Introduction

Given the power of the FGK formalism it is natural to generalize it to

- Higher dimensions $\quad d \geq 4$
- Extended objects (branes) $\quad p \geq 0$

We are going to follow the same steps as in the $d=4$, $p=0$ case:

1. Study the theories (e.o.m.)
2. Study the metric Ansatz
3. Effective action $\to$ Consequences

The d>4 theories with p>0 branes

p-branes sweep $(p+1)$-dimensional worldvolumes and couple to $(p+1)$-form potentials $A_{(p+1)\mu_1 \cdots \mu_{p+1}}$:

Weiss-Zumino terms $q\int_{\text{Worldvolume}} A_{(p+1)\mu_1 \cdots \mu_{p+1}} dX^{\mu_1} \wedge \cdots \wedge dX^{\mu_{p+1}}$

We are interested in theories with several $\hat{A}_{(p+1)}$ appearing only through the gauge-invariant $(p+2)$-form

$$\hat{F}_{(p+2)} = d\hat{A}_{(p+1)}$$

$$\hat{F}_{(p+2)} = (p+2)\,\partial_{[\mu_1} \hat{A}_{(p+1)|\mu_2 \cdots \mu_{p+2}]}$$

relax condition for certain kinds of solutions.

In d dimensions $*\hat{F}_{(p+2)}$ is a $(d-p-2)$-form

$\rightarrow$ dual $\tilde{p} = (d-p-4)$ - branes

$\Rightarrow$ Simplicity (no intersecting branes)

Only when $\tilde{p} = p$ we consider magnetic p-branes (dual electric)

$$\boxed{d = 2(p+2)}$$

Thus, we consider actions $S[g_{\mu\nu}, A^{\Lambda}_{(p+1)}, \varphi^i]$

$$\boxed{S = \int d^d x \sqrt{|g|}\left\{ R + \mathcal{G}_{ij}\partial^\mu\varphi^i\partial_\mu\varphi^j + \frac{4(-1)^p}{(p+2)!} I_{\Lambda\Sigma} F^{\Lambda}_{(p+2)}\cdot F^{\Sigma}_{(p+2)} + \frac{4\xi^2(-1)^p}{(p+2)!} R_{\Lambda\Sigma} F^{\Lambda}\cdot * F^{\Sigma} \right\}}$$

$$\xi^2 = (-1)^{p+1} \qquad *^2 F_{(p+2)} = \xi^2 F_{(p+2)}$$

$$I_{\Lambda\Sigma} = I_{\Lambda\Sigma}(\varphi)\,; \quad R_{\Lambda\Sigma} = R_{\Lambda\Sigma}(\varphi) = -\xi^2 R_{\Sigma\Lambda}$$

$d \neq 2(p+2) \Rightarrow R_{\Lambda\Sigma} = 0$ (last term would make no sense)

$\left\{ \begin{array}{l} d = 2(p+2) \\ \xi^2 = +1 \rightarrow \end{array} \right.$ Real (anti-) selfduality possible

Equations of motion and symmetries

We start with those of the $\hat{A}_{(p+1)}$

$$\frac{\delta S}{\delta \hat{A}_{(p+1)\mu_1 \cdots \mu_{p+1}}} = -(p+2)\,\partial_\nu \frac{\delta S}{\delta \hat{F}_{(p+2)\nu\mu_1\cdots\mu_{p+1}}} \sim \sqrt{|g|}\,\nabla_\nu {*G}_\Lambda{}^{\nu\mu_1\cdots\mu_{p+1}} = 0$$

$$\parallel\!\!\mid$$

$$\xi^2 \sqrt{|g|}\,\frac{8(-1)^p}{(p+2)!}\,({*G}_\Lambda)_{\nu\mu_1\cdots\mu_{p+1}}$$

$$\Rightarrow \quad \xi^2 {*G}_\Lambda = I_{\Lambda\Sigma} F^\Sigma + \xi^2 R_{\Lambda\Sigma} {*F}^\Sigma$$

$$* \Rightarrow \quad \cancel{\xi^4}\, G_\Lambda = I_{\Lambda\Sigma} {*F}^\Sigma + \cancel{\xi^4}\, R_{\Lambda\Sigma} F^\Sigma$$

$$\frac{1}{2}\left(G_\Lambda + \xi {*G}_\Lambda\right) = I_{\Lambda\Sigma}\frac{1}{2}\left({*F}^\Sigma + \xi^3 F^\Sigma\right) + R_{\Lambda\Sigma}\frac{1}{2}\left(F^\Sigma + \xi {*F}^\Sigma\right)$$

$$= \left(R_{\Lambda\Sigma} + \xi^3 I_{\Lambda\Sigma}\right)\frac{1}{2}\left(F^\Sigma + \xi {*F}^\Sigma\right)$$

$$\underbrace{* \tfrac{1}{2}\left(G_\Lambda \pm \xi * G_\Lambda\right)}_{\displaystyle \begin{array}{c} \parallel \\ G^{\pm}_\Lambda \end{array}} = \pm \tfrac{1}{2}\, \xi^3 \left(G_\Lambda \pm \xi * G_\Lambda\right)$$

$$* G^{\pm}_\Lambda = \pm \xi^3 G^{\pm}_\Lambda$$

Define, as well,

$$\boxed{\begin{aligned} \overline{\mathcal{N}}_{\Lambda\Sigma} &\equiv R_{\Lambda\Sigma} + \xi^3 I_{\Lambda\Sigma} \\ \mathcal{N}_{\Lambda\Sigma} &\equiv R_{\Lambda\Sigma} + \xi I_{\Lambda\Sigma} \end{aligned}}$$

Linear, twisted self-duality constraint

$$\boxed{G_\Lambda{}^{+} = \overline{\mathcal{N}}_{\Lambda\Sigma} F^{\Sigma +}}$$

Define $\left(\mathcal{F}^M\right) \equiv \begin{pmatrix} F^a \\ G_a \end{pmatrix}$

Define $\left(\mathcal{M}_{MN}\right) \equiv \begin{pmatrix} I - \xi^2 R I^{-1} R & \xi^2 R I^{-1} \\ -I^{-1} R & I^{-1} \end{pmatrix}$

$\quad\quad\quad\quad\ \ ||$

$\quad\quad\quad\ \ \mathcal{M}_{NM}$

and

Define $\left(\Omega_{MN}\right) = \left(\begin{array}{c|c} & \mathbb{1} \\ \hline \xi^2 \mathbb{1} & \end{array}\right)$

$\xi^2 = -1$; $Sp(2\bar{m}, \mathbb{R})$

$\xi^2 = +1$; $SO(\bar{m}, \bar{m})$

Ω^T

$||$

$$\boxed{\Omega^{-1} \mathcal{M}\, \Omega = \mathcal{M}^{-1} \text{ or } \mathcal{M}\,\Omega\,\mathcal{M}^T = \Omega}$$

$\mathcal{M}$ is a symplectic or $SO(\bar{m}, \bar{m})$ matrix

With these definitions, the linear, twisted, selfduality constraint always ($d=2(p+2)$) takes the form

$$\boxed{*F^M = \Omega^{MN} M_{NP} F^P}$$

The Bianchi identities & Maxwell equations $dF^M=0$ are left invariant by

$$F' = S F \; ; \; S \in GL(2\bar{n}, \mathbb{R})$$

The selfduality constraint $*F = \Omega M F$ invariant if

$$\boxed{\begin{array}{l} M' = \kappa S M S^{-1} \; ; \; S \Omega S^{-1} = \kappa \Omega \; ; \; \kappa^2 = 1 \\ M'(\varphi) = M(\varphi'(\varphi)) \; ; \; \varphi'(\varphi) \text{ isometry of } G_{ij} \end{array}}$$

Next, the Einstein equations

$$\frac{\delta S}{\delta g^{\mu\nu}} = \sqrt{|g|}\Big\{ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$+ \mathcal{G}_{ij}\left[\partial_\mu \varphi^i \partial_\nu \varphi^j - \frac{1}{2} g_{\mu\nu} (\partial\varphi)^2\right]$$

$$\underbrace{+ \frac{8(-1)^{p+1}}{(p+1)!} I_{\Lambda\Sigma}\left[F^{\Lambda}{}_{\mu}{}^{\rho_1\cdots} F^{\Sigma}{}_{\nu\rho_1\cdots} - \frac{1}{2(p+2)} g_{\mu\nu} F^{\Lambda}\cdot F^{\Sigma}\right]\Big\}}_{\displaystyle \frac{1}{2}\mathcal{M}_{MN}\mathcal{F}^{M}{}_{\mu}{}^{\rho_1\cdots}\mathcal{F}^{N}{}_{\nu\rho_1\cdots}}$$

self duality constraint $\Longrightarrow$ $\shortparallel$

$$\frac{1}{2}\xi^2 \Omega_{MN} \mathcal{F}^{M}{}_{\mu}{}^{\rho_1\cdots} \star\mathcal{F}_{\nu\rho_1\cdots}$$

$\Longrightarrow \; K = +1 \; \Longrightarrow \; Sp(2\bar{n}, \mathbb{R})$

T-duality for $d=2, p=0$ $\longrightarrow$ $SO(\bar{n}, \bar{n})$

The equations of motion of our generic theory are invariant under isometries $\varphi'^i(\varphi)$ of $g_{ij}(\varphi)$ such that

$$\boxed{\begin{gathered} \mathcal{M}(\varphi') = S\,\mathcal{M}(\varphi)\,S^{-1} \\ \text{with} \\ S\,\Omega\,S^{-1} = \Omega \end{gathered}}$$

Only when $d = 2(n+2)$, $R_{n2} \neq 0$

⊕ Isometries of scalars not coupled to vectors!

Metrics for static, flat branes

Gibbons did not solve this problem for us.

Inspiration from known black p-brane solutions

Ansatz:

$(p+1)$ worldvolume coordinates

$$\boxed{\begin{aligned} ds^2_{(d)} &= e^{\frac{2}{p+1}U}\left[W^{\frac{p}{p+1}}dt^2 - W^{-\frac{1}{p+1}}d\vec{y}_p^{\,2}\right] - e^{-\frac{2}{\tilde{p}+1}\tilde{U}}\, d\sigma^2_{\tilde{p}+3}\,; \\ d\sigma^2_{(\tilde{p}+3)} &= f(\rho)^{\frac{2}{\tilde{p}+1}}\left[f(\rho)^2\frac{d\rho^2}{(\tilde{p}+1)^2} + d\Omega^2_{(\tilde{p}+2)}\right] \end{aligned}}$$

transverse metric

$\tilde{p}+3 = d-(p+1)$

radial coordinate in transverse space

$U = U(\rho)\;;\quad W = W(\rho)\;;\longleftarrow$ 2 indep. functions

Before we study these metrics, we are going to perform the reduction because that will determine $W(\rho)$ up to integration consts.

Reduction and effective theories

The Ansatz that we are going to substitute consists in the metric presented before plus

1.- Scalars $\varphi^i = \varphi^i(\rho)$

2.- $(p+1)$-form potentials

3.- Dual $(p+1)$-form pots.

when $d = 2(p+2)$

$$\boxed{\begin{aligned} A^{n}{}_{(p+1)t y_1 \dots y_p} &= \psi^{n}(\rho) \\ A_{n\,(p+1)\,t y_1 \dots y_p} &= \psi_{n}(\rho) \end{aligned}}$$

Same procedure, similar equations plus one more equation for $W(\rho)$:

$$\boxed{\frac{d^2 \ln W}{d\rho^2} = 0} \Rightarrow \boxed{W = e^{\beta + \gamma\rho}}$$

$W(\rho)$'s functional form is theory-independent (like $f(\rho)$)

$\Rightarrow$ add it to the metric Ansatz

(with $\beta = 0$ to have asymptotic flatness)

Ansatz:

$$ds^2_{(d)} = e^{\frac{2}{\tilde{p}+1}\tilde{U}} \left[e^{\frac{\tilde{p}}{\tilde{p}+1}\gamma\rho} dt^2 - e^{-\frac{1}{\tilde{p}+1}\gamma\rho} d\vec{y}_p^{\,2} \right] - e^{-\frac{2}{\tilde{p}+1}\tilde{U}} d\sigma^2_{\tilde{p}+3};$$

$$d\sigma^2_{(\tilde{p}+3)} = f(\rho)^{\frac{2}{\tilde{p}+1}} \left[f(\rho)^2 \frac{d\rho^2}{(\tilde{p}+1)^2} + d\Omega^2_{(\tilde{p}+2)} \right];$$

$$f(\rho) = \frac{\omega/2}{\sinh \frac{\omega}{2} \rho}$$

Now, let us study this metric!

1 arbitrary function: $\tilde{U}$; 2 arbitrary constants : γ, $\omega < 0$

convention

In the near-horizon limit $\rho \longrightarrow +\infty$

$\tilde{p} \neq 0$, $d > 4$ convention

$$f(\rho) \sim \frac{\omega/2}{-\frac{1}{2} e^{-\omega/2 \rho}} \sim \omega e^{\frac{\omega}{2}\rho}$$

$$d\sigma^2_{(\tilde{p}+3)} \sim \left(-\omega e^{\frac{\omega}{2}\rho}\right)^{\frac{2}{\tilde{p}+1}} \left[\left(\omega e^{\frac{\omega}{2}\rho}\right)^2 \frac{d\rho^2}{(\tilde{p}+1)^2} + d\Omega^2_{(\tilde{p}+2)} \right]$$

$$\Rightarrow e^{-\frac{2}{\tilde{p}+1}\tilde{U}} \left(-\omega e^{\frac{\omega}{2}\rho}\right)^{\frac{2}{\tilde{p}+1}} d\Omega^2_{(\tilde{p}+2)}$$ will behave well if

$$\boxed{\tilde{U} \underset{\rho \to +\infty}{\sim} c + \frac{\omega}{2}\rho}$$

Then, the worldvolume metric

$$\Rightarrow \quad ds^2_{(d)}\Big| \sim e^{\frac{2}{\tilde{p}+1}\left(c + \frac{\omega}{2}\rho\right)} \left[e^{\frac{\tilde{p}}{\tilde{p}+1}\gamma\rho} dt^2 - e^{-\frac{1}{\tilde{p}+1}\gamma\rho} d\vec{y}_p^{\,2} \right]$$

$$\Rightarrow \quad ds^2_{(d)}\Big|_{WV} \sim e^{h+1 \cdots} \left[e^{h+1 \cdots} dt^2 - e^{h+1 \cdots} d\vec{y}_r^{\,2} \right]$$

Demanding regularity $\Rightarrow$ $\boxed{\gamma = \omega}$

(It is OK if $g_{tt} \xrightarrow[\rho \to \infty]{} 0$ because we expect a horizon there!)

$$\Rightarrow ds^2_{(d)} \sim e^{\frac{2}{\tilde{p}+1}(c + \frac{\omega}{2}\rho)} \left[e^{\frac{\tilde{p}}{\tilde{p}+1}\omega\rho} dt^2 - e^{-\frac{1}{\tilde{p}+1}\omega\rho} d\vec{y}_p^{\,2} \right]$$

$$- e^{-\frac{2}{\tilde{p}+1}(c + \frac{\omega}{2}\rho)} \left[\left(-\omega e^{\frac{\omega}{2}\rho}\right)^{\frac{2}{\tilde{p}+1}+2} \frac{d\rho^2}{(\tilde{p}+1)^2} + \left(-\omega e^{\frac{\omega}{2}\rho}\right)^{\frac{2}{\tilde{p}+1}} d\Omega^2_{(\tilde{p}+2)} \right]$$

$$\sim e^{\frac{2c}{p+1}} \left[e^{\omega\rho} dt^2 - d\vec{y}_p^{\,2} \right] - \frac{(-\omega)^{\frac{2}{\tilde{p}+1}+2} e^{-\frac{2c}{\tilde{p}+1}}}{(\tilde{p}+1)^2} e^{\omega\rho} d\rho^2$$

$$\underbrace{-(-\omega)^{\frac{2}{\tilde{p}+1}} e^{\frac{-2c}{\tilde{p}+1}} d\Omega^2_{(\tilde{p}+2)}}_{S^{\tilde{p}+2} \text{ with radius } \left[-\omega e^{-c}\right]^{\frac{1}{\tilde{p}+1}}}$$

Rindler

$\mathbb{R}^p$

The near-horizon geometry is

$$\underbrace{\mathrm{Rindler}_2 \times \mathbb{R}^p \times S^{\tilde{p}+2}}$$

constant-time sections of
the event horizon

The horizon volume is infinite unless we compactify the brane's worldvolume $y^i_p \sim y^i_p + 2\pi R^i$

$$V_{(d-2)} = \left(2\pi e^{\frac{c}{\tilde{p}+1}}\right)^p R^1 \cdots R^p \; R_h^{\tilde{p}+2} \; \omega_{(\tilde{p}+2)}$$

volume of $S^{\tilde{p}+2}$ $R=1$

Define the entropy density per unit worldvolume

$$\boxed{\tilde{S} = R_h^{\tilde{p}+2}} \qquad \left(\tilde{S} = S/\pi \text{ for } d=4,\ p=0\right)$$

$$\Rightarrow \quad \boxed{\tilde{S} = \left(-\omega e^{-c}\right)^{\frac{\tilde{p}+2}{\tilde{p}+1}}} \quad \text{or} \quad \boxed{e^{c} = -\omega\, \tilde{S}^{\frac{\tilde{p}+1}{\tilde{p}+2}}}$$

From the acceleration of the Rindler space we find

$$T = \frac{(-\omega)^{\frac{1}{\tilde{p}+1}}}{\tilde{p}+1} e^{-Cc} \quad ; \quad c \equiv \frac{1}{p+1} + \frac{1}{\tilde{p}+1} ;$$

$$\Rightarrow \boxed{(-\omega)^{\frac{1}{p+1}} = \frac{4\pi}{\tilde{p}+1} T \, \tilde{S}^{\frac{(d-2)}{(p+1)(\tilde{p}+2)}}}$$

non-extremality parameter again.

Finally, let us find the p-brane's tension T_p

Let r be a radial coordinate such that spatial infinity is at $r \longrightarrow \infty$ and in that limit the angular part of the metric $\sim r^2 \, d\Omega^2_{(\tilde{p}+2)}$

Define $\quad g_{\mu\nu} \sim \eta_{\mu\nu} + \frac{C_{\mu\nu}}{r^{\tilde{p}+1}}$

Then, the energy-momentum tensor t_{ab} of the p-brane

$$\boxed{t_{ab} = -\frac{\omega_{(\tilde{p}+2)}}{16\pi G_N^{(d)}}\left[(\tilde{p}+1)\,c_{ab} + \eta_{ab}\,\eta^{cd}c_{cd}\right]}$$

The tension is just the tt component:

$$\boxed{T_p = t_{tt} = -\frac{\omega_{(\tilde{p}+2)}}{16\pi G_N^{(d)}}\left[(\tilde{p}+1)\,c_{tt} + \eta^{ab}c_{ab}\right]}$$

when $\rho \longrightarrow 0$ (spatial infinity) $e^{\tilde{U}} \sim 1 + \tilde{u}\rho$

$$ds^2_{(d)} \sim \left(1+\frac{2\tilde{u}}{p+1}\rho\right)\left[\left(1+\frac{p}{p+1}\omega\rho\right)dt^2 - \left(1-\frac{1}{p+1}\omega\rho\right)d\vec{y}_p^{\,2}\right]$$

$$-\left(\quad\right)d\rho^2 - \left(1-\frac{2}{\tilde{p}+1}\tilde{u}\rho\right)\rho^{-\frac{2}{\tilde{p}+1}}\,d\Omega^2_{(\tilde{p}+2)} \rightarrow r^2$$

$$ds^2_{(d)} \sim \Big(1 + \overbrace{\frac{2\tilde{u}+pw}{p+1}}^{c_{tt}} \frac{1}{r^{\tilde{p}+1}}\Big) dt^2 - \Big(1 + \overbrace{\frac{2\tilde{u}-w}{p+1}}^{c_{ii}} \frac{1}{r^{\tilde{p}+1}}\Big) d\vec{y}_p^{\,2}$$

$$-\Big(\quad \Big) dr^2 - r^2 \, d\Omega^2_{(\tilde{p}+2)}$$

$$\eta^{ab} C_{ab} = \frac{2\tilde{u}+pw}{p+1} - p\left(-\frac{(2\tilde{u}-w)}{p+1}\right) = 2\tilde{u}$$

$$\boxed{T_p = -\frac{w_{(\tilde{p}+2)}}{16\pi G_N^{(d)}} \left[\frac{(\tilde{p}+1)(2\tilde{u}+pw)}{(p+1)} + 2\tilde{u}\right]}$$

$p=0, \; d=4, \; \tilde{p}=0 \qquad T_p = M$

$$M = -\frac{4\pi}{16\pi G_N^{(4)}} 4\tilde{u} = -\frac{\tilde{u}}{G_N^{(4)}} \Rightarrow e^{\tilde{U}} \sim e^{-\frac{M}{G}\rho}$$

(We have set $G_N^{(4)} = 1$ before)

Ready to replace $W(\xi) = e^{\omega \xi}$

in the remaining e.o.m.

(finally!) $\longrightarrow$

The e.o.m. for $U(\rho)$, $\varphi^i(\rho)$ are

$$\boxed{\begin{aligned} &\ddot{U} + e^{2U} V_{bb} = 0; \\ &\ddot{\varphi}^i + \Gamma_{jk}{}^{i} \dot{\varphi}^j \dot{\varphi}^k + \frac{c}{2} e^{2\tilde{U}} \partial^i V_{bb} = 0; \\ &(\dot{\tilde{U}})^2 + \frac{1}{c} g_{ij} \dot{\varphi}^i \dot{\varphi}^j + e^{2\tilde{U}} V_{bb} = (\omega/2)^2; \end{aligned}}$$

$$c = \frac{1}{p+1} + \frac{1}{\tilde{p}+1};$$

$$\boxed{V_{bb} \sim \frac{2\alpha^2}{c} Q^M \mathcal{M}_{MN} Q^N;}$$

"Black-brane potential"

The first two equations can be derived from the FGK-like effective action

$$\boxed{S[\tilde{U}, \varphi^i] = \int d\rho \left\{ (\dot{\tilde{U}})^2 + \frac{1}{c} \mathcal{G}_{ij} \dot{\varphi}^i \dot{\varphi}^j - e^{2\tilde{U}} V_{bb} \right\}}$$

while the third is the Hamiltonian constraint

$$\boxed{H = (\omega/2)^2}$$

Very similar systems but very different solutions (metrics).

FGK theorems in general dimensions

Not surprisingly one can show that, for extremal branes

$(\omega \longrightarrow 0, \ \tilde{S} \text{ finite}, \ T \longrightarrow 0)$

1 $$\boxed{\tilde{S} = \left[-V_{bb}(\varphi_h, Q) \right]^{\frac{\tilde{p}+2}{2(\tilde{p}+1)}}}$$

2 $$\boxed{\partial_i V_{bb} \Big|_{\varphi_h} = 0}$$

and, in the general case

3 $$\boxed{\tilde{u}^2 + \frac{1}{c} \mathcal{G}_{ij}(\varphi_\infty) \Sigma^i \Sigma^j + V_{bb}(\varphi_\infty, Q) = (\omega/2)^2 \geq 0}$$

FGK formalism for the black holes of N=1, d=5 SUGRA

$d \neq 2(p+2)$

Our generalization can deal with this case (already treated in the literature (?!)).

vector multiplets

We just need

1. The matter content $\begin{cases} n \text{ real scalars } \phi^x \\ n+1 \text{ vector fields } A^I{}_\mu = A^0{}_\mu, A^x{}_\mu \end{cases}$

graviphoton

2. The couplings $\left\{ \begin{array}{l} g_{xy}(\phi) \\ a_{IJ}(\phi) \end{array} \right\} \rightarrow$ Real special geometry

$I \wedge \Sigma$?

The Chern-Simons term immaterial for BHs

The FGK effective action and Hamiltonian constraint are

$$S = \int d\rho \left\{ \dot{U}^2 + \tfrac{1}{3} g_{xy} \dot{\phi}^x \dot{\phi}^y - e^{2U} V_{bh} \right\};$$

$$\dot{U}^2 + \tfrac{1}{3} g_{xy} \dot{\phi}^x \dot{\phi}^y + e^{2U} V_{bh} = (\omega/2)^2$$

Real special geometry

where

$$-V_{bh}(\phi, q) = a^{IJ} q_I q_J = Z_e^2 + 3 g^{xy} \partial_x Z_e \partial_y Z_e;$$

$$Z_e \equiv q_I h^I(\phi); \quad \text{"electric" central charge}$$

Again $\partial_x Z_e \big|_{\hat{\phi}} = 0 \Rightarrow \partial_x V_{bh} \big|_{\hat{\phi}} = 0 \Rightarrow \hat{\phi}^x = \phi^x_h$

SUSY "ATTRACTORS" $\not\Leftarrow$ ← not true in general

The FGK effective action of $N=1, d=5$ SUGRA can be rewritten *à la Bogomol'nyi*:

$$S = \int d\rho \left\{ (\dot{U} \pm e^U \mathcal{Z}_e)^2 + \frac{1}{3} g_{xy} (\dot{\phi}^x \pm 3 e^U \partial^x \mathcal{Z}_e)(\quad)^y \mp \frac{d}{d\rho}(2 e^U \mathcal{Z}_e) \right\}$$

$\Rightarrow$ Flow equations

$$\boxed{\begin{aligned} \dot{U} &= \mp e^U \mathcal{Z}_e\,; \\ \dot{\phi}^x &= \mp 3 e^U \partial^x \mathcal{Z}_e\,; \end{aligned}}$$

But also in this way

$$S = \int d\rho \left\{ e^{2U} a^{IJ} \left[\frac{d}{d\rho}(e^{-U} h_I) \pm q_I \right] \left[\quad \right]_J \pm \frac{d}{d\rho}(2 e^U \mathcal{Z}_e) \right\}$$

$\Rightarrow$ Flow equations

$$\boxed{\frac{d}{d\rho}(e^{-U} h_I) = \mp q_I}$$

$$\boxed{e^{-U} h_I = A_I \mp q_I \rho}$$

FGK formalism for the black strings of N=1, d=5 SUGRA

The 1-forms $A^I{}_\mu$ of these theories can be dualized into 2-forms $B_{I\,\mu\nu}$:

$$S \sim \int d^5x \sqrt{|g|} \left\{ -\frac{1}{4} a_{IJ} F^I \cdot F^J \right\}$$

$$\Rightarrow \quad d\left(\underbrace{a_{IJ} \star F^J}_{\parallel} \right) = 0 \; ; \text{(e.o.m.)} \qquad dF^I = 0 \; ; \text{(Bianchi)}$$

$$H_I \equiv dB_I \quad \text{(local solution)} \qquad \Downarrow$$

$$\Rightarrow \quad F^I = \star\, a^{IJ} H_J \qquad \boxed{d\left(a^{IJ} \star H_J\right) = 0} \; ; \text{(e.o.m.)}$$

$$\boxed{S \sim \int d^5x \sqrt{|g|} \left\{ -\frac{1}{12} a^{IJ} H_I \cdot H_J \right\}} \quad \Leftarrow \text{Dual action}$$

We expect string solutions ($p=1$) associated to the $B_{I\,\mu\nu}$

The FGK effective action is

$$S = \int d\rho \left\{ \dot{U}^2 + \tfrac{1}{3} g_{xy} \dot{\phi}^x \dot{\phi}^y - e^{2\tilde{U}} V_{bs} \right\};$$

$$\dot{U}^2 + \tfrac{1}{3} g_{xy} \dot{\phi}^x \dot{\phi}^y + e^{2\tilde{U}} V_{bs} = (\omega/2)^2$$

where, now

Real special geometry

$$-V_{bs} = a_{IJ}\, p^I p^J = Z_m^2 + 3 g^{xy} \partial_x Z_m \partial_y Z_m;$$

$Z_m = p^I h_I(\phi)$: "magnetic" central charge

Z_m's properties are similar to those of Z_e (w.r.t V_{bh})

But the physical fields are very different.