Non-Abelian Black Holes in String Theory

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In collaboration with

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Introduction/Motivation

- Many interactions in Nature are non-Abelian.
- String Theory describes non-Abelian interactions:

  → Fenomenology

  → Anomaly cancellation

  and Gravity.
- At some level the interplay between Gravity and non-Abelian fields must be relevant.

  Black holes...?
- Very little is known: the focus, so far, has been on gravitating solutions carrying Abelian charges.

- Working with non-Abelian fields is much more complicated.
- Previous work in Einstein-Yang-Mills (EYM) and EYM-Higgs: interesting (non-Abelian hair) but discouraging (only numerical solutions) (Bartnik-McKinnon's particle, Bizon's black hole etc.)
- Supersymmetry, the main tool in the construction of gravitating solutions with Abelian fields has not been used in EYM or EYMH

$\rightsquigarrow$ Consider the supersymmetrization

$\cancel{N=1}$

$N=2$

SEYM Theories

(gauged SUGRAS)

SEYM Theories:

- Supersymmetric solution-generating techniques ("Classifications")
- Embedding in Superstring Theories.

Plan of the talk

1.– Which SEYM theories?

2.– Equations for timelike susy solutions.

3.– Examples of solutions.

4.– Embedding in String Theory.

N=2 SEYM Theories

($d = 4, 5$)

Simplest supersymmetrization of EYM theories (Higgses are determined by SUSY)

$\mathcal{N}=1$: No "timelike" (black holes, monopoles) solutions.

$\mathcal{N}=2$: Simplest with timelike supersymmetric solutions.

Options

a) NA gauging of isometries
iso hypers ⇒ iso vectors

b) NA gauging of R-symmetry (SU(2))
Fayet-Iliopoulos ⇒ iso vectors

$\Rightarrow$ Simplest possibility:

gauge $SU(2) \subset$ iso vectors

w/o $\left\{ \begin{array}{l} \text{FI terms} \\ \text{hypers} \end{array} \right.$

$\mathcal{N}=2,\ d=4,5,\ SU(2)$ SEYM Theories

Scalar potentials?

$d=4$; $\quad V(z,\bar{z}) = -\frac{1}{4} g^2\, \operatorname{Im} \mathcal{N}^{\Lambda\Sigma} P_\Lambda P_\Sigma \geqslant 0$

$d=5$; $\quad V(\phi) = 0 \geqslant 0$

$\Rightarrow$ Asymptotically-flat susy solutions.

Equations for timelike susy solutions

Hübscher, Meessen, O., Vaulà 2008
Bellorín, O., 2007, Bellorín 2008
Bueno, Meessen, O., Ramírez 2015
Meessen, O., Ramírez 2015

1 Timelike susy solutions of $\mathcal{N}=2, d=4$, SEYM

2 Timelike & 3 Null } susy solutions of $\mathcal{N}=2, d=5$, SEYM with one additional isometry

SAME EQUATIONS!

+ 3 sets of rules

.

Bogomol'nyi eqs: $$\tfrac{1}{2}\varepsilon_{\underline{rsw}}\breve{F}^{\Lambda}{}_{\underline{sw}} - \breve{\mathfrak{D}}_{\underline{r}}\Phi^{\Lambda} = 0\,,$$

Dyon eqs: $$\breve{\mathfrak{D}}_{\underline{r}}\breve{\mathfrak{D}}_{\underline{r}}\Phi_{\Lambda} - g^{2} f_{\Lambda\Sigma}{}^{\Omega} f_{\Delta\Omega}{}^{\Gamma}\Phi^{\Sigma}\Phi^{\Delta}\Phi_{\Gamma} = 0\,,$$

Bubble eqs: $$\Phi_{\Lambda}\breve{\mathfrak{D}}_{\underline{r}}\breve{\mathfrak{D}}_{\underline{r}}\Phi^{\Lambda} - \Phi^{\Lambda}\breve{\mathfrak{D}}_{\underline{r}}\breve{\mathfrak{D}}_{\underline{r}}\Phi_{\Lambda} = 0\,,$$

Variables: $\Phi^{\Lambda}\,,\ \Phi_{\Lambda}\,,\ \breve{A}^{\Lambda}{}_{\underline{r}} \longrightarrow \breve{\mathfrak{D}}_{\underline{r}}$

$\breve{F}_{\underline{r}}{}^{\Lambda}$

$\underline{r},\underline{s} = 1,2,3$

$\mathbb{E}^3$ indices

$\Lambda = 0,1,\ldots,n_{V4}$

$n_{V_4} = n_{V_5} + 1$

① Bogomol'nyi Equations

$$\frac{1}{2}\varepsilon_{\underline{rsw}}\breve{F}^{\Lambda}{}_{\underline{sw}} - \breve{\mathfrak{D}}_{\underline{r}}\Phi^{\Lambda} = 0,$$

Magnetic gauge field in Minkowski 1+3

Time-independent adjoint Higgs field

YMH action in 1+3: $\int d^4x \left\{ -\frac{1}{4}\breve{F}^{\Lambda}\breve{F}^{\Lambda} + \frac{1}{2}\breve{\mathfrak{D}}\Phi^{\Lambda}\breve{\mathfrak{D}}\Phi^{\Lambda} \right\}$

Time-independent magnetic configuration

$$\| \quad -\frac{1}{2}\int d^4x \left[*_3\breve{F}^{\Lambda} \pm \breve{\mathfrak{D}}\Phi^{\Lambda} \right]^2$$

1st order B. eqs $\Rightarrow$ 2nd order YMH e.o.m.

The solutions are BPS magnetic monopoles.

Relation to $d=5$:

a) Kronheimer 1985 : selfdual instantons in GH spaces

$\|$

BPS monopoles in $\mathbb{E}^3$

GH spaces :
$$ds^2 = H^{-1}(dz+\alpha)^2 + H\, d\vec{x}^{\,3}$$
$$dH = *_3\, d\alpha$$
$$\downarrow \qquad\qquad \downarrow$$
$$\tilde{\Phi}^0 \qquad\qquad \tilde{A}^0$$

(Abelian B. eqs)

b) Gauntlett et al. 2002
Bellorín, O. 2007

$\mathcal{N}=2$, $d=5$, SEYM timelike susy solutions with one additional isometry have GH base spaces and selfdual YM fields

the SU(2) Bogomol'nyi Eqs.

a) Spherically symmetric (Protogenov 1977)

General form
$$\begin{cases} \breve{A}^A = -h(r)\, \varepsilon^A{}_{rs}\, x^r\, dx^s ; \\ \breve{\Phi}^A = -f(r)\, \delta^A{}_r\, x^r ; \end{cases}$$

BPS 't Hooft-Polyakov magnetic monopole

$$f = -\frac{1}{g r^2}\left[1 - \mu r \coth(\mu r + s)\right];$$

$$h = \frac{1}{g r^2}\left[\frac{\mu r}{\sinh(\mu r + s)} - 1\right];$$

Protogenov's hair parameter

Coloured monopoles → BPST instantons $\left(H = \frac{1}{2}\right)$

$$f = -\frac{1}{g r^2(1+\lambda^2 r)} ; \quad h = -f ;$$

f)

Ramirez's multimonopole solution 2015

$$\Phi^A = -\delta^{Ar}\frac{1}{gP}\partial_{\underline{r}}P\ ; \qquad \breve{A}^A{}_{\underline{r}} = -\varepsilon^A{}_{rs}\frac{1}{gP}\partial_{\underline{s}}P\ ;$$

$$\partial_{\underline{r}}\partial_{\underline{r}}P = 0 \quad ; \qquad P = \lambda^2 + \frac{1}{r}$$ → Coloured monopole

No more simple solutions known

2 Dyon Equations

$$\breve{\mathfrak{D}}_{\underline{r}}\breve{\mathfrak{D}}_{\underline{r}}\Phi_\Lambda - g^2 f_{\Lambda\Sigma}{}^{\Omega} f_{\Delta\Omega}{}^{\Gamma}\Phi^\Sigma\Phi^\Delta\Phi_\Gamma = 0\,,$$

Determined by the B. Eqs.

a) Trivial solution: $\Phi_\Lambda = 0$;

b) Dyon solution: $\Phi_\Lambda = \kappa\, \Phi^\Lambda$; (compact groups)

c) Ramírez's dyon:

$$\begin{pmatrix} \Phi^A = -\delta^{Ar}\dfrac{1}{gP}\partial_{\underline{r}} P; \\ \breve{A}^A{}_{\underline{r}} = -\varepsilon^A{}_{rs}\dfrac{1}{gP}\partial_{\underline{s}} P; \end{pmatrix}$$

$$\boxed{\begin{array}{l} \Phi_A = -\dfrac{1}{gP}\delta_A{}^{\underline{r}}\partial_{\underline{r}} Q; \\ \dfrac{1}{P}\partial_{\underline{r}}\partial_{\underline{r}} Q \doteq 0, \end{array}}$$

③ Bubble Equations

$$\underbrace{\Phi_\Lambda \breve{\mathfrak{D}}_{\underline{r}} \breve{\mathfrak{D}}_{\underline{r}} \Phi^\Lambda}_{=0} - \underbrace{\Phi^\Lambda \breve{\mathfrak{D}}_{\underline{r}} \breve{\mathfrak{D}}_{\underline{r}} \Phi_\Lambda}_{=0} \;=\; 0\,,$$

$$\tfrac{1}{2}\varepsilon_{\underline{rsw}} \breve{F}^{\Lambda}{}_{\underline{sw}} - \breve{\mathfrak{D}}_{\underline{r}} \Phi^\Lambda \;=\; 0\,, \quad \xrightarrow{\text{Bianchi YM}} \quad \breve{\mathfrak{D}}_{\underline{r}} \breve{\mathfrak{D}}_{\underline{r}} \Phi^\Lambda = 0\,;$$

$$\breve{\mathfrak{D}}_{\underline{r}} \breve{\mathfrak{D}}_{\underline{r}} \Phi_\Lambda - g^2 f_{\Lambda\Sigma}{}^{\Omega} f_{\Delta\Omega}{}^{\Gamma} \Phi^\Sigma \Phi^\Delta \Phi_\Gamma \;=\; 0\,, \quad \xrightarrow{f_{(\Lambda\Sigma)}{}^{\Gamma}=0} \quad \Phi^\Lambda \breve{\mathfrak{D}}_{\underline{r}} \breve{\mathfrak{D}}_{\underline{r}} \Phi_\Lambda = 0;$$

except at the singularities:

Fixed relative positions in Abelian multicenter (Denef, Bates)

For Ramirez's multicenter dyon

$$P = P_0 + \sum_\alpha \frac{P_\alpha}{|\vec{x}-\vec{x}_\alpha|}\;; \quad Q = Q_0 + \sum_\alpha \frac{Q_\alpha}{|\vec{x}-\vec{x}_\alpha|}$$

NO RESTRICTIONS

The bubble eqs are the integrability conditions of:

$$\partial_{[\underline{r}}\omega_{\underline{s}]} = 2\varepsilon_{rsw}\left(\Phi_\Lambda \breve{\mathfrak{D}}_{\underline{w}}\Phi^\Lambda - \Phi^\Lambda \breve{\mathfrak{D}}_{\underline{w}}\Phi_\Lambda\right)$$

For Ramirez's multicenter dyon $\quad \omega^{NA}_{\underline{r}} = -4\varepsilon_{rsw}\frac{\partial_{\underline{s}}P}{P}\frac{\partial_{\underline{w}}Q}{P}$

and it does not contribute at the horizons $r \to 0$

Asymptotically $\omega^{NA}_{\underline{r}} \sim 1/r^5$

The solutions $\Phi^{\wedge}$, $\bar{\Phi}_{\wedge}$, $\breve{A}^{\wedge}{}_{\underline{n}}$ of these equations are the **building blocks** of the SEYM solutions

Let's build some!

N=2,d=4 SEYM solutions

Rules to construct solutions of the $\mathbb{CP}^3$ model:

Metric: $$ds^2 = e^{2U}(dt+\omega)^2 - e^{-2U}dx^r dx^r\,,$$

where
$$\begin{cases} e^{-2U} = \mathsf{W}(\mathcal{I})\,. \qquad \mathsf{W}(\mathcal{I}) = \tfrac{1}{2}\eta_{\Lambda\Sigma}\mathcal{I}^\Lambda\mathcal{I}^\Sigma + 2\eta^{\Lambda\Sigma}\mathcal{I}_\Lambda\mathcal{I}_\Sigma\,. \\ \partial_{[\underline{r}}\omega_{\underline{s}]} = 2\varepsilon_{rsw}\left(\Phi_\Lambda\breve{\mathfrak{D}}_{\underline{w}}\Phi^\Lambda - \Phi^\Lambda\breve{\mathfrak{D}}_{\underline{w}}\Phi_\Lambda\right) \end{cases}$$

$$\Big(\overset{0}{+}\ \underbrace{\overbrace{-\ -\ -}^{A}}_{SO(3)}\Big)$$

and
$$\mathcal{I}^\Lambda = -\sqrt{2}\Phi^\Lambda\,, \qquad \mathcal{I}_\Lambda = -\sqrt{2}\Phi_\Lambda\,,$$

(To simplify the presentation we focus on the metrics and ignore scalars and vector fields)

1) Spherically-symmetric solutions: monopole
Abelian BH + monopole
coloured BH

2) Multicenter coloured BHs

1.- Global monopole

Abelian sector $\Lambda = 0$ $\rightarrow$ $\Phi^0 =$ constant, $\Phi_0 = 0$

Non-Abelian sector $\Lambda = A$ $\rightarrow$ $\begin{cases} \Phi^A \text{ Higgs field of BPS 't Hooft-Polyakov} \\ \Phi_A = 0 \end{cases}$

$$\Rightarrow \quad \omega = 0; \quad \boxed{e^{-2U} = 1 + (\mu/g)^2 - \frac{1}{g^2 r^2}\left[1 - \frac{\mu r}{\sinh \mu r}\cosh \mu r\right]^2}$$

$$e^{-2U} \in \Big[\underbrace{1 + \tfrac{1}{2}(\mu/g)^2}_{r=0}, \underset{\substack{\uparrow \\ r\sim\infty}}{1}\Big), \quad M = \frac{\mu}{g^2 G_N^{(4)}};$$

Globally regular, horizonless solution.

2.- Global monopole + RN BH

$\Phi_0 = 0$

Abelian sector $\Lambda = 0$ $\rightarrow$ $\Phi^0 = \text{constant} + \frac{\overset{\nearrow 0}{p^0/2}}{r}$;

Non-Abelian sector $\Lambda = A$ $\rightarrow$ $\begin{cases} \Phi^A \text{ Higgs field of BPS 't Hooft-Polyakov} \\ \Phi_A = 0 \end{cases}$

$$\Rightarrow \omega = 0; \quad \boxed{e^{-2U} = \left[\sqrt{1+(\mu/g)^2} + \frac{p^0/2}{r}\right]^2 - \frac{1}{g^2 r^2}\left[1 - \frac{\mu r}{\sinh \mu r}\cosh \mu r\right]^2}$$

$$e^{-2U} \in (\underset{\substack{\uparrow \\ r\to 0}}{\infty}, \underset{\substack{\uparrow \\ r\sim\infty}}{1}), \quad G_N^{(4)} M = \sqrt{\frac{1+(\mu/g)^2}{4}}\, p^0 + \frac{\mu}{g^2};$$

$$e^{-2U} \underset{r\sim 0}{\sim} \frac{(p^0)^2/4}{r^2}, \quad S = \pi (p^0)^2/4;$$

No contribution to the entropy.

3.- Coloured Black Hole

Non-Abelian sector: $\begin{cases} \Phi^A = \dfrac{1}{g r^2(1+\lambda^2 r)}\delta^A{}_r x^r ; \\ \Phi_A = 0 ; \end{cases}$

Since $\Phi^A \Phi^A \underset{r\sim 0}{\sim} \dfrac{1}{g^2 r^2}$, we need a charge in the Abelian sector

Abelian sector: $\Phi^0 = 1 + \dfrac{p^0/2}{r}$; $\Phi_0 = 0$;

$$\Rightarrow \omega = 0 ; \quad \boxed{e^{-2U} = \left(1 + \frac{p^0/2}{r}\right)^2 - \frac{1}{g^2 r^2 (1+\lambda^2 r)^2} ;}$$

$$e^{-2U} \underset{r\to\infty}{\sim} 1 + \frac{p^0}{r} ; \quad M = \frac{p^0}{2 G_N^{(4)}} ;$$

The non-Abelian field only at horizon?

$$e^{-2U} \sim \left[(p^0/2)^2 - \frac{1}{g^2}\right]\frac{1}{r^2} ; \quad S = \frac{\pi}{G_N^{(4)}}\left[(p^0/2)^2 - \frac{1}{g^2}\right] ;$$

4.- Dumbbell solution

Non-Abelian sector : $\begin{cases} \Phi^A = \dfrac{1}{g r^2(1+\lambda^2 r)}\,\delta^A{}_r x^r ; \\ \Phi_A = 0 ; \end{cases}$

Will not be asymptotically flat

Abelian sector : $\Phi^0 \doteq \cancel{1} + \dfrac{p^0/2}{r}$; $\Phi_0 = 0$;

$$\Rightarrow \omega = 0 ; \quad \boxed{e^{-2U} = \frac{(p^0/2)^2}{r^2} - \frac{1}{g^2 r^2 (1+\lambda^2 r)^2} ;}$$

$$e^{-2U} \underset{r\sim\infty}{\sim} \frac{(p^0/2)^2}{r^2} \Rightarrow \text{AdS}_2 \times S^2 \text{ at } r=\infty$$

Different radii

$$e^{-2U} \underset{r\sim 0}{\sim} \left[(p^0/2)^2 - \frac{1}{g^2} \right] \frac{1}{r^2} ; \Rightarrow \text{AdS}_2 \times S^2 \text{ at } r = 0$$

5.- Multicenter Coloured Black Hole

$$H = h + \sum_{\alpha=1}^{N} \frac{p_\alpha}{r_\alpha}\,, \qquad P = \lambda + \sum_{\alpha=1}^{N} \frac{s_\alpha}{r_\alpha}\,, \qquad Q = -\sum_{\alpha=1}^{N} \frac{\eta_\alpha s_\alpha/2}{r_\alpha}\,,$$

$$\Phi^0 = -H\,, \qquad \vec{\Phi} = -\frac{1}{gP}\vec{\nabla}P\,, \qquad \vec{\mathcal{J}} = \frac{2}{gP}\vec{\nabla}Q\,,$$

$\vec{\Phi} = (\Phi^A)$

$\vec{\mathcal{J}} = 2(\Phi_A)$

Non-Abelian sector: Ramírez's dyon

Abelian sector: Papapetrou-Majumdar (multi-Reissner-Nordström)

$$e^{-2U} = H^2 - \vec{\Phi}^2 - \vec{\mathcal{J}}^2\,, \qquad \vec{Z} = e^{-i\gamma}\frac{\vec{\Phi} + i\vec{\mathcal{J}}}{H}\,,$$

$$\vec{\omega} = 2g^2\,\vec{\Phi} \times \vec{\mathcal{J}}\,, \qquad V = 2g^2 e^{4U}\,|\vec{\Phi} \times \vec{\mathcal{J}}|^2$$

The metric function e^{-2U} can be written like this:

$$e^{-2U} = h + \sum_{\alpha=1}^{N} \frac{2M_\alpha}{r_\alpha} + \sum_{\alpha=1}^{N} \left[E_\alpha + (1+\eta_\alpha^2)\, R_\alpha\right] \frac{1}{r_\alpha^2}$$

$$+ \sum_{\alpha>\beta}^{N} \left[E_{\alpha\beta} - E_\alpha - E_\beta + 2(1+\eta_\alpha\eta_\beta)\, R_{\alpha\beta}\right] \frac{1}{r_\alpha r_\beta}$$

Manifestly positive functions

where

Mass of α^{th} BH

$$M_\alpha \equiv h p_\alpha\,, \quad > 0$$

Entropy of α^{th} BH

$$E_\alpha \equiv p_\alpha^2 - (1+\eta_\alpha^2)/g^2\,, \quad > 0$$

$E(\alpha+\beta)$

$$E_{\alpha\beta} \equiv (p_\alpha + p_\beta)^2 - 4/g^2 - (\eta_\alpha+\eta_\beta)^2/g^2 \quad > E_\alpha + E_\beta$$

$\Rightarrow e^{-2U} > 0$

$\vec{\omega}$ is regular at each r_α and there are no CTCs.

N=2,d=5 SEYM solutions

$C_{0xy} = \frac{1}{6}\eta_{xy}$

Rules to construct timelike solutions of the $ST[2,m]$ model:

Metric:

$$ds^2 = \hat{f}^{\,2}(dt+\hat{\omega})^2 - \hat{f}^{\,-1}\left[H^{-1}(dz+\chi)^2 + H dx^r dx^r\right]$$

where

$$\left\{\begin{aligned}
\hat{f}^{\,-1} \;&=\; H^{-1}\Big\{\tfrac{1}{4}\left(6HL_0+8\eta_{xy}K^xK^y\right)\big[9H^2\eta^{xy}L_xL_y+48HK^0L_xK^x \\
&\qquad +64(K^0)^2\eta_{xy}K^xK^y\big]\Big\}^{1/3}\,. \\
\hat{\omega} \;&=\; \omega_5(dz+\chi)+\omega\,, \\
\omega_5 \;&=\; M+16\sqrt{2}H^{-2}C_{IJK}K^IK^JK^K+3\sqrt{2}H^{-1}L_IK^I\,, \\
\partial_{[\underline{r}}\omega_{\underline{s}]} &= 2\varepsilon_{rsw}\left(\Phi_\Lambda\breve{\mathfrak{D}}_{\underline{w}}\Phi^\Lambda-\Phi^\Lambda\breve{\mathfrak{D}}_{\underline{w}}\Phi_\Lambda\right)
\end{aligned}\right.$$

and $\quad K^I \;=\; \delta^I{}_\Lambda\Phi^{\Lambda+1}\,, \qquad L_I = -\frac{2\sqrt{2}}{3}\delta_I{}^\Lambda\Phi_{\Lambda+1}\,, \qquad H = -2\sqrt{2}\Phi^0\,, \qquad M = +\sqrt{2}\Phi_0\,,$

Simplest non-Abelian Black Hole

Abelian sector:
(3-charge BH)

$$\begin{cases} L_0 = -\frac{2\sqrt{2}}{3}\,\Phi_1 = B_0 + q_0/\rho^2\,; \\ L_\pm = L_1 \pm L_2 = -\frac{2\sqrt{2}}{3}\left(\Phi_2 \pm \Phi_3\right) = B_\pm + q_\pm/\rho^2\,; \end{cases}$$

Non-Abelian sector:
("Coloured monopole")

$$\Phi^A = \frac{1}{g r^2 (1 + \lambda^2 r)}\,\delta^A{}_r\, x^r$$

GH metric:
$(\mathbb{R}^4 - \{0\})$

$$H = 1/r\,; \qquad r = \rho^2/4\,;$$

$$\mathbb{R}^3 \qquad \mathbb{R}^4$$

Kronheimer

BPST instanton in $\mathbb{R}^4$

$$\Phi^2 = \Phi^A \Phi^A = \frac{2\kappa^4}{3 g^2 \rho^4 (\rho^2 + \kappa^2)^2}\,;$$

$$\hat{\omega} = 0 \; ; \quad \hat{f}^{-3} = \underbrace{\left(L_0 - \frac{\rho^2}{3}\Phi^2 \right)}_{\tilde{L}_0} L_+ L_- \; ;$$

$$\tilde{L}_0 B_0 + \boxed{\frac{q_0}{\rho^2}} - \boxed{\frac{2}{9g^2}\frac{\kappa^2}{\rho^2(\rho^2+\kappa^2)^2}} \; ;$$

$O\left(\frac{1}{\rho^2}\right)$ on the horizon $\rho \to 0$

$O\left(\frac{1}{\rho^2}\right)$ at $\rho \to \infty$

$O\left(\frac{1}{\rho^6}\right)$ at $\rho \to \infty$

$\longrightarrow$ Same puzzle as in $d=4$.

BUT

$$\tilde{L}_0 = B_0 + \boxed{\left(q_0 - \frac{2}{9g^2}\right)\frac{1}{\rho^2}} + \boxed{\frac{2}{9g^2}\frac{\rho^2+2\kappa^2}{(\rho^2+\kappa^2)^2}},$$

!!

It is remarkable that we can rewrite $\tilde{L}_0$ like this:

$$\tilde{L}_0 = B_0 + (q_0 - \frac{2}{9g^2})\frac{1}{\rho^2} + \frac{2}{9g^2}\frac{\rho^2 + 2\kappa^2}{(\rho^2+\kappa^2)^2},$$

Suggests: $q_0 - \frac{2}{9g^2}$ some standard brane charge $\frac{1}{\rho^2}$

What is $\frac{2}{9g^2}$?

Let's switch off everything else: $\begin{cases} q_0 - \frac{2}{9g^2} = 0 \\ q_\pm = 0 \end{cases}$

What do we get?

The full solution has this form:

$$\boxed{\begin{aligned}
ds^2 &= \hat{f}^2 dt^2 - \hat{f}^{-1}(d\rho^2 + \rho^2 d\Omega^2_{(3)})\,,\\
\hat{f}^{-3} &= 1 + \frac{2e^{-\phi_\infty}k_\infty^{2/3}}{3g^2}\frac{\rho^2+2\kappa^2}{(\rho^2+\kappa^2)^2}\,,\\
A^0 &= -\frac{1}{\sqrt{3}}\hat{f}^3 dt\,, \qquad A^A = \frac{\kappa^2}{g(\rho^2+\kappa^2)}v_L^A\,,\\
e^{2\phi} &= e^{2\phi_\infty}\hat{f}^{-3}\,, \qquad k = k_\infty \hat{f}^{3/4}\,,
\end{aligned}}$$

Spherically symmetric, globally regular, horizon-less, asymptotically flat

"GLOBAL INSTANTON"

What is a "global instanton"?

let's uplift the solution to d=10 Heterotic Supergravity (other uplifts more difficult or impossible)

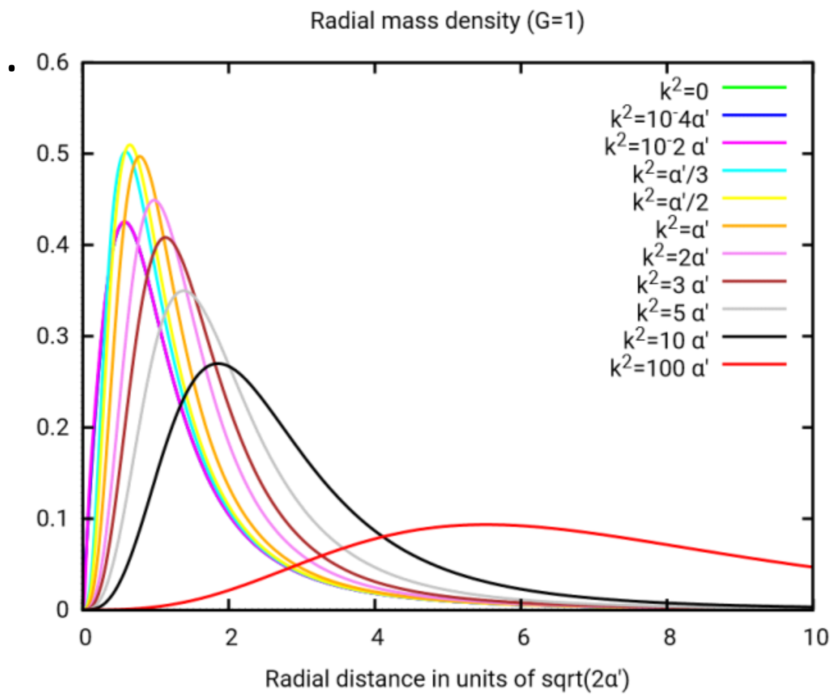
$g_5 = k_\infty^{1/3} e^{-\phi_\infty/2} / \sqrt{12\alpha'}$;

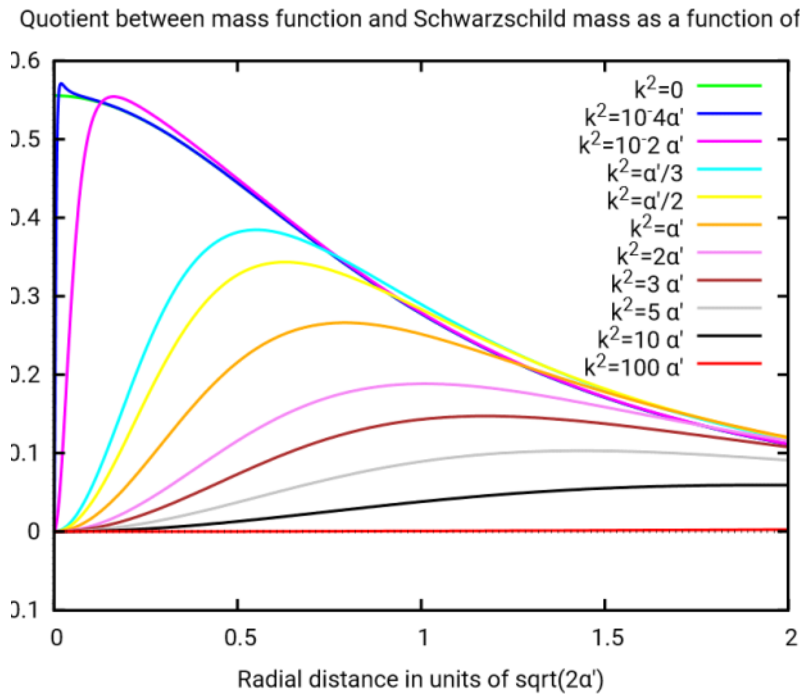
$g_s = e^{\phi_\infty}$;

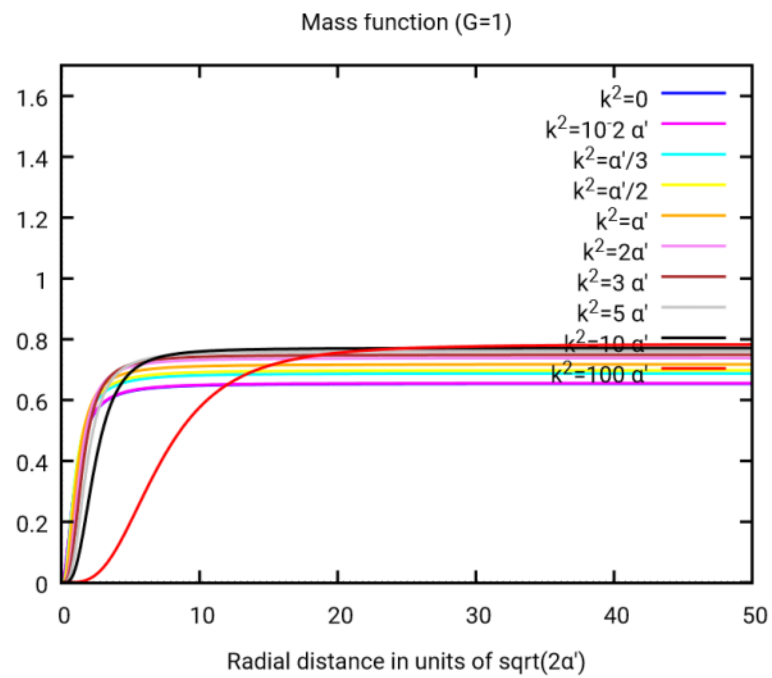
$l_s = \sqrt{\alpha'}$;

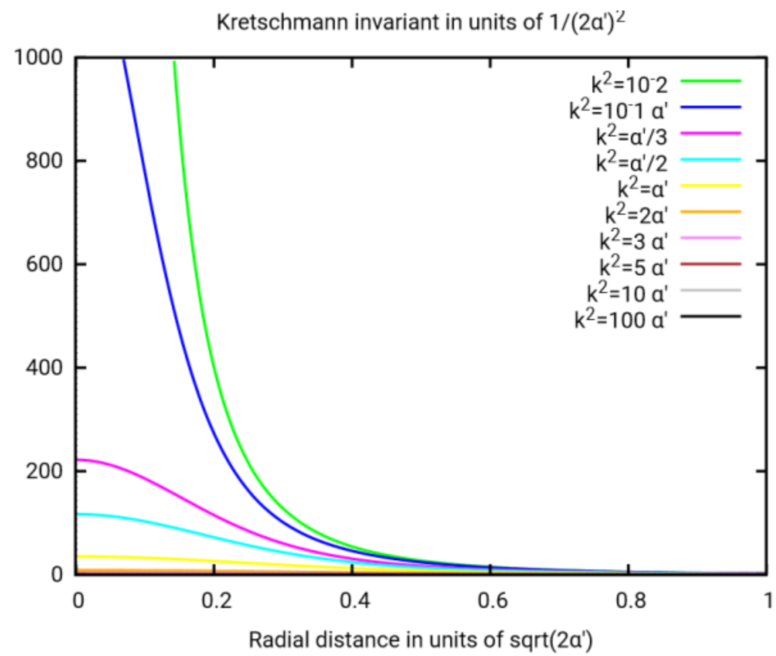
$$\Rightarrow \quad e^{2\phi} = e^{2\phi_\infty} \hat{f}^{-3} = e^{2\phi_\infty} \left\{ 1 + 8\alpha' \frac{\rho^2 + 2\kappa^2}{(\rho^2 + \kappa^2)^2} \right\}$$

Characteristic of the GAUGE FIVEBRANE


Radial mass density (G=1)
$k^2=0$
$k^2=10^{-4}\alpha'$
$k^2=10^{-2}\ \alpha'$
$k^2=\alpha'/3$
$k^2=\alpha'/2$
$k^2=\alpha'$
$k^2=2\alpha'$
$k^2=3\ \alpha'$
$k^2=5\ \alpha'$
$k^2=10\ \alpha'$
$k^2=100\ \alpha'$
Radial distance in units of sqrt(2α')


Quotient between mass function and Schwarzschild mass as a function of
$k^2=0$
$k^2=10^{-4}\alpha'$
$k^2=10^{-2}\ \alpha'$
$k^2=\alpha'/3$
$k^2=\alpha'/2$
$k^2=\alpha'$
$k^2=2\alpha'$
$k^2=3\ \alpha'$
$k^2=5\ \alpha'$
$k^2=10\ \alpha'$
$k^2=100\ \alpha'$
Radial distance in units of sqrt(2α')


Mass function (G=1)
$k^2=0$
$k^2=10^{-2}\ \alpha'$
$k^2=\alpha'/3$
$k^2=\alpha'/2$
$k^2=\alpha'$
$k^2=2\alpha'$
$k^2=3\ \alpha'$
$k^2=5\ \alpha'$
$k^2=10\ \alpha'$
$k^2=100\ \alpha'$
Radial distance in units of sqrt(2α')


Kretschmann invariant in units of $1/(2\alpha')^2$
$k^2=10^{-2}$
$k^2=10^{-1}\ \alpha'$
$k^2=\alpha'/3$
$k^2=\alpha'/2$
$k^2=\alpha'$
$k^2=2\alpha'$
$k^2=3\ \alpha'$
$k^2=5\ \alpha'$
$k^2=10\ \alpha'$
$k^2=100\ \alpha'$
Radial distance in units of sqrt(2α')

These non-Abelian black holes consist of $\underbrace{\text{3 standard branes}}$ plus a $\underbrace{\text{gauge 5-brane.}}$

Standard 3-charge BH

D1 D5 W up to dualities

(Strominger-Vafa)

Contributes to the mass but not to the entropy

$\Rightarrow$ We can explain the entropy.

Correct identification of charges is essential.

After some redefinitions, this is the full solution

$$
\begin{aligned}
ds^2 &= f^2 dt^2 - f^{-1}(d\rho^2 + \rho^2 d\Omega^2_{(3)})\,, \\
A^0 &= -\sqrt{3}e^{-\phi_\infty} k_\infty^{2/3} \frac{dt}{\tilde{\mathcal{Z}}_0}\,, & A^1 + A^2 &= -\sqrt{3}e^{\phi_\infty} k_\infty^{2/3} \frac{dt}{\mathcal{Z}_+}\,, \\
A^A &= -\frac{1}{g}\frac{\rho^2}{(\kappa^2+\rho^2)} v_R^A\,, & A^1 - A^2 &= -2\sqrt{3} k_\infty^{-4/3} \frac{dt}{\mathcal{Z}_-}\,, \\
e^{2\phi} &= e^{2\phi_\infty} \frac{\tilde{\mathcal{Z}}_0}{\mathcal{Z}_+}\,, & k &= k_\infty (f\mathcal{Z}_-)^{3/4}\,,
\end{aligned}
$$

$$
f^{-3} = \tilde{\mathcal{Z}}_0\, \mathcal{Z}_+\, \mathcal{Z}_-\,,
$$

$(q_0 - \frac{q}{2g^2})$

$$
\tilde{\mathcal{Z}}_0 = 1 + \frac{\tilde{\mathcal{Q}}_0}{\rho^2} + \frac{2e^{-\phi_\infty} k_\infty^{2/3}}{3g^2} \frac{\rho^2 + 2\kappa^2}{(\rho^2+\kappa^2)^2}\,, \quad \mathcal{Z}_\pm = 1 + \frac{\mathcal{Q}_\pm}{\rho^2}.
$$

.

$$
\begin{aligned}
M &= \frac{\pi}{4G_N^{(5)}}\left[\tilde{\mathcal{Q}}_0 + \frac{2e^{-\phi_\infty}k_\infty^{2/3}}{3g^2} + \mathcal{Q}_+ + \mathcal{Q}_-\right] \\
S &= \frac{\pi^2}{2G_N^{(5)}}\sqrt{\tilde{\mathcal{Q}}_0\mathcal{Q}_+\mathcal{Q}_-}\,.
\end{aligned}
$$

5-brane contribution

absent

$$\tilde{Q}_0 \sim \int_{S^3_\infty} (\star F^0 - \omega_{CS}); \quad \left(d\star F^0 - F^A \wedge F^A = 0\right)$$

In d = 10 Heterotic Supergravity

$$d\hat{s}^2 = \frac{2}{\mathcal{Z}_+} du \left(dv - \tfrac{1}{2}\mathcal{Z}_- du\right) - \tilde{\mathcal{Z}}_0 (d\rho^2 + \rho^2 d\Omega^2_{(3)}) - dz^i dz^i \,,$$

F1

S5 + S5

$$\hat{B} = -\frac{1}{\mathcal{Z}_+} dv \wedge du + \tfrac{1}{4} \mathcal{Q}_0 \cos\theta d\psi \wedge d\phi \,,$$

$$\hat{A}^A = -\frac{\rho^2}{(\kappa^2 + \rho^2)} v^A_R \,, \longrightarrow \text{BPST instanton}$$

$$e^{-2\hat{\phi}} = e^{-2\hat{\phi}_\infty} \frac{\mathcal{Z}_+}{\tilde{\mathcal{Z}}_0} \,,$$

$Q_0 = \tilde{Q}_0 + 8\alpha'$

Conclusions

- Non-Abelian solutions are interesting but still poorly understood.
- The $d=5$ case is very rich and easier to understand than the $d=4$ (which are the right charges?)
- non-extremal
  Asymptotically AdS
  ⋮
  ?
- Type II embeddings?
- Dumbbells & RG flows?

Thanks!