Extensions

and new supersymmetric solutions

of N=2,d=4 supergravities

Tomás Ortín (I.F.T. UAM/CSIC, Madrid)

Seminar given onDecember 4th 2007 at the University of Torino

Based on hep-th/0601128, hep-th/0602280, hep-th/0611036, hep-th/0612072 arXiv:0711.0857 and on work in preparation.

Work done in collaboration with *E. Bergshoeff, M. de Roo, J. Hartong, S. Kerstan* (U. of Groningen, The Netherlands) *F. Riccioni* (King's College, London, UK), *M. Hübscher, P. Meessen and S. Vaulà* (IFT UAM/CSIC, Madrid, Spain)

Plan of the Talk:

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- ☞ For $p = d-2$ one has to dualize constants (coupling or gauge constants, masses etc.). This has been done only in the simplest cases.
- ☞ For $p = d-1$ there is nothing to be dualized and we have no idea of which $(d-1)$- (*spacetime filling*) branes the theory may contain.

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This program has been carried out for $N = 2A, B, d = 10$ Supergravities in Bergshoeff, de Roo, Kerstan & Riccioni, `hep-th/0506013` and Bergshoeff, de Roo, Kerstan, O. & Riccioni, `hep-th/0602280`. New extensions have been found, all of them fitting in the proposed E_{11} symmetry of M-Theory.

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In this talk I am going to briefly review new (just published) results on extensions of matter-coupled $N = 2, d = 4$ Supergravity theories.

2 – Extensions of $N = 2A, d = 10$ SUGRA

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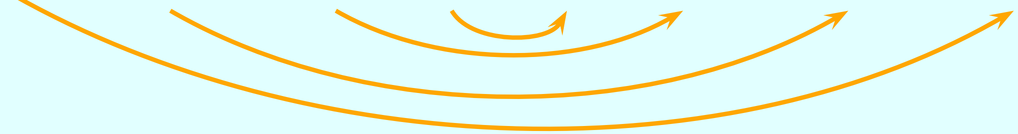

With the supersymmetry transformation (no gravitino in the r.h.s.!)

$$\delta_\epsilon B^{(8)}{}_{\mu_1 \cdots \mu_8} = \tfrac{1}{2} e^{-2\phi} \bar{\epsilon} \Gamma_{\mu_1 \cdots \mu_8} \Gamma_{11} \lambda + (\text{gauge} - \text{field dependent terms}) \ .$$

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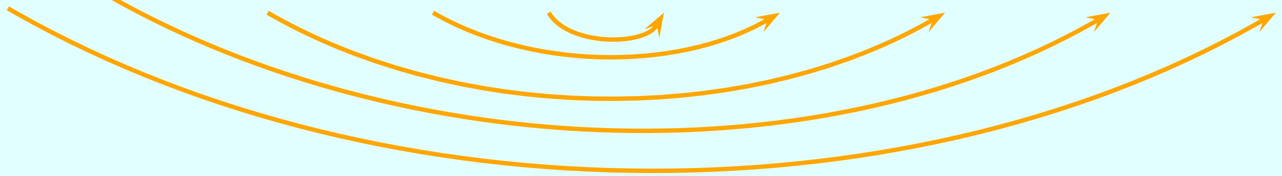

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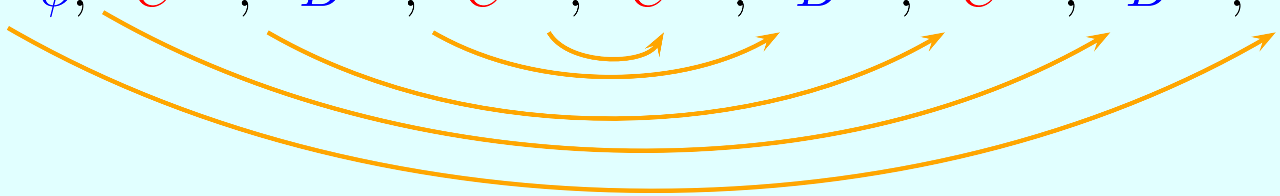

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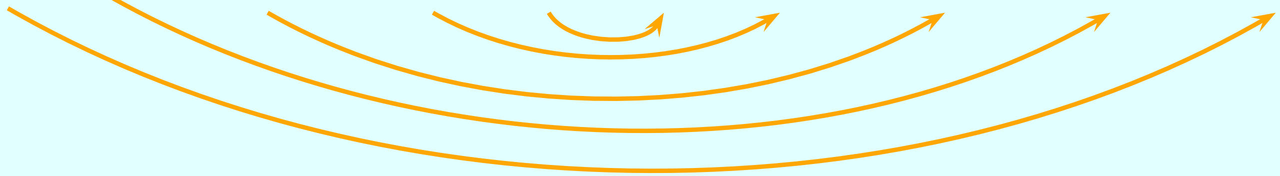

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If they did, they would do it via κ-invariant Lagrangians of the form

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For half-supersymmetric branes, the Lagrangians must be invariant under 16 linearly realized supersymmetries of the form

$$\delta_\epsilon \, g_{\mu\nu} = 2i\bar{\epsilon}\Gamma_{(\mu}\psi_{\nu)} + \text{h.c.}\,, \qquad \delta_\epsilon A^{(p+1)}{}_{\mu_1\cdots\mu_{p+1}} \sim f(\phi)\,\bar{\epsilon}\Gamma_{[\mu_1\cdots\mu_p}\psi_{\mu_{p+1}]}\,.$$

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One finds that

$$\delta_\epsilon\mathcal{L}_{\text{brane}} \sim \big(\tau_{\text{brane}} + f(\phi)\Gamma_{01\cdots p}\big)\epsilon\,,$$

and, thus,

$$\tau_{\text{brane}}(\phi) = f(\phi)\,,$$

and the Lagrangian is invariant under the the 16 independent transformations satisfying the projection

$$\tfrac{1}{2}(1 + \Gamma_{01\cdots p})\epsilon = 0\,.$$

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By simple inspection we conclude that the IIA supersymmetric branes and their tensions are

Potential	Brane	Tension	Projection operator
$C^{(1)}$	D0	$e^{-\phi}$	$\frac{1}{2}(1+\Gamma_0)$
$B^{(2)}$	F1	1	$\frac{1}{2}(1+\Gamma_{01}\Gamma_{11})$
$C^{(3)}$	D2	$e^{-\phi}$	$\frac{1}{2}(1+\Gamma_{012})$
$C^{(5)}$	D4	$e^{-\phi}$	$\frac{1}{2}(1+\Gamma_{01\ldots4}\Gamma_{11})$
$B^{(6)}$	NS5	$e^{-2\phi}$	$\frac{1}{2}(1+\Gamma_{01\ldots5})$
$C^{(7)}$	D6	$e^{-\phi}$	$\frac{1}{2}(1+\Gamma_{01\ldots6})$
$C^{(9)}$	D8	$e^{-\phi}$	$\frac{1}{2}(1+\Gamma_{01\ldots8}\Gamma_{11})$
$\mathcal{D}^{(10)}$	NS9	$e^{-2\phi}$	$\frac{1}{2}(1+\Gamma_{11})$

3 – Extensions of $N = 2B, d = 10$ SUGRA

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The relation with the $SL(2,\mathbb{R})$ fields that have a String Theory interpretation (dilaton,Kalb-Ramond 2-form, Ramond-Ramond forms) has to be found *a posteriori*.

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The following form-fields realizing the local supersymmetry algebra were found:

$$\left\{ U^{\alpha\beta}, A^{(2)\,\alpha}, A^{(4)}, A^{(6)\,\alpha}, A^{(8)\,\alpha\beta}, A^{(10)\,\alpha}, A^{(10)\,\alpha\beta\gamma} \right\} ,$$

$$\alpha, \beta, \gamma = 1, 2, \quad SU(1,1) \text{ indices}$$

The scalars:

$$\{U^{\alpha\beta},$$

$$\delta_\epsilon V_+^\alpha = V_-^\alpha \; \bar{\epsilon}_C \lambda \quad , \quad \delta_\epsilon V_-^\alpha = V_+^\alpha \; \bar{\epsilon} \lambda_C \quad ,$$

$U^{\alpha\beta} = V_+^\alpha, V_-^\alpha$ is an $SU(1,1)$ matrix that parametrizes the $SU(1,1)/U(1)$ coset. It describes two real degrees of freedom:

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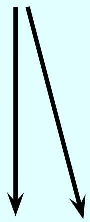

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The precise relation between $U^{\alpha\beta}$, ϕ and ℓ is not unique and amounts to a choice of basis.

Observe that they do not transform into the gravitino and, therefore, cannot couple to dynamical branes (but they can couple to instantons).

The doublet of 2-forms:

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$$\delta_\epsilon A^{(2)\,\alpha}_{\mu\nu} = V^\alpha_- \,\bar\epsilon\Gamma_{\mu\nu}\lambda + V^\alpha_+ \,\bar\epsilon_C\Gamma_{\mu\nu}\lambda_C + 4iV^\alpha_- \,\bar\epsilon_C\Gamma_{[\mu}\psi_{\nu]} + 4iV^\alpha_+ \,\bar\epsilon\Gamma_{[\mu}\psi_{\nu]C}\,.$$

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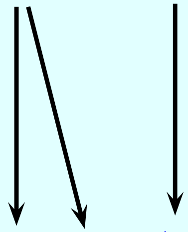

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$A^{(2)\,\alpha}$ is an $SU(1,1)$ doublet that describes two real 2-forms: the NS-NS 2-form which couples to the F1 and the RR 2-form which couples to the D1.

The doublet of 2-forms:

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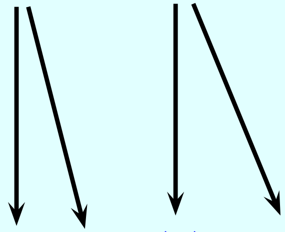

$$\{\phi, \ell, B^{(2)}, C^{(2)},$$

$$\delta_\epsilon A^{(2)\,\alpha}_{\mu\nu} = V_-^\alpha\ \bar\epsilon\Gamma_{\mu\nu}\lambda + V_+^\alpha\ \bar\epsilon_C\Gamma_{\mu\nu}\lambda_C + 4iV_-^\alpha\ \bar\epsilon_C\Gamma_{[\mu}\psi_{\nu]} + 4iV_+^\alpha\ \bar\epsilon\Gamma_{[\mu}\psi_{\nu]C}\,.$$

$A^{(2)\,\alpha}$ is an $SU(1,1)$ doublet that describes two real 2-forms: the NS-NS 2-form which couples to the F1 and the RR 2-form which couples to the D1.

The precise relation between them depends on the same choice of basis.

The 4-form:

$$\{U^{\alpha\beta}, A^{(2)\,\alpha}, \qquad A^{(4)},$$

$$\{\phi, \ell, B^{(2)}, C^{(2)},$$

$$\delta_\epsilon A^{(4)}{}_{\mu\nu\rho\sigma} = \bar{\epsilon}\Gamma_{[\mu\nu\rho}\psi_{\sigma]} - \bar{\epsilon}_C\Gamma_{[\mu\nu\rho}\psi_{\sigma]C} - \frac{3i}{8}\epsilon_{\alpha\beta}A^{(2)\,\alpha}{}_{[\mu\nu}\delta_\epsilon A^{(2)\,\beta}{}_{\rho\sigma]}\,.$$

$A^{(4)}$ is an $SU(1,1)$ singlet.

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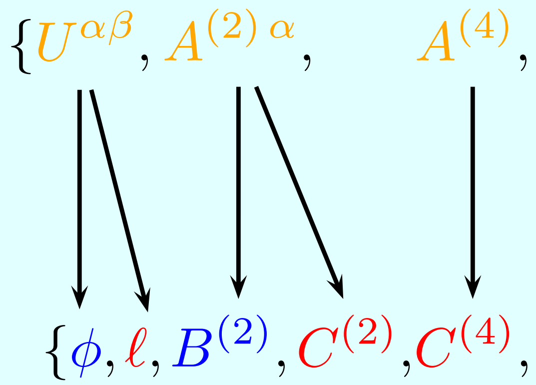

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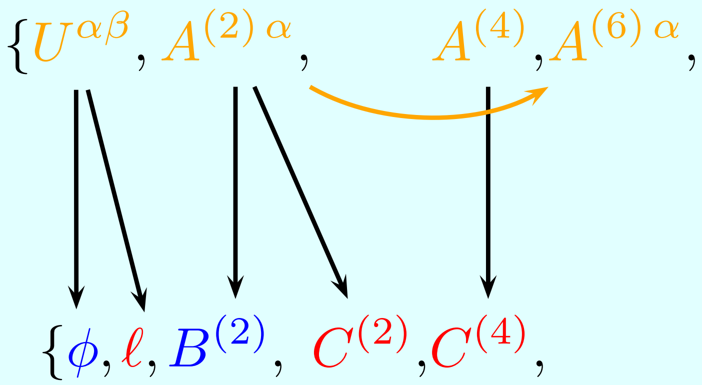
$A^{(4)}$ is an $SU(1,1)$ singlet. It describes the RR 4-form which couples to the D3. The precise relation between them depends on the same choice of basis. It is important to notice that $C^{(4)}$ is not S-duality-invariant.

The doublet of 6-forms:



$$
\begin{aligned}
\delta_\epsilon A^{(6)\,\alpha}{}_{\mu_1\ldots\mu_6} \quad = \quad & iV_-^\alpha \bar\epsilon \Gamma_{\mu_1\ldots\mu_6}\lambda - iV_+^\alpha \bar\epsilon_C \Gamma_{\mu_1\ldots\mu_6}\lambda_C \\
& +12\left(V_-^\alpha \bar\epsilon_C \Gamma_{[\mu_1\ldots\mu_5}\psi_{\mu_6]} - V_+^\alpha \bar\epsilon \Gamma_{[\mu_1\ldots\mu_5}\psi_{C\,\mu_6]}\right) \\
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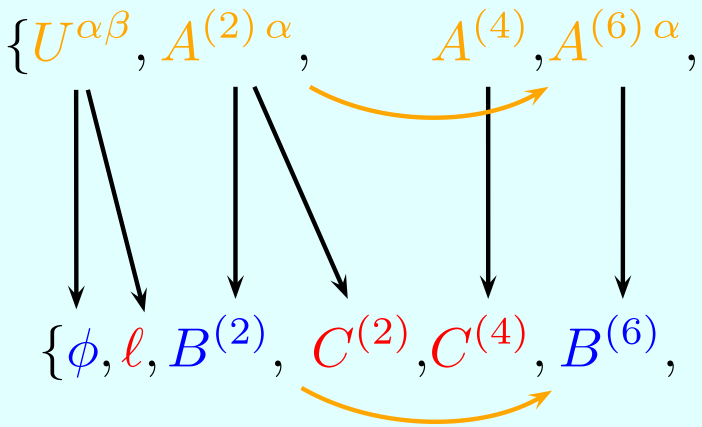

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$A^{(6)\,\alpha}$ is an $SU(1,1)$ doublet that can be obtained by Hodge-dualizing $A^{(2)\,\alpha}$. It describes the NS-NS 6-form dual to the Kalb-Ramond 2-form which couples to the solitonic 5-brane

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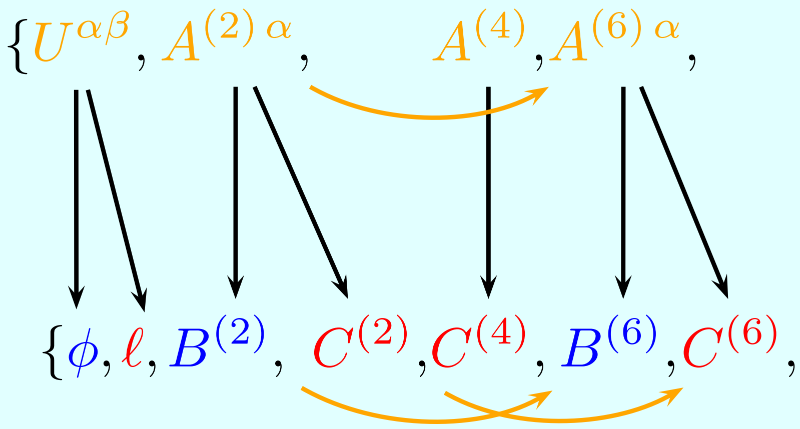

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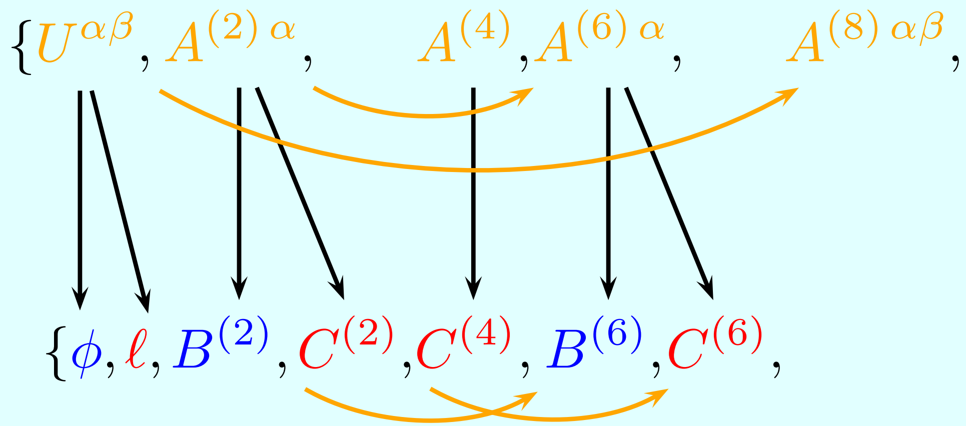
The triplet of 8-forms:



$$
\begin{aligned}
\delta_\epsilon A^{(8)\,\alpha\beta}{}_{\mu_1\ldots\mu_8} \quad = \quad & +iV_-^{(\alpha}V_-^{\beta)}\bar{\epsilon}_C\Gamma_{\mu_1\ldots\mu_8}\lambda - iV_+^{(\alpha}V_+^{\beta)}\bar{\epsilon}\Gamma_{\mu_1\ldots\mu_8}\lambda_C \\
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$A^{(8)\,\alpha\beta}$ is an $SU(1,1)$ triplet that can be obtained by Hodge-dualizing the 3 Noether currents associated to the global $SU(1,1)$ invariance (Meessen & O. `hep-th/9806120`, Dall'Agata Lechner & Tonin `hep-th/9806140`) .

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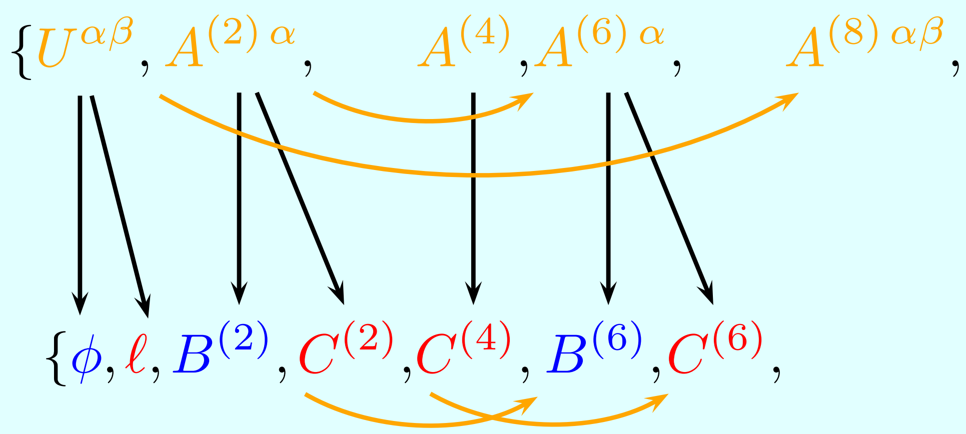

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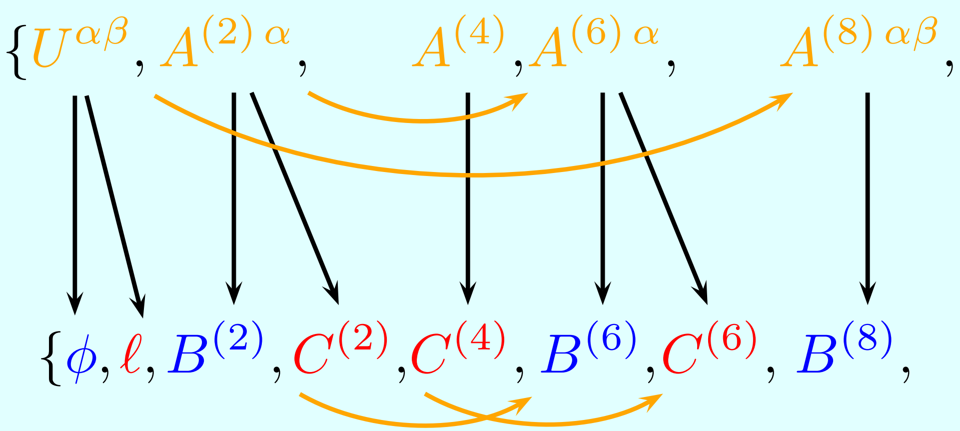

$$\{U^{\alpha\beta}, A^{(2)\,\alpha}, \quad A^{(4)}, A^{(6)\,\alpha}, \quad A^{(8)\,\alpha\beta},$$

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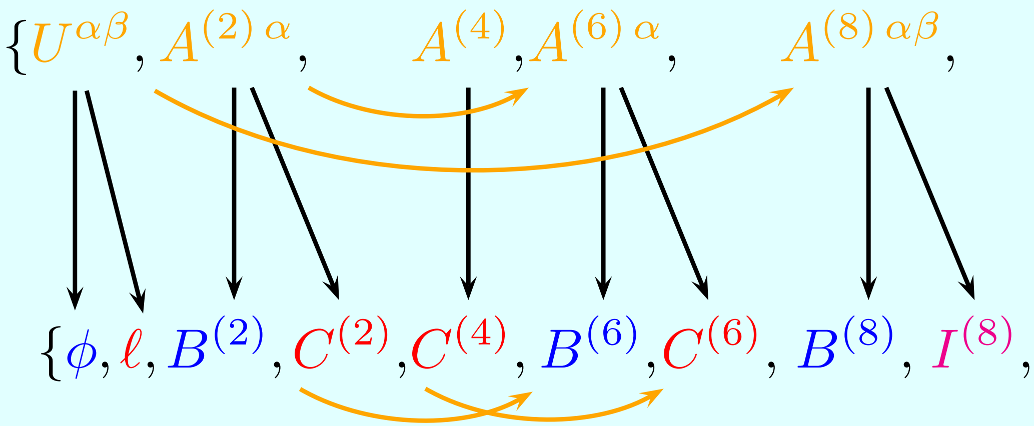

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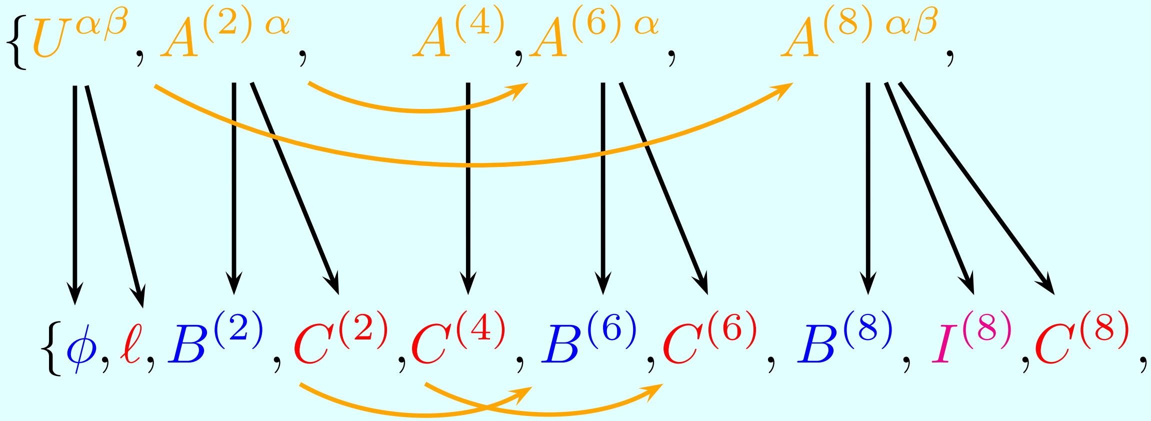

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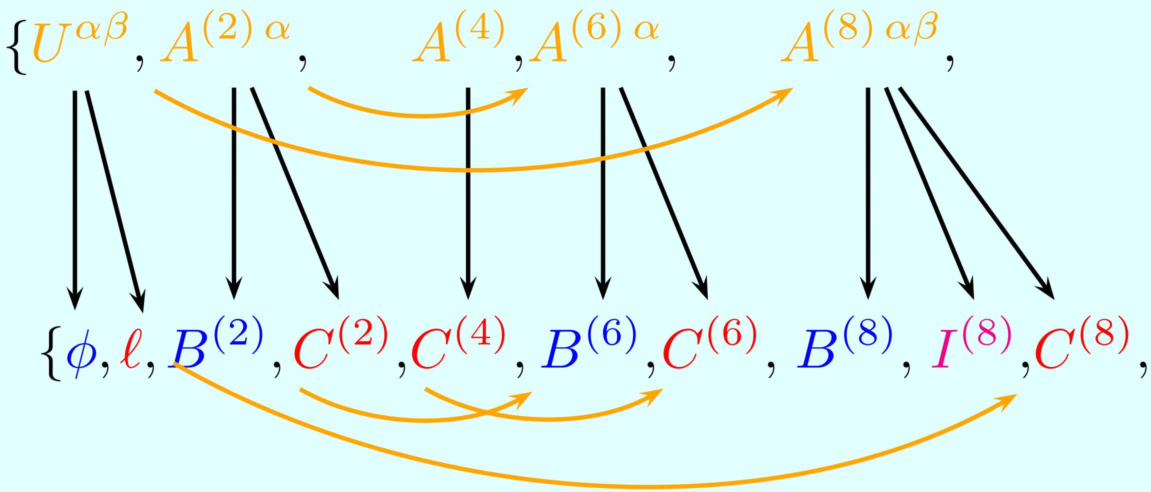

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How many 7-branes are there?

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A 7-brane is characterized by the 3 charges p, r, q that weight its coupling to each of the 3 8-form potentials. The leading terms of its worldvolume action are:

$$\mathcal{L}_{(p,r,q)} \sim \tau_{(p,r,q)} \sqrt{-g} + \epsilon^{\mu_1 \cdots \mu_8} \left(p \, C^{(8)}{}_{\mu_1 \cdots \mu_8} + r D^{(8)}{}_{\mu_1 \cdots \mu_8} + q B^{(8)}{}_{\mu_1 \cdots \mu_8} \right) .$$

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$$\tau_{(p,r,q)} = |p\,e^{-\phi} + r\,\ell e^{-\phi} + q\,(e^{-3\phi} + \ell^2 e^{-\phi})|\,.$$

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The tension (as the Lagrangian) also has manifestly $SL(2, \mathbb{R})$-invariant form in the Einstein frame:

$$\tau^E{}_{(p,r,Q)} = Q_{\alpha\beta} \mathcal{M}^{\alpha\beta} , \qquad Q_{\alpha\beta} = \begin{pmatrix} q & r/2 \\ r/2 & p \end{pmatrix} , \qquad \mathcal{M} = e^{\phi} \begin{pmatrix} \ell^2 + e^{-2\phi} & \ell \\ \ell & 1 \end{pmatrix} .$$

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$\det Q = pq - r^2/4$ is an $SL(2,\mathbb{R})$ invariant which labels different conjugacy classes of 7-brane charges. Each element of a conjugacy class is a non-linear doublet.

For instance, the standard D7-brane $(p, r, q) = (1, 0, 0)$ belongs to the $\det Q = 0$ class of "pq-7-branes" which transform in the simple non-linear form

$$(\sqrt{q'}, \sqrt{p'}) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} (\sqrt{q}, \sqrt{p}) \, .$$

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This implies that the third possible kind of 7-brane $(p, r, q) = (0, 1, 0)$ cannot exist independently and be supersymmetric

Are there also as many 7-brane solutions?

7-brane configurations are supersymmetric solutions of the gravity+scalar part of the $N = 2B, d = 10$ SUGRA action:

$$S = \int d^{10}x \sqrt{|g|} \left[R - \frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{2 \left(\Im\text{m}\, \tau \right)^2} \right] , \qquad \tau \equiv \ell + i e^{-\phi}, ,$$

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Similar solutions exist for any d. In $d = 4$ they are strings.

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☞ The transformation $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ satisfies $S^2 = 1$ when acting on τ, $S^4 = 1$ when acting on $f(z)$ and $S^8 = 1$ when acting on ϵ.

We say that $\tau(z)$ describes a (p, r, q) 7-brane at $z = z_0$ if the monodromy of $\tau(z)$ around $z = z_0$ is

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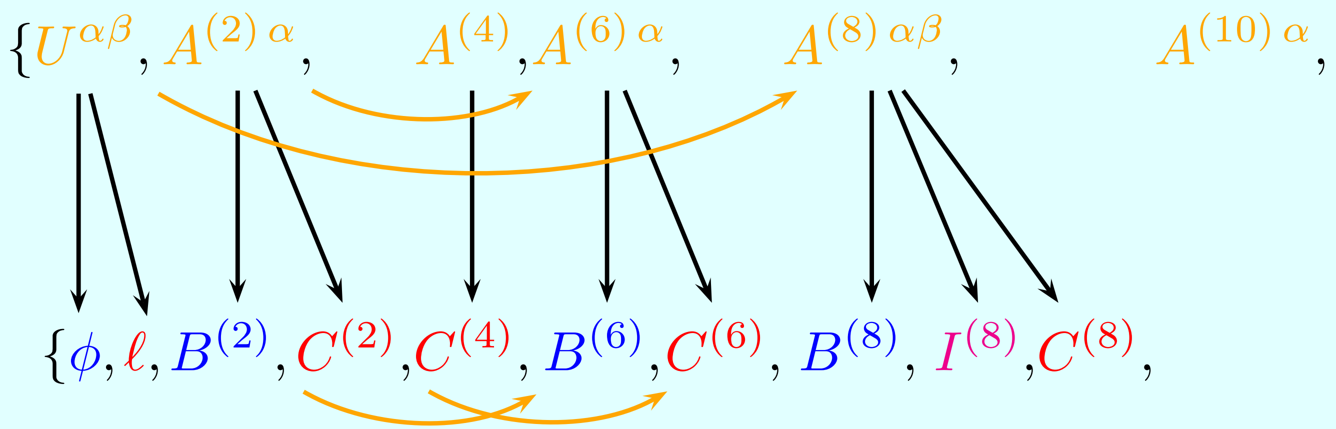
The doublet of 10-forms:



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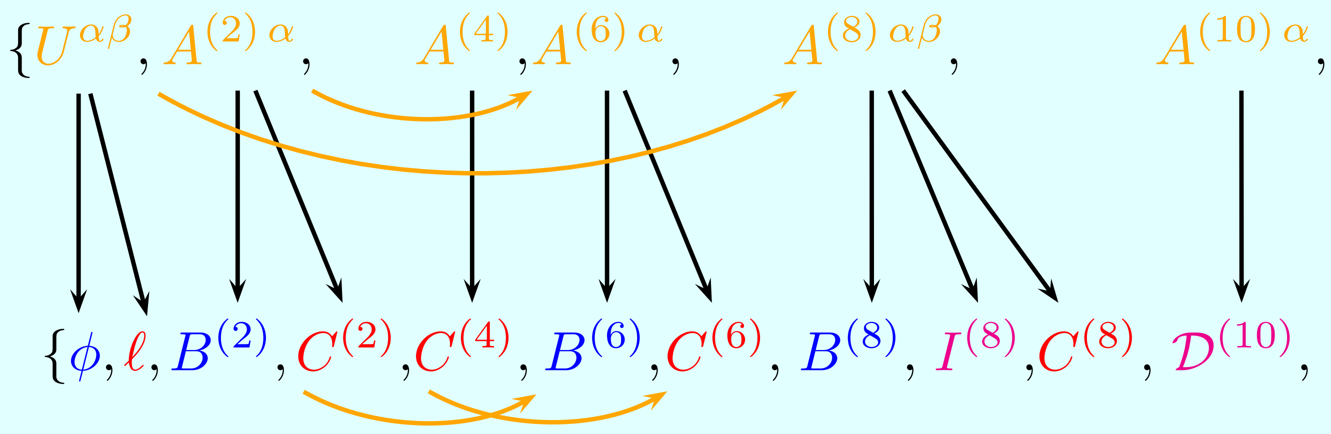

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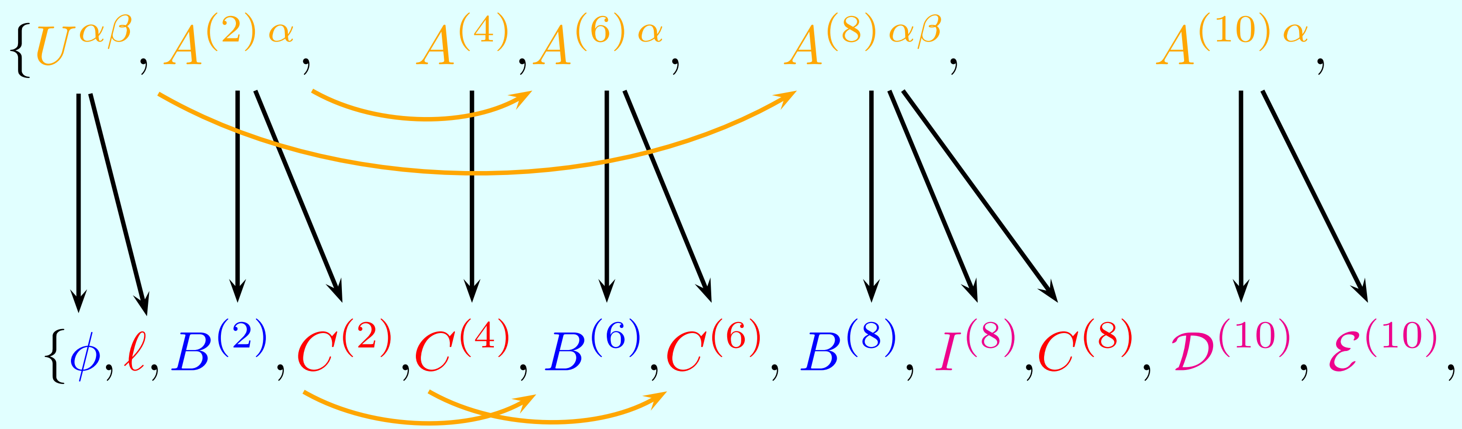

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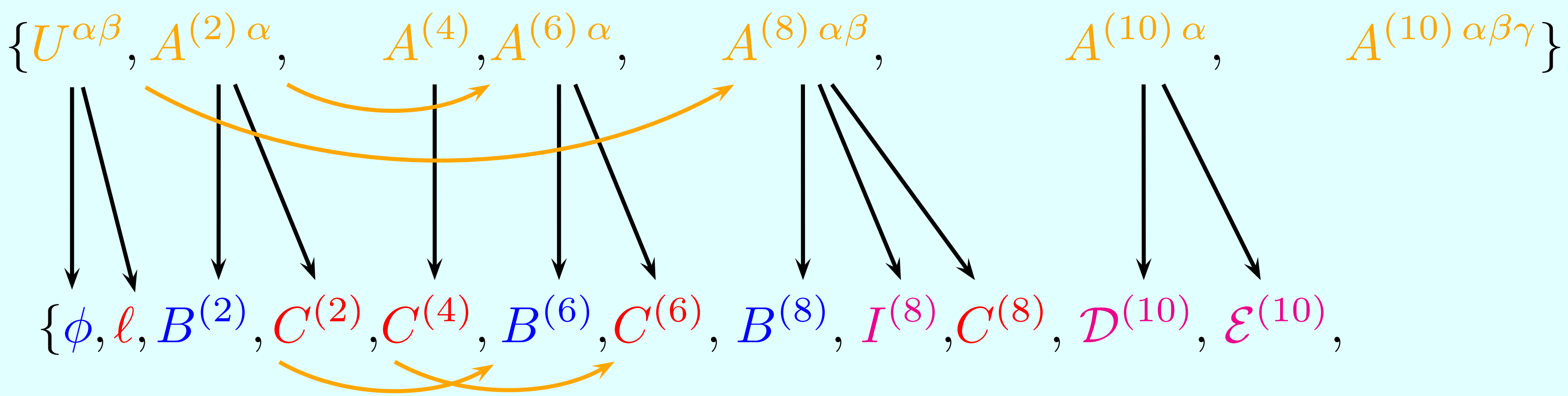

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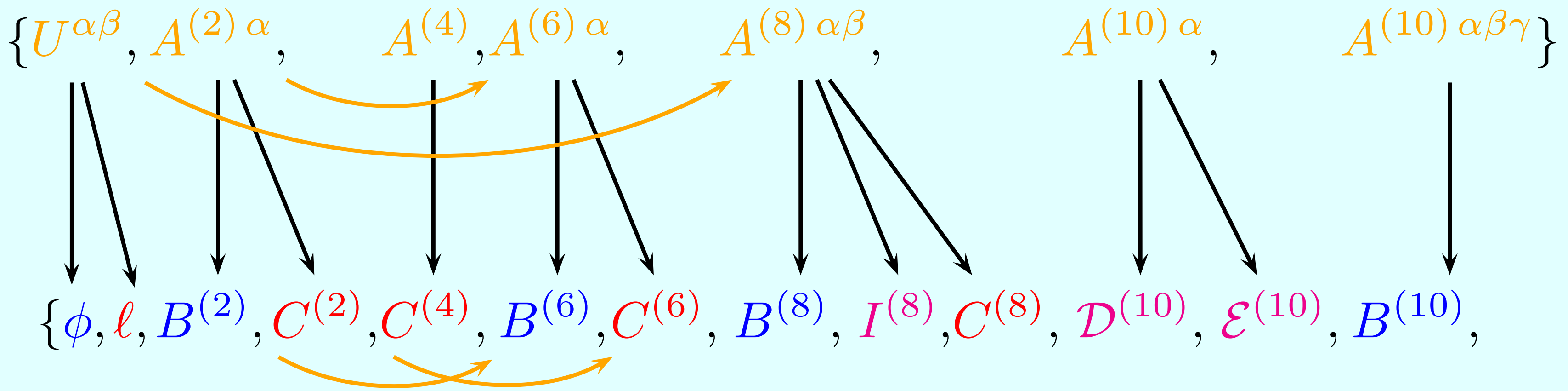

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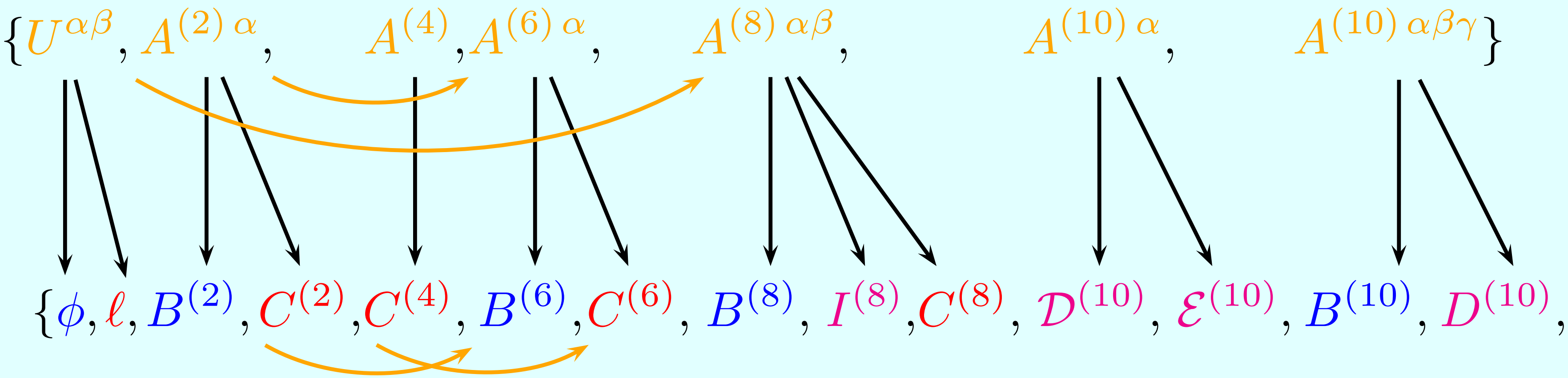

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$$\{\phi, \ell, B^{(2)}, C^{(2)}, C^{(4)}, B^{(6)}, C^{(6)}, B^{(8)}, I^{(8)}, C^{(8)}, \mathcal{D}^{(10)}, \mathcal{E}^{(10)}, B^{(10)}, D^{(10)}, E^{(10)},$$

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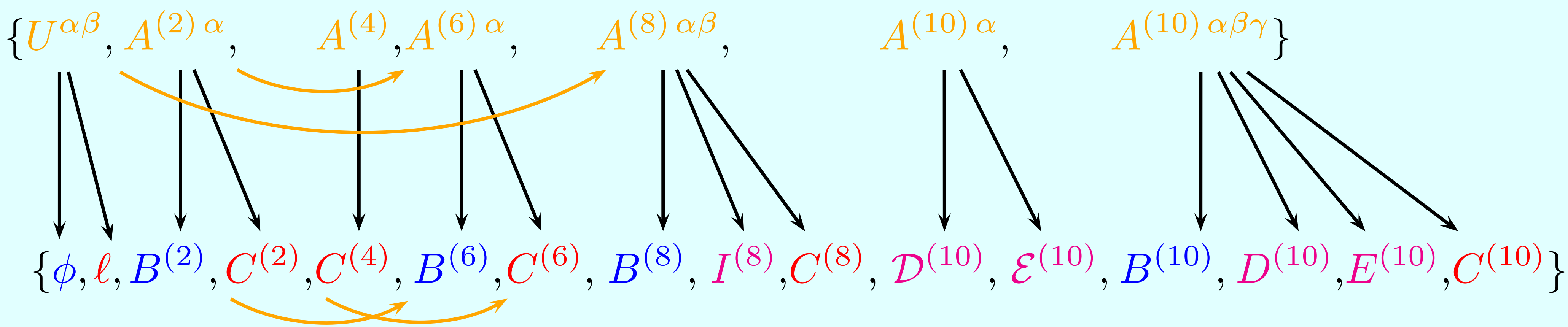

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→ Supersymmetry leads to the following $SU(1,1)$-covariant restriction on the coupling to the quadruplet of 9-branes

$$Q^{\alpha\beta} = q_{\alpha\gamma\delta}\, q_{\beta\epsilon\zeta} \epsilon^{\gamma\epsilon} \epsilon^{\delta\zeta} = 0\,,$$

so in the quadruplet there are only two independent 9-brane charges that transform, again, as a non-linear doublet. The standard D9-brane belongs to it.

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→ The Wess-Zumino term of the linear doublet of 9-branes does not contain couplings to any Born-Infeld field, which is, however, naively required for κ-symmetry.

The branes of $N = 2B$ SUGRA

Potential	Brane	Tension	Projection operator
$B^{(2)}$	F1	1	$\frac{1}{2}\left(1+\sigma_3\Gamma_{01}\right)$
$C^{(2)}$	D1	$\sqrt{e^{-2\phi}+\ell^2}$	$\frac{1}{2}\left(1+\frac{-e^{-\phi}\sigma_1+\ell\sigma_3}{\sqrt{e^{-2\phi}+\ell^2}}\Gamma_{01}\right)$
$C^{(4)}$	D3	$e^{-\phi}$	$\frac{1}{2}(1+i\sigma_2\Gamma_{0123})$
$B^{(6)}$	NS5	$e^{-\phi}\sqrt{e^{-2\phi}+\ell^2}$	$\frac{1}{2}\left(1+\frac{e^{-\phi}\sigma_3+\ell\sigma_1}{\sqrt{e^{-2\phi}+\ell^2}}\Gamma_{01\cdots5}\right)$
$C^{(6)}$	D5	$e^{-\phi}$	$\frac{1}{2}(1+\sigma_1\Gamma_{01\cdots5})$
$B^{(8)}$	$\widetilde{\mathrm{D7}}$	$e^{-3\phi}+\ell^2 e^{-\phi}$	$\frac{1}{2}\left(1+i\sigma_2\Gamma_{01\cdots7}\right)$
$C^{(8)}$	D7	$e^{-\phi}$	$\frac{1}{2}\left(1+i\sigma_2\Gamma_{01\cdots7}\right)$
$\mathcal{D}^{(10)}$	S9	$e^{-2\phi}$	$\frac{1}{2}(1+\sigma_3)$
$\mathcal{E}^{(10)}$	$\widetilde{\mathrm{S9}}$	$e^{-2\phi}\sqrt{e^{-2\phi}+\ell^2}$	$\frac{1}{2}\left(1+\frac{-e^{-\phi}\sigma_1+\ell\sigma_3}{\sqrt{e^{-2\phi}+\ell^2}}\right)$
$B^{(10)}$	$\widetilde{\mathrm{D9}}$	$e^{-\phi}\left(e^{-2\phi}+\ell^2\right)^{3/2}$	$\frac{1}{2}\left(1-\frac{\ell\sigma_1+e^{-\phi}\sigma_3}{\sqrt{e^{-2\phi}+\ell^2}}\right)$
$C^{(10)}$	D9	$e^{-\phi}$	$\frac{1}{2}(1+\sigma_1)$

4 – Extensions of $N = 2, d = 4$ SUGRA: supersymmetric solutions

N=2,d=4 SUGRA admits electrically and magnetically charged 1/2 supersymmetric black-hole solutions (Ferrara, Kallosh & Strominger, hep-th/9508072, Behrndt, Lüst & Sabra hep-th/9705169).

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Those in the vector multiplet sector have the form:

$$\left\{ \begin{array}{rcl} ds^2 & = & dt^2 - dy^2 - 2e^{-\mathcal{K}(Z,Z^*)}|f|^2 dz dz^* \,, \\ \\ Z^i & = & Z^i(z)\,, \qquad\qquad f = f(z)\,, \end{array} \right.$$

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and their Killing spinors take the general form

$$\epsilon_I = (f/f^*)^{1/4}\epsilon_{I\,0}\,, \qquad \gamma_{\underline{z}^*}\epsilon_{I\,0} = 0\,.$$

In general, the holomorphic functions $Z^i(z)$ will have have singularities and branch cuts and, when crossing a branch cut, their value may be transformed by an element of the isometry group of the Kähler metric $G_V \subseteq Sp(2\bar{n}, \mathbb{R})$.

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So this is basically the (local) story concerning the solutions. Now the question is:

Are there 2-forms in N=2,d=4 SUGRA to which we can couple these strings?

5 – Extensions of $N = 2, d = 4$ SUGRA: 1.- vector fields

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It is easy to introduce the $\bar{n}$ dual vector fields $A_{\Lambda\,\mu}$ that couple to the magnetic charges p^{Λ} of supersymmetric black holes.

5 – Extensions of $N = 2, d = 4$ SUGRA: 1.- vector fields

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All these vectors can be combined into an $Sp(2\bar{n}, \mathbb{R})$ vector

$$\mathcal{A}_{\mu} \equiv \begin{pmatrix} A^{\Lambda}{}_{\mu} \\ A_{\Lambda\,\mu} \end{pmatrix} ,$$

with supersymmetry transformation rule

$$\delta_{\epsilon}\mathcal{A}_{\mu} \; = \; \tfrac{1}{4}\mathcal{V}\,\epsilon_{IJ}\bar{\psi}^{I}_{\mu}\epsilon^{J} + \tfrac{i}{8}\mathfrak{D}_{i}\mathcal{V}\,\epsilon_{IJ}\bar{\lambda}^{Ii}\gamma_{\mu}\epsilon^{J} + \text{c.c.}\,, \qquad \mathcal{V} = \begin{pmatrix} \mathcal{L}^{\Lambda} \\ \mathcal{M}_{\Lambda} \end{pmatrix} , \qquad \mathfrak{D}_{i}\mathcal{V} = \begin{pmatrix} f^{\Lambda}{}_{i} \\ h_{\Lambda\,i} \end{pmatrix} ,$$

The supersymmetric, gauge and symplectic-invariant coupling to electric and magnetically charged black holes (0-branes) is given by the worldline action

$$S = \int d\xi \; |\mathcal{Z}| \; \sqrt{\frac{dX^\mu}{d\xi}\frac{dX^\nu}{d\xi} g_{\mu\nu}(X)} + \int d\xi \langle\, q \mid \mathcal{A}_\mu \,\rangle \frac{dX^\mu}{d\xi} \,.$$

where $\mathcal{Z}$ is the central charge

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We are now prepared to search for the 2-forms.

6 – Extensions of $N = 2, d = 4$ SUGRA: 2.- 2-form fields

The main lesson we learned from the $N = 2B, d = 10$ 7-branes is that the $(d-2)$-form potentials are dual to the isometries of the scalar manifold. Thus, we have to Hodge-dualize the Noether currents associated to the transformations

$$\delta_\alpha Z^i = \alpha^A k_A{}^i(Z)\,, \qquad \delta_\alpha \mathcal{A}_\mu = \alpha^A T_A \mathcal{A}_\mu\,,$$

with $T_A \in \mathfrak{g}_v \subseteq \mathfrak{sp}(2\bar{n}, \mathbb{R})$ (Gaillard & Zumino, Nucl. Phys. B **193** (1981) 221).

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we can write these currents as symplectic-invariant 1-forms

$$J_{N,A} = 2i\langle\, \mathfrak{D}\mathcal{V}^* \mid T_A \mathcal{V}\,\rangle + \text{c.c.} + 4 \star \langle\, \mathcal{F} \mid T_A \mathcal{A}\,\rangle\,.$$

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They are not gauge-invariant due to the last term. (On-shell) current conservation

$$d \star J_{N,A} = 0 \;\Rightarrow dB_A \equiv \star J_{N,A} = \star[2i\langle\, \mathfrak{D}\mathcal{V}^* \mid T_A \mathcal{V}\,\rangle + \text{c.c.}] - 4\langle\, \mathcal{F} \mid T_A \mathcal{A}\,\rangle\,.$$

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And then we define the gauge-invariant 3-form field-strength

$$H_A \equiv dB_A + 4\langle\, \mathcal{F} \mid T_A \mathcal{A}\,\rangle\,.$$

The B_As are the 2-forms to which the strings of $N = 2, d = 4$ SUGRA will couple to. Their supersymmetry transformation rules are found to be

$$\begin{aligned} \delta_\epsilon B_{A\,\mu\nu} &= -\tfrac{1}{2}\langle\, \mathfrak{D}_i \mathcal{V} \mid T_A \mathcal{V}^* \,\rangle\, \bar{\epsilon}_I \gamma_{\mu\nu} \lambda^{iI} + \text{c.c.} \\ &\quad -i\langle\, \mathcal{V} \mid T_A \mathcal{V}^* \,\rangle\, \bar{\epsilon}^I \gamma_{[\mu} \psi_{I\nu]} + \text{c.c.} \\ &\quad +8\langle\, \mathcal{A}_{[\mu} \mid T_A \delta_\epsilon \mathcal{A}_{\nu]} \,\rangle\,. \end{aligned}$$

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Then, the leading term of the Wess-Zumino term must be the pullback of $q^A B_{A\,\mu\nu}$. The action them is supersymmetric with the standard projection on ϵ, but it is not gauge-invariant and it is impossible to add any term constructed with the vector fields to restore gauge-invariance.

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Exactly the same problem arises in the construction of a κ-symmetric worldsheet action for heterotic strings propagating in the background of Yang-Mills fields. The solution in that case is the addition of heterotic fermions whose gauge transformations cancel those of the 2-form (Atick, Dhar & Ratra, Phys. Lett. B **169** (1986) 54).

7 – Some new supersymmetric solutions of $N = 2, d = 4$ supergravity

Once the form of all the supersymmetric solutions of all ungauged $N = 2, d = 4$ SUGRAs is known (Meessen & O. `hep-th/0603099`, Hübscher, Meessen & O., `hep-th/0606281`) it is natural to ask what happens in the gauged theories.

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As a first step in this direction we are studying $N=2, d=4$ Einstein-Yang-Mills theories: $N=2, d=4$ SUGRAcoupled to non-Abelian vector fields. In these theories, only the isometries of the special-Kähler manifold are gauged and the scalar potential is $V \geq 0$.

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The form of all the supersymmetric solutions in the timelike class has been completely determined (Hübscher, Meessen, O. & Vaulà, in preparation). They can be constructed as follows:

Recipe:

☞ Find a set of Yang-Mills $A^{\Lambda}{}_{m}$ and functions $\mathcal{I}^{\Lambda}$ in flat 3-d space satisfying

$$\frac{1}{2}\,\epsilon_{pmn}\,F^{\Lambda}_{mn} \;=\; -\frac{1}{\sqrt{2}}\,\mathfrak{D}_p\mathcal{I}^{\Lambda}\;,$$

which is the Bogomol'nyi equation satisfied by known magnetic monopole solutions.

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  which is the Bogomol'nyi equation satisfied by known magnetic monopole solutions.

- ☞ Use the above solution to find a solution of

$$\mathfrak{D}_m\mathfrak{D}_m\mathcal{I}_{\Lambda} = \frac{1}{2}g^2\,\left[f_{\Lambda(\Sigma}{}^{\Gamma} f_{\Delta)\Gamma}{}^{\Omega}\,\mathcal{I}^{\Sigma}\mathcal{I}^{\Delta}\right]\,\mathcal{I}_{\Omega}\,,$$

  so that

$$\langle\,\mathcal{I}\mid\mathfrak{D}_m\mathcal{I}\rangle = \mathcal{I}_{\Lambda}\mathfrak{D}_m\mathcal{I}^{\Lambda} - \mathcal{I}^{\Lambda}\mathfrak{D}_m\mathcal{I}_{\Lambda} = 0\,.$$

☞ Solve the stabilization equations to find $\mathcal{R}^\Lambda$ and $\mathcal{R}_\Lambda$. N.B.:

$$\mathcal{I}^\Lambda \equiv \Im\mathrm{m}(\mathcal{L}^\Lambda/X)\,, \qquad \mathcal{I}_\Lambda \equiv \Im\mathrm{m}(\mathcal{M}_\Lambda/X)\,,$$

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$$2|X|^2 = \langle\, \mathcal{R} \mid \mathcal{I}\,\rangle^{-1}\,,$$

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and the symplectic vector of 2-form field strengths

$$\mathcal{F} = -\sqrt{2}\,\mathfrak{D}\left(\,|X|^2 \mathcal{R}\; dt\right) - \sqrt{2}\,|X|^2\, \star (dt \wedge\, \mathfrak{D}\mathcal{I})\,.$$

$SO(3)$ Examples:

Let us consider $N = 2$ EYM systems containing an $SO(3)$ gauge group, with indices $a = 1, 2, 3$.

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A 2-parameter (μ and ρ) family of solutions is given by

$$\mathcal{I}(r) = \frac{\sqrt{2}\mu}{g}\mathsf{H}_\rho(\mu r)\,, \quad \mathsf{H}_\rho(r) = \coth(r+\rho) - \frac{1}{r}\,,$$

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The two most interesting cases are $\rho = 0, \infty$.

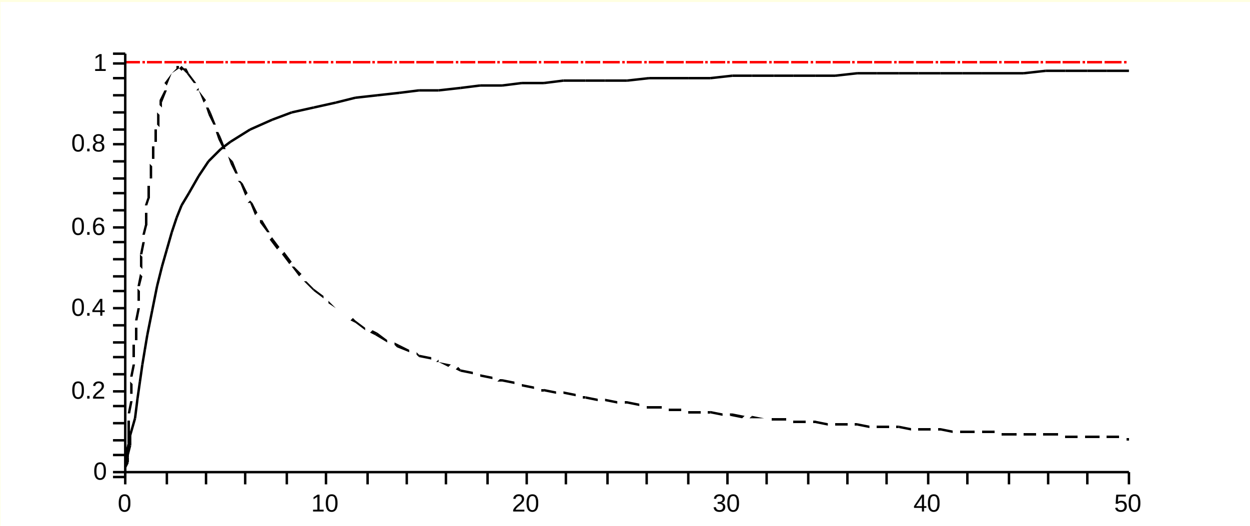
't Hooft-Polyakov Monopoles

The $\rho = 0$ solution can be written in the form

$$A^a{}_m = \varepsilon_{mb}{}^a \, n^b \, \frac{\mu}{g} \, \mathsf{G}_0(\mu r) \,, \qquad \mathsf{G}_0(r) = \frac{1}{r} - \frac{1}{\sinh r} \,,$$

$$\mathcal{I}^a = \frac{\sqrt{2}\mu}{g} \, \mathsf{H}_0(\mu r) \, n^a \,, \qquad \mathsf{H}_0(r) = \coth r - \frac{1}{r} \,.$$

The profiles of the functions G and H are



$\mathcal{I}^a$ is regular at $r = 0$ for $\rho = 0$, and describes the 't Hooft-Polyakov monopole.

Black Hedgehogs

In the limit $\rho \to \infty$ we find the "black hedgehog" solution

$$\mathcal{I}^a = -\sqrt{2}\left(\mathcal{I}_\infty + \frac{1}{gr}\right) n^a \,,$$

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The possible existence of an event horizon covering the singularity at $r = 0$ has to be studied in specific models.

Before finding $\mathcal{R}$ and $|X|$ we have to find the $\mathcal{I}_a$s solving

$$\mathfrak{D}_m \mathfrak{D}_m \mathcal{I}_\Lambda \;=\; \tfrac{1}{2} g^2 \;\left[f_{\Lambda(\Sigma}{}^{\Gamma} f_{\Delta)\Gamma}{}^{\Omega}\; \mathcal{I}^{\Sigma} \mathcal{I}^{\Delta} \right]\; \mathcal{I}_\Omega \;,$$

and solve the staticity constraint

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In this simple case

$$\mathcal{I}_a \;=\; \frac{g}{2} \mathcal{J}\, \mathcal{I}^a \;,$$

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where $\mathcal{J}$ is an arbitrary constant.

If we split the index Λ into an a-index and an u-index labeling the ungauged directions, the staticity constraint only acts non-trivially on the ungauged part:

$$\mathcal{I}_u \, d\mathcal{I}^u \; - \; \mathcal{I}^u \, d\mathcal{I}_u \; + \; \mathcal{I}_a \, \mathfrak{D}\mathcal{I}^a \; - \; \mathcal{I}^a \, \mathfrak{D}\mathcal{I}_a \; = \; \mathcal{I}_u \, d\mathcal{I}^u \; - \; \mathcal{I}^u \, d\mathcal{I}_u \; = \; 0 \; ,$$

which we can solve as in the Abelian case or just set to zero.

Before finding $\mathcal{R}$ and $|X|$ we have to find the $\mathcal{I}_a$s solving

$$\mathfrak{D}_m \mathfrak{D}_m \mathcal{I}_\Lambda \;=\; \tfrac{1}{2} g^2 \;\left[f_{\Lambda(\Sigma}{}^{\Gamma} f_{\Delta)\Gamma}{}^{\Omega}\, \mathcal{I}^\Sigma \mathcal{I}^\Delta \right]\, \mathcal{I}_\Omega \;,$$

and solve the staticity constraint

$$\langle\, \mathcal{I} \mid \mathfrak{D}_m \mathcal{I} \rangle = 0 \,.$$

In this simple case

$$\mathcal{I}_a \;=\; \frac{g}{2} \mathcal{J}\, \mathcal{I}^a \;,$$

where $\mathcal{J}$ is an arbitrary constant.

If we split the index Λ into an a-index and an u-index labeling the ungauged directions, the staticity constraint only acts non-trivially on the ungauged part:

$$\mathcal{I}_u\, d\mathcal{I}^u \;-\; \mathcal{I}^u\, d\mathcal{I}_u \;+\; \mathcal{I}_a\, \mathfrak{D}\mathcal{I}^a \;-\; \mathcal{I}^a\, \mathfrak{D}\mathcal{I}_a \;=\; \mathcal{I}_u\, d\mathcal{I}^u \;-\; \mathcal{I}^u\, d\mathcal{I}_u \;=\; 0 \,,$$

which we can solve as in the Abelian case or just set to zero.

This determines completely the family of solutions but, in order to find explicit expressions for $\mathcal{R}$ and $|X|$ and the spacetime metric we must solve the stabilization equations which depend on the specific model considered.

Metrics

For simplicity let us consider a $\overline{\mathbb{CP}}^3$ model whose prepotential reads

$$\mathcal{F} \;=\; \frac{i}{4}\,\eta_{\Lambda\Sigma}\,\mathcal{X}^{\Lambda}\,\mathcal{X}^{\Sigma}\,, \qquad \eta \;=\; \mathrm{diag}\,(\,-\,,\;[+]^{n}\,)\;.$$

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With the hedgehog Ansatz $\mathcal{I}^{a\,2} = \mathcal{I}^2$ and $SU(2)$ effectively reduces to a $U(1)$ in the metric! For black holes with finite entropy (attractor) we need at least two $U(1)$s. However, since $\mathcal{I}^a$ is bound in the monopole, we do not need $\mathcal{I}^0, \mathcal{I}_0$ and we can set them to constants.

Normalizing to have asymptotic flatness, we get, for the monopole

$$-g_{rr} = 1 \; + \; \mu^2 \left[\frac{1}{g^2} \; + \; \mathcal{J}^2\right] \; \left[1 \; - \; \mathsf{H}^2(\mu r)\right] \; ,$$

which is completely regular and describes an object of mass

$$\mathsf{M} \; = \; \mu \left[1/g^2 \; + \; \mathcal{J}^2\right] \; .$$

(related to Harvey & Liu (1991) and Chamseddine & Volkov (1997) monopole solutions.)

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To embed the black hedgehog into this model and get a regular solution ($|Z|^2 < 1$) we need non-trivial $\mathcal{I}^0$ or $\mathcal{I}_0$. The conditions for regularity are the same as in an standard, Abelian $U(1) \times U(1)$ black hole of this model:

$$\mathsf{M} \quad = \quad \mathcal{I}^0_\infty p^0 + \mathcal{I}_{0\infty} q_0 - 2\mu \left[1/g^2 \ + \ \mathcal{J}^2\right] > 0 \, ,$$

$$\frac{A}{4\pi} \quad = \quad \tfrac{1}{2}[(p^0)^2 + 4(q_0)^2] - 2\frac{\mu^2}{g^2} \left[1/g^2 \ + \ \mathcal{J}^2\right] > 0 \, ,$$

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How does the attractor mechanism work in this solution?

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  - Relation between strings (in general $(d-3)$-branes) and Scherk-Schwarz reductions...