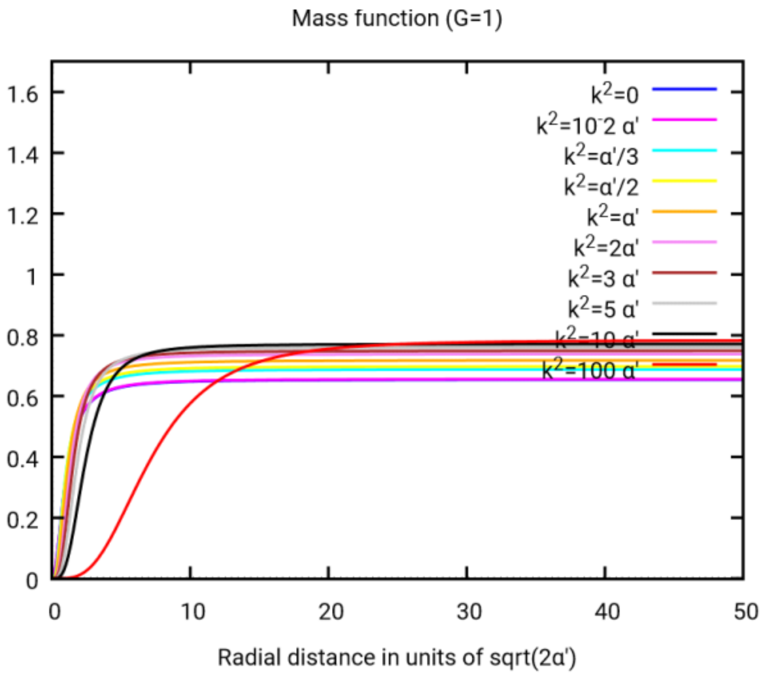
$$l_s = \sqrt{\alpha'} \;;$$

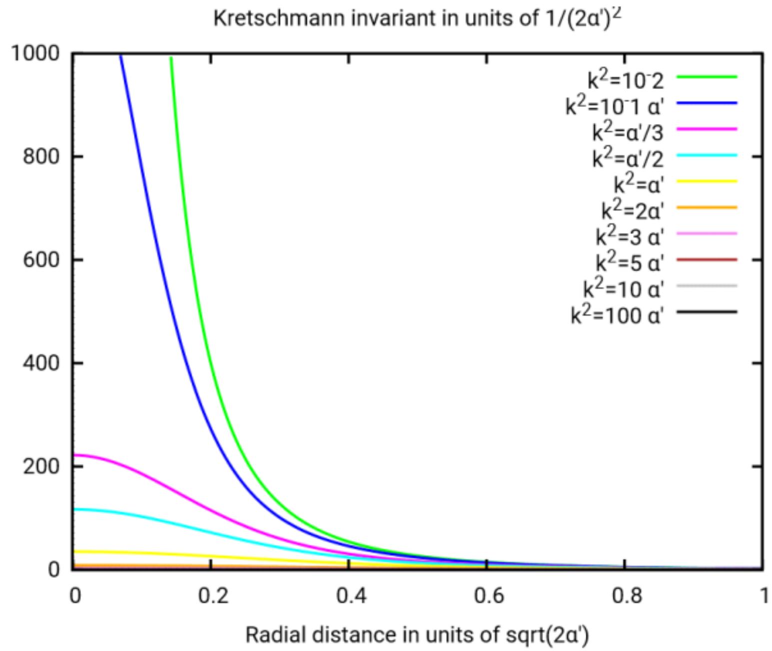
$$\Rightarrow \quad e^{2\phi} = e^{2\phi_\infty} \hat{f}^{-3} = e^{2\phi_\infty} \left\{ 1 + 8\alpha' \frac{\rho^2 + 2\kappa^2}{(\rho^2+\kappa^2)^2} \right\}$$

Characteristic of the GAUGE FIVEBRANE


Radial mass density (G=1)
Radial distance in units of sqrt(2α')
$k^2=0$
$k^2=10^{-4}\alpha'$
$k^2=10^{-2}\alpha'$
$k^2=\alpha'/3$
$k^2=\alpha'/2$
$k^2=\alpha'$
$k^2=2\alpha'$
$k^2=3\alpha'$
$k^2=5\alpha'$
$k^2=10\alpha'$
$k^2=100\alpha'$


Quotient between mass function and Schwarzschild mass as a function of
Radial distance in units of sqrt(2α')
$k^2=0$
$k^2=10^{-4}\alpha'$
$k^2=10^{-2}\alpha'$
$k^2=\alpha'/3$
$k^2=\alpha'/2$
$k^2=\alpha'$
$k^2=2\alpha'$
$k^2=3\alpha'$
$k^2=5\alpha'$
$k^2=10\alpha'$
$k^2=100\alpha'$


Mass function (G=1)
Radial distance in units of sqrt(2α')
$k^2=0$
$k^2=10^{-2}\alpha'$
$k^2=\alpha'/3$
$k^2=\alpha'/2$
$k^2=\alpha'$
$k^2=2\alpha'$
$k^2=3\alpha'$
$k^2=5\alpha'$
$k^2=10\alpha'$
$k^2=100\alpha'$


Kretschmann invariant in units of $1/(2\alpha')^2$
Radial distance in units of sqrt(2α')
$k^2=10^{-2}$
$k^2=10^{-1}\alpha'$
$k^2=\alpha'/3$
$k^2=\alpha'/2$
$k^2=\alpha'$
$k^2=2\alpha'$
$k^2=3\alpha'$
$k^2=5\alpha'$
$k^2=10\alpha'$
$k^2=100\alpha'$

These non-Abelian black holes consist of $\underbrace{\text{3 standard branes}}$ plus a $\underbrace{\text{gauge 5-brane.}}$

↓ Standard 3-charge BH

↓ D1 D5 W up to dualities

(Strominger-Vafa)

↓ Contributes to the mass but not to the entropy

$\Rightarrow$ We can explain the entropy.

Correct identification of charges is essential.

After some redefinitions, this is the full solution

$$ds^2 = f^2 dt^2 - f^{-1}(d\rho^2 + \rho^2 d\Omega^2_{(3)}) \,,$$

$$A^0 = -\sqrt{3}e^{-\phi_\infty}k_\infty^{2/3}\frac{dt}{\tilde{\mathcal{Z}}_0} \,, \qquad A^1 + A^2 = -\sqrt{3}e^{\phi_\infty}k_\infty^{2/3}\frac{dt}{\mathcal{Z}_+} \,,$$

$$A^A = -\frac{1}{g}\frac{\rho^2}{(\kappa^2+\rho^2)}v_R^A \,, \qquad A^1 - A^2 = -2\sqrt{3}k_\infty^{-4/3}\frac{dt}{\mathcal{Z}_-} \,,$$

$$e^{2\phi} = e^{2\phi_\infty}\frac{\tilde{\mathcal{Z}}_0}{\mathcal{Z}_+} \,, \qquad k = k_\infty (f\mathcal{Z}_-)^{3/4} \,,$$

$$f^{-3} = \tilde{\mathcal{Z}}_0\, \mathcal{Z}_+\, \mathcal{Z}_- \,,$$

$$\left(q_0 - \frac{q}{2g^2}\right)$$

$$\tilde{\mathcal{Z}}_0 = 1 + \frac{\tilde{\mathcal{Q}}_0}{\rho^2} + \frac{2e^{-\phi_\infty}k_\infty^{2/3}}{3g^2}\frac{\rho^2 + 2\kappa^2}{(\rho^2+\kappa^2)^2} \,, \qquad \mathcal{Z}_\pm = 1 + \frac{\mathcal{Q}_\pm}{\rho^2} \,.$$

.

$$M = \frac{\pi}{4G_N^{(5)}}\left[\tilde{\mathcal{Q}}_0 + \frac{2e^{-\phi_\infty}k_\infty^{2/3}}{3g^2} + \mathcal{Q}_+ + \mathcal{Q}_-\right]$$

$$S = \frac{\pi^2}{2G_N^{(5)}}\sqrt{\tilde{\mathcal{Q}}_0\mathcal{Q}_+\mathcal{Q}_-}\,.$$

5-brane contribution

absent

$$\tilde{Q}_0 \sim \int_{S^3_\infty} \left(*F^0 - \omega_{CS}\right); \qquad \left(d*F^0 - F^A\wedge F^A = 0\right)$$

. In $d=10$ Heterotic Supergravity

$$d\hat{s}^2 = \frac{2}{\mathcal{Z}_+}du\left(dv - \tfrac{1}{2}\mathcal{Z}_- du\right) - \tilde{\mathcal{Z}}_0(d\rho^2 + \rho^2 d\Omega^2_{(3)}) - dz^i dz^i \,,$$

F1

S5 + G5

$$\hat{B} = -\frac{1}{\mathcal{Z}_+}dv \wedge du + \tfrac{1}{4}\mathcal{Q}_0 \cos\theta d\psi \wedge d\phi \,,$$

$$\hat{A}^A = -\frac{\rho^2}{(\kappa^2+\rho^2)}v^A_R \,, \longrightarrow \text{BPST instanton}$$

$$e^{-2\hat{\phi}} = e^{-2\hat{\phi}_\infty}\frac{\mathcal{Z}_+}{\tilde{\mathcal{Z}}_0} \,,$$

$Q_0 = \tilde{Q}_0 + 8\alpha'$

Pure gauged N=2,d=5

1 vector field (graviphoton) $\Rightarrow$ Abelian gauging (F-I)

0 scalars

$$S = \int d^5x\sqrt{g}\left\{R + 4g^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{12\sqrt{3}}\frac{\varepsilon^{\mu\nu\rho\sigma\alpha}}{\sqrt{g}}F_{\mu\nu}F_{\rho\sigma}A_{\alpha}\right\},$$

Cosmological Einstein-Maxwell + Chern-Simons

$\Lambda = -\frac{4}{3}g^2 < 0$ AdS-type

Finding supersymmetric solutions is much more difficult: coupled differential equations.

(No known multicenter solutions)

The supersymmetric solutions have the

(Gauntlett & Gutowski, 2003)

$$ds^2 = \hat{f}^{\,2}(dt+\hat{\omega})^2 - \hat{f}^{\,-1}\underline{h_{mn}}\,dx^m dx^n$$

metric function

Kähler metric

1-forms

$$A = -\sqrt{3}\,\hat{f}\,(dt+\hat{\omega}) + \hat{A}$$

$\hat{f}$, $\hat{\omega}$, $\hat{A}$ time-independent defined only on $\underline{h_{mn}}$ satisfying the following conditions

- $$\hat{F}^{+} = \frac{2}{\sqrt{3}}(\hat{f}d\hat{\omega})^{+}$$

Kähler 2-form

- $$\hat{F}^{-} = -2g\hat{f}^{-1}\hat{J}$$

Ricci 2-form

- $$\hat{\mathfrak{R}}_{mn} = -g\hat{F}_{mn} \longrightarrow \hat{R} = 8g^{2}\hat{f}^{-1}$$

1 Kähler metric for each solution

- $$\hat{\nabla}^{2}\hat{f}^{-1} - \frac{1}{6}\hat{F}\cdot\hat{\star}\hat{F} + \frac{1}{\sqrt{3}}g\hat{J}\cdot(d\hat{\omega}) = 0$$

We need a method to construct Kähler metrics which give interesting solutions.

efficient!

(6th-order equations?)

(Figueras, Herdeiro, Pacetti, Correia 2006)

(Cassani, Lorenzen, Quartelli 2016)

. How can we "parametrize" 4-d Kähler manifolds?

Figueras et al. (2006) : cones over Sasaki manifolds

Cassani et al. (2015) : orthotoric Kähler

Chimento & O. (2016) : 1 holomorphic isometry

$$ds^2 = H^{-1}(dz+\chi)^2 + H\left\{(dx^2)^2 + W^2(\vec{x})[(dx^1)^2 + (dx^3)^2]\right\}$$

(Le Brun 1991 ?
Tod 1995
Chimento & O. 2016)

$$\left.\begin{aligned}(d\chi)_{\underline{12}} &= \partial_{\underline{3}} H\,,\\(d\chi)_{\underline{23}} &= \partial_{\underline{1}} H\,,\\(d\chi)_{\underline{31}} &= \partial_{\underline{2}}\left(W^2 H\right)\,,\end{aligned}\right\} \rightarrow \partial_{\underline{1}}\partial_{\underline{1}} H + \partial_{\underline{2}}\partial_{\underline{2}}\left(W^2 H\right) + \partial_{\underline{3}}\partial_{\underline{3}} H = 0\,.$$

$W = 1 \Rightarrow$ hyperKähler with triholomorphic isometry.

Example

$x^2 \to \rho$; $\quad x^1 \to x$; $\quad x^3 \to y$;

Assume $H = H(\rho) \longrightarrow W^2 = \frac{\rho}{H(\rho)} \Phi(x,y) + \frac{1}{H(\rho)} \Sigma(x,y)$

A different Kähler space for each choice of H, Φ, Σ !

Simple choice $\Sigma = 0$; $\quad W^2 = \frac{\rho}{H(\rho)} \Phi(x,y)$;

$$(\partial_x^2 + \partial_y^2) \log \Phi = -2k\Phi \; ; \quad \text{(Liouville eq.)}$$

$$ds^2 = H^{-1}(dz + \chi_{(k)})^2 + H d\rho^2 + \rho \, d\Omega^2_{(2,k)} ;$$

$$H^{-1}(\rho) = \rho\left(k + \tfrac{4}{3} g^2 \rho\right) ; \to \overline{CP}^2 \quad k = 0, \pm 1$$

(Gutowski & Reall 2004) $\boxed{AdS_5} \longleftarrow \hat{\rho} = 1$; $\hat{\omega} = \frac{2}{\sqrt{3}} g (dz + \chi_{(k)})$

N=2,d=5 AdS SEYM (work in progress)

Combine SEYM (ST[2,n]) with F.I gauging in another direction.

$\rightarrow \hat{F}^A = \star \hat{F}^A$ in Kähler base space

Assume a holomorphic isometry and generalise Kronheimer:

$$\hat{A}^I = -H^{-1}\Phi^I(dz+\chi) + \breve{A}^I$$

$$\Downarrow$$

Generalized Bogomolu'yi eq. $\rightarrow$

$$\breve{\mathfrak{D}}\Phi^I = \star_3 \breve{F}^I - \Phi^I \partial_2 \log W^2 dx^2 ,$$

on

$$d\breve{s}_3 = (dx^2)^2 + W^2\left[(dx^1)^2 + (dx^3)^2\right] .$$

. In the case $W^2 = \psi(\rho)\, \Phi(x,y)$

$$\breve{\mathfrak{D}}\left(\underbrace{\Psi\Phi^I}_{\tilde{\Phi}^I}\right) = \Psi \star_3 \breve{F}^I$$

If $\Phi(x,y)\left[dx^2 + dy^2\right] = d\Omega^2_{(2)}$ (S^2) we can define Cartesian coordinates y^1, y^2, y^3 $\quad y^A y^A = \rho^2$ etc.

$\Rightarrow$ Hedgehog Ansatz
$$\begin{cases} \Phi^A = f(\rho)\, y^A; \\ A^A = h(\rho)\, \varepsilon^A{}_{BC}\, y^B dy^C; \end{cases}$$

$$\Psi f\left(1 + h\rho^2\right) = -\Psi\left(\rho h' + 2h\right),$$

$$\frac{\Psi}{\rho} h' + \rho^2 h^2 - 2h\frac{\rho^2 - \Psi}{\rho^2} = \left(\frac{f'}{\rho} - hf\right)\Psi + \frac{\Psi'}{\rho} f.$$

A Protogenov problem for each $\psi(\rho)$

Conclusions

- $d = 4, 5$ supersymmetric solutions with n-A fields are very interesting and deserve more attention.
- Interesting geometrical problems: instantons on 4-d Kähler spaces (non-compact!)
- From the physical point of view important problems remain to be solved: non-extremal?
- Solutions of SU(2) F-I case to be found.
  Hypers?

Thanks!