The tensor hierarchy of N=1,d=4 gauged supergravity

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Talk given on the 25th of March 2009 at the *Institute of Theoretical Physics, ETH, Zürich*

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Work done in collaboration with *E. Bergshoeff, O. Hohm* (U. Groningen) *J. Hartong* (U. Bern) and *M. Hübscher* (IFT UAM/CSIC, Madrid)

Plan of the Talk:

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3. The *embedding tensor method* (Cordaro, Fré, Gualtieri,Termonia & Trigiante, arXiv:hep-th/9804056.) can be used to construct systematically the most general gauged supergravities . This construction requires the introduction of additional $(p+1)$-form potentials.

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We are going to use the embedding tensor method to find all the $(p+1)$-form potentials and the corresponding democratic formulations of any 4-dimensional field theory with gauge symmetry and we are going to apply the general results to the particular case of $N=1$ supergravity .

What we are going to do in this seminar:

[a]So far, only maximal and half-maximal supergravities have been studied from this point of view de Wit, Samtleben & Trigiante, arXiv:hep-th/0412173, Samtleben & Weidner arXiv:hep-th/0506237, Schon & Weidner, arXiv:hep-th/0602024, de Wit, Samtleben & Trigiante, arXiv:0705.2101, Bergshoeff, Gomis, Nutma & Roest, arXiv:0711.2035, de Wit, Nicolai & Samtleben, arXiv:0801.1294. The only exception is de Vroome & de Wit arXiv:0707.2717, but the $U(2)$ R-symmetry group has not been properly taken into account.

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Next, we will extend the formalism to electric and magnetic gaugings of general (perturbative and non-perturbative) symmetries. We will find the need to introduce higher-rank form potentials defining a structure called *tensor hierarchy* de Wit & Samtleben, arXiv:hep-th/0501243, de Wit, Samtleben & Trigiante, arXiv:hep-th/0507289.

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Finally, we are going to apply the general results to the particular case of $N = 1$ supergravity taking special care of the existence of $U(1)_R$ symmetry and a superpotential [a].

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2 – The embedding tensor method: electric gaugings

Consider a general ($N = 1$ supergravity -inspired) 4-dimensional ungauged theory with bosonic fields $\{Z^i, A^\Lambda\}$ (the metric plays no relevant role here)

$$S_{\rm u}[Z^i, A^\Lambda] = \int \{-2\mathcal{G}_{ij^*} dZ^i \wedge \star dZ^{*\,j^*} - 2\Im{\rm m} f_{\Lambda\Sigma} F^\Lambda \wedge \star F^\Sigma + 2\Re{\rm e} f_{\Lambda\Sigma} F^\Lambda \wedge F^\Sigma - \star V_{\rm u}(Z, Z^*)\}\,.$$

with $F^\Lambda \equiv dA^\Lambda$, the fundamental (electric) field strengths and $f_{\Lambda\Sigma}(Z)$.

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Let us assume this action is invariant under the global transformations

$$\begin{aligned}
\delta_\alpha Z^i &= \alpha^A k_A{}^i(Z)\,,\\
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Each embedding tensor $\vartheta_\Lambda{}^A$ defines a possible identification:

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Now we construct derivatives $\mathfrak{D}$

$$\mathfrak{D} Z^i \equiv dZ^i + A^\Lambda \vartheta_\Lambda{}^A k_A{}^i \,,$$

covariant under

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This only works if $\vartheta_{\Lambda}{}^{A}$ is an invariant tensor

$$\delta_{\Lambda}\vartheta_{\Sigma}{}^{A} = -\Lambda^{\Omega} Q_{\Omega\Sigma}{}^{A} = 0\,, \qquad Q_{\Sigma\Lambda}{}^{A} \equiv \vartheta_{\Sigma}{}^{B} T_{B\,\Lambda}{}^{\Omega}\vartheta_{\Omega}{}^{A} - \vartheta_{\Sigma}{}^{B}\vartheta_{\Lambda}{}^{C} f_{BC}{}^{A}\,.$$

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$$[T_{A}, T_{B}] = -f_{AB}{}^{C}\,, \;\Rightarrow\; [X_{\Sigma}, X_{\Lambda}] = -X_{\Sigma\Lambda}{}^{\Omega}X_{\Omega}\,,$$

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Then we construct the covariant 2-form field strengths

$$F^\Lambda = dA^\Lambda + \tfrac{1}{2} {X_{\Sigma\Omega}}^\Lambda A^\Sigma \wedge A^\Omega\,,$$

and the gauge -invariant action of the electrically gauged theory takes the form

$$S_{\rm eg}[Z^i, A^\Lambda] = \int \{-2\mathcal{G}_{ij^*}\mathfrak{D}Z^i \wedge \star \mathfrak{D}Z^{*\,j^*} - 2\Im\mathrm{m} f_{\Lambda\Sigma} F^\Lambda \wedge \star F^\Sigma + 2\Re\mathrm{e} f_{\Lambda\Sigma} F^\Lambda \wedge F^\Sigma - \star V_{\rm eg}(Z, Z^*)\}$$

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- The theory (equations of motion) has more non-perturbative global symmetries that can be gauged . They include electric -magnetic duality rotations:

$$\begin{aligned}
\delta_\alpha Z^i &= \alpha^A k_A{}^i(Z)\,,\\
\delta_\alpha f_{\Lambda\Sigma} &= \alpha^A\{-T_{A\,\Lambda\Sigma} + 2T_{A\,(\Lambda}{}^{\Omega} f_{\Sigma)\Omega} - T_A{}^{\Omega\Gamma} f_{\Omega\Lambda} f_{\Gamma\Sigma}\}\,,\\
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Knowing (Gaillard & Zumino) that the T_A matrices either belong to $\mathfrak{sp}(2n_V,\mathbb{R})$ or vanish, we introduce the symplectic notation

$$A^M \equiv \begin{pmatrix} A^\Sigma \\ A_\Sigma \end{pmatrix} \qquad \vartheta_M{}^A \equiv (\vartheta_\Sigma{}^A, \vartheta^{\Sigma\, A}) \qquad \alpha^A(x) \equiv \Lambda^M \vartheta_M{}^A\,,$$

$$(T_{A\, M}{}^N) \equiv \begin{pmatrix} T_{A\,\Sigma}{}^\Lambda & T_A{}^{\Sigma\Lambda} \\ T_{A\,\Sigma\Lambda} & T_A{}^\Sigma{}_\Lambda \end{pmatrix}\,.$$

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The electric and magnetic charges must by mutually local (de Wit, Samtleben & Trigiante, arXiv:`hep-th/0507289`):

$$Q^{AB} \equiv \tfrac{1}{4}\vartheta^{M\, A}\vartheta_M{}^B = 0\,.$$

Now we can repeat the procedure of the electric case:

First we construct derivatives $\mathfrak{D}$

$$\mathfrak{D}Z^{i} \equiv dZ^{i} + A^{M}\vartheta_{M}{}^{A}k_{A}{}^{i}\,,$$

covariant under

$$\delta_{\Lambda}Z^{i} \;=\; \Lambda^{M}\vartheta_{M}{}^{A}k_{A}{}^{i}(Z)\,,$$

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which only works if $\vartheta_{M}{}^{A}$ is an invariant tensor

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Before moving forward, we must impose another constraint on the embedding tensor on top of the two quadratic ones $Q_{MN}{}^A = Q^{AB} = 0$:

$$L_{MNP} \equiv X_{(MNP)} = \vartheta_{(M}{}^A T_{A\,NP)} = 0 \,.$$

This *linear* or *representation constraint* is based on supergravity and eliminates certain possible representations of the embedding tensor . On the other hand, we cannot construct gauge -covariant 2-form field strengths F^M without it!

4 – The 4-d tensor hierarchy

To construct the gauge -covariant 2-form field strengths F^{M} we take the covariant derivative of the gauge -covariant "field strength" $\mathcal{D}Z^{i}$:

$$\mathfrak{D}\mathfrak{D}Z^{i} = [dA^{M} + \tfrac{1}{2}X_{NP}{}^{M}A^{N}\wedge A^{P}]\vartheta_{M}{}^{A}k_{A}{}^{i}\,,$$

which suggests the definition

$$F^{M} \equiv dA^{M} + \tfrac{1}{2}X_{NP}{}^{M}A^{N}\wedge A^{P} + \Delta F^{M}\,, \qquad \vartheta_{M}{}^{A}\Delta F^{M} = 0\,,$$

so we have the Bianchi identity

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$\delta_\Lambda B_A$ is determined by the gauge -covariance of F^M <u>plus</u> $\delta B_A \sim d\Lambda_A$.

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$$F^M \equiv dA^M + \tfrac{1}{2}X_{NP}{}^M A^N \wedge A^P + \Delta F^M \,, \qquad \vartheta_M{}^A \Delta F^M = 0 \,,$$

so we have the Bianchi identity

$$\boxed{\mathfrak{D}\mathfrak{D}Z^i = F^M \vartheta_M{}^A k_A{}^i \;.}$$

Using the constraint $Q^{AB} \equiv \frac{1}{4}\vartheta^{MA}\vartheta_M{}^B = 0$ the natural solution is

$$\Delta F^M = -\tfrac{1}{2}\vartheta^{MA}B_A \equiv Z^{MA}B_A \,.$$

$\delta_\Lambda B_A$ is determined by the gauge -covariance of F^M plus $\delta B_A \sim d\Lambda_A$.

But we do not need it to move forward.

If we take the covariant derivative of the gauge -covariant 2-form field strength F^{M} we find

$$\mathfrak{D}F^{M} = Z^{MA}\{\mathfrak{D}B_{A} + T_{A\,RS}A^{R} \wedge [dA^{S} + \tfrac{1}{3}X_{NP}{}^{S}A^{N} \wedge A^{P}]\}\,.$$

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$$H_{A} = \mathfrak{D}B_{A} + T_{A\,RS}A^{R}\wedge[dA^{S} + \tfrac{1}{3}X_{NP}{}^{S}A^{N}\wedge A^{P}] + \Delta H_{A}\,, \qquad \text{where} \qquad Z^{MA}\Delta H_{A} = 0\,.$$

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$$Q_{MN}{}^A = \vartheta_M{}^B(T_{B\,N}{}^P\vartheta_P{}^A - \vartheta_N{}^C f_{BC}{}^A) \equiv 2Z_M{}^A Y_{AN}{}^P = 0$$

the natural solution for $Z^{MA}\Delta H_A = Z^{MA}\Delta B_A = 0$ is

$$\Delta H_A \equiv Y_{AM}{}^C C_C{}^M\,.$$

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$$G_C{}^M = \mathfrak{D}C_C{}^M + F^M \wedge B_C + \cdots + \Delta G_C{}^M\,, \qquad \text{where} \qquad Y_{AM}{}^C \Delta G_C{}^M = 0\,.$$

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To determine $\Delta G_C{}^M$ we need to find invariant tensors that vanish upon contraction with $Y_{AM}{}^C$. They appear automatically when we take the gauge -covariant derivative of the Bianchi identity and $G_C{}^M$ (if we “forget” we are in 4 dimensions!).

Acting with $\mathfrak{D}$ on the Bianchi identity of H_A we find

$$Y_{AM}{}^{C}\{\mathfrak{D}G_{C}{}^{M} - F^{M} \wedge H_{A}\} = 0\,, \;\; \Rightarrow \;\; \mathfrak{D}G_{C}{}^{M} = F^{M} \wedge H_{A} + \Delta\mathfrak{D}G_{C}{}^{M}\,,$$

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$$\mathfrak{D}\Delta\mathfrak{D}G_C{}^M = W_C{}^{MAB} H_A \wedge H_B + W_{CNPQ}{}^M F^N \wedge F^P \wedge F^Q + W_{CNP}{}^{EM} F^N \wedge G_E{}^P\,.$$

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This implies that there are 3 such tensors $W_C{}^{MAB}, W_{CNPQ}{}^{M}, W_{CNP}{}^{EM}$ that vanish contracted with $Y_{AM}{}^{C}$ and which we can use to build $\Delta G_C{}^{M}$.

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This implies that there are 3 such tensors $W_C{}^{MAB}, W_{CNPQ}{}^{M}, W_{CNP}{}^{EM}$ that vanish contracted with $Y_{AM}{}^{C}$ and which we can use to build $\Delta G_C{}^{M}$.
The natural solution is

$$\Delta G_C{}^{M} = W_C{}^{MAB}D_{AB} + W_{CNPQ}{}^{M}D^{NPQ} + W_{CNP}{}^{EM}D_E{}^{NP}\,,$$

and $\delta_\Lambda D_{AB}, \delta_\Lambda D^{NPQ}, \delta_\Lambda D_E{}^{NP}$ will follow from the gauge -covariance of $G_C{}^{M}$.

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\delta_\Lambda A^M &= -\mathfrak{D}\Lambda^M - Z^{MA}\Lambda_A\,,\\
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But, what does it mean?
What is the meaning of the additional fields?

5 – The meaning of the $d = 4$ tensor hierarchy

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- ☞ These two duality relations together with the Bianchi identity $\mathfrak{D}F^M = Z^{MA} H_A$ give a set of electric -magnetic duality -covariant Maxwell equations:

$$\mathfrak{D}F^\Lambda = -\tfrac{1}{4}\vartheta_\Lambda{}^A \star j_A \ , \qquad \mathfrak{D}G_\Lambda = \tfrac{1}{4}\vartheta^{\Lambda\, A} \star j_A \ .$$

☞ The 3-forms $C_C{}^M$ must be dual to constants: the embedding tensor $\vartheta_M{}^C$. This duality is expressed through the formula

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This equation is similar to the consistency condition (gauge or Noether identity) that Noether currents must satisfy off-shell in theories with gauge invariance:

$$\mathfrak{D} \star j_A = -2(k_A{}^i \mathcal{E}_i + \text{c.c.}) + 4T_{A\,MN} G^M \wedge G^N + \star Y_A{}^{MC} \frac{\partial V}{\partial \vartheta_M{}^C} \,,$$

where $\mathcal{E}_i$ is the e.o.m. of Z^i. Both equations, together, imply

$$k_A{}^i \mathcal{E}_i + \text{c.c.} = 0 \quad ,$$

which is equivalent to the scalar e.o.m. for symmetric σ-models.

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To show that this interpretation is right, we must construct a gauge -invariant action for these fields, including the embedding tensor .

The gauge -invariant action is

$$
\begin{aligned}
S \;=\; & \int \{-2\mathcal{G}_{ij^*}\mathfrak{D}Z^i \wedge \star\mathfrak{D}Z^{*\,j^*} + 2F^{\Sigma}\wedge G_{\Sigma} - \star V \\
& -4Z^{\Sigma A}B_A \wedge \left(F_{\Sigma} - \tfrac{1}{2}Z_{\Sigma}{}^{B}B_B\right) - \tfrac{4}{3}X_{[MN]\Sigma}A^M \wedge A^N \wedge \left(F^{\Sigma} - Z^{\Sigma B}B_B\right) \\
& -\tfrac{2}{3}X_{[MN]}{}^{\Sigma}A^M \wedge A^N \wedge \left(dA_{\Sigma} - \tfrac{1}{4}X_{[PQ]\Sigma}A^P \wedge A^Q\right) \\
& -2\mathfrak{D}\vartheta_M{}^A \wedge (C_A{}^M + A^M \wedge B_A) \\
& +2Q_{NP}{}^{E}(D_E{}^{NP} - \tfrac{1}{2}A^N \wedge A^P \wedge B_E) + 2Q^{AB}D_{AB} + 2L_{MNP}D^{MNP}\}\,,
\end{aligned}
$$

And the e.o.m. in full glory are....

$$\tfrac{1}{2}\frac{\delta S}{\delta Z^i} = \mathcal{G}_{ij^*}\mathfrak{D}\star\mathfrak{D}Z^{*\,j^*} - \partial_i G_M{}^+ \wedge G^{M+} - \star\tfrac{1}{2}\partial_i V\,,$$

$$\begin{aligned}-\tfrac{1}{4}\star\frac{\delta S}{\delta A^M} =\; & \mathfrak{D}F_M - \tfrac{1}{4}\vartheta_M{}^A \star j_A - \tfrac{1}{3}dX_{[PQ]M}\wedge A^P\wedge A^Q - \tfrac{1}{2}Q_{(NM)}{}^E A^N\wedge B_E \\ & -L_{MNP}A^N\wedge\left(dA^P + \tfrac{3}{8}X_{[RS]}{}^P A^R\wedge A^S\right) + \tfrac{1}{8}Q_{NP}{}^E T_{E\,QM}A^N\wedge A^P\wedge A^Q \\ & -d(F_M - G_M) - X_{[MN]}{}^P A^N\wedge(F_P - G_P) + \tfrac{1}{2}\mathfrak{D}\vartheta_M{}^A\wedge B_A + \tfrac{1}{2}Q_{MP}{}^E C_E{}^P\,,\end{aligned}$$

$$\star\frac{\delta S}{\delta B_A} = \vartheta^{PA}(F_P - G_P) + Q^{AB}B_B - \mathfrak{D}\vartheta_M{}^A\wedge A^M - \tfrac{1}{2}Q_{NP}{}^A A^N\wedge A^P\,,$$

$$\begin{aligned}\tfrac{1}{2}\frac{\delta S}{\delta\vartheta_M{}^A} =\; & (G_A{}^M - \tfrac{1}{2}\star\partial V/\partial\vartheta_M{}^A) - A^M\wedge(H_A + \tfrac{1}{2}\star j_A) \\ & +\tfrac{1}{2}T_{A\,NP}A^M\wedge A^N\wedge(F^P - G^P) - (F^M - G^M)\wedge B_A\,,\end{aligned}$$

$$\frac{\delta S}{\delta D_{AB}} = Q_{AB}\,, \qquad \frac{\delta S}{\delta D_E{}^{NP}} = Q_{NP}{}^E\,, \qquad \frac{\delta S}{\delta D^{MNP}} = L_{MNP}\,.$$

6 – Application: general gaugings of $N = 1, d = 4$ supergravity

Now we want to apply our results to gauge $N = 1 \;\; d = 4$ supergravity with generic matter content and couplings.

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We are going to review ungauged $N = 1$ supergravity and its global symmetries and then we are going to gauge them using the embedding tensor formalism.

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All fermions are represented by chiral 4-component spinors:

$$\gamma_5 \psi_{\mu} = -\psi_{\mu}\,, \text{etc.}$$

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The spinors transform as *sections* of the bundle: under Kähler transformations

$$\delta_\lambda \mathcal{K} = \lambda(Z) + \lambda^*(Z^*)\, , \qquad \delta_\lambda \psi_\mu = -\tfrac{1}{4}[\lambda(Z) - \lambda^*(Z^*)]\psi_\mu \, ,$$

and their covariant derivatives contain the pullback of the Kähler connection 1-form $\mathcal{Q} \equiv \mathcal{Q}_i dZ^i + \mathcal{Q}_{i^*} dZ^{*i^*}$ e.g.

$$\mathcal{D}_\mu \psi_\nu = \{\nabla_\mu + \tfrac{i}{2}\mathcal{Q}_\mu\}\psi_\nu \, .$$

$N = 1$ supergravity allows for a holomorphic but otherwise arbitrary kinetic matrix $f_{\Lambda\Sigma}(Z)$ for the vector fields, so they appear in the action through the term

$$-2\Im\mathrm{m} f_{\Lambda\Sigma} F^{\Lambda} \wedge \star F^{\Sigma} + 2\Re\mathrm{e} f_{\Lambda\Sigma} F^{\Lambda} \wedge F^{\Sigma}\,, \qquad F^{\Lambda} \equiv dA^{\Lambda}\,.$$

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Finally, ungauged $N=1$ supergravity allows for a holomorphic superpotential $\mathcal{W}(Z)$ which appears through the covariantly holomorphic section of Kähler weight $(1,-1)$ $\mathcal{L}(Z,Z^*)$:

$$\mathcal{L}(Z,Z^*) = \mathcal{W}(Z) e^{\mathcal{K}/2} \,, \qquad \mathcal{D}_{i^*}\mathcal{L} = 0 \,,$$

which couples to the fermions in various ways and gives rise to the scalar potential

$$V_{\mathrm{u}}(Z,Z^*) = -24|\mathcal{L}|^2 + 8\mathcal{G}^{ij^*}\mathcal{D}_i\mathcal{L}\mathcal{D}_{j^*}\mathcal{L}^* \,.$$

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The bosonic action is

$$\begin{aligned}S_{\mathrm{u}}[g_{\mu\nu},Z^i,A^{\Lambda}] \;=\; & \int\{\star R-2\mathcal{G}_{ij^*}dZ^i\wedge\star dZ^{*\,j^*}-2\Im\mathrm{m} f_{\Lambda\Sigma}F^{\Lambda}\wedge\star F^{\Sigma}\\ & +2\Re\mathrm{e} f_{\Lambda\Sigma}F^{\Lambda}\wedge F^{\Sigma}-\star V_{\mathrm{u}}(Z,Z^*)\}\,.\end{aligned}$$

To first order in fermions , the supersymmetry transformation rules for the fermions are

$$\begin{aligned}
\delta_\epsilon \psi_\mu &= \mathcal{D}_\mu \epsilon + i\mathcal{L}\gamma_\mu \epsilon^* = \left[\nabla_\mu + \tfrac{i}{2}\mathcal{Q}_\mu\right]\epsilon + i\mathcal{L}\gamma_\mu\epsilon^* \,, \\
\delta_\epsilon \lambda^\Lambda &= \tfrac{1}{2}\not{F}^{\Lambda\,+}\epsilon \,, \\
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Even though this is not the full theory, these expressions (bosonic action and supersymmetry transformations) contain all the information necessary to reconstruct it.

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☞ $U(1)_R$ only acts on the spinors as a multiplication by $e^{-iq\alpha^{\#}}$, where q is the Kähler weight. Then $A = \mathrm{a}, \#$ where the symmetries labeled by a, act on scalars, and/or 1-forms.

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- ☞ At this point we are dealing with a $A = \mathrm{a}$ symmetry and we can say that a non-vanishing superpotential breaks $U(1)_R$ and we cannot gauge it.

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- According to the previous discussion, the symmetries $A = \underline{\mathrm{a}}, \#$ are broken and cannot be gauged if $\mathcal{L} \neq 0$.

$$\mathcal{L} \neq 0\,,\quad \Rightarrow \vartheta_M{}^A(\delta_A{}^{\underline{\mathrm{a}}}\mathcal{P}_{\underline{\mathrm{a}}} + \delta_A{}^{\#}\mathcal{P}_{\#}) = 0\ .$$

9 – The $N = 1, d = 4$ bosonic tensor hierarchy

We have found that, for non-vanishing superpotential , the embedding tensor must satisfy another constraint

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☞ Also the constraint $Y_{AM}{}^C \Delta G_C{}^M = 0$ can be solved in a more general way:

$$\Delta' G_C{}^M = \Delta G_C{}^M + Y_C D^M\,.$$

9 – The $N=1, d=4$ bosonic tensor hierarchy

We have found that, for non-vanishing superpotential , the embedding tensor must satisfy another constraint

$$Q_M \equiv \vartheta_M{}^A(\delta_A{}^{\underline{a}}\mathcal{P}_{\underline{a}} + \delta_A{}^{\#}\mathcal{P}_{\#}) = 0\,,$$

and, therefore, in that case we expect changes in the standard $d=4$ tensor hierarchy which have to be confirmed by checking supersymmetry .

- ☞ Now ($\mathcal{L} \neq 0$) the constraint $Z^{MA}\Delta H_A = 0$ can be solved in a more general form:

$$\Delta' H_A \equiv \Delta H_A + Y_A C\,, \qquad Y_A \equiv (\delta_A{}^{\underline{a}}\mathcal{P}_{\underline{a}} + \delta_A{}^{\#}\mathcal{P}_{\#})\,.$$

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This will happen in $N=1$ supergravity if we find new Stückelberg shifts

$$\delta' B_A \sim \delta_h B_A + Y_A \Lambda \quad \text{and} \quad \delta' C_C{}^M = \delta_h C_C{}^M + Y_C \Lambda^M\,.$$

10 – The $N = 1, d = 4$ supersymmetric tensor hierarchy

As a first step to include the tensor hierarchy fields into $N = 1$ supergravity we are going to construct supersymmetry transformation rules such that the local supersymmetry algebra, to leading order in fermions , closes on the new fields up to duality relations.

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This new construction requires new duality rules for the supersymmetric partners as well.

This construction gives, independently, the gauge transformations of the fields and will confirm or refute the hierarchy's results.

The scalars Z^i

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At leading order in fermions $\delta_\eta \delta_\epsilon Z^i = \frac{1}{4}\overline{(\delta_\eta \chi^i)}\epsilon$, where now

$$\delta_\eta \chi^i = i \not{\mathfrak{D}} Z^i \eta^* + 2\mathcal{G}^{ij^*}\mathcal{D}_{j^*}\mathcal{L}^*\eta\,, \qquad \mathfrak{D}Z^i = dZ^i + A^M \vartheta_M{}^A k_A{}^i\,.$$

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We find the expected result

$$\begin{aligned}
[\delta_\eta\,,\,\delta_\epsilon] Z^i &= \delta_{\text{g.c.t.}} Z^i + \delta_h Z^i\,,\\
\delta_{\text{g.c.t.}} Z^i &= \pounds_\xi Z^i = +\xi^\mu \partial_\mu Z^i\,,\\
\delta_h Z^i &= \Lambda^M \vartheta_M{}^A k_A{}^i\,,\\
\xi^\mu &\equiv \tfrac{i}{4}(\bar{\epsilon}\gamma^\mu \eta^* - \bar{\eta}\gamma^\mu \epsilon^*)\,,\\
\Lambda^M &\equiv \xi^\mu A^M{}_\mu\,.
\end{aligned}$$

The 1-forms A^M

We introduce supersymmetric partners λ_Σ for the magnetic 1-forms A_Σ and make the symplectic -covariant Ansatz

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\begin{aligned}
\delta_\epsilon A^M{}_\mu &= -\tfrac{i}{8}\bar{\epsilon}^*\gamma_\mu\lambda^M + \text{c.c.}\,,\\
\delta_\epsilon \lambda^M &= \tfrac{1}{2}\left[\not{F}^{M+} + i\mathcal{D}^M\right]\epsilon\,,
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where we have defined the symplectic vector

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\mathcal{D}^M \equiv \begin{pmatrix} \mathcal{D}^\Lambda \\ \mathcal{D}_\Lambda \end{pmatrix} \equiv \begin{pmatrix} \mathcal{D}_\Lambda \\ f_{\Lambda\Sigma}\mathcal{D}^\Sigma \end{pmatrix}\,, \qquad \mathcal{D}^\Lambda = -\Im\text{m} f^{\Lambda\Sigma}\,(\vartheta_\Sigma{}^A + f^*_{\Sigma\Omega}\vartheta^{\Omega\,A})\mathcal{P}_A\,.
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but <u>we do not need them</u> to show that

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where

$$\Lambda_A \equiv -T_{A\,MN}A^N\Lambda^M + b_A - \mathcal{P}_A\xi\,, \qquad b_{A\,\mu} \equiv B_{A\,\mu\nu}\xi^\nu\,.$$

The 2-forms B_A

We introduce the supersymmetric partners ζ_A, φ_A (linear supermultiplets)

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\begin{aligned}
\delta_\epsilon \zeta_A &= -i[\tfrac{1}{12}\, \not{H}'_A + \not{\mathfrak{D}} \varphi_A]\epsilon^* - 4\delta_A{}^{\mathbf{a}} \varphi_{\mathbf{a}} \mathcal{L}^* \epsilon\,, \\
\delta_\epsilon B_{A\,\mu\nu} &= \tfrac{1}{4}[\bar{\epsilon}\gamma_{\mu\nu}\zeta_A + \text{c.c.}] - i[\varphi_A \bar{\epsilon}^* \gamma_{[\mu}\psi_{\nu]} - \text{c.c.}] + 2T_{A\,MN} A^M{}_{[\mu}\delta_\epsilon A^N{}_{\nu]}\,, \\
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where now

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but, again, we do not need them to show that

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[\delta_\eta\,,\,\delta_\epsilon]B_A = \delta_{\text{g.c.t.}}B_A + \delta'_h B_A\,,
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which shows that there is indeed an extra Stückelberg shift in B_A.

The 3-forms $C_A{}^M$

In this case we do not introduce any supersymmetric partners. We just make the Ansatz

$$\delta_\epsilon C_A{}^M{}_{\mu\nu\rho} = -\tfrac{i}{8}[\mathcal{P}_A \bar{\epsilon}^* \gamma_{\mu\nu\rho} \lambda^M - \text{c.c.}] - 3B_{A\,[\mu\nu|} \delta_\epsilon A^M{}_{|\rho]} - 2T_{A\,PQ} A^M{}_{[\mu} A^P{}_{\nu|} \delta_\epsilon A^Q{}_{|\rho]}\,.$$

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This corresponds to a scalar potential of the form

$$V_{\text{e-mg}} = V_{\text{u}} - \tfrac{1}{2}\Re\text{e}\,\mathcal{D}^M\vartheta_M{}^A\mathcal{P}_A = V_{\text{u}} + \tfrac{1}{2}\mathcal{M}^{MN}\vartheta_M{}^A\vartheta_N{}^B\mathcal{P}_A\mathcal{P}_B\,,$$

where

$$(\mathcal{M}^{MN}) \equiv \begin{pmatrix} I^{\Lambda\Sigma} & I^{\Lambda\Omega}R_{\Omega\Sigma} \\ R_{\Lambda\Omega}I^{\Omega\Sigma} & I_{\Lambda\Sigma} + R_{\Lambda\Omega}I^{\Omega\Gamma}R_{\Gamma\Sigma} \end{pmatrix}, \qquad \begin{array}{rcl} f_{\Lambda\Sigma} & \equiv & R_{\Lambda\Sigma} + iI_{\Lambda\Sigma}\,, \\ I^{\Lambda\Omega}I_{\Omega\Sigma} & \equiv & \delta^\Lambda{}_\Sigma\,, \end{array}$$

so it is manifestly symplectic -invariant, as it must.

The 3-forms C, C'

The consistency of the previous results requires the existence of a 3-form C transforming under the extra Stückelberg shift of B_A.

$$\delta_\epsilon C_{\mu\nu\rho} = -3i\eta\mathcal{L}\,\bar{\epsilon}^*\gamma_{[\mu\nu}\psi^*{}_{\rho]} - \tfrac{1}{2}\eta\mathcal{D}_i\mathcal{L}\bar{\epsilon}^*\gamma_{\mu\nu\rho}\chi^i + \text{c.c.}\,,$$

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If we rescale the superpotential by $\mathcal{L} \to \eta\mathcal{L}$, the above duality relation takes the standard form

$$G' = \tfrac{1}{2}\star\frac{\partial V_{\text{e-mg}}}{\partial\eta}\,,$$

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So, what is the 3-form C dual to?

The 4-forms $D_{AB}, D^{NPQ}, D_E{}^{NP}, D^M$

The calculations become horribly complicated and we only check the closure of the local supersymmetry algebra in the ungauged $\vartheta_M{}^A = 0$ case when there are no symmetries acting on the 1-forms i.e. $T_{A\,M}{}^N = 0$.

The supersymmetry transformations are

$$\begin{aligned}
\delta_\epsilon D_{AB} &= -\tfrac{i}{2} \star \mathcal{P}_{[A}\partial_i \mathcal{P}_{B]}\bar{\epsilon}\chi^i + \text{c.c.} - B_{[A} \wedge \delta_\epsilon B_{B]}\,, \\
\delta_\epsilon D^{NPQ} &= 10 A^{(N} \wedge F^P \wedge \delta_\epsilon A^{Q)}\,, \\
\delta_\epsilon D_E{}^{NP} &= C_E{}^P \wedge \delta_\epsilon A^N\,. \\
\delta_\epsilon D^M &= -\tfrac{i}{2} \star \mathcal{L}^* \bar{\epsilon}\lambda^M + \text{c.c.} + C \wedge \delta_\epsilon A^M\,.
\end{aligned}$$

This proves that D^M can be consistently added to the supersymmetric theory. Its role in the action will be that of Lagrange multiplier of the constraint Q_M.

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- What happens in higher dimensions? (work in progress)