SUSY cosmic strings in N=2,d=4 supergravities

Tomás Ortín (I.F.T., Madrid)

Seminar given on October 1st 2007 at the SITP

Based on hep-th/0601128, hep-th/0602280, hep-th/0611036, hep-th/0612072 and on work in preparation.

Work done in collaboration with *E. Bergshoeff, M. de Roo, J. Hartong, S. Kerstan* (U. of Groningen, The Netherlands) *F. Riccioni* (King's College, London, UK), *and M. Hübscher* (IFT, Madrid, Spain)

Plan of the Talk:

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- ☞ For $p = d-2$ one has to dualize constants (coupling or gauge constants, masses etc.). This has been done only in the simplest cases.
- ☞ For $p = d-1$ there is nothing to be dualized and we have no idea of which $(d-1)$- (*spacetime filling*) branes the theory may contain.

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This program has been carried out for $N = 2A, B, d = 10$ Supergravities in Bergshoeff, de Roo, Kerstan & Riccioni, `hep-th/0506013` and Bergshoeff, de Roo, Kerstan, O. & Riccioni, `hep-th/0602280`. New extensions have been found, all of them fitting in the proposed E_{11} symmetry of M-Theory.

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In this talk I am going to briefly review these results and new (still unpublished) results on extensions of matter-coupled $N = 2, d = 4$ Supergravity theories.

2 – Extensions of $N = 2A, d = 10$ SUGRA

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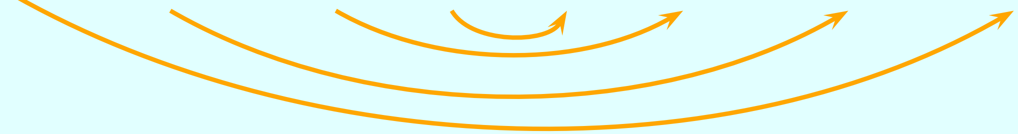

With the supersymmetry transformation (no gravitino in the r.h.s.!)

$$\delta_\epsilon B^{(8)}{}_{\mu_1 \cdots \mu_8} = \frac{1}{2} e^{-2\phi} \bar{\epsilon} \Gamma_{\mu_1 \cdots \mu_8} \Gamma_{11} \lambda + (\text{gauge} - \text{field dependent terms}) \ .$$

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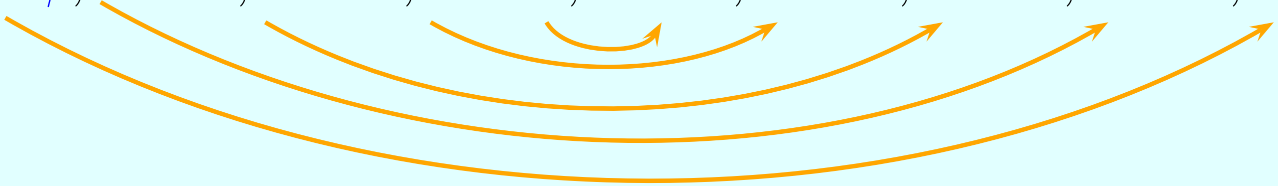

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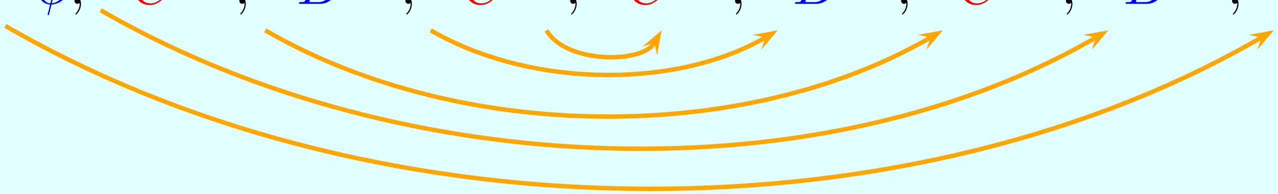

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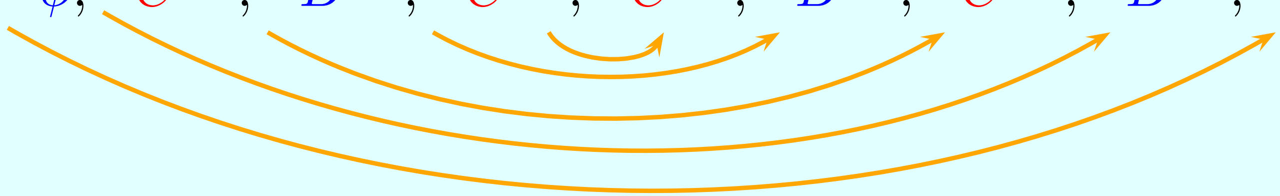

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If they did, they would do it via κ-invariant Lagrangians of the form

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For half-supersymmetric branes, the Lagrangians must be invariant under 16 linearly realized supersymmetries of the form

$$\delta_\epsilon\, g_{\mu\nu} = 2i\bar{\epsilon}\Gamma_{(\mu}\psi_{\nu)} + \text{h.c.}\,, \qquad \delta_\epsilon A^{(p+1)}{}_{\mu_1\cdots\mu_{p+1}} \sim f(\phi)\,\bar{\epsilon}\Gamma_{[\mu_1\cdots\mu_p}\psi_{\mu_{p+1}]} .$$

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One finds that

$$\delta_\epsilon \mathcal{L}_{\text{brane}} \sim \left(\tau_{\text{brane}} + f(\phi)\Gamma_{01\cdots p}\right)\epsilon\,,$$

and, thus,

$$\tau_{\text{brane}}(\phi) = f(\phi)\,,$$

and the Lagrangian is invariant under the the 16 independent transformations satisfying the projection

$$\tfrac{1}{2}(1 + \Gamma_{01\cdots p})\epsilon = 0\,.$$

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By simple inspection we conclude that the IIA supersymmetric branes and their tensions are

Potential	Brane	Tension	Projection operator
$C^{(1)}$	D0	$e^{-\phi}$	$\frac{1}{2}(1+\Gamma_0)$
$B^{(2)}$	F1	1	$\frac{1}{2}(1+\Gamma_{01}\Gamma_{11})$
$C^{(3)}$	D2	$e^{-\phi}$	$\frac{1}{2}(1+\Gamma_{012})$
$C^{(5)}$	D4	$e^{-\phi}$	$\frac{1}{2}(1+\Gamma_{01\cdots 4}\Gamma_{11})$
$B^{(6)}$	NS5	$e^{-2\phi}$	$\frac{1}{2}(1+\Gamma_{01\cdots 5})$
$C^{(7)}$	D6	$e^{-\phi}$	$\frac{1}{2}(1+\Gamma_{01\cdots 6})$
$C^{(9)}$	D8	$e^{-\phi}$	$\frac{1}{2}(1+\Gamma_{01\cdots 8}\Gamma_{11})$
$\mathcal{D}^{(10)}$	NS9	$e^{-2\phi}$	$\frac{1}{2}(1+\Gamma_{11})$

3 – Extensions of $N = 2B, d = 10$ SUGRA

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The relation with the $SL(2,\mathbb{R})$ fields that have a String Theory interpretation (dilaton,Kalb-Ramond 2-form, Ramond-Ramond forms) has to be found *a posteriori*.

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The following form-fields realizing the local supersymmetry algebra were found:

$$\left\{U^{\alpha\beta},\, A^{(2)\,\alpha},\, A^{(4)},\, A^{(6)\,\alpha},\, A^{(8)\,\alpha\beta},\, A^{(10)\,\alpha},\, A^{(10)\,\alpha\beta\gamma}\right\}\,,$$

$$\alpha,\beta,\gamma = 1,2, \quad SU(1,1) \text{ indices}$$

The scalars:

$$\{U^{\alpha\beta},$$

$$\delta_\epsilon V_+^\alpha = V_-^\alpha \; \bar{\epsilon}_C \lambda \quad , \quad \delta_\epsilon V_-^\alpha = V_+^\alpha \; \bar{\epsilon} \lambda_C \quad ,$$

$U^{\alpha\beta} = V_+^\alpha, V_-^\alpha$ is an $SU(1,1)$ matrix that parametrizes the $SU(1,1)/U(1)$ coset. It describes two real degrees of freedom:

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The precise relation between $U^{\alpha\beta}$, ϕ and ℓ is not unique and amounts to a choice of basis.

Observe that they do not transform into the gravitino and, therefore, cannot couple to dynamical branes (but they can couple to instantons).

The doublet of 2-forms:

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$$\delta_\epsilon A^{(2)\,\alpha}_{\mu\nu} = V_-^\alpha \; \bar{\epsilon}\Gamma_{\mu\nu}\lambda + V_+^\alpha \; \bar{\epsilon}_C\Gamma_{\mu\nu}\lambda_C + 4iV_-^\alpha \; \bar{\epsilon}_C\Gamma_{[\mu}\psi_{\nu]} + 4iV_+^\alpha \; \bar{\epsilon}\Gamma_{[\mu}\psi_{\nu]C} \, .$$

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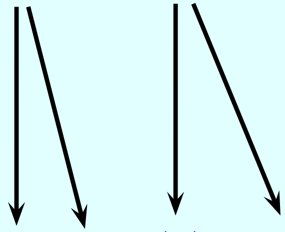

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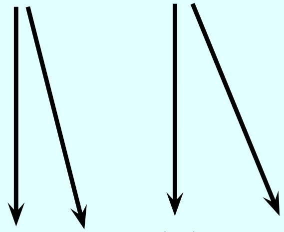
The precise relation between them depends on the same choice of basis.

The 4-form:

$$\{U^{\alpha\beta}, A^{(2)\,\alpha}, \qquad A^{(4)},$$



$$\{\phi, \ell, B^{(2)}, C^{(2)},$$

$$\delta_\epsilon A^{(4)}{}_{\mu\nu\rho\sigma} = \bar{\epsilon}\Gamma_{[\mu\nu\rho}\psi_{\sigma]} - \bar{\epsilon}_C\Gamma_{[\mu\nu\rho}\psi_{\sigma]C} - \tfrac{3i}{8}\epsilon_{\alpha\beta}A^{(2)\,\alpha}{}_{[\mu\nu}\delta_\epsilon A^{(2)\,\beta}{}_{\rho\sigma]}\,.$$

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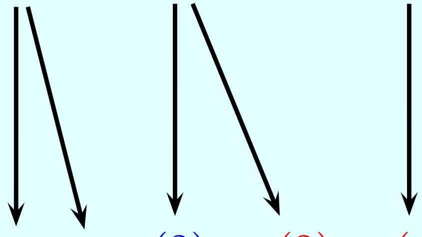

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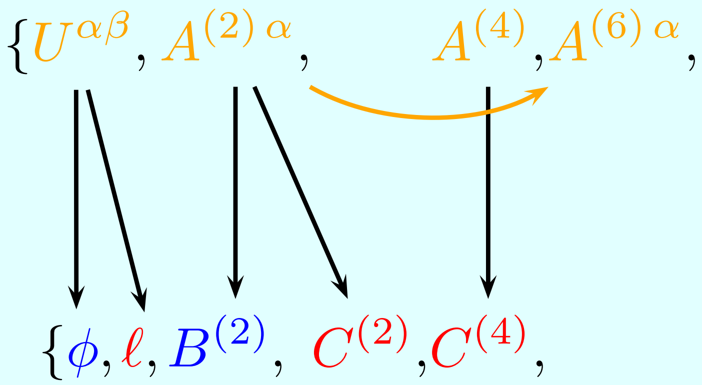
The doublet of 6-forms:



$$\begin{aligned}\delta_\epsilon A^{(6)\,\alpha}{}_{\mu_1\ldots\mu_6} \quad = \quad & iV_-^\alpha\bar\epsilon\Gamma_{\mu_1\ldots\mu_6}\lambda - iV_+^\alpha\bar\epsilon_C\Gamma_{\mu_1\ldots\mu_6}\lambda_C \\ & +12\left(V_-^\alpha\bar\epsilon_C\Gamma_{[\mu_1\ldots\mu_5}\psi_{\mu_6]} - V_+^\alpha\bar\epsilon\Gamma_{[\mu_1\ldots\mu_5}\psi_{C\,\mu_6]}\right) \\ & +\mathrm{gauge-field-dependent\ \ terms}\,.\end{aligned}$$

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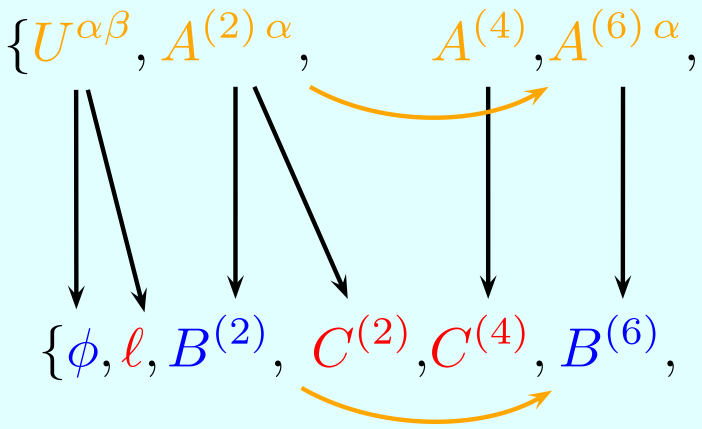

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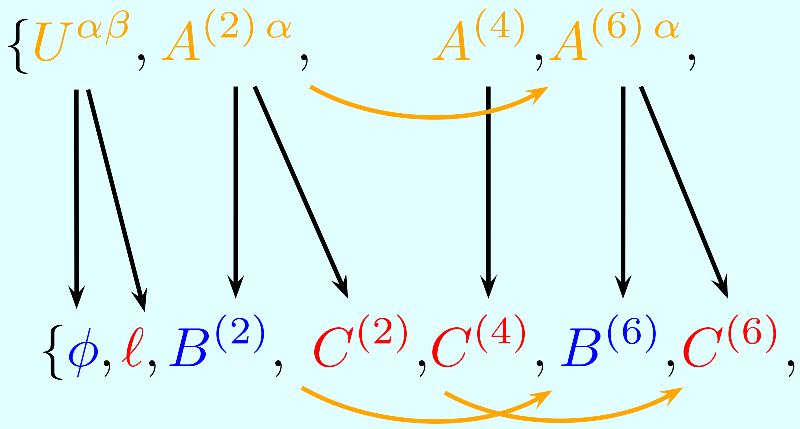

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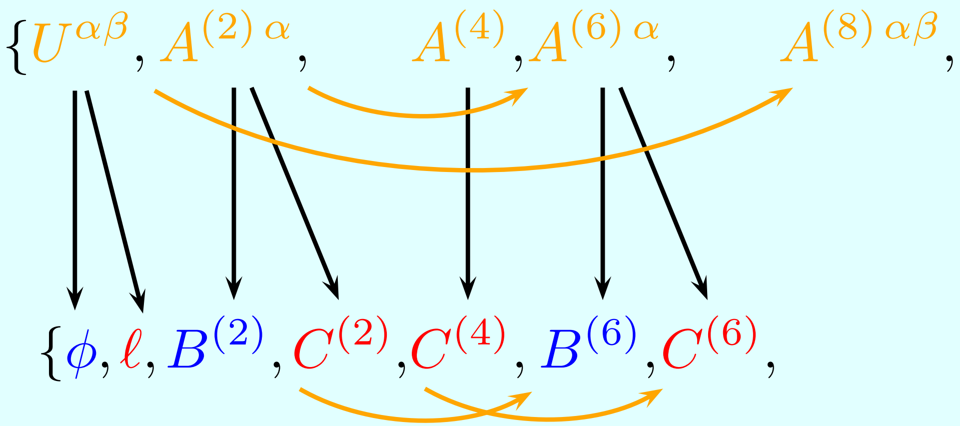
The triplet of 8-forms:



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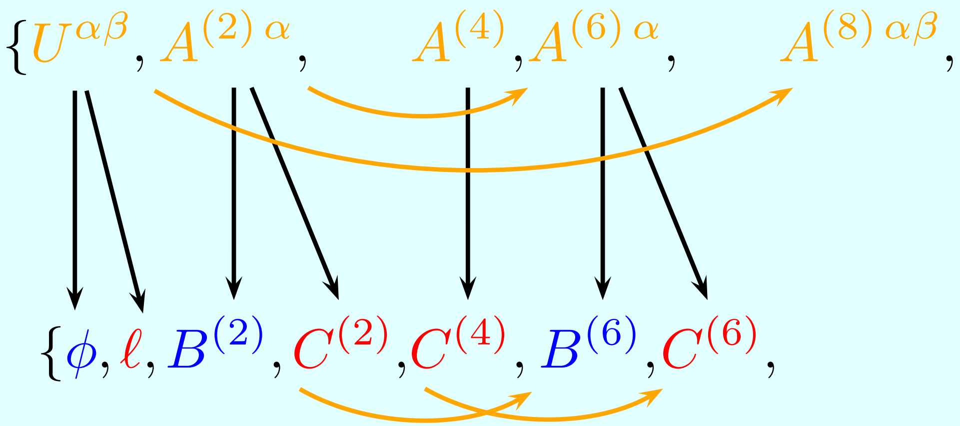

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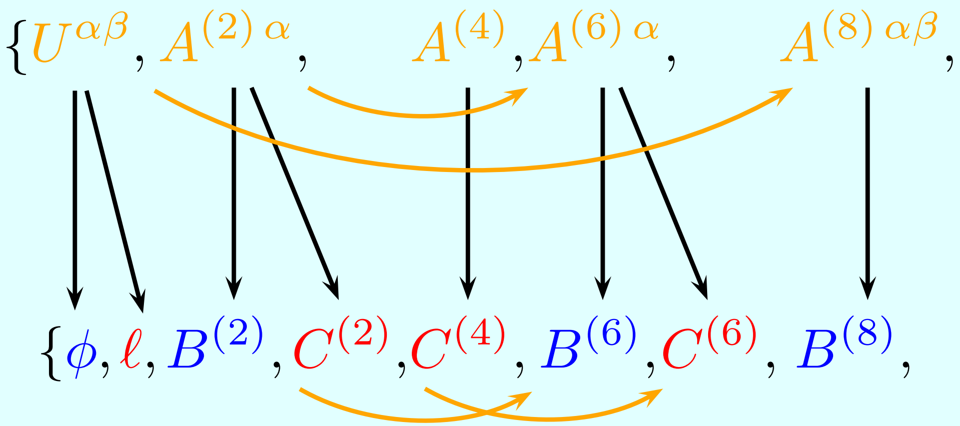

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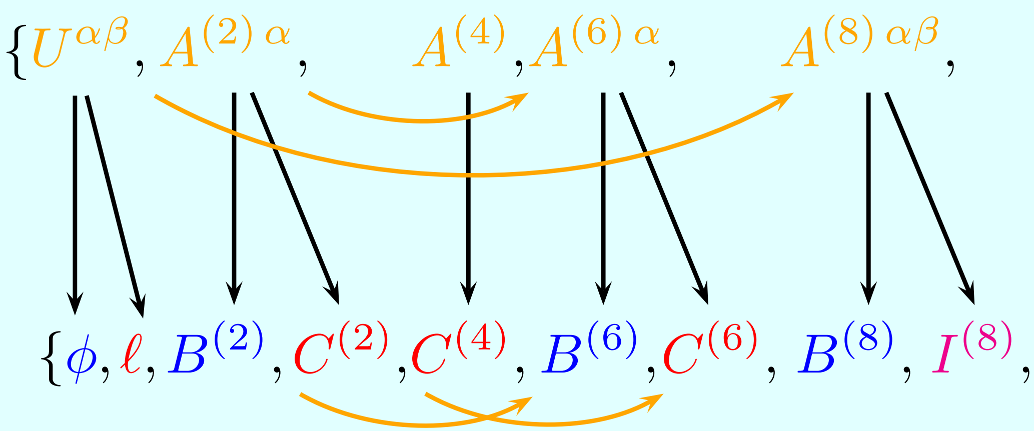

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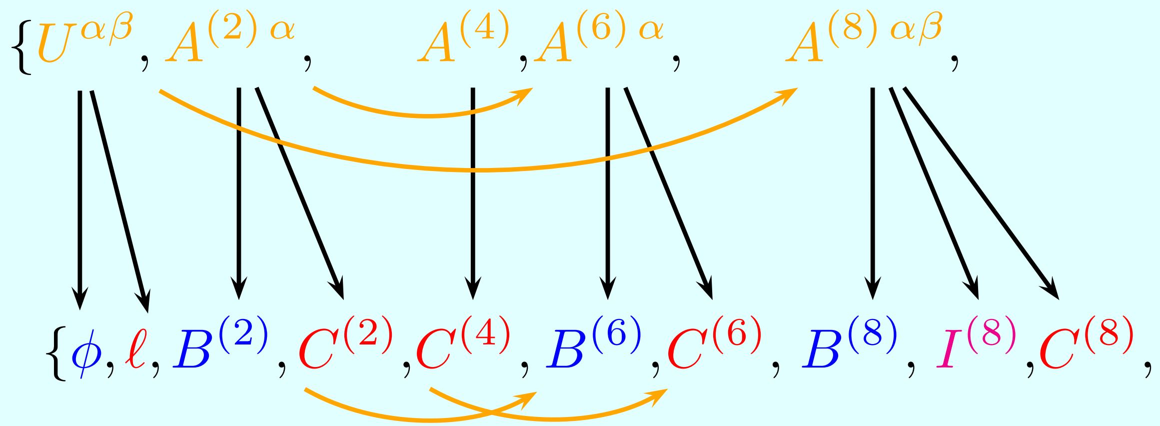

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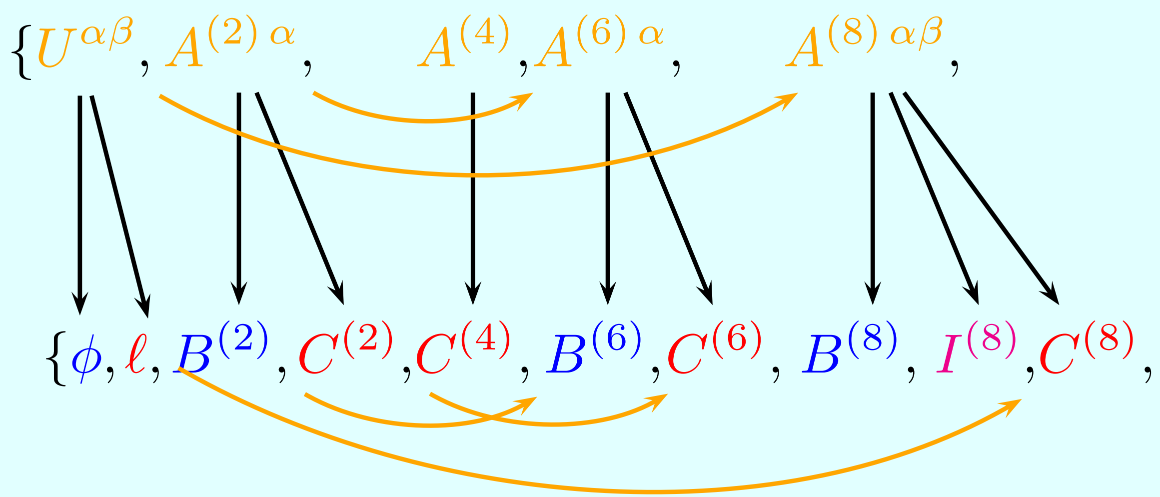

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A 7-brane is characterized by the 3 charges p, r, q that weight its coupling to each of the 3 8-form potentials. The leading terms of its worldvolume action are:

$$\mathcal{L}_{(p,r,q)} \sim \tau_{(p,r,q)} \sqrt{-g} + \epsilon^{\mu_1 \cdots \mu_8} \left(p\, C^{(8)}{}_{\mu_1 \cdots \mu_8} + r D^{(8)}{}_{\mu_1 \cdots \mu_8} + q B^{(8)}{}_{\mu_1 \cdots \mu_8} \right) .$$

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The tension (as the Lagrangian) also has manifestly $SL(2,\mathbb{R})$-invariant form in the Einstein frame:

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$\det Q = pq - r^2/4$ is an $SL(2,\mathbb{R})$ invariant which labels different conjugacy classes of 7-brane charges. Each element of a conjugacy class is a non-linear doublet.

For instance, the standard D7-brane $(p, r, q) = (1, 0, 0)$ belongs to the $\det Q = 0$ class of "pq-7-branes" which transform in the simple non-linear form

$$(\sqrt{q'}, \sqrt{p'}) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} (\sqrt{q}, \sqrt{p}) \,.$$

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This implies that the third possible kind of 7-brane $(p, r, q) = (0, 1, 0)$ cannot exist independently and be supersymmetric

Are there also as many 7-brane solutions?

7-brane configurations are supersymmetric solutions of the gravity+scalar part of the $N = 2B, d = 10$ SUGRA action:

$$S = \int d^{10}x \sqrt{|g|} \left[R - \frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{2\,(\Im\mathrm{m}\,\tau)^2} \right] , \qquad \tau \equiv \ell + ie^{-\phi},,$$

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Similar solutions exist for any d. In $d = 4$ they are strings.

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☞ The transformation $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ satisfies $S^2 = 1$ when acting on τ, $S^4 = 1$ when acting on $f(z)$ and $S^8 = 1$ when acting on ϵ.

We say that $\tau(z)$ describes a (p, r, q) 7-brane at $z = z_0$ if the monodromy of $\tau(z)$ around $z = z_0$ is

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Monodromy = 7-brane charge

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For the standard D7-brane

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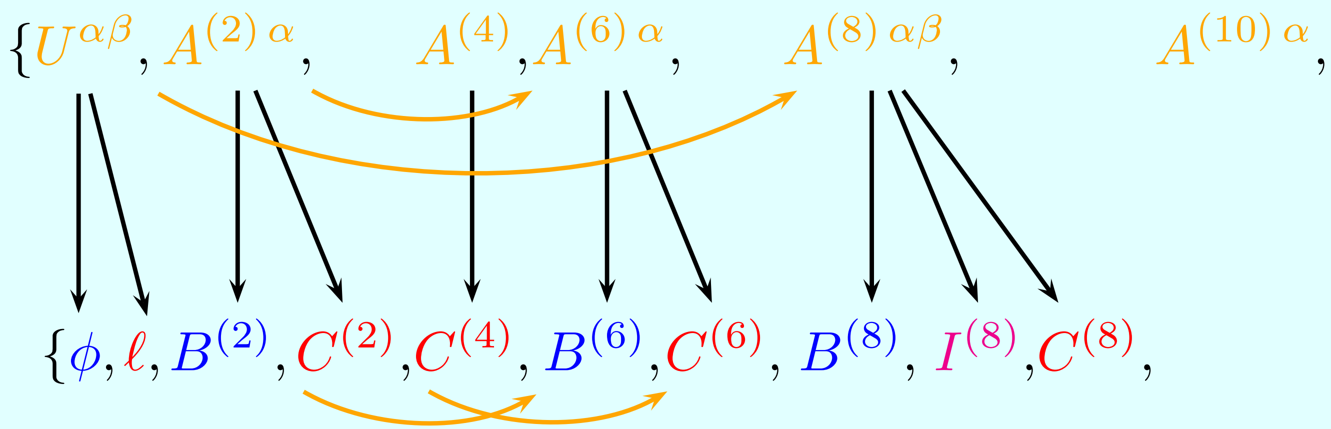
The doublet of 10-forms:



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\begin{aligned}
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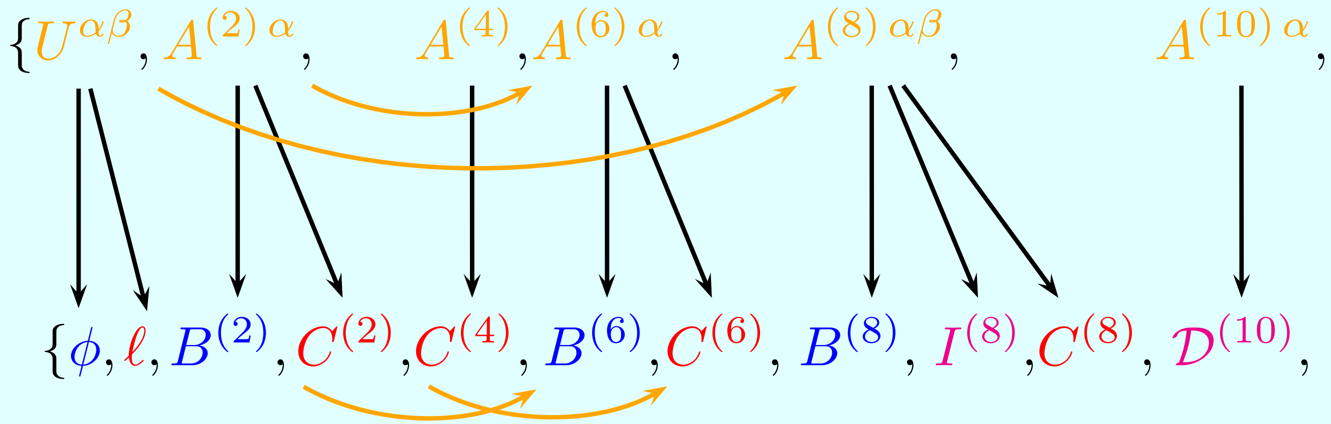

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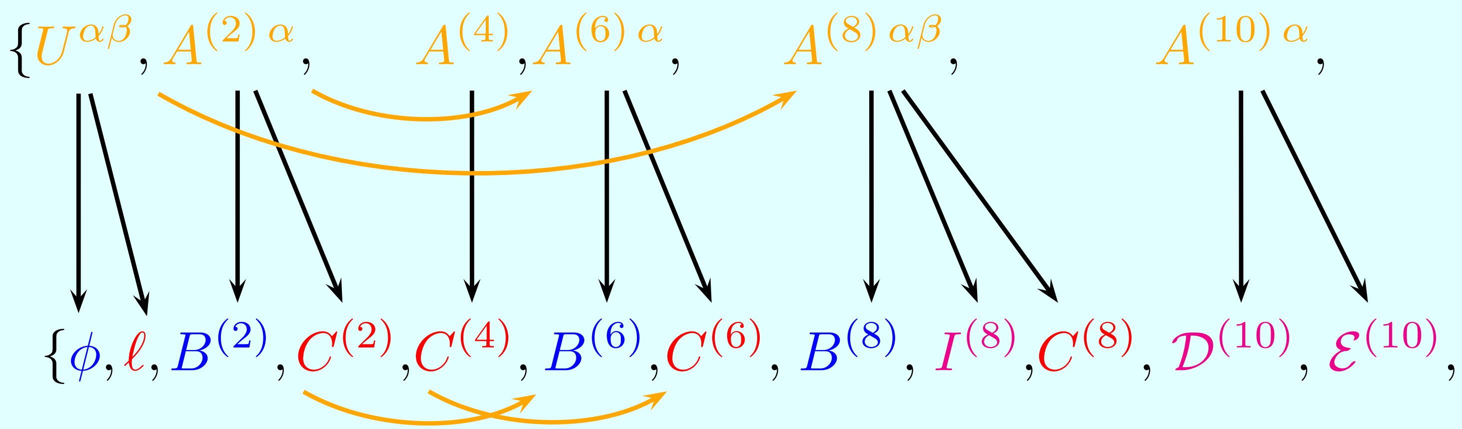

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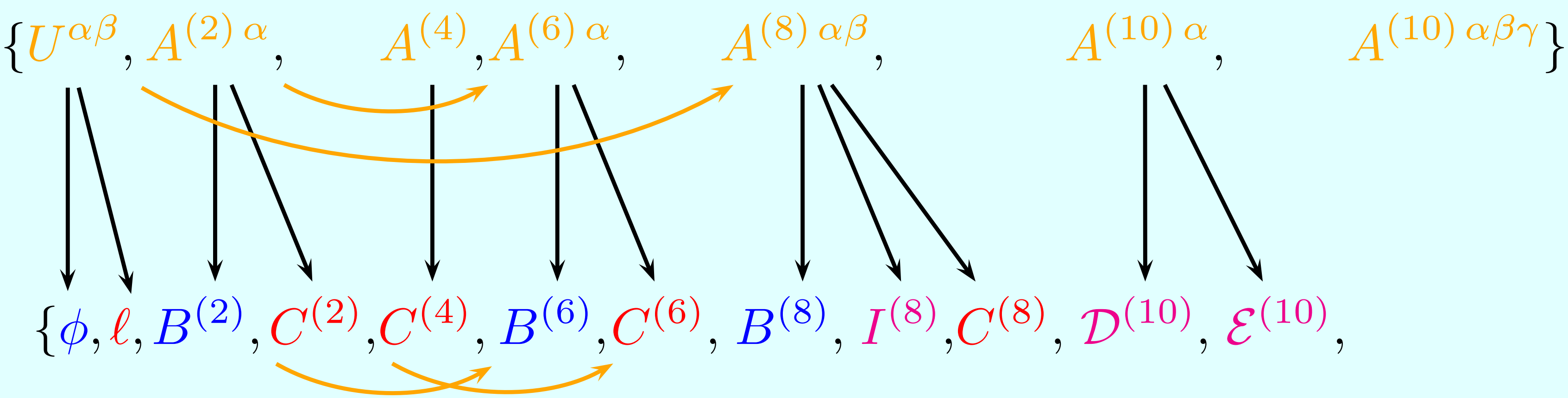

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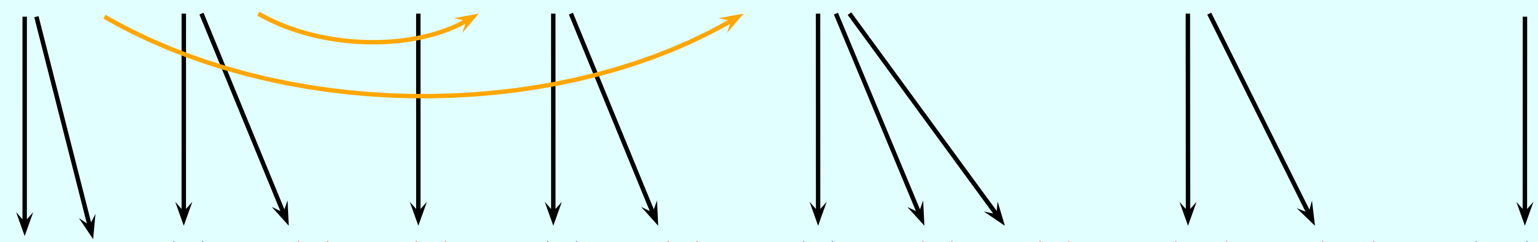

$$\{\phi, \ell, B^{(2)}, C^{(2)}, C^{(4)}, B^{(6)}, C^{(6)}, B^{(8)}, I^{(8)}, C^{(8)}, \mathcal{D}^{(10)}, \mathcal{E}^{(10)}, B^{(10)},$$

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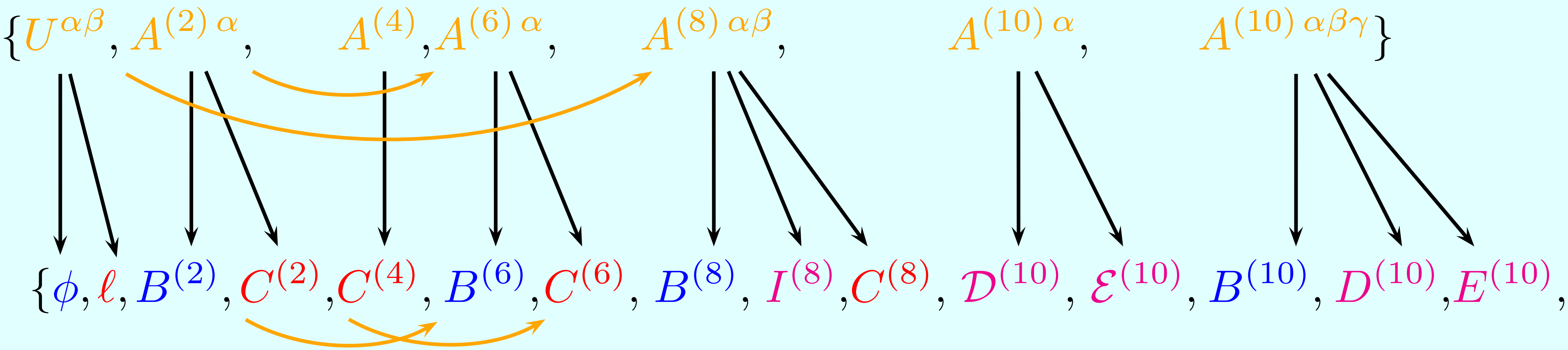

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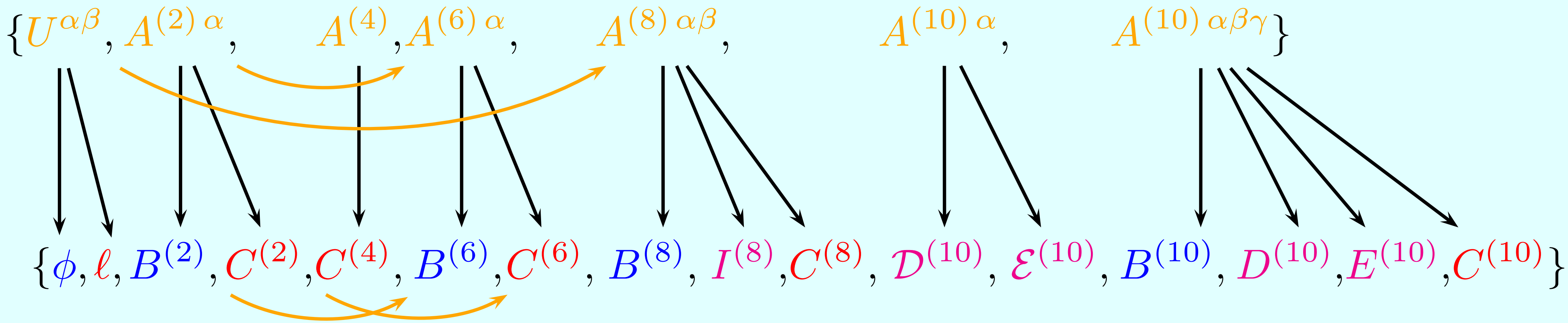

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→ Supersymmetry leads to the following $SU(1,1)$-covariant restriction on the coupling to the quadruplet of 9-branes

$$Q^{\alpha\beta} = q_{\alpha\gamma\delta}\, q_{\beta\epsilon\zeta}\epsilon^{\gamma\epsilon}\epsilon^{\delta\zeta} = 0\,,$$

so in the quadruplet there are only two independent 9-brane charges that transform, again, as a non-linear doublet. The standard D9-brane belongs to it.

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→ The Wess-Zumino term of the linear doublet of 9-branes does not contain couplings to any Born-Infeld field, which is, however, naively required for κ-symmetry.

The branes of $N = 2B$ SUGRA

Potential	Brane	Tension	Projection operator
$B^{(2)}$	F1	1	$\frac{1}{2}\left(1+\sigma_3\Gamma_{01}\right)$
$C^{(2)}$	D1	$\sqrt{e^{-2\phi}+\ell^2}$	$\frac{1}{2}\left(1+\frac{-e^{-\phi}\sigma_1+\ell\sigma_3}{\sqrt{e^{-2\phi}+\ell^2}}\Gamma_{01}\right)$
$C^{(4)}$	D3	$e^{-\phi}$	$\frac{1}{2}\left(1+i\sigma_2\Gamma_{0123}\right)$
$B^{(6)}$	NS5	$e^{-\phi}\sqrt{e^{-2\phi}+\ell^2}$	$\frac{1}{2}\left(1+\frac{e^{-\phi}\sigma_3+\ell\sigma_1}{\sqrt{e^{-2\phi}+\ell^2}}\Gamma_{01\cdots 5}\right)$
$C^{(6)}$	D5	$e^{-\phi}$	$\frac{1}{2}\left(1+\sigma_1\Gamma_{01\cdots 5}\right)$
$B^{(8)}$	$\widetilde{\mathrm{D7}}$	$e^{-3\phi}+\ell^2 e^{-\phi}$	$\frac{1}{2}\left(1+i\sigma_2\Gamma_{01\cdots 7}\right)$
$C^{(8)}$	D7	$e^{-\phi}$	$\frac{1}{2}\left(1+i\sigma_2\Gamma_{01\cdots 7}\right)$
$\mathcal{D}^{(10)}$	S9	$e^{-2\phi}$	$\frac{1}{2}\left(1+\sigma_3\right)$
$\mathcal{E}^{(10)}$	$\widetilde{\mathrm{S9}}$	$e^{-2\phi}\sqrt{e^{-2\phi}+\ell^2}$	$\frac{1}{2}\left(1+\frac{-e^{-\phi}\sigma_1+\ell\sigma_3}{\sqrt{e^{-2\phi}+\ell^2}}\right)$
$B^{(10)}$	$\widetilde{\mathrm{D9}}$	$e^{-\phi}\left(e^{-2\phi}+\ell^2\right)^{3/2}$	$\frac{1}{2}\left(1-\frac{\ell\sigma_1+e^{-\phi}\sigma_3}{\sqrt{e^{-2\phi}+\ell^2}}\right)$
$C^{(10)}$	D9	$e^{-\phi}$	$\frac{1}{2}\left(1+\sigma_1\right)$

4 – Extensions of $N = 2, d = 4$ SUGRA: supersymmetric solutions

N=2,d=4 SUGRA admits electrically and magnetically charged 1/2 supersymmetric black-hole solutions (Ferrara, Kallosh & Strominger, `hep-th/9508072`, Behrndt, Lüst & Sabra `hep-th/9705169`).

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Those in the vector multiplet sector have the form:

$$\left\{ \begin{array}{rcl} ds^2 & = & dt^2 - dy^2 - 2e^{-\mathcal{K}(Z,Z^*)}|f|^2 dz dz^* \,, \\ \\ Z^i & = & Z^i(z)\,, \qquad\qquad f = f(z)\,, \end{array} \right.$$

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and their Killing spinors take the general form

$$\epsilon_I = (f/f^*)^{1/4}\epsilon_{I\,0}\,, \qquad\qquad \gamma_{\underline{z}^*}\epsilon_{I\,0} = 0\,.$$

In general, the holomorphic functions $Z^i(z)$ will have have singularities and branch cuts and, when crossing a branch cut, their value may be transformed by an element of the isometry group of the Kähler metric $G_V \subseteq Sp(2\bar{n}, \mathbb{R})$.

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So this is basically the (local) story concerning the solutions. Now the question is:

Are there 2-forms in N=2,d=4 SUGRA to which we can couple these strings?

5 – Extensions of $N = 2, d = 4$ SUGRA: 1.- vector fields

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It is easy to introduce the $\bar{n}$ dual vector fields $A_{\Lambda\,\mu}$ that couple to the magnetic charges p^{Λ} of supersymmetric black holes.

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It is easy to introduce the $\bar{n}$ dual vector fields $A_{\Lambda\,\mu}$ that couple to the magnetic charges p^{Λ} of supersymmetric black holes.

All these vectors can be combined into an $Sp(2\bar{n}, \mathbb{R})$ vector

$$\mathcal{A}_{\mu} \equiv \begin{pmatrix} A^{\Lambda}{}_{\mu} \\ A_{\Lambda\,\mu} \end{pmatrix},$$

with supersymmetry transformation rule

$$\delta_{\epsilon}\mathcal{A}_{\mu} \;=\; \tfrac{1}{4}\mathcal{V}\,\epsilon_{IJ}\bar{\psi}^{I}_{\mu}\epsilon^{J} + \tfrac{i}{8}\mathfrak{D}_{i}\mathcal{V}\,\epsilon_{IJ}\bar{\lambda}^{Ii}\gamma_{\mu}\epsilon^{J} + \text{c.c.}\,, \qquad \mathcal{V} = \begin{pmatrix} \mathcal{L}^{\Lambda} \\ \mathcal{M}_{\Lambda} \end{pmatrix}, \qquad \mathfrak{D}_{i}\mathcal{V} = \begin{pmatrix} f^{\Lambda}{}_{i} \\ h_{\Lambda\,i} \end{pmatrix},$$

The supersymmetric, gauge and symplectic-invariant coupling to electric and magnetically charged black holes (0-branes) is given by the worldline action

$$S = \int d\xi \; |\mathcal{Z}| \sqrt{\frac{dX^\mu}{d\xi}\frac{dX^\nu}{d\xi} g_{\mu\nu}(X)} + \int d\xi \langle\, q \mid \mathcal{A}_\mu \,\rangle \frac{dX^\mu}{d\xi} \,.$$

where $\mathcal{Z}$ is the central charge

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We are now prepared to search for the 2-forms.

6 – Extensions of $N = 2, d = 4$ SUGRA: 2.- 2-form fields

The main lesson we learned from the $N = 2B, d = 10$ 7-branes is that the $(d-2)$-form potentials are dual to the isometries of the scalar manifold. Thus, we have to Hodge-dualize the Noether currents associated to the transformations

$$\delta_\alpha Z^i = \alpha^A k_A{}^i(Z)\,, \qquad\qquad \delta_\alpha \mathcal{A}_\mu = \alpha^A T_A \mathcal{A}_\mu\,,$$

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we can write these currents as symplectic-invariant 1-forms

$$J_{N,A} = 2i\langle \mathfrak{D}\mathcal{V}^* \mid T_A \mathcal{V} \rangle + \text{c.c.} + 4 \star \langle \mathcal{F} \mid T_A \mathcal{A} \rangle\,.$$

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$$d \star J_{N,A} = 0 \;\Rightarrow dB_A \equiv \star J_{N,A} = \star[2i\langle\, \mathfrak{D}\mathcal{V}^* \mid T_A \mathcal{V}\,\rangle + \text{c.c.}] - 4\langle\, \mathcal{F} \mid T_A \mathcal{A}\,\rangle\,.$$

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And then we define the gauge-invariant 3-form field-strength

$$H_A \equiv dB_A + 4\langle\, \mathcal{F} \mid T_A \mathcal{A}\,\rangle\,.$$

The B_As are the 2-forms to which the strings of $N = 2, d = 4$ SUGRA will couple to. Their supersymmetry transformation rules are found to be

$$\begin{aligned}\delta_\epsilon B_{A\,\mu\nu} &= -\tfrac{1}{2}\langle\,\mathfrak{D}_i\mathcal{V}\mid T_A\mathcal{V}^*\,\rangle\,\bar{\epsilon}_I\gamma_{\mu\nu}\lambda^{iI} + \text{c.c.}\\ &\quad -i\langle\,\mathcal{V}\mid T_A\mathcal{V}^*\,\rangle\,\bar{\epsilon}^I\gamma_{[\mu}\psi_{I\nu]} + \text{c.c.}\\ &\quad +8\langle\,\mathcal{A}_{[\mu}\mid T_A\delta_\epsilon\mathcal{A}_{\nu]}\,\rangle\,.\end{aligned}$$

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Exactly the same problem arises in the construction of a κ-symmetric worldsheet action for heterotic strings propagating in the background of Yang-Mills fields. The solution in that case is the addition of heterotic fermions whose gauge transformations cancel those of the 2-form (Atick, Dhar & Ratra, Phys. Lett. B **169** (1986) 54).

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