Tomás Ortín

(I.F.T. UAM/CSIC, Madrid)

Seminar given on December 15th, 2016 at the APCTP 2016 Workshop on Frontiers of Physics

Based on 1503.01044 1512.07131 1605.00005 and work in preparation.

Work done in collaboration with P.F. Ramírez (IFT UAM/CSIC, Madrid) and P. Meessen (U. Oviedo)

Plan of the Talk:

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- 3 Generalized Bogomol'nyi equations
- 6 Solutions to the SU(2) Bogomol'nyi equations: Protogenov's
- 8 Solutions to the SU(2) Bogomol'nyi equations: Ramírez's
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The timelike supersymmetric solutions of $\mathcal{N} = 1, d = 5$ SEYM theories were classified in 0705.2567 (earlier) but no non-Abelian black-hole solutions were constructed until very recently (1512.07131).

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We are going to present these equations and some relevant solutions.



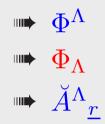
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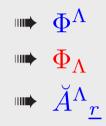


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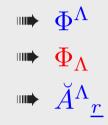
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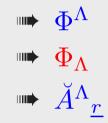


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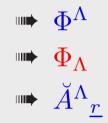
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$$\Phi_{\Lambda} \breve{\mathfrak{D}}_{\underline{r}} \breve{\mathfrak{D}}_{\underline{r}} \Phi^{\Lambda} - \Phi^{\Lambda} \breve{\mathfrak{D}}_{\underline{r}} \breve{\mathfrak{D}}_{\underline{r}} \Phi_{\Lambda} = 0 \,.$$

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In general there will be a Abelian sector (λ) and a non-Abelian sector (A) which will always be SU(2) in this talk:

$$\begin{split} \frac{1}{2} \varepsilon_{\underline{r}\underline{s}\underline{w}} \breve{F}^{\lambda}{}_{\underline{s}\underline{w}} &- \partial_{\underline{r}} \Phi^{\lambda} &= 0 \,, \quad \Rightarrow \quad \partial_{\underline{r}} \partial_{\underline{r}} \Phi^{\lambda} = 0 \,, \\ \frac{1}{2} \varepsilon_{\underline{r}\underline{s}\underline{w}} \breve{F}^{A}{}_{\underline{s}\underline{w}} &- \breve{\mathfrak{D}}_{\underline{r}} \Phi^{A} &= 0 \,, \\ \partial_{\underline{r}} \partial_{\underline{r}} \Phi_{\lambda} &= 0 \,, \quad \Rightarrow \quad \frac{1}{2} \varepsilon_{\underline{r}\underline{s}\underline{w}} \breve{F}_{A\underline{s}\underline{w}} - \partial_{\underline{r}} \Phi_{A} = 0 \,, \\ \breve{\mathfrak{D}}_{\underline{r}} \breve{\mathfrak{D}}_{\underline{r}} \Phi_{A} - g^{2} \left(\Phi^{B} \Phi^{B} \Phi_{A} - \Phi^{A} \Phi^{B} \Phi_{B} \right) &= 0 \,, \\ \left(\Phi_{\lambda} \partial_{\underline{r}} \partial_{\underline{r}} \Phi^{\lambda} - \Phi^{\lambda} \partial_{\underline{r}} \partial_{\underline{r}} \Phi_{\lambda} \right) + \\ \left(\Phi_{A} \breve{\mathfrak{D}}_{\underline{r}} \breve{\mathfrak{D}}_{\underline{r}} \Phi^{A} - \Phi^{A} \breve{\mathfrak{D}}_{\underline{r}} \breve{\mathfrak{D}}_{\underline{r}} \Phi_{A} \right) &= 0 \,. \end{split}$$

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The solutions of the Abelian sector are completely determined by a choice of harmonic functions Φ^{λ} , Φ_{λ} in \mathbb{E}^{3} . What happens in the non-Abelian sector?

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The last set of equation mixing Φ^A , Φ_A , $\breve{A}_{\underline{r}}$ is automatically solved except at the singularities, where one has to impose conditions on the integration constant (Denef, Bates.)

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3 - Solutions to the SU(2) Bogomol'nyi equations: Protogenov's

All the spherically-symmetric configurations $\Phi^A, \breve{A}_{\underline{r}}$ can be brought to the form (*hedgehog ansatz*)

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The Bogomol'nyi equations become an system of ODFs for f(r) and h(r)

$$\begin{cases} r\partial_r h + 2h + f(1 + gr^2 h) = 0, \\ r\partial_r (h - f) - gr^2 h(h - f) = 0. \end{cases}$$

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$$f_{\mu,s} = \frac{1}{gr^2} \left[1 - \mu r \coth(\mu r + s) \right], \quad h_{\mu,s} = \frac{1}{gr^2} \left[1 - \frac{\mu r}{\sinh(\mu r + s)} \right],$$

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The *coloured* monopoles are very interesting solutions: their charge is screened at infinity and they can be generalized to multicenter solutions.

4 -Solutions to the SU(2) Bogomol'nyi equations: Ramírez's

Recently (1608.01330), Ramírez has shown that the SU(2) Bogomol'nyi equations are solved by

$$\Phi^{A} = \delta^{A\underline{r}} \frac{1}{gP} \partial_{\underline{r}} P , \qquad \qquad \breve{A}^{A}{}_{\underline{r}} = \varepsilon^{A}{}_{rs} \frac{1}{gP} \partial_{\underline{s}} P ,$$

where P is any real function satisfying

$$\frac{1}{P}\partial_{\underline{r}}\partial_{\underline{r}}P = 0, \text{ like, for instance, } P = P_0 + \sum_{\alpha} \frac{P_{\alpha}}{|\vec{x} - \vec{x}_{\alpha}|}.$$

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For just one pole, this is the coloured monopole with $\lambda^2 = P_0/P_1$. Many poles: many coloured monopoles in equilibrium. All the coefficients of the poles must have the same sign.



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The last equation is solved quite non-trivially everywhere: no constraints on the integration constants!

Now, given a solution

$$\Phi^{\lambda}, \Phi_{\lambda}, \Phi^{A}, \Phi_{A}, \breve{A}_{\underline{r}}$$

to the equations, we construct supergravity solutions

AS FOLLOWS:

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The last is the most appropriate for us because

$$\left(\begin{array}{c} \mathcal{I}^{\Lambda} \\ \mathcal{I}_{\Lambda} \end{array}\right) = -\sqrt{2} \left(\begin{array}{c} \Phi^{\Lambda} \\ \Phi_{\Lambda} \end{array}\right) \,,$$

and the \check{A}_r^{Λ} are the corresponding part of the $\mathcal{N} = 2, d = 4$ supergravity vector fields.

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The metric has the form

$$ds^2 = \mathbf{W}^{-1} (dt + \boldsymbol{\omega})^2 - \mathbf{W} dx^r dx^r \,,$$

where the 1-form $\omega = \omega_{\underline{r}\leq} dx^r$ on \mathbb{R}^3 is found by solving the equation

$$(d\omega)_{\underline{rs}} = 2\epsilon_{\underline{rst}}\mathcal{I}_M \,\breve{\mathfrak{D}}_{\underline{t}}\mathcal{I}^M = 2\epsilon_{\underline{rst}} \left[\mathcal{I}_{\Lambda} \,\breve{\mathfrak{D}}_{\underline{t}}\mathcal{I}^{\Lambda} - \mathcal{I}^{\Lambda} \,\breve{\mathfrak{D}}_{\underline{t}}\mathcal{I}_{\Lambda} \right] \,.$$

The last equation of Φ^{Λ} , Φ_{Λ} , $\check{A}^{\Lambda}{}_{\underline{r}}$ implies the integrability condition of this equation. ω is trivial when the integrability condition is satisfied trivially (static solutions).

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The simplest model that admits a SU(2) gauging is the $\overline{\mathbb{CP}}^3$ model, with Hesse potential

$$\mathsf{W} = \frac{1}{2} \eta_{\Lambda \Sigma} \mathcal{I}^{\Lambda} \mathcal{I}^{\Sigma} + 2 \eta^{\Lambda \Sigma} \mathcal{I}_{\Lambda} \mathcal{I}_{\Sigma}, \text{ with } \eta = \operatorname{diag}(+ - - -).$$

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Let us see some choices with good properties (we focus on the metric only for the sake of simplicity).

Global monopole: H = 1 + BPS 't Hooft-Polyakov monopole

$$ds^{2} = \mathsf{W}^{-1}dt^{2} - \mathsf{W}(dr^{2} + r^{2}d\Omega_{(2)}^{2}), \text{ where } \mathsf{W} = 1 - \frac{1}{g^{2}r^{2}}\left[1 - \mu r \coth(\mu r)\right]^{2}$$

Globally regular. Mass but no horizon nor entropy. (BPS 't Hooft-Polyakov monopoles always do this when combined with other fields).

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Horizon at r = 0. The monopole contributes to the entropy but not to the mass: **HAIR!** **Global monopole:** H = 1 + BPS 't Hooft-Polyakov monopole

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$$ds^{2} = \mathsf{W}^{-1}dt^{2} - \mathsf{W}(dr^{2} + r^{2}d\Omega_{(2)}^{2}), \text{ where } \mathsf{W} = \frac{(p^{0})^{2}}{r^{2}} - \frac{1}{g^{2}r^{2}} \left[\frac{1}{1 + \lambda^{2}r}\right]^{2}$$

Flows fom one $AdS_2 \times S^2$ to another $AdS_2 \times S^2$ of different radius!

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These are the simplest, but more general solutions are possible (dyonic, with objects of different types, black hedgehogs, etc.).

8 – 5-dimensional non-Abelian black holes

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We are interested in a special class of solutions that can be described in terms of functions M, H, Φ^I, L_I , which are related to the building blocks $\Phi^{\Lambda}, \Phi_{\Lambda}, \Lambda = 0, 1, \dots, n_{V5} + 1$ by

$$\Phi^{I} = \Phi^{I+1}, \quad L_{I} = -\frac{2\sqrt{2}}{3} \Phi_{I+1}, \quad H = -2\sqrt{2} \Phi^{0}, \quad M = +\sqrt{2} \Phi_{0} \cdot I = 1, \cdots, n_{V5}.$$

The 5-dimensional metric has the form

$$ds^{2} = (\mathbf{W}/2)^{-4/3} (dt + \hat{\boldsymbol{\omega}})^{2} - (\mathbf{W}/2)^{2/3} \left[\mathbf{H}^{-1} (dz + \boldsymbol{\chi})^{2} + \mathbf{H} dx^{r} dx^{r} \right] ,$$

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and can be reconstructed from the above functions as follows:

$$d\chi = \star_3 dH,$$

$$dH_I = L_I + 8C_{IJK} \Phi^J \Phi^K / H,$$

$$\hat{\omega} = \omega_5 (dz + \chi) + \omega.$$

where ω is the same one would find for the 4-dimensional solution.

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We may obtain black holes, but beware of the singularities!!.

December 15th 2016

9 – A simple example with gauge group SU(2)

It is given by $C_{0\Lambda\Sigma} = \frac{1}{3!}\eta_{\Lambda\Sigma} \Lambda\Sigma = 1, x x, y = A + 1$ or by

$$\mathsf{W} = \left\{ \frac{27}{2} H_0 \eta^{\Lambda \Sigma} H_{\Lambda} H_{\Sigma} \right\}^{1/2} \,,$$

which gives

$$(\mathsf{W}/2)^{2/3} = H^{-1} \left\{ \frac{1}{4} \left(6HL_0 + 8\eta_{xy} \Phi^x \Phi^y \right) \left[9H^2 \eta^{xy} L_x L_y + 48H \Phi^0 L_x \Phi^x \right. \right. \\ \left. + 64(\Phi^0)^2 \eta_{xy} \Phi^x \Phi^y \right] \right\}^{1/3} .$$

The simplest solution has just H, L_0, L_1, Φ^{A+1}

$$(\mathsf{W}/2)^{2/3} = \left\{ \frac{27}{2} \left(L_0 - \frac{4}{3} \Phi^{A+1} \Phi^{A+1} \right) (L_1)^2 \right\}^{1/3}$$

and it is just a D1D5W black hole with a non-Abelian contribution which has to be the BPST instanton for one center (more centers are under investigation)



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- They present interesting new features that, for the first time, can be studied analytically.
- Explaining these entropies from a mircorscopic point of view presents a new challenge to superstring theory.
- More general non-Abelian solutions can be obtained: black rings (Ortín, Ramírez, 1605.00005), microstate geometries (Ramírez, 1608.01330), and non-extremal black holes (work in progress).

Kronheimer, MSc Thesis, 1985:

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The metric of a 4-d HK space admitting a free U(1) action shifting $z \sim z + 4\pi$ by an arbitrary constant is of the form (Gibbons, Hawking, 1979)

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where (unhatted $\Rightarrow \mathbb{E}^3$)

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Then, the 3-dimensional gauge and Higgs fields A and Φ defined by

$$\Phi \equiv -H\hat{A}_z \,,$$

$$A_{\underline{r}} \equiv \hat{A}_{\underline{r}} - \chi_{\underline{r}} \hat{A}_{z} \,,$$

satisfy the Bogomol'nyi equation in $\mathbb{E}^3 \mathfrak{D}_{\underline{r}} \Phi = \frac{1}{2} \epsilon_{\underline{rst}} F_{\underline{st}}$.