SELF-DUAL NON-LINEAR Electrodynamics WITH HIGHER DERIVATIVES

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SELF-DUALITY : INTRODUCTION

(We only consider the $d=4$ case)

$$S_{Max}[F] = \int d^4x \left\{ -\frac{1}{4} F^2 \right\} \; ; \qquad F = dA \; ;$$

$$\left.\begin{array}{ll} F = dA \Rightarrow dF = 0 ; \longrightarrow & \partial_\nu \tilde{F}^{\nu\mu} = 0 ; \\ \dfrac{\delta S}{\delta A_\mu} = 0 \; ; \longrightarrow & \partial_\nu F^{\nu\mu} = 0 ; \end{array}\right\}$$

This set of eqs + energy-momentum tensor are invariant under the $U(1)$ e-m duality group

$$\Rightarrow \begin{array}{l} F' = \cos\alpha\, F + \sin\alpha\, \tilde{F} ; \\ \tilde{F}' = -\sin\alpha\, F + \cos\alpha\, \tilde{F} ; \end{array} \qquad \left(\begin{array}{l} \tilde{F}^{\mu\nu} \equiv \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} ; \\ \tilde{\tilde{F}} = -F ; \end{array} \right)$$

The action is not invariant: $(F')^2 = (\tilde{F})^2 = -F^2$;

However, the new eqs. can be obtained from an action which is identical $S[F'] = S[F']$

$\longrightarrow$ Selfduality .

In more general theories (of just 1 Abelian field) $S[F]$ has terms of higher order in F and $\partial^\mu F$

Define $\tilde{G}^{\mu\nu} \equiv 2 \dfrac{\delta S}{\delta F_{\mu\nu}}$;

$\Rightarrow$ The Maxwell equation becomes $\begin{cases} \partial_\mu \tilde{F}^{\mu\nu} = 0; & \text{(Bianchi)} \\ \partial_\mu \tilde{G}^{\mu\nu} = 0; & \end{cases}$

These equations still have the same invariance, but

- Is the energy-momentum tensor invariant too?
- Is the theory self-dual?

$\rightarrow$ If $S'[F']$ is such that $\tilde{G}'^{\mu\nu} = 2 \dfrac{\delta S'[F']}{\delta F'^{\mu\nu}}$;

$$\boxed{\text{WHEN IS } S'[F'] = S[F'] \text{ ?}}$$

ANSWER: When the NGZ identity is satisfied.. ③

Consistency between
$$\begin{cases} \delta\begin{pmatrix} F \\ G \end{pmatrix} = \alpha \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} F \\ G \end{pmatrix}; \\ \tilde{G}^{\mu\nu} = 2\,\dfrac{\delta S[F]}{\delta F_{\mu\nu}}; \end{cases}$$
by assumption!

$$\delta\tilde{G}^{\mu\nu} = 2\,\delta\left(\frac{\delta S[F]}{\delta F_{\mu\nu}}\right) = 2\left\{\frac{\delta \overbrace{S'[F']}^{=S[F']}}{\delta F'_{\mu\nu}} - \frac{\delta S[F]}{\delta F_{\mu\nu}}\right\}$$

$$= 2\left\{\frac{\delta S[F']}{\delta F_{\alpha\beta}}\frac{\delta F_{\alpha\beta}}{\delta F'_{\mu\nu}} - \frac{\delta S[F]}{\delta F_{\mu\nu}}\right\} = 2\left\{\frac{\delta S[F']}{\delta F_{\alpha\beta}}\frac{\delta}{\delta F_{\mu\nu}}\left(F_{\alpha\beta} - \alpha G_{\alpha\beta}\right) - \frac{\delta S[F]}{\delta F_{\mu\nu}}\right\}$$

$$= 2\left\{\frac{\delta}{\delta F_{\mu\nu}}\delta S - \alpha\,\underbrace{\frac{\delta S[F']}{\delta F_{\alpha\beta}}}_{\sim\frac{1}{2}\tilde{G}^{\alpha\beta}}\frac{\delta G_{\alpha\beta}}{\delta F_{\mu\nu}}\right\} = 2\,\frac{\delta}{\delta F_{\mu\nu}}\left\{\underbrace{\delta S}_{\frac{\delta S}{\delta F}\delta F} - \frac{1}{4}\alpha\,\tilde{G}^{\alpha\beta}G_{\alpha\beta}\right\}$$

$$\parallel$$

$$-\alpha\,\tilde{F}^{\mu\nu}$$

$$\parallel$$

$$-\frac{\alpha}{2}\frac{\delta}{\delta F_{\mu\nu}}\left(\tilde{F}^{\alpha\beta}F_{\alpha\beta}\right) = -\frac{\alpha}{2}\frac{\delta}{\delta F_{\mu\nu}}\left(\tilde{G}^{\alpha\beta}G_{\alpha\beta}\right);$$

$$\frac{\delta}{\delta F_{\mu\nu}}\left\{\tilde{F}F + \tilde{G}G\right\} = 0; \longrightarrow \boxed{\tilde{F}^{\alpha\beta}F_{\alpha\beta} + \tilde{G}^{\alpha\beta}G_{\alpha\beta} + \cancel{C} = 0;}$$

$\delta J_{\mu\nu} = 0$

Maxwell: $G = \tilde{F}$, $\tilde{G} = -F$ $\Rightarrow$ NGZ satisfied

On shell

$$\partial_\alpha \tilde{F}^{\alpha\beta} = 0 \quad \overset{\text{locally}}{\Longleftrightarrow} \quad F_{\alpha\beta} = 2\partial_{[\alpha} A_{\beta]} ;$$

$$\partial_\alpha \tilde{G}^{\alpha\beta} = 0 \quad \overset{\text{locally}}{\Longleftrightarrow} \quad G_{\alpha\beta} = 2\partial_{[\alpha} B_{\beta]} ;$$

NGZ :

$$0 = \tilde{F}^{\alpha\beta} F_{\alpha\beta} + \tilde{G}^{\alpha\beta} G_{\alpha\beta} = 2\left\{ \tilde{F}^{\alpha\beta} \partial_\alpha A_\beta + \tilde{G}^{\alpha\beta} \partial_\alpha B_\beta \right\} =$$

$$= \partial_\alpha \underbrace{\left\{ \tilde{F}^{\alpha\beta} A_\beta + \tilde{G}^{\alpha\beta} B_\beta \right\}}_{J^\alpha_{NGZ}} - \underbrace{\cancel{2\left\{ \partial_\alpha \tilde{F}^{\alpha\beta} A_\beta + \partial_\alpha \tilde{G}^{\alpha\beta} B_\beta \right\}}}_{\text{on shell}}$$

$$\Longleftrightarrow \quad \boxed{\partial_\alpha J^\alpha_{NGZ} = 0 ;}$$

FURTHERMORE:

$$\boxed{S_{inv} \equiv S - \frac{1}{4}\int d^4x \, F \cdot \tilde{G} ;}$$

$$\delta S_{inv} = \frac{\delta S}{\delta F_{\mu\nu}} \delta F_{\mu\nu} - \frac{1}{4} \tilde{G}^{\mu\nu} \delta F_{\mu\nu} - \frac{1}{4} \tilde{F}^{\mu\nu} \delta G_{\mu\nu} =$$

$$= \frac{1}{2} \tilde{G}^{\mu\nu} \delta F_{\mu\nu} - \frac{1}{4} \tilde{G}^{\mu\nu} \delta F_{\mu\nu} - \frac{1}{4} \tilde{F}^{\mu\nu} \delta G_{\mu\nu} =$$

$$= \frac{1}{4}\Big(\tilde{G}^{\mu\nu} \underset{\alpha G_{\mu\nu}}{\underset{\parallel}{\delta F_{\mu\nu}}} - \tilde{F}^{\mu\nu} \underset{-\alpha F_{\mu\nu}}{\underset{\parallel}{\delta G_{\mu\nu}}} \Big) = \frac{\alpha}{4}\left(\tilde{G}^{\mu\nu} G_{\mu\nu} + \tilde{F}^{\mu\nu} F_{\mu\nu} \right) \overset{NGZ}{=} 0$$

(Maxwell $S_{inv} = 0$;)

FINALLY, $\boxed{\text{IF} \quad S = S[\lambda, F] \Rightarrow S_{inv}[\lambda, F] = -\lambda \frac{d}{d\lambda} S[\lambda, F]}$

$$\delta S_{inv}[\lambda, F] = -\lambda \frac{d}{d\lambda} \delta S[\lambda, F] = -\lambda \frac{d}{d\lambda}\left(\frac{\alpha}{2} \tilde{G}^{\mu\nu} G_{\mu\nu}\right) =$$

$$= -\frac{\alpha}{2} \lambda \frac{d}{d\lambda} \left\{ \tilde{G}^{\mu\nu} G_{\mu\nu} + \underbrace{\tilde{F}^{\mu\nu} F_{\mu\nu}}_{\lambda\text{-indep}} \right\} \underset{NG2}{=} 0 ;$$

(The proportionality factor follows from comparison with known cases, like BORN-INFELD'S)

This is a "RECONSTRUCTIVE IDENTITY" for, if we know $G(\lambda, F)$

$$S = S_{inv} + \frac{1}{4}\int d^4x\, F \tilde{G} = -\lambda \frac{d}{d\lambda} S + \frac{1}{4}\int d^4x\, F \cdot \tilde{G} ;$$

$$\underbrace{S + \lambda \frac{d}{d\lambda} S}_{= d(\lambda S)/d\lambda} = +\frac{1}{4}\int d^4x\, F \cdot \tilde{G} ; \longrightarrow \boxed{S = \frac{1}{4\lambda}\int d\lambda\, d^4x\, F \cdot \tilde{G}}$$

Using $S = \frac{1}{4\lambda}\int d\lambda\, d^4x\; F\cdot\tilde{G}(\lambda,F)$; we can "RECONSTRUCT" a self dual action, provided that we have a $G(\lambda,F)$ that satisfies the NGZ identity $\tilde{G}\cdot G + \tilde{F}F = 0$;

Now, the question is

$$\boxed{\text{HOW DO WE FIND } G(\lambda,F)\text{s THAT SATISFY NGZ?}}$$

We should mention that non-trivial $G(\lambda,F)$s do exist:

$$G_{\mu\nu}(\lambda,F) = \Delta^{-1/2}\left[\tilde{F}_{\mu\nu} + \lambda\,(F\cdot\tilde{F})\,F_{\mu\nu}\right];$$

$$\Delta = -\det\left(\eta_{\alpha\beta} + \lambda^{1/2}F_{\alpha\beta}\right) = 1 + \frac{1}{2}\lambda F^2 - \frac{\lambda^2}{16}(F\cdot\tilde{F})^2;$$

$$S = \frac{1}{\lambda}\int d^4x\left\{1 - \Delta^{1/2}\right\}$$

BORN-INFELD

THE GRAVIPHOTON AND TWISTED SELFDUALITY CONSTRAINTS ⑦

IN $N \geq 2$ SUGRAS $\quad T_{IJ\mu\nu} \equiv \langle V_{IJ} | \mathcal{F}_{\mu\nu} \rangle = F^{\Lambda}_{\mu\nu} h_{\Lambda IJ} - G_{\Lambda\mu\nu} f^{\Lambda}{}_{IJ}$;

$N=2 \Rightarrow \quad h_{\Lambda IJ} \to M_{\Lambda}\epsilon_{IJ}, \quad f^{\Lambda}{}_{IJ} \to L^{\Lambda}\epsilon_{IJ}$

4 OBJECTS:

$$\boxed{\begin{array}{cccc} T_{IJ}{}^{+} & ; & T_{IJ}{}^{-} & ; \\ \downarrow & & \downarrow & \\ T^{IJ-} & ; & T^{IJ+} & \end{array}}$$

complex conj.

The twisted self-duality constraint is $\quad \boxed{G^{+}_{\Lambda} = \overline{\mathcal{N}}_{\Lambda\Sigma} F^{\Sigma +}}$

$$\Rightarrow \boxed{T^{IJ+} = \left(T_{IJ}{}^{-}\right)^{*} = 0}$$

IN THE U(1) CASE AT HANDS $\quad T_{\mu\nu} \equiv F - i\,G \quad$ (NO TWIST!)

4 OBJECTS

$$\boxed{\begin{array}{cccc} T^{+} & ; & T^{-} & \\ \downarrow & & \downarrow & \\ T^{*-} & ; & T^{*+} & \end{array}}$$

complex conj.

$G = \tilde{F} \Rightarrow G^{+} = \tilde{F}^{+} = -i\,F^{+}$

$$\Rightarrow \boxed{T^{+} = T^{*-} = 0}$$

Under duality $\quad \delta T^{(\pm)} = i\alpha\, T^{(\pm)}$;

$\delta T^{*(\pm)} = -i\alpha\, T^{*(\pm)}$;

Now if we are given a **MANIFESTLY DUALITY-INVARIANT**

$I^{(1)}[T^-, T^{*+}]$ $\quad$ (T^-, T^{*+} treated as independent)

we can **DEFORM** the (twisted) self duality constraint

$$T^+_{\mu\nu} = 0 \longrightarrow \boxed{T^{+\,\mu\nu} = \frac{\delta I^{(1)}}{\delta T^{*+}_{\mu\nu}}[T^-, T^{*+}]}$$

$\uparrow$

$I^{(1)} = 0$

INITIAL SOURCE OF DEFORMATION

$$\rightarrow G^+ = -i F^+ + i \frac{\delta I^{(1)}}{\delta T^{*+}}[T, T^{*+}] \rightarrow \text{non-linearities in } G[F]$$

WHY IS THIS DEFORMATION GOOD?

BECAUSE THE NGZ IDENTITY IS AUTOMATICALLY SATISFIED : (CARRASCO, KALLOSH, ROIBAN)

$$\hat{G}G + \tilde{F}F \sim T^{*+}T^{+} - T^{*-}T^{-} = T^{*+}\frac{\delta I^{(1)}}{\delta T^{*+}} - T^{-}\frac{\delta I^{(1)}}{\delta T^{-}} =$$

$$= \frac{1}{i\alpha}\left\{ \delta T^{*+}\frac{\delta I^{(1)}}{\delta T^{*+}} + \delta T^{-}\frac{\delta I^{(1)}}{\delta T^{-}} \right\} = \frac{1}{i\alpha}\,\delta I^{(1)} \underset{\text{by hypothesis}}{=} 0;$$

AND BECAUSE IT CAN BE SOLVED ITERATIVELY AS A POWER SERIES IN SOME PARAMETER $\lambda \longrightarrow G[\lambda, F]$

$\boxed{\text{WHICH } I^{(1)} \text{ SHOULD WE TAKE?}}$

Assume $I^{(1)}[T^-, T^{*+}] \sim \mathcal{O}(\lambda)$

$$T^+ = -2\sum_{n=1}^{\infty} \lambda^n T^{(n)+} \equiv \left.\frac{\delta I^{(1)}}{\delta T^{*+}}\right|_{T^+ = -2\sum \lambda^n T^{(n)+}}$$

$$\Rightarrow -2\lambda T^{(1)+} = \left.\frac{\delta I^{(1)}}{\delta T^{*+}}\right|_{T^+=0}$$

$$\Rightarrow T^+ = \left.\frac{\delta I^{(1)}}{\delta T^{*+}}\right|_{T^+=0} + \mathcal{O}(\lambda^2),$$

$$-i G^+ = -F^+ + \left.\frac{\delta I^{(1)}}{\delta T^{*+}}\right|_{T^+=0} + \mathcal{O}(\lambda^2),$$

$$\tilde{G} = -F + \left.\left(\frac{\delta I^{(1)}}{\delta T^{*+}} + \frac{\delta I^{(1)}}{\delta T^-}\right)\right|_{T^+=0};$$

$$S = +\frac{1}{4\lambda}\int d\lambda\, d^4x\; F\cdot\tilde{G} = +\frac{1}{4\lambda}\int d\lambda\, d^4x \left\{-F^2 + F\left.\left(\frac{\delta I^{(1)}}{\delta T^{*+}} + c.c.\right)\right|_{T^+=0}\right\}$$

$$= \int d^4x\left\{-\frac{1}{4}F^2\right\} + \frac{1}{4\lambda}\int d\lambda\, d^4x \underbrace{\left\{F^+ \left.\frac{\delta I^{(1)}}{\delta T^{*+}}\right|_{T^+=0} + c.c.\right\}}_{\left\{T^{*+}\frac{\delta I^{(1)}}{\delta T^{*+}} + c.c.\right\}\Big|_{T^+=0}}$$

$$S = \int d^4x \left\{ -\frac{1}{4} F^2 + \boxed{\frac{1}{4} I^{(1)}(T^-, T^{*+}) \Big|_{T^+=0}} + \mathcal{O}(\chi^2) \right\}$$

1st correction

We are interested in sources of deformation $I^{(1)}$ which reduce to some kind of higher-order corrections for $T^+ = 0$.

$F^4 \longrightarrow$ BORN-INFELD-TYPE CORRECTIONS

$\partial^n F^4 \longrightarrow$ Derivative corrections.

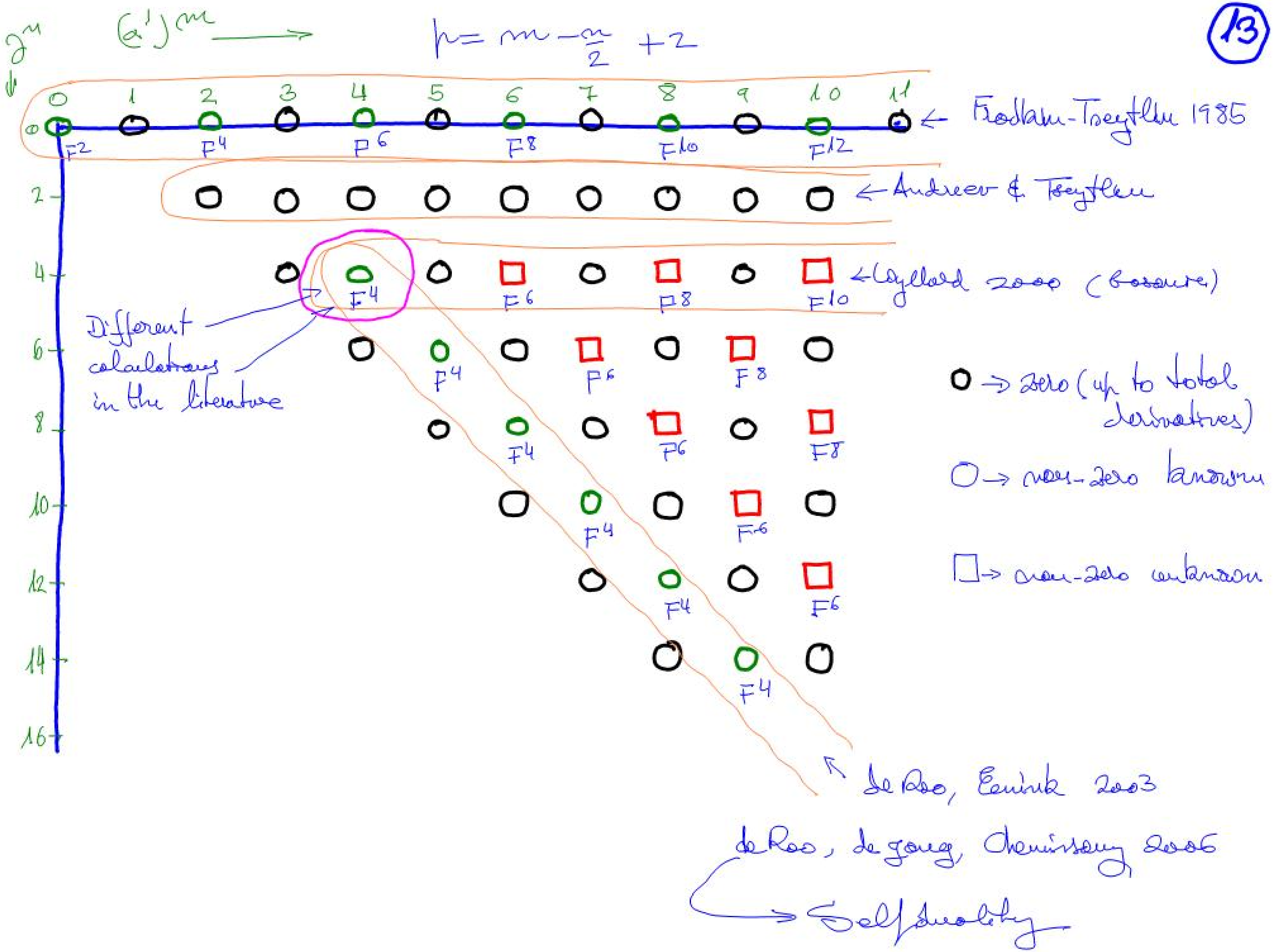
Open superstring

The open superstring effective action has terms of the generic form (bosonic)

$$(\alpha')^m \partial^n F^p$$

$$p = m + 2 - \frac{n}{2} \; ; \text{ for dimensional reasons}$$

WHAT DO WE KNOW ABOUT THEM?



∂^n
$(\alpha')^m$
$p = m - \frac{n}{2} + 2$
0 1 2 3 4 5 6 7 8 9 10 11
0 2 4 6 8 10 12 14 16
F^2 F^4 F^6 F^8 F^{10} F^{12}
← Fradkin-Tseytlin 1985
← Andreev & Tseytlin
← Wyllard 2000 (bosonic)
Different calculations in the literature
de Roo, Eenink 2003
de Roo, de Jong, Chemissany 2006
→ Selfduality
○ → zero (up to total derivatives)
○ → non-zero known
□ → non-zero unknown

In 2006, de Roo, de Jong & Chemissany proved that the series of F^4 terms $(\alpha')^m \partial^{2-m} F^4$ is selfdual (NGZ satisfied)

Then we could try to use a correction as initial source of deformation $I^{(1)}$ and find more **ASSUMING** that the open superstring effective action is **SELFDUAL**

$I^{(1)}\Big|_{T^+=0} \sim F^4$ Carrasco, Kallosh & Roiban

$I^{(1)}\Big|_{T^+=0} \sim \partial^4 F^4$ Chemissany, Kallosh, O.

The $(\alpha')^2 \partial^4 F^4$ correction has this well-known form

i) Define the $t^{(8)}$ tensor (Schwarz):

$$t^{(8)}_{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4} = t^{(8)}_{\underbrace{[\mu_1\nu_1]}_{a}\underbrace{[\mu_2\nu_2]}_{b}\underbrace{[\mu_3\nu_3]}_{c}\underbrace{[\mu_4\nu_4]}_{d}}$$

$$t^{(8)}_{abcd} = t^{(8)}_{(abcd)};$$

$$t^{(8)}_{abcd} A^a B^b C^c D^d = 8\,\mathrm{Tr}(ABCD + ACBD + ACDB) - 2\left[\mathrm{Tr}(AB)\,\mathrm{Tr}(CD) + \mathrm{Tr}(AC)\,\mathrm{Tr}(BD) + \mathrm{Tr}(AD)\,\mathrm{Tr}(BC)\right];$$

$$\ldots = 4\,\mathrm{Tr}(A^+B^+)\,\mathrm{Tr}(C^-D^-) + 4\,\mathrm{Tr}(A^+C^+)\,\mathrm{Tr}(B^-D^-) + 4\,\mathrm{Tr}(A^+D^+)\,\mathrm{Tr}(B^-C^-) + (+ \Leftrightarrow -)$$

ii) Then, the correction is $\boxed{\lambda\, t^{(8)}_{abcd}\, \partial_\mu F^a \partial^\mu F^b \partial_\nu F^c \partial^\nu F^d}$

iii) It can be rewritten in the form

$$\lambda t^{(8)}_{abcd}\, \partial_\mu F^a \partial^\mu F^b \partial_\nu F^c \partial^\nu F^d =$$

$$= 8\lambda\, \mathrm{Tr}(\partial_\alpha F^+ \partial^\alpha F^+)\, \mathrm{Tr}(\partial_\beta F^- \partial^\beta F^-) + 16\lambda\, \mathrm{Tr}(\partial_\alpha F^+ \partial_\beta F^+)\, \mathrm{Tr}(\partial^\alpha F^- \partial^\beta F^-);$$

iv) To reproduce this term we need **two sources**:

$$\boxed{\begin{array}{l} A:\; I^{(1)}_A = \frac{\lambda}{2^3}\, t^{(8)}_{abcd}\, \partial_\alpha T^{*+} \partial^\alpha T^- \partial_\beta T^{*+} \partial^\beta T^- ; \\ B:\; I^{(1)}_B = \frac{\lambda}{2^3}\, t^{(8)}_{abcd}\, \partial_\alpha T^{*+} \partial_\beta T^- \partial^\alpha T^{*+} \partial^\beta T^- ; \end{array}}$$

$$\bar{I}^{(1)}_A \Big|_{T^+=0} = 8\lambda\, \overline{\mathrm{Tr}}(\partial_\alpha \bar{F}^+ \partial_\beta \bar{F}^-)\, \overline{\mathrm{Tr}}(\partial^\alpha \bar{F}^- \partial^\beta \bar{F}^-);$$

$$I^{(1)}_B \Big|_{T^+=0} = 8\lambda\, \mathrm{Tr}(\partial_\alpha F^+ \partial^\alpha F^+)\, \mathrm{Tr}(\partial_\beta F^- \partial^\beta F^-),$$

The $(\alpha')^2 \partial^4 F^4$ term is related to $I_A^{(1)}$ and $I_B^{(1)}$ by

$$\boxed{\lambda\, t^{(8)}_{abcd}\, \partial_\alpha F^a \partial^\alpha F^b \partial_\beta F^c \partial^\beta F^d = \left\{ 2\, I_A^{(1)} + I_B^{(1)} \right\}\Big|_{T^\dagger = 0}}$$

We are going to construct a selfdual completion of this term. We solve the A and B models separately and then we combine the results.

For these two models the deformed (non-linear) self-duality constraint can be solved order by order in λ

MODEL A:

$$T^{+}_{a} = \frac{\delta I^{(1)}_{A}}{\delta T^{*+a}} = -\frac{\lambda}{4} t^{(8)}_{abcd}\, \partial_\alpha \left[\partial^\alpha T^{-b}\, \partial_\beta T^{*+c}\, \partial^\beta T^{-d} \right]$$

$$\left.\begin{array}{ll} T^{*+} \equiv 2 \sum_{n=0}^{\infty} \lambda^n T^{(n)+} \;; & T^{(0)+} \equiv F^{+} \;; \\ T^{-} \equiv 2 \sum_{n=0}^{\infty} \lambda^n T^{(n)+*} \;; & T^{(0)+} \equiv F^{-} \;; \end{array}\right\}$$

$$\boxed{T^{(n)+}_{a} = t^{(8)}_{abcd} \sum_{p,q,r=0} \delta_{p+q+r+1,n}\, \partial_\alpha \left[\partial^\alpha T^{(p)-b}\, \partial_\beta T^{(q)+c}\, \partial^\beta T^{(r)-d} \right]}$$

The coefficient of order n depends on those of order $< n$

$\Rightarrow$ It can be solved **RECURSIVELY.**

$$T_a^{(0)+} = F_a{}^+ ;$$

$$T_a^{(1)+} = t^{(8)}{}_{abcd}\, \partial_\alpha \left[\partial^\alpha T^{(0)-b}\, \partial_\beta T^{(0)+c}\, \partial^\beta T^{(0)-d} \right]$$

$$= t^{(8)}{}_{abcd}\, \partial_\alpha \left[\partial^\alpha F^{-b}\, \partial_\beta F^{+c}\, \partial^\beta F^{-d} \right] ;$$

$$T_a^{(2)+} = t^{(8)}{}_{abcd}\, \partial_\alpha \Big\{ \partial^\alpha T^{(0)-b}\, \partial_\beta T^{(0)+c}\, \partial^\beta T^{(1)-d} + \partial^\alpha T^{(0)-b}\, \partial_\beta T^{(1)+c}\, \partial^\beta T^{(0)-d} + \partial^\alpha T^{(1)-b}\, \partial_\beta T^{(0)+c}\, \partial^\beta T^{(0)-d} \Big\}$$

$$= \dots ;$$

Defining the expansion $S = \int d^4x \left\{ -\frac{1}{4} F^2 \right\} + 2 \sum_{n=1}^{\infty} \lambda^n S^{(n)}$,

$$S^{(n)} = -\frac{1}{4(n+1)} \int d^4x \left\{ F^+ T^{(n)+} + \text{c.c.} \right\} ;$$

$$S^{(1)} = \int d^4x\, \frac{1}{4}\, t^{(8)}{}_{abcd}\, \partial_\alpha F^{+a}\, \partial^\alpha F^{-b}\, \partial_\beta F^{+c}\, \partial^\beta F^{-d} ; \quad \mathcal{O}(\partial^4 F^4)$$

$$S^{(2)} = \int d^4x \left\{ -\frac{1}{4} T^{(1)+}{}_a T^{(1)+a} + \text{c.c.} \right\} ; \quad \mathcal{O}(\partial^8 F^6) \quad \text{etc.}$$

Model B is solved in the same way $(+-+- \longrightarrow ++--)$

The linear combination of models can be solved in the same way.

One should (or just may) add the source corresponding to the Born-Infeld corrections:

$$S_{BI} = \frac{1}{\lambda}\int d^4x \left\{ 1 - \sqrt{1 + \frac{1}{2}\lambda F^2 - \frac{\lambda^2}{16}(F\cdot\tilde{F})^2} \right\}$$

$$\sim \frac{1}{\lambda}\int d^4x \left\{ 1 - \left\{ 1 + \frac{1}{4}\lambda F^2 - \frac{\lambda^2}{32}\left[(F^2)^2 + (F\cdot\tilde{F})^2\right]\right\} + \mathcal{O}(\lambda^3) \right\}$$

$$\sim \int d^4x \left\{ -\frac{1}{4}F^2 + \frac{\lambda}{32}\underbrace{\left[(F^2)^2 + (F\cdot\tilde{F})^2\right]}_{4(F^+)^2(F^-)^2} + \mathcal{O}(\lambda^2) \right\}$$

We need $I^{(1)}_{BI}(T^-, T^{*+})\Big|_{T^+=0} = \frac{\lambda}{2^3}(F^+)^2(F^-)^2$,

$$\Downarrow$$

$$I^{(1)}_{BI}(T^-, T^{*+}) = \frac{\lambda}{2^7}(T^{*+})^2(T^-)^2,$$

We can also use the $t^{(8)}$ tensor to linearise the expression:

$$I^{(1)}_{BI}(T^-, T^{*+}) = \frac{\lambda}{2^8 \cdot 4!}\, t^{(8)}_{abcd}\, T^{-a}\, T^{*+b}\, T^{-c}\, T^{*d},$$

and proceed as before

$$\boxed{\text{THE PROCEDURE DOES NOT GIVE BORN-INFELD!}}$$

(Carrasco, Kallosh, Roiban)

Born-Infeld is not the only self-dual completion of Maxwell (Gibbons & Rasheed 1995)
It is not the only one with the given $\mathcal{O}(\lambda)$ term:

$$I^{(1)}_{BI}(T^{-},T^{*+}) \longrightarrow \sum_{n=1}^{\infty} f_n \left[I^{(1)}_{BI}(T^{-},T^{*+}) \right]^{n}$$

$$f_1 = 1$$

DOES THE SAME JOB TO ORDER λ

The standard Born-Infeld theory is recovered by this method with

$$\boxed{I^{(1)}(T^{-},T^{*+}) = \frac{n}{\lambda}\left\{1 - {}_3F_2\left(-\tfrac{1}{2}, -\tfrac{1}{4}, \tfrac{1}{4}; \tfrac{1}{3}, \tfrac{2}{3}; -\tfrac{\lambda}{27}(T^{*+})^2 (T^{-})^2\right)\right\}}$$

(CKR)

In our case we can combine $I^{(1)}_A$, $I^{(1)}_B$, $I^{(1)}_{BI}$ in infinitely many ways with the same $\mathcal{O}(\lambda)$ term

$$\boxed{\text{Is there a master source of deformation that codifies all the open superstring corrections? (or any given selfdual model?)}}$$

Is there a generalisation for non-Abelian duality groups?

Can we construct $E_{7(7)}$-invariant counterterms for $N=8$ SUGRA?

(The $\partial^4 F^4$ terms computed by Freedman & Tonni (2012) coincide with those of the open superstring)

THANKS!

APPLICATION TO N=8 SUGRA

(Kallosh & O., to appear)

i

Can we reproduce the series of quantum corrections by this method?

1- We need $E_{7(7)}$-invariant sources of deformation $I^{(1)}$ for unconstrained doublets $\begin{pmatrix} F \\ G \end{pmatrix}$ combined (or not) into graviphoton field strengths

$$T_{IJ} \equiv \left\langle V_{IJ} \middle| \begin{pmatrix} F^{ij} \\ G_{ij} \end{pmatrix} \right\rangle$$

2- $I^{(1)}$ should reproduce the known corrections when we impose the linear, twisted, selfduality constraint

$$G_{ij}^{+} = \frac{1}{2}\bar{\mathcal{N}}_{ij\,kl}\, F^{kl+} \iff T^{IJ+} = \left(T_{IJ}^{-}\right)^{*} = 0 .$$

It can be argued that treating F and G as independent fields is incompatible with SUSY in $N=8$.

- We can, nevertheless, construct bosonic $E_{7(7)}$ invariants but it can be shown that they cannot reproduce the known corrections which are incompatible with full $E_{7(7)}$ invariance

(They are invariant in the limit of vanishing scalars only.)

The same is true in pure $N=4$ SUGRA

$\boxed{\text{IS THIS RELATED TO THE ABSENCE OF UV DIVERGENCES?}}$

$E_{7(7)}$ INVARIANTS FOR AMPLITUDES

We need invariants for 4 different fundamentals $\mathcal{F}(\mathbf{56})$

$$\begin{pmatrix} F^{ij} \\ G_{ij} \end{pmatrix} \longrightarrow \begin{pmatrix} F^{\Lambda} \\ G_{\Lambda} \end{pmatrix} \equiv \mathcal{F}^{M}$$

(symmetric in the 4)

They can be constructed with the "$\mathbb{K}$-tensor"

$$J_{(8)}(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4) \equiv \mathbb{K}_{MNPQ}\, \mathcal{F}_1^{M} \mathcal{F}_2^{N} \mathcal{F}_3^{P} \mathcal{F}_4^{Q}$$

(8 Lorentz indices)

(For charges $\mathbb{K}_{MNPQ}\, Q^M Q^N Q^P Q^Q = J_4(Q)$, Cartan)

We need to express it in the complex basis

$$\mathcal{F}^{AB} \equiv \frac{1}{2\sqrt{2}} \left(F^{ij} - i\, G_{ij}\right) \Gamma^{ij}{}_{AB} \longleftarrow SO(8)\ \gamma\text{-matrices}$$

$\diamondsuit_{(8)}(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4)$ is the "linearization" of $\diamondsuit$ (Cremmer-Julia)

$$x^2 \longrightarrow \frac{1}{2}(x^1 x^2 + x^2 x^1) \quad \ldots$$

$$\diamondsuit_{(8)}(\mathcal{F}_1,\mathcal{F}_2,\mathcal{F}_3,\mathcal{F}_4) = \frac{1}{6}\mathrm{Tr}_{SU(8)}\Big\{\overline{\mathcal{F}}_1\cdot\mathcal{F}_2\cdot\overline{\mathcal{F}}_3\cdot\mathcal{F}_4 + \overline{\mathcal{F}}_1\cdot\mathcal{F}_3\cdot\overline{\mathcal{F}}_4\cdot\mathcal{F}_2 + \overline{\mathcal{F}}_1\cdot\mathcal{F}_4\cdot\overline{\mathcal{F}}_2\cdot\mathcal{F}_3 + \text{c.c.}\Big\}$$

$$-\frac{1}{12}\Big\{[\mathcal{F}_1|\mathcal{F}_2][\mathcal{F}_3|\mathcal{F}_4] + [\mathcal{F}_1|\mathcal{F}_3][\mathcal{F}_2|\mathcal{F}_4] + [\mathcal{F}_1|\mathcal{F}_4][\mathcal{F}_2|\mathcal{F}_3]\Big\}$$

$$+\frac{1}{4}\Big[\mathrm{Pf}\,\|\mathcal{F}_1\mathcal{F}_2\mathcal{F}_3\mathcal{F}_4\| + \text{c.c.}\Big].$$

$$[\mathcal{F}_1|\mathcal{F}_2] \equiv -\frac{1}{2}\mathrm{Tr}_{SU(8)}\{\overline{\mathcal{F}}_1\cdot\mathcal{F}_2\} + \text{c.c.}$$

To construct a Lorentz scalar we use $t^{(8)}$:

$$\boxed{t^{(8)}\cdot\diamondsuit_{(8)}(\mathcal{F}_1,\mathcal{F}_2,\mathcal{F}_3,\mathcal{F}_4)\ f(s,t,u)}$$