Supersymmetry and the Supergravity Landscape

Tomás Ortín (I.F.T., Madrid)

Seminar given on October 6th 2005 at the University of Groningen

Based on hep-th/0506056 and on work in preparation. Work done in collaboration with

Jorge Bellorín and Mechthild Hübscher (I.F.T., Madrid)

Introduction/Motivation

Introduction/Motivation

☞ The vacuum selection problem is the most important problem in Superstring Theory and similar unification schemes that include gravity.

Introduction/Motivation

- The vacuum selection problem is the most important problem in Superstring Theory and similar unification schemes that include gravity.
- The vacuum state is the most important state of any Quantum Field Theory: its symmetries determine all the kinematic properties: the possible conserved charges and the spectrum of allowed particles.

Introduction/Motivation

- The vacuum selection problem is the most important problem in Superstring Theory and similar unification schemes that include gravity.
- The vacuum state is the most important state of any Quantum Field Theory: its symmetries determine all the kinematic properties: the possible conserved charges and the spectrum of allowed particles.
- In a theory that includes gravity the same is still true and, further, it fixes the scale of energy.

Introduction/Motivation

- The vacuum selection problem is the most important problem in Superstring Theory and similar unification schemes that include gravity.
- The vacuum state is the most important state of any Quantum Field Theory: its symmetries determine all the kinematic properties: the possible conserved charges and the spectrum of allowed particles.
- In a theory that includes gravity the same is still true and, further, it fixes the scale of energy.
- In Kaluza-Klein theories, the symmetries of the vacuum state also determine the interactions.

Introduction/Motivation

- The vacuum selection problem is the most important problem in Superstring Theory and similar unification schemes that include gravity.
- The vacuum state is the most important state of any Quantum Field Theory: its symmetries determine all the kinematic properties: the possible conserved charges and the spectrum of allowed particles.
- In a theory that includes gravity the same is still true and, further, it fixes the scale of energy.
- In Kaluza-Klein theories, the symmetries of the vacuum state also determine the interactions.
- However, in theories that include gravity, the energies of different vacua cannot be compared and it is not known how *the* vacuum is chosen, and, therefore, why our Universe is the way it is.

Introduction/Motivation

- The vacuum selection problem is the most important problem in Superstring Theory and similar unification schemes that include gravity.
- The vacuum state is the most important state of any Quantum Field Theory: its symmetries determine all the kinematic properties: the possible conserved charges and the spectrum of allowed particles.
- In a theory that includes gravity the same is still true and, further, it fixes the scale of energy.
- In Kaluza-Klein theories, the symmetries of the vacuum state also determine the interactions.
- However, in theories that include gravity, the energies of different vacua cannot be compared and it is not known how *the* vacuum is chosen, and, therefore, why our Universe is the way it is.
- This is an old and very well known problem. It is also of crucial importance. And it is still **UNSOLVED**.

The failure to solve the vacuum selection problem through some dynamical mechanism has favored recently a purely statistical approach in which one first has to explore and chart ("classify") the space of vacua

The failure to solve the vacuum selection problem through some dynamical mechanism has favored recently a purely statistical approach in which one first has to explore and chart ("classify") the space of vacua

a.k.a.

Landscape

The failure to solve the vacuum selection problem through some dynamical mechanism has favored recently a purely statistical approach in which one first has to explore and chart ("classify") the space of vacua

a.k.a.

Landscape

In this approach, our Universe is the way it is because the probability of this kind of Universe is overwhelming.

The failure to solve the vacuum selection problem through some dynamical mechanism has favored recently a purely statistical approach in which one first has to explore and chart ("classify") the space of vacua

a.k.a.

Landscape

In this approach, our Universe is the way it is because the probability of this kind of Universe is overwhelming.

Of course, this way of thinking can be combined with different forms of the Anthropic Principle.

The failure to solve the vacuum selection problem through some dynamical mechanism has favored recently a purely statistical approach in which one first has to explore and chart ("classify") the space of vacua

a.k.a.

Landscape

In this approach, our Universe is the way it is because the probability of this kind of Universe is overwhelming.

Of course, this way of thinking can be combined with different forms of the Anthropic Principle.

In this talk we are going to review some recent general results on the classification of supersymmetric String Theory vacua and new techniques that can be used to find them, with $N = 1, 2, d = 4$ SUGRA examples.

The failure to solve the vacuum selection problem through some dynamical mechanism has favored recently a purely statistical approach in which one first has to explore and chart ("classify") the space of vacua

a.k.a.

Landscape

In this approach, our Universe is the way it is because the probability of this kind of Universe is overwhelming.

Of course, this way of thinking can be combined with different forms of the Anthropic Principle.

In this talk we are going to review some recent general results on the classification of supersymmetric String Theory vacua and new techniques that can be used to find them, with $N = 1, 2, d = 4$ SUGRA examples.

But, first, we are going to review briefly how we have come to consider this scenario in our quest for UNIFICATION.

Plan of the Talk:

1 – Unification & Landscape

or

"How We Got Into This Mess"

Index

Slide 1 / 20

1 – Unification & Landscape

or

"How We Got Into This Mess"

Unification has been one of the most fruitful guiding principles in our search for the fundamental components and forces if the Universe. It is also a logical necessity for understanding it.

1 – Unification & Landscape

or

"How We Got Into This Mess"

Unification has been one of the most fruitful guiding principles in our search for the fundamental components and forces if the Universe. It is also a logical necessity for understanding it.

Let's review first how the pursuit of unification has led to the (key, but yet unsolved) vacuum selection problem and this to the idea of landscape.
There have been many instances of unification:

1 – Unification & Landscape

or

"How We Got Into This Mess"

Unification has been one of the most fruitful guiding principles in our search for the fundamental components and forces if the Universe. It is also a logical necessity for understanding it.

Let's review first how the pursuit of unification has led to the (key, but yet unsolved) vacuum selection problem and this to the idea of landscape.
There have been many instances of unification:

1 Electricity $\bigoplus$ Magnetism $\stackrel{\text{Faraday,Maxwell}}{\Longrightarrow}$ Electromagnetism

$$\vec{E}, \vec{B} \longrightarrow (F_{\mu\nu}) \equiv \left(\begin{array}{c|c} 0 & -\vec{E}^T \\ \hline \vec{E} & {}^\star\vec{B} \end{array}\right)$$

Required by the Special Theory of Relativity just as Newtonian gravity and gravitomagnetism are combined in General Relativity.

2 Space $\bigoplus$ Time $\stackrel{\text{Einstein,Minkowski}}{\Longrightarrow}$ Spacetime

$$t, \vec{x} \;\longrightarrow\; (x^\mu) \equiv (ct, \vec{x})\,.$$

Strongly related to the former, is associated to an enhancement of symmetry from the Galileo to the Poincaré group which is not apparent at low speeds, but is never broken.

2 Space $\bigoplus$ Time $\stackrel{\text{Einstein,Minkowski}}{\Longrightarrow}$ Spacetime

$$t, \vec{x} \longrightarrow (x^{\mu}) \equiv (ct, \vec{x}) \, .$$

Strongly related to the former, is associated to an enhancement of symmetry from the Galileo to the Poincaré group which is not apparent at low speeds, but is never broken.

3 Waves $\bigoplus$ Particles $\stackrel{\text{deBroglie}}{\Longrightarrow}$ Quantum particles

Required by the Quantum Mechanics, it is of a completely different nature. There is not enhancement of symmetry involved.

2 Space $\bigoplus$ Time $\stackrel{\text{Einstein,Minkowski}}{\Longrightarrow}$ Spacetime

$$t, \vec{x} \longrightarrow (x^\mu) \equiv (ct, \vec{x})\,.$$

Strongly related to the former, is associated to an enhancement of symmetry from the Galileo to the Poincaré group which is not apparent at low speeds, but is never broken.

3 Waves $\bigoplus$ Particles $\stackrel{\text{deBroglie}}{\Longrightarrow}$ Quantum particles

Required by the Quantum Mechanics, it is of a completely different nature. There is not enhancement of symmetry involved.

4 Gravity (GR) $\bigoplus$ Electromagnetism $\stackrel{\text{Kaluza,Klein,Einstein}}{\Longrightarrow}$ Gravity in higher dimensions

$$g_{\mu\nu}, A_\mu \longrightarrow (\hat{g}_{\hat{\mu}\hat{\nu}}) \equiv \left(\begin{array}{c|c} k^2 & A_\nu \\ \hline A_\mu & g_{\mu\nu} \end{array}\right)$$

This unsuccessful attempt has some differences with the electromagnetic unification:

2 Space $\bigoplus$ Time $\stackrel{\text{Einstein,Minkowski}}{\Longrightarrow}$ Spacetime

$$t, \vec{x} \longrightarrow (x^{\mu}) \equiv (ct, \vec{x})\,.$$

Strongly related to the former, is associated to an enhancement of symmetry from the Galileo to the Poincaré group which is not apparent at low speeds, but is never broken.

3 Waves $\bigoplus$ Particles $\stackrel{\text{deBroglie}}{\Longrightarrow}$ Quantum particles

Required by the Quantum Mechanics, it is of a completely different nature. There is not enhancement of symmetry involved.

4 Gravity (GR) $\bigoplus$ Electromagnetism $\stackrel{\text{Kaluza,Klein,Einstein}}{\Longrightarrow}$ Gravity in higher dimensions

$$g_{\mu\nu}, A_{\mu} \longrightarrow (\hat{g}_{\hat{\mu}\hat{\nu}}) \equiv \left(\begin{array}{c|c} k^2 & A_{\nu} \\ \hline A_{\mu} & g_{\mu\nu} \end{array}\right)$$

This unsuccessful attempt has some differences with the electromagnetic unification:

- There is enhancement of <u>local</u> symmetry from g.c.t.'s in $d = 4$ to g.c.t.'s in $d = 5$, but this symmetry is spontaneously broken (in modern parlance) to g.c.t.'s in $d = 4$ and $U(1)$ due to the (completely arbitrary) choice of vacuum. The rule is always:

global symmetry of the vacuum $\sim$ local symmetry of the reduced theory.

➠ A new massless field is predicted: the Kaluza-Klein scalar k.

- A new massless field is predicted: the Kaluza-Klein scalar k.
- The attempt was unsuccessful (it was perhaps too early) but the ideas involved have stayed around until now.

- A new massless field is predicted: the Kaluza-Klein scalar k.
- The attempt was unsuccessful (it was perhaps too early) but the ideas involved have stayed around until now.

5 Quantum Mechanics $\bigoplus$ Relativistic Field Theory $\overset{\text{Many people...}}{\Longrightarrow}$ Quantum Field Theory

A difficult but fruitful marriage.

- A new massless field is predicted: the Kaluza-Klein scalar k.
- The attempt was unsuccessful (it was perhaps too early) but the ideas involved have stayed around until now.

5 Quantum Mechanics $\bigoplus$ Relativistic Field Theory $\stackrel{\text{Many people...}}{\Longrightarrow}$ Quantum Field Theory

A difficult but fruitful marriage.

6 Weak interactions $\bigoplus$ Electromagnetism $\stackrel{\text{Glashow,Salam,Weinberg}}{\Longrightarrow}$ Electroweak interaction

In this case

- A new massless field is predicted: the Kaluza-Klein scalar k.
- The attempt was unsuccessful (it was perhaps too early) but the ideas involved have stayed around until now.

5 Quantum Mechanics $\bigoplus$ Relativistic Field Theory $\overset{\text{Many people...}}{\Longrightarrow}$ Quantum Field Theory

A difficult but fruitful marriage.

6 Weak interactions $\bigoplus$ Electromagnetism $\overset{\text{Glashow,Salam,Weinberg}}{\Longrightarrow}$ Electroweak interaction

In this case

- Unification is achieved by an enhancement of local (Yang-Mills-type) symmetry, from $U(1)$ to $SU(2) \times U(1)$.

- A new massless field is predicted: the Kaluza-Klein scalar k.
- The attempt was unsuccessful (it was perhaps too early) but the ideas involved have stayed around until now.

5 Quantum Mechanics $\bigoplus$ Relativistic Field Theory $\overset{\text{Many people...}}{\Longrightarrow}$ Quantum Field Theory

A difficult but fruitful marriage.

6 Weak interactions $\bigoplus$ Electromagnetism $\overset{\text{Glashow,Salam,Weinberg}}{\Longrightarrow}$ Electroweak interaction

In this case

- Unification is achieved by an enhancement of <u>local</u> (Yang-Mills-type) symmetry, from $U(1)$ to $SU(2) \times U(1)$.
- The symmetry is spontaneously broken by the Higgs mechanism: choice of vacuum by energetic reasons (minimization of the *ad hoc* Higgs potential). (This is the main difference with Kaluza-Klein and other theories including gravity.)

- A new massless field is predicted: the Kaluza-Klein scalar k.
- The attempt was unsuccessful (it was perhaps too early) but the ideas involved have stayed around until now.

5 Quantum Mechanics $\bigoplus$ Relativistic Field Theory $\overset{\text{Many people...}}{\Longrightarrow}$ Quantum Field Theory

A difficult but fruitful marriage.

6 Weak interactions $\bigoplus$ Electromagnetism $\overset{\text{Glashow,Salam,Weinberg}}{\Longrightarrow}$ Electroweak interaction

In this case

- Unification is achieved by an enhancement of <u>local</u> (Yang-Mills-type) symmetry, from $U(1)$ to $SU(2) \times U(1)$.
- The symmetry is spontaneously broken by the Higgs mechanism: choice of vacuum by energetic reasons (minimization of the *ad hoc* Higgs potential). (This is the main difference with Kaluza-Klein and other theories including gravity.)
- The spontaneous breaking of the symmetry renders the model renormalizable.

- A new massless field is predicted: the Kaluza-Klein scalar k.
- The attempt was unsuccessful (it was perhaps too early) but the ideas involved have stayed around until now.

5 Quantum Mechanics $\bigoplus$ Relativistic Field Theory $\overset{\text{Many people...}}{\Longrightarrow}$ Quantum Field Theory

A difficult but fruitful marriage.

6 Weak interactions $\bigoplus$ Electromagnetism $\overset{\text{Glashow, Salam, Weinberg}}{\Longrightarrow}$ Electroweak interaction

In this case

- Unification is achieved by an enhancement of local (Yang-Mills-type) symmetry, from $U(1)$ to $SU(2) \times U(1)$.
- The symmetry is spontaneously broken by the Higgs mechanism: choice of vacuum by energetic reasons (minimization of the *ad hoc* Higgs potential). (This is the main difference with Kaluza-Klein and other theories including gravity.)
- The spontaneous breaking of the symmetry renders the model renormalizable.
- The symmetry is restored at high energies.

- A new massless field is predicted: the Kaluza-Klein scalar k.
- The attempt was unsuccessful (it was perhaps too early) but the ideas involved have stayed around until now.

5 Quantum Mechanics $\bigoplus$ Relativistic Field Theory $\stackrel{\text{Many people...}}{\Longrightarrow}$ Quantum Field Theory

A difficult but fruitful marriage.

6 Weak interactions $\bigoplus$ Electromagnetism $\stackrel{\text{Glashow,Salam,Weinberg}}{\Longrightarrow}$ Electroweak interaction

In this case

- Unification is achieved by an enhancement of local (Yang-Mills-type) symmetry, from $U(1)$ to $SU(2) \times U(1)$.
- The symmetry is spontaneously broken by the Higgs mechanism: choice of vacuum by energetic reasons (minimization of the *ad hoc* Higgs potential). (This is the main difference with Kaluza-Klein and other theories including gravity.)
- The spontaneous breaking of the symmetry renders the model renormalizable.
- The symmetry is restored at high energies.
- New massive particles are predicted associated to the enhanced symmetry (gauge bosons, found) and a new massless spin-0 particle is also predicted (Higgs boson, not yet found).

- A new massless field is predicted: the Kaluza-Klein scalar k.
- The attempt was unsuccessful (it was perhaps too early) but the ideas involved have stayed around until now.

5 Quantum Mechanics $\bigoplus$ Relativistic Field Theory $\overset{\text{Many people...}}{\Longrightarrow}$ Quantum Field Theory

A difficult but fruitful marriage.

6 Weak interactions $\bigoplus$ Electromagnetism $\overset{\text{Glashow,Salam,Weinberg}}{\Longrightarrow}$ Electroweak interaction

In this case

- Unification is achieved by an enhancement of local (Yang-Mills-type) symmetry, from $U(1)$ to $SU(2) \times U(1)$.
- The symmetry is spontaneously broken by the Higgs mechanism: choice of vacuum by energetic reasons (minimization of the *ad hoc* Higgs potential). (This is the main difference with Kaluza-Klein and other theories including gravity.)
- The spontaneous breaking of the symmetry renders the model renormalizable.
- The symmetry is restored at high energies.
- New massive particles are predicted associated to the enhanced symmetry (gauge bosons, found) and a new massless spin-0 particle is also predicted (Higgs boson, not yet found).

The extraordinary success of this model has made of it the paradigm of unification schemes.

7 Electroweak interaction $\bigoplus$ Strong interactions $\stackrel{\text{Many people...}}{\Longrightarrow}$ Grand Unified Theory

An unsuccessful generalization of the electroweak unification scheme based on a semisimple gauge group $(SO(10), SU(5), \cdots)$ spontaneously broken by a generalized Higgs mechanism to $SU(3) \times U(1)$

7 Electroweak interaction $\bigoplus$ Strong interactions $\overset{\text{Many people...}}{\Longrightarrow}$ Grand Unified Theory

An unsuccessful generalization of the electroweak unification scheme based on a semisimple gauge group $(SO(10), SU(5), \cdots)$ spontaneously broken by a generalized Higgs mechanism to $SU(3) \times U(1)$

- New massive and massless particles predicted may mediate proton desintegration (not observed).

7 Electroweak interaction $\bigoplus$ Strong interactions $\overset{\text{Many people...}}{\Longrightarrow}$ Grand Unified Theory

An unsuccessful generalization of the electroweak unification scheme based on a semisimple gauge group $(SO(10), SU(5), \cdots)$ spontaneously broken by a generalized Higgs mechanism to $SU(3) \times U(1)$

- New massive and massless particles predicted may mediate proton desintegration (not observed).
- Unification of coupling constants should occur at the energy at which the symmetry is restored, but this does not seems to work.

7 Electroweak interaction $\bigoplus$ Strong interactions $\stackrel{\text{Many people...}}{\Longrightarrow}$ Grand Unified Theory

An unsuccessful generalization of the electroweak unification scheme based on a semisimple gauge group $(SO(10), SU(5), \cdots)$ spontaneously broken by a generalized Higgs mechanism to $SU(3) \times U(1)$

- New massive and massless particles predicted may mediate proton desintegration (not observed).
- Unification of coupling constants should occur at the energy at which the symmetry is restored, but this does not seems to work.

8 Bosons $\bigoplus$ Fermions $\stackrel{\text{Golfand,Likhtman,Volkov,Akulov,Soroka,Wess and Zumino}}{\Longrightarrow}$ Superfields

This is a new kind of unification based in an enhancement of (global spacetime) symmetry to supersymmetry, which should also be spontaneously broken by a yet unknown super-Higgs mechanism.

7 Electroweak interaction $\bigoplus$ Strong interactions $\overset{\text{Many people...}}{\Longrightarrow}$ Grand Unified Theory

An unsuccessful generalization of the electroweak unification scheme based on a semisimple gauge group $(SO(10), SU(5), \cdots)$ spontaneously broken by a generalized Higgs mechanism to $SU(3) \times U(1)$

- New massive and massless particles predicted may mediate proton desintegration (not observed).
- Unification of coupling constants should occur at the energy at which the symmetry is restored, but this does not seems to work.

8 Bosons $\bigoplus$ Fermions $\overset{\text{Golfand, Likhtman, Volkov, Akulov, Soroka, Wess and Zumino}}{\Longrightarrow}$ Superfields

This is a new kind of unification based in an enhancement of (global spacetime) symmetry to supersymmetry, which should also be spontaneously broken by a yet unknown super-Higgs mechanism.

- This new symmetry can be combined with Yang-Mills-type symmetries (super-Yang-Mills theories) and with GUT models in which, in some cases, unification of coupling constants can be achieved.

7 Electroweak interaction $\bigoplus$ Strong interactions $\stackrel{\text{Many people...}}{\Longrightarrow}$ Grand Unified Theory

An unsuccessful generalization of the electroweak unification scheme based on a semisimple gauge group $(SO(10), SU(5), \cdots)$ spontaneously broken by a generalized Higgs mechanism to $SU(3) \times U(1)$

- New massive and massless particles predicted may mediate proton desintegration (not observed).
- Unification of coupling constants should occur at the energy at which the symmetry is restored, but this does not seems to work.

8 Bosons $\bigoplus$ Fermions $\stackrel{\text{Golfand,Likhtman,Volkov,Akulov,Soroka,Wess and Zumino}}{\Longrightarrow}$ Superfields

This is a new kind of unification based in an enhancement of (global spacetime) symmetry to supersymmetry, which should also be spontaneously broken by a yet unknown super-Higgs mechanism.

- This new symmetry can be combined with Yang-Mills-type symmetries (super-Yang-Mills theories) and with GUT models in which, in some cases, unification of coupling constants can be achieved.
- It is the most general extension of the Poincaré and Yang-Mills symmetries of the S-matrix (Haag-Lopuszanski-Sohnius).

7 Electroweak interaction $\bigoplus$ Strong interactions $\overset{\text{Many people...}}{\Longrightarrow}$ Grand Unified Theory

An unsuccessful generalization of the electroweak unification scheme based on a semisimple gauge group $(SO(10), SU(5), \cdots)$ spontaneously broken by a generalized Higgs mechanism to $SU(3) \times U(1)$

- New massive and massless particles predicted may mediate proton desintegration (not observed).
- Unification of coupling constants should occur at the energy at which the symmetry is restored, but this does not seems to work.

8 Bosons $\bigoplus$ Fermions $\overset{\text{Golfand,Likhtman,Volkov,Akulov,Soroka,Wess and Zumino}}{\Longrightarrow}$ Superfields

This is a new kind of unification based in an enhancement of (global spacetime) symmetry to supersymmetry, which should also be spontaneously broken by a yet unknown super-Higgs mechanism.

- This new symmetry can be combined with Yang-Mills-type symmetries (super-Yang-Mills theories) and with GUT models in which, in some cases, unification of coupling constants can be achieved.
- It is the most general extension of the Poincaré and Yang-Mills symmetries of the S-matrix (Haag-Lopuszanski-Sohnius).
- It can also be combined with g.c.t.'s, making it local (supergravity theories). We can have supergravity theories with Yang-Mills fields etc. etc. But in most of these theories gravity is not unified with the other interactions.

- However, extended $(N > 1)$ supergravities contain in the same supermultiplet of the graviton additional bosonic fields that may describe the other interactions. In this scheme all interactions would be described in a unified way.

- However, extended ($N > 1$) supergravities contain in the same supermultiplet of the graviton additional bosonic fields that may describe the other interactions. In this scheme all interactions would be described in a unified way.
- These extended supergravities can in general be obtained from compactification of simpler higher-dimensional supergravities. It was also discovered that many $N = 1$ supergravities coupled to Yang-Mills fields could also be obtained in the same way, by a careful choice of compact manifold (i.e. Kaluza-Klein vacuum). This lead to a new brand of unified theories which could describe everything: Theories Of Everything.

- However, extended ($N > 1$) supergravities contain in the same supermultiplet of the graviton additional bosonic fields that may describe the other interactions. In this scheme all interactions would be described in a unified way.
- These extended supergravities can in general be obtained from compactification of simpler higher-dimensional supergravities. It was also discovered that many $N = 1$ supergravities coupled to Yang-Mills fields could also be obtained in the same way, by a careful choice of compact manifold (i.e. Kaluza-Klein vacuum). This lead to a new brand of unified theories which could describe everything: Theories Of Everything.

9 Kaluza-Klein Supergravity

Based in compactifications of $N = 1, d = 11$ supergravity, the unique supergravity that can be constructed in the highest dimension in which a supergravity can be constructed. It can accomodate the bosonic part of the Standard Model.

- However, extended ($N > 1$) supergravities contain in the same supermultiplet of the graviton additional bosonic fields that may describe the other interactions. In this scheme all interactions would be described in a unified way.
- These extended supergravities can in general be obtained from compactification of simpler higher-dimensional supergravities. It was also discovered that many $N = 1$ supergravities coupled to Yang-Mills fields could also be obtained in the same way, by a careful choice of compact manifold (i.e. Kaluza-Klein vacuum). This lead to a new brand of unified theories which could describe everything: Theories Of Everything.

9 Kaluza-Klein Supergravity

Based in compactifications of $N = 1, d = 11$ supergravity, the unique supergravity that can be constructed in the highest dimension in which a supergravity can be constructed. It can accomodate the bosonic part of the Standard Model.

- But these theories are anomalous and it is impossible to obtain the chiral structure of the Standard Model by compactification on smooth manifolds (Witten).

- However, extended $(N > 1)$ supergravities contain in the same supermultiplet of the graviton additional bosonic fields that may describe the other interactions. In this scheme all interactions would be described in a unified way.
- These extended supergravities can in general be obtained from compactification of simpler higher-dimensional supergravities. It was also discovered that many $N = 1$ supergravities coupled to Yang-Mills fields could also be obtained in the same way, by a careful choice of compact manifold (i.e. Kaluza-Klein vacuum). This lead to a new brand of unified theories which could describe everything: Theories Of Everything.

9 Kaluza-Klein Supergravity

Based in compactifications of $N = 1, d = 11$ supergravity, the unique supergravity that can be constructed in the highest dimension in which a supergravity can be constructed. It can accomodate the bosonic part of the Standard Model.

- But these theories are anomalous and it is impossible to obtain the chiral structure of the Standard Model by compactification on smooth manifolds (Witten).
- These problems and the advent of String Theory, anomaly-free and with chiral fermions were possible, killed these theories, although they have resurrected again.

- However, extended ($N > 1$) supergravities contain in the same supermultiplet of the graviton additional bosonic fields that may describe the other interactions. In this scheme all interactions would be described in a unified way.
- These extended supergravities can in general be obtained from compactification of simpler higher-dimensional supergravities. It was also discovered that many $N = 1$ supergravities coupled to Yang-Mills fields could also be obtained in the same way, by a careful choice of compact manifold (i.e. Kaluza-Klein vacuum). This lead to a new brand of unified theories which could describe everything: Theories Of Everything.

9 Kaluza-Klein Supergravity

Based in compactifications of $N = 1, d = 11$ supergravity, the unique supergravity that can be constructed in the highest dimension in which a supergravity can be constructed. It can accomodate the bosonic part of the Standard Model.

- But these theories are anomalous and it is impossible to obtain the chiral structure of the Standard Model by compactification on smooth manifolds (Witten).
- These problems and the advent of String Theory, anomaly-free and with chiral fermions were possible, killed these theories, although they have resurrected again.
- The vacuum of this theory was arbitrarily chosen to recover the Standard Model. Conceptually, the arbitrariness in the choice of vacuum replaces that of the choice of Higgs field and potential (and gauge interactions, dimensionality...).

- However, extended ($N > 1$) supergravities contain in the same supermultiplet of the graviton additional bosonic fields that may describe the other interactions. In this scheme all interactions would be described in a unified way.
- These extended supergravities can in general be obtained from compactification of simpler higher-dimensional supergravities. It was also discovered that many $N = 1$ supergravities coupled to Yang-Mills fields could also be obtained in the same way, by a careful choice of compact manifold (i.e. Kaluza-Klein vacuum). This lead to a new brand of unified theories which could describe everything: Theories Of Everything.

9 Kaluza-Klein Supergravity

Based in compactifications of $N = 1, d = 11$ supergravity, the unique supergravity that can be constructed in the highest dimension in which a supergravity can be constructed. It can accomodate the bosonic part of the Standard Model.

- But these theories are anomalous and it is impossible to obtain the chiral structure of the Standard Model by compactification on smooth manifolds (Witten).
- These problems and the advent of String Theory, anomaly-free and with chiral fermions were possible, killed these theories, although they have resurrected again.
- The vacuum of this theory was arbitrarily chosen to recover the Standard Model. Conceptually, the arbitrariness in the choice of vacuum replaces that of the choice of Higgs field and potential (and gauge interactions, dimensionality...).

The rule of this game is:

global supersymmetry of the vacuum $\sim$ local supersymmetry of the reduced theory.

10 Superstring Theories

All particles are different vibration states of a single physical entity: the superstring. All known interactions can be described in this way. At low energies, one recovers an anomaly-free supergravity theory theory. However

10 Superstring Theories

All particles are different vibration states of a single physical entity: the superstring. All known interactions can be described in this way. At low energies, one recovers an anomaly-free supergravity theory theory. However

- They are 10-dimensional, and require compactification. At low energies we are faced with 10-dimensional Kaluza-Klein supergravity and the vacuum selection problem.

10 Superstring Theories

All particles are different vibration states of a single physical entity: the superstring. All known interactions can be described in this way. At low energies, one recovers an anomaly-free supergravity theory theory. However

- They are 10-dimensional, and require compactification. At low energies we are faced with 10-dimensional Kaluza-Klein supergravity and the vacuum selection problem.
- There are at least five superstring theories: Types IIA, Type IIB, Heterotic $SO(32)$, Heterotic $E(8) \times E(8)$ and Type I $SO(32)$. Which one to take?

10 Superstring Theories

All particles are different vibration states of a single physical entity: the superstring. All known interactions can be described in this way. At low energies, one recovers an anomaly-free supergravity theory theory. However

- They are 10-dimensional, and require compactification. At low energies we are faced with 10-dimensional Kaluza-Klein supergravity and the vacuum selection problem.
- There are at least five superstring theories: Types IIA, Type IIB, Heterotic $SO(32)$, Heterotic $E(8) \times E(8)$ and Type I $SO(32)$. Which one to take?
- The theory seems to contain other extended objects besides strings: D-branes, NSNS-branes... Why should strings be fundamental?

10 Superstring Theories

All particles are different vibration states of a single physical entity: the superstring. All known interactions can be described in this way. At low energies, one recovers an anomaly-free supergravity theory theory. However

- They are 10-dimensional, and require compactification. At low energies we are faced with 10-dimensional Kaluza-Klein supergravity and the vacuum selection problem.
- There are at least five superstring theories: Types IIA, Type IIB, Heterotic $SO(32)$, Heterotic $E(8) \times E(8)$ and Type I $SO(32)$. Which one to take?
- The theory seems to contain other extended objects besides strings: D-branes, NSNS-branes... Why should strings be fundamental?

11 $\bigoplus$ Superstring Theories $\stackrel{\text{Witten et al.}}{\Longrightarrow}$ M theory

All the superstring theories are understood as different duality-related vacua of an unknown theory, one of whose low-energy limits is $N = 1, d = 11$ supergravity.

10 Superstring Theories

All particles are different vibration states of a single physical entity: the superstring. All known interactions can be described in this way. At low energies, one recovers an anomaly-free supergravity theory theory. However

- They are 10-dimensional, and require compactification. At low energies we are faced with 10-dimensional Kaluza-Klein supergravity and the vacuum selection problem.
- There are at least five superstring theories: Types IIA, Type IIB, Heterotic $SO(32)$, Heterotic $E(8) \times E(8)$ and Type I $SO(32)$. Which one to take?
- The theory seems to contain other extended objects besides strings: D-branes, NSNS-branes... Why should strings be fundamental?

11 $\bigoplus$ Superstring Theories $\stackrel{\text{Witten et al.}}{\Longrightarrow}$ M theory

All the superstring theories are understood as different duality-related vacua of an unknown theory, one of whose low-energy limits is $N = 1, d = 11$ supergravity.

- Now we are back into the old Kaluza-Klein supergravity scenario. The chirality problem can be solved by considering non-smooth manifolds (orbifolds).

10 Superstring Theories

All particles are different vibration states of a single physical entity: the superstring. All known interactions can be described in this way. At low energies, one recovers an anomaly-free supergravity theory theory. However

- They are 10-dimensional, and require compactification. At low energies we are faced with 10-dimensional Kaluza-Klein supergravity and the vacuum selection problem.
- There are at least five superstring theories: Types IIA, Type IIB, Heterotic $SO(32)$, Heterotic $E(8) \times E(8)$ and Type I $SO(32)$. Which one to take?
- The theory seems to contain other extended objects besides strings: D-branes, NSNS-branes... Why should strings be fundamental?

11 $\bigoplus$ Superstring Theories $\stackrel{\text{Witten et al.}}{\Longrightarrow}$ M theory

All the superstring theories are understood as different duality-related vacua of an unknown theory, one of whose low-energy limits is $N = 1, d = 11$ supergravity.

- Now we are back into the old Kaluza-Klein supergravity scenario. The chirality problem can be solved by considering non-smooth manifolds (orbifolds).
- The vacuum selection problem remains, although with some improvements because many vacua are related by dualities.

10 Superstring Theories

All particles are different vibration states of a single physical entity: the superstring. All known interactions can be described in this way. At low energies, one recovers an anomaly-free supergravity theory theory. However

- They are 10-dimensional, and require compactification. At low energies we are faced with 10-dimensional Kaluza-Klein supergravity and the vacuum selection problem.
- There are at least five superstring theories: Types IIA, Type IIB, Heterotic $SO(32)$, Heterotic $E(8) \times E(8)$ and Type I $SO(32)$. Which one to take?
- The theory seems to contain other extended objects besides strings: D-branes, NSNS-branes... Why should strings be fundamental?

11 $\bigoplus$ Superstring Theories $\overset{\text{Witten et al.}}{\Longrightarrow}$ M theory

All the superstring theories are understood as different duality-related vacua of an unknown theory, one of whose low-energy limits is $N = 1, d = 11$ supergravity.

- Now we are back into the old Kaluza-Klein supergravity scenario. The chirality problem can be solved by considering non-smooth manifolds (orbifolds).
- The vacuum selection problem remains, although with some improvements because many vacua are related by dualities.
- Strings, D-branes etc. are related by dualities and they are on the same footing.

10 Superstring Theories

All particles are different vibration states of a single physical entity: the superstring. All known interactions can be described in this way. At low energies, one recovers an anomaly-free supergravity theory theory. However

- They are 10-dimensional, and require compactification. At low energies we are faced with 10-dimensional Kaluza-Klein supergravity and the vacuum selection problem.
- There are at least five superstring theories: Types IIA, Type IIB, Heterotic $SO(32)$, Heterotic $E(8) \times E(8)$ and Type I $SO(32)$. Which one to take?
- The theory seems to contain other extended objects besides strings: D-branes, NSNS-branes... Why should strings be fundamental?

11 $\bigoplus$ Superstring Theories $\stackrel{\text{Witten et al.}}{\Longrightarrow}$ M theory

All the superstring theories are understood as different duality-related vacua of an unknown theory, one of whose low-energy limits is $N = 1, d = 11$ supergravity.

- Now we are back into the old Kaluza-Klein supergravity scenario. The chirality problem can be solved by considering non-smooth manifolds (orbifolds).
- The vacuum selection problem remains, although with some improvements because many vacua are related by dualities.
- Strings, D-branes etc. are related by dualities and they are on the same footing.

> This theory could satisfy all our desires for unification, but we have to find in it our Universe's vacuum and explain why and how it is selected.

Since many things seem to work, the vacuum-selection problem (of which the moduli estabilization problem is just another manifestation) becomes more acute.

Since many things seem to work, the vacuum-selection problem (of which the moduli estabilization problem is just another manifestation) becomes more acute.

Further, nowadays we also ask more from vacua than just the Standard Model:

Since many things seem to work, the vacuum-selection problem (of which the moduli estabilization problem is just another manifestation) becomes more acute.

Further, nowadays we also ask more from vacua than just the Standard Model:

- They should support INFLATION.

Since many things seem to work, the vacuum-selection problem (of which the moduli estabilization problem is just another manifestation) becomes more acute.

Further, nowadays we also ask more from vacua than just the Standard Model:

- They should support INFLATION.
- They should explain in a fundamental way DARK ENERGY.

Since many things seem to work, the vacuum-selection problem (of which the moduli estabilization problem is just another manifestation) becomes more acute.

Further, nowadays we also ask more from vacua than just the Standard Model:

- They should support INFLATION.
- They should explain in a fundamental way DARK ENERGY.
- ...

Since many things seem to work, the vacuum-selection problem (of which the moduli estabilization problem is just another manifestation) becomes more acute.

Further, nowadays we also ask more from vacua than just the Standard Model:

- They should support INFLATION.
- They should explain in a fundamental way DARK ENERGY.
- ...

Two directions of work:

Since many things seem to work, the vacuum-selection problem (of which the moduli estabilization problem is just another manifestation) becomes more acute.

Further, nowadays we also ask more from vacua than just the Standard Model:

- They should support INFLATION.
- They should explain in a fundamental way DARK ENERGY.
- ...

Two directions of work: $\Bigg\{$

Since many things seem to work, the vacuum-selection problem (of which the moduli estabilization problem is just another manifestation) becomes more acute.

Further, nowadays we also ask more from vacua than just the Standard Model:

- They should support INFLATION.
- They should explain in a fundamental way DARK ENERGY.
- ...

Two directions of work: { Find phenomenologically viable vacua.

Since many things seem to work, the vacuum-selection problem (of which the moduli estabilization problem is just another manifestation) becomes more acute.

Further, nowadays we also ask more from vacua than just the Standard Model:

- They should support INFLATION.
- They should explain in a fundamental way DARK ENERGY.
- ...

Two directions of work:
- Find phenomenologically viable vacua.
- Find a vacuum-selection mechanism.

Since many things seem to work, the vacuum-selection problem (of which the moduli estabilization problem is just another manifestation) becomes more acute.

Further, nowadays we also ask more from vacua than just the Standard Model:

- They should support INFLATION.
- They should explain in a fundamental way DARK ENERGY.
- ...

Two directions of work: $\left\{ \begin{array}{l} \text{Find phenomenologically viable vacua.} \\ \\ \text{Find a vacuum-selection mechanism.} \end{array} \right.$

There has not been real progress in the second direction for many years. This has lead to an statistical/anthropical approach to the problem which requires the knowledge of the *space of M theory vacua* a.k.a. *landscape*.

Since many things seem to work, the vacuum-selection problem (of which the moduli estabilization problem is just another manifestation) becomes more acute.

Further, nowadays we also ask more from vacua than just the Standard Model:

- They should support INFLATION.
- They should explain in a fundamental way DARK ENERGY.
- ...

Two directions of work: { Find phenomenologically viable vacua.
Find a vacuum-selection mechanism.

There has not been real progress in the second direction for many years. This has lead to an statistical/anthropical approach to the problem which requires the knowledge of the *space of M theory vacua* a.k.a. *landscape*.

In the original proposal, only vacua with 4 noncompact spacetime dimensions and 6 space dimensions compactified in a Calabi-Yau space (which gives $N = 1, d = 4$ supergravities) were considered.

Since many things seem to work, the vacuum-selection problem (of which the moduli estabilization problem is just another manifestation) becomes more acute.

Further, nowadays we also ask more from vacua than just the Standard Model:

- They should support INFLATION.
- They should explain in a fundamental way DARK ENERGY.
- ...

Two directions of work: $\left\{ \begin{array}{l} \text{Find phenomenologically viable vacua.} \\ \\ \text{Find a vacuum-selection mechanism.} \end{array} \right.$

There has not been real progress in the second direction for many years. This has lead to an statistical/anthropical approach to the problem which requires the knowledge of the *space of M theory vacua* a.k.a. *landscape*.

In the original proposal, only vacua with 4 noncompact spacetime dimensions and 6 space dimensions compactified in a Calabi-Yau space (which gives $N = 1, d = 4$ supergravities) were considered.

But this is only a (computationally necessary) simplification of the genuine problem in which all possible compactifications should be considered.

One can consider other simplified formulations of the same problem:

One can consider other simplified formulations of the same problem:

- Supergravity landscape (Van Proeyen): the space of all possible supergravities covers all possible low-energy limits of supersymmetric M theory vacua. It is not known if all supergravities can be given an M theory origin, but the problem could be treated in a systematic way.

One can consider other simplified formulations of the same problem:

- Supergravity landscape (Van Proeyen): the space of all possible supergravities covers all possible low-energy limits of supersymmetric M theory vacua. It is not known if all supergravities can be given an M theory origin, but the problem could be treated in a systematic way.
- Landscape of supersymmetric vacua: the space of all supersymmetric solutions of 11- and 10-dimensional supergravities covers all possible supersymmetric M theory compactification vacua plus the supersymmetric solutions of the corresponding lower-dimensional supergravities. It also covers all the supersymmetric objects (black holes, p-branes...) of M theory.

One can consider other simplified formulations of the same problem:

- Supergravity landscape (Van Proeyen): the space of all possible supergravities covers all possible low-energy limits of supersymmetric M theory vacua. It is not known if all supergravities can be given an M theory origin, but the problem could be treated in a systematic way.
- Landscape of supersymmetric vacua: the space of all supersymmetric solutions of 11- and 10-dimensional supergravities covers all possible supersymmetric M theory compactification vacua plus the supersymmetric solutions of the corresponding lower-dimensional supergravities. It also covers all the supersymmetric objects (black holes, p-branes...) of M theory.

Finding and classifying all these supersymmetric solutions is a tractable but very complicated problem, at it is the subject of this talk.

One can consider other simplified formulations of the same problem:

- Supergravity landscape (Van Proeyen): the space of all possible supergravities covers all possible low-energy limits of supersymmetric M theory vacua. It is not known if all supergravities can be given an M theory origin, but the problem could be treated in a systematic way.
- Landscape of supersymmetric vacua: the space of all supersymmetric solutions of 11- and 10-dimensional supergravities covers all possible supersymmetric M theory compactification vacua plus the supersymmetric solutions of the corresponding lower-dimensional supergravities. It also covers all the supersymmetric objects (black holes, p-branes...) of M theory.

Finding and classifying all these supersymmetric solutions is a tractable but very complicated problem, at it is the subject of this talk.

First, we are going to define what we mean by supersymmetric solutions and we are going to see

One can consider other simplified formulations of the same problem:

- Supergravity landscape (Van Proeyen): the space of all possible supergravities covers all possible low-energy limits of supersymmetric M theory vacua. It is not known if all supergravities can be given an M theory origin, but the problem could be treated in a systematic way.
- Landscape of supersymmetric vacua: the space of all supersymmetric solutions of 11- and 10-dimensional supergravities covers all possible supersymmetric M theory compactification vacua plus the supersymmetric solutions of the corresponding lower-dimensional supergravities. It also covers all the supersymmetric objects (black holes, p-branes...) of M theory.

Finding and classifying all these supersymmetric solutions is a tractable but very complicated problem, at it is the subject of this talk.

First, we are going to define what we mean by supersymmetric solutions and we are going to see

- How to characterize them.

One can consider other simplified formulations of the same problem:

- Supergravity landscape (Van Proeyen): the space of all possible supergravities covers all possible low-energy limits of supersymmetric M theory vacua. It is not known if all supergravities can be given an M theory origin, but the problem could be treated in a systematic way.
- Landscape of supersymmetric vacua: the space of all supersymmetric solutions of 11- and 10-dimensional supergravities covers all possible supersymmetric M theory compactification vacua plus the supersymmetric solutions of the corresponding lower-dimensional supergravities. It also covers all the supersymmetric objects (black holes, p-branes...) of M theory.

Finding and classifying all these supersymmetric solutions is a tractable but very complicated problem, at it is the subject of this talk.

First, we are going to define what we mean by supersymmetric solutions and we are going to see

- How to characterize them.
- How they lead to lower-dimensional supergravities.

One can consider other simplified formulations of the same problem:

- Supergravity landscape (Van Proeyen): the space of all possible supergravities covers all possible low-energy limits of supersymmetric M theory vacua. It is not known if all supergravities can be given an M theory origin, but the problem could be treated in a systematic way.

- Landscape of supersymmetric vacua: the space of all supersymmetric solutions of 11- and 10-dimensional supergravities covers all possible supersymmetric M theory compactification vacua plus the supersymmetric solutions of the corresponding lower-dimensional supergravities. It also covers all the supersymmetric objects (black holes, p-branes...) of M theory.

Finding and classifying all these supersymmetric solutions is a tractable but very complicated problem, at it is the subject of this talk.

First, we are going to define what we mean by supersymmetric solutions and we are going to see

- How to characterize them.

- How they lead to lower-dimensional supergravities.

- The relation between the supersymmetries of the solution and those of the supergravities.

One can consider other simplified formulations of the same problem:

- Supergravity landscape (Van Proeyen): the space of all possible supergravities covers all possible low-energy limits of supersymmetric M theory vacua. It is not known if all supergravities can be given an M theory origin, but the problem could be treated in a systematic way.
- Landscape of supersymmetric vacua: the space of all supersymmetric solutions of 11- and 10-dimensional supergravities covers all possible supersymmetric M theory compactification vacua plus the supersymmetric solutions of the corresponding lower-dimensional supergravities. It also covers all the supersymmetric objects (black holes, p-branes...) of M theory.

Finding and classifying all these supersymmetric solutions is a tractable but very complicated problem, at it is the subject of this talk.

First, we are going to define what we mean by supersymmetric solutions and we are going to see

- How to characterize them.
- How they lead to lower-dimensional supergravities.
- The relation between the supersymmetries of the solution and those of the supergravities.
- How to find all of them (*Tod's problem*).

One can consider other simplified formulations of the same problem:

- Supergravity landscape (Van Proeyen): the space of all possible supergravities covers all possible low-energy limits of supersymmetric M theory vacua. It is not known if all supergravities can be given an M theory origin, but the problem could be treated in a systematic way.
- Landscape of supersymmetric vacua: the space of all supersymmetric solutions of 11- and 10-dimensional supergravities covers all possible supersymmetric M theory compactification vacua plus the supersymmetric solutions of the corresponding lower-dimensional supergravities. It also covers all the supersymmetric objects (black holes, p-branes...) of M theory.

Finding and classifying all these supersymmetric solutions is a tractable but very complicated problem, at it is the subject of this talk.

First, we are going to define what we mean by supersymmetric solutions and we are going to see

- How to characterize them.
- How they lead to lower-dimensional supergravities.
- The relation between the supersymmetries of the solution and those of the supergravities.
- How to find all of them (*Tod's problem*).
- Some useful identities that they always satisfy (Killing spinor identities).

One can consider other simplified formulations of the same problem:

- Supergravity landscape (Van Proeyen): the space of all possible supergravities covers all possible low-energy limits of supersymmetric M theory vacua. It is not known if all supergravities can be given an M theory origin, but the problem could be treated in a systematic way.
- Landscape of supersymmetric vacua: the space of all supersymmetric solutions of 11- and 10-dimensional supergravities covers all possible supersymmetric M theory compactification vacua plus the supersymmetric solutions of the corresponding lower-dimensional supergravities. It also covers all the supersymmetric objects (black holes, p-branes...) of M theory.

Finding and classifying all these supersymmetric solutions is a tractable but very complicated problem, at it is the subject of this talk.

First, we are going to define what we mean by supersymmetric solutions and we are going to see

- How to characterize them.
- How they lead to lower-dimensional supergravities.
- The relation between the supersymmetries of the solution and those of the supergravities.
- How to find all of them (*Tod's problem*).
- Some useful identities that they always satisfy (Killing spinor identities).
- Application to $N = 1, 2, d = 4$ supergravity.

2 – Susy Solutions

Index

Slide 9 / 20

2 – Susy Solutions

Supersymmetric solutions (a.k.a. solutions with residual or unbroken or preserved supersymmetry) are classical bosonic solutions of supergravity (SUGRA) theories which are invariant under some supersymmetry transformations.

2 – Susy Solutions

Index

Slide 9 / 20

Supersymmetric solutions (a.k.a. solutions with residual or unbroken or preserved supersymmetry) are classical bosonic solutions of supergravity (SUGRA) theories which are invariant under some supersymmetry transformations.

Generically, the supersymmetry transformations take the form

$$\delta_\epsilon \phi^b \sim \bar{\epsilon}\phi^f \,, \qquad \delta_\epsilon \phi^f \sim \partial\epsilon + \phi^b \epsilon \,. \qquad (1)$$

2 – Susy Solutions

Supersymmetric solutions (a.k.a. solutions with residual or unbroken or preserved supersymmetry) are classical bosonic solutions of supergravity (SUGRA) theories which are invariant under some supersymmetry transformations.

Generically, the supersymmetry transformations take the form

$$\delta_\epsilon \phi^b \sim \bar{\epsilon}\phi^f \,, \qquad \delta_\epsilon \phi^f \sim \partial\epsilon + \phi^b \epsilon \,. \tag{1}$$

Then, a bosonic configuration ($\phi^f = 0$) will be invariant under the infinitesimal supersymmetry transformation generated by the parameter $\epsilon^\alpha(x)$ if it satisfies the *Killing spinor equations* (one for each f)

$$\delta_\epsilon \phi^f \sim \partial\epsilon + \phi^b \epsilon = 0 \,. \tag{2}$$

2 – Susy Solutions

Supersymmetric solutions (a.k.a. solutions with residual or unbroken or preserved supersymmetry) are classical bosonic solutions of supergravity (SUGRA) theories which are invariant under some supersymmetry transformations.

Generically, the supersymmetry transformations take the form

$$\delta_\epsilon \phi^b \sim \bar{\epsilon}\phi^f \,, \qquad \delta_\epsilon \phi^f \sim \partial\epsilon + \phi^b \epsilon \,. \tag{1}$$

Then, a bosonic configuration ($\phi^f = 0$) will be invariant under the infinitesimal supersymmetry transformation generated by the parameter $\epsilon^\alpha(x)$ if it satisfies the *Killing spinor equations* (one for each f)

$$\delta_\epsilon \phi^f \sim \partial\epsilon + \phi^b \epsilon = 0 \,. \tag{2}$$

This is a generalization of the concept of isometry, an infinitesimal general coordinate transformation generated by $\xi^\mu(x)$ that leaves the metric $g_{\mu\nu}$ invariant because it satisfies the *Killing (vector) equation*

$$\delta_\xi g_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)} = 0 \,. \tag{3}$$

To each bosonic symmetry we associate a generator

$$\xi^{\mu}_{(I)}(x) \to P_I \, ,$$

of a symmetry algebra

$$[P_I, P_J] = f_{IJ}{}^{K} P_K \, .$$

To each bosonic symmetry we associate a generator

$$\xi^{\mu}_{(I)}(x) \rightarrow P_I \,,$$

of a symmetry algebra

$$[P_I, P_J] = f_{IJ}{}^{K} P_K \,.$$

The supersymmetries are associated to the odd generators

$$\epsilon^{\alpha}_{(n)}(x) \rightarrow \mathcal{Q}_n \,,$$

of a superalgebra

$$[\mathcal{Q}_n, P_I] = f_{nI}{}^{m} \mathcal{Q}_m \,, \qquad \{\mathcal{Q}_n, \mathcal{Q}_m\} = f_{nm}{}^{I} P_I \,.$$

To each bosonic symmetry we associate a generator

$$\xi^{\mu}_{(I)}(x) \rightarrow P_I \,,$$

of a symmetry algebra

$$[P_I, P_J] = f_{IJ}{}^{K} P_K \,.$$

The supersymmetries are associated to the odd generators

$$\epsilon^{\alpha}_{(n)}(x) \rightarrow \mathcal{Q}_n \,,$$

of a superalgebra

$$[\mathcal{Q}_n, P_I] = f_{nI}{}^{m} \mathcal{Q}_m \,, \qquad \{\mathcal{Q}_n, \mathcal{Q}_m\} = f_{nm}{}^{I} P_I \,.$$

Kaluza-Klein principle:
These global supersymmetries of the vacuum solution become the local supersymmetries of the supergravity built on it.

To each bosonic symmetry we associate a generator

$$\xi^{\mu}_{(I)}(x) \rightarrow P_I \,,$$

of a symmetry algebra

$$[P_I, P_J] = f_{IJ}{}^{K} P_K \,.$$

The supersymmetries are associated to the odd generators

$$\epsilon^{\alpha}_{(n)}(x) \rightarrow \mathcal{Q}_n \,,$$

of a superalgebra

$$[\mathcal{Q}_n, P_I] = f_{nI}{}^{m} \mathcal{Q}_m \,, \qquad \{\mathcal{Q}_n, \mathcal{Q}_m\} = f_{nm}{}^{I} P_I \,.$$

Kaluza-Klein principle:
These global supersymmetries of the vacuum solution become the local supersymmetries of the supergravity built on it.

When the supersymmetric vacuum solution has a clear (possibly warped) product structure we distinguish internal and spacetime symmetries
$\longrightarrow$ spontaneous compactification.

3 – Tod's problem

This is the problem of finding **all** the bosonic field configurations ϕ^b for which a SUGRA's Killing spinor equations

$$\delta_\epsilon \phi^f \Big|_{\phi^f=0} \sim \partial\epsilon + \phi^b \epsilon = 0\,,$$

have a solution ϵ, (i.e. all the possible supersymmetric bosonic field configurations ϕ^b), which includes all the possible supersymmetric vacua and compactifications.

a

3 – Tod's problem

This is the problem of finding **all** the bosonic field configurations ϕ^b for which a SUGRA's Killing spinor equations

$$\delta_\epsilon \phi^f \Big|_{\phi^f=0} \sim \partial\epsilon + \phi^b \epsilon = 0\,,$$

have a solution ϵ, (i.e. all the possible supersymmetric bosonic field configurations ϕ^b), which includes all the possible supersymmetric vacua and compactifications.

N.B. Not all supersymmetric bosonic field configurations satisfy the classical bosonic equations of motion $\frac{\delta S}{\delta \phi^b}\Big|_{\phi^f=0} \equiv S_{,b}|_{\phi^f=0} \equiv \mathcal{E}(\phi^b)$.

a

3 – Tod's problem

This is the problem of finding **all** the bosonic field configurations ϕ^b for which a SUGRA's Killing spinor equations

$$\delta_\epsilon \phi^f \Big|_{\phi^f=0} \sim \partial\epsilon + \phi^b \epsilon = 0 \,,$$

have a solution ϵ, (i.e. all the possible supersymmetric bosonic field configurations ϕ^b), which includes all the possible supersymmetric vacua and compactifications.

N.B. Not all supersymmetric bosonic field configurations satisfy the classical bosonic equations of motion $\frac{\delta S}{\delta \phi^b}\Big|_{\phi^f=0} \equiv S_{,b}|_{\phi^f=0} \equiv \mathcal{E}(\phi^b)$.

Actually, the bosonic equations of motion of supersymmetric bosonic field configurations satisfy the so-called *Killing spinor identities*[a].

a

3 – Tod's problem

This is the problem of finding **all** the bosonic field configurations ϕ^b for which a SUGRA's Killing spinor equations

$$\delta_\epsilon \phi^f \big|_{\phi^f=0} \sim \partial\epsilon + \phi^b \epsilon = 0\,,$$

have a solution ϵ, (i.e. all the possible supersymmetric bosonic field configurations ϕ^b), which includes all the possible supersymmetric vacua and compactifications.

N.B. Not all supersymmetric bosonic field configurations satisfy the classical bosonic equations of motion $\frac{\delta S}{\delta \phi^b}\Big|_{\phi^f=0} \equiv S_{,b}|_{\phi^f=0} \equiv \mathcal{E}(\phi^b)$.

Actually, the bosonic equations of motion of supersymmetric bosonic field configurations satisfy the so-called *Killing spinor identities*[a].

The supersymmetry invariance of the action implies

$$\delta_\epsilon S \qquad = \int d^dx\,(S_{,b}\,\delta_\epsilon\phi^b + S_{,f}\,\delta_\epsilon\phi^f) \qquad = 0\,,$$

[a] R. Kallosh & T.O. (1993), J. Bellorín & T.O. (2005)

3 – Tod's problem

This is the problem of finding **all** the bosonic field configurations ϕ^b for which a SUGRA's Killing spinor equations

$$\delta_\epsilon \phi^f \big|_{\phi^f=0} \sim \partial\epsilon + \phi^b \epsilon = 0\,,$$

have a solution ϵ, (i.e. all the possible supersymmetric bosonic field configurations ϕ^b), which includes all the possible supersymmetric vacua and compactifications.

N.B. Not all supersymmetric bosonic field configurations satisfy the classical bosonic equations of motion $\frac{\delta S}{\delta \phi^b}\Big|_{\phi^f=0} \equiv S_{,b}|_{\phi^f=0} \equiv \mathcal{E}(\phi^b)$.

Actually, the bosonic equations of motion of supersymmetric bosonic field configurations satisfy the so-called *Killing spinor identities*[a].

The supersymmetry invariance of the action implies after taking the functional derivative w.r.t. fermions and setting them to zero

$$(\delta_\epsilon S\ )_{,f_1}\Big|_{\phi^f=0} = \left\{ \int d^dx\,(S_{,b}\,\delta_\epsilon\phi^b + S_{,f}\,\delta_\epsilon\phi^f)\ \right\}_{,f_1}\Bigg|_{\phi^f=0} = 0\,,$$

[a] R. Kallosh & T.O. (1993), J. Bellorín & T.O. (2005)

Many terms vanish automatically because they are odd in fermion fields ϕ^f

$$\delta_\epsilon \phi^b\big|_{\phi^f=0} = S_{,f}\big|_{\phi^f=0} = (\delta_\epsilon \phi^f)_{,f_1}\big|_{\phi^f=0} = 0\,,$$

Many terms vanish automatically because they are odd in fermion fields ϕ^f

$$\delta_\epsilon \phi^b\big|_{\phi^f=0} = S_{,f}\big|_{\phi^f=0} = (\delta_\epsilon \phi^f)_{,f_1}\big|_{\phi^f=0} = 0\,,$$

and we get

$$\left\{S_{,b}\,(\delta_\epsilon \phi^b)_{,f_1} + S_{,f\,f_1}\,\delta_\epsilon \phi^f\right\}\Big|_{\phi^f=0} = 0\,.$$

Many terms vanish automatically because they are odd in fermion fields ϕ^f

$$\delta_\epsilon \phi^b \big|_{\phi^f=0} = S_{,f}\big|_{\phi^f=0} = (\delta_\epsilon \phi^f)_{,f_1}\big|_{\phi^f=0} = 0\,,$$

and we get

$$\left\{ S_{,b}\,(\delta_\epsilon \phi^b)_{,f_1} + S_{,f\,f_1}\,\delta_\epsilon \phi^f \right\}\big|_{\phi^f=0} = 0\,.$$

This is valid for any fields ϕ^b and any supersymmetry parameter ϵ. For a supersymmetric field configuration ϵ is a Killing spinor $\delta_\epsilon \phi^f\big|_{\phi^f=0}$ and we obtain the Killing spinor identities

$$\mathcal{E}(\phi^b)\ (\delta_\epsilon \phi^b)_{,f_1}\big|_{\phi^f=0} = 0\,.$$

Many terms vanish automatically because they are odd in fermion fields ϕ^f

$$\delta_\epsilon \phi^b \big|_{\phi^f=0} = S_{,f}\big|_{\phi^f=0} = (\delta_\epsilon \phi^f)_{,f_1}\big|_{\phi^f=0} = 0\,,$$

and we get

$$\left\{ S_{,b}\,(\delta_\epsilon \phi^b)_{,f_1} + S_{,f\,f_1}\,\delta_\epsilon \phi^f \right\}\big|_{\phi^f=0} = 0\,.$$

This is valid for any fields ϕ^b and any supersymmetry parameter ϵ. For a supersymmetric field configuration ϵ is a Killing spinor $\delta_\epsilon \phi^f\big|_{\phi^f=0}$ and we obtain the Killing spinor identities

$$\boxed{\mathcal{E}(\phi^b)\;(\delta_\epsilon \phi^b)_{,f_1}\big|_{\phi^f=0} = 0\,.}$$

These non-trivial identities are linear relations between the bosonic equations of motion and can be used to solve Tod's problem, obtain BPS bounds etc. Let's see some examples.

$N = 1, d = 4$ supergravity

$N = 1, d = 4$ supergravity

Its field content is $\{e^a{}_\mu, \psi_\mu\}$. The bosonic action is just the Einstein-Hilbert action

$$S|_{\psi_\mu=0} = \int d^4x \sqrt{|g|}\, R\,, \;\Rightarrow\; \mathcal{E}_a{}^\mu(e) \sim G_a{}^\mu\,,$$

$N = 1, d = 4$ supergravity

Its field content is $\{e^{a}{}_{\mu}, \psi_{\mu}\}$. The bosonic action is just the Einstein-Hilbert action

$$S|_{\psi_\mu=0} = \int d^4x \sqrt{|g|}\, R\,, \;\Rightarrow\; \mathcal{E}_a{}^{\mu}(e) \sim G_a{}^{\mu}\,,$$

and the supersymmetry transformations are

$$\delta_\epsilon e^{a}{}_{\mu} = -i\bar{\epsilon}\gamma^{a}\psi_{\mu}\,, \qquad \delta_\epsilon \psi_\mu = \nabla_\mu \epsilon = \partial_\mu \epsilon - \tfrac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\epsilon\,.$$

The K.S.I.s are

$$-i\bar{\epsilon}\gamma^{a}G_a{}^{\mu} = 0\,, \;\Rightarrow R = 0\,, \;\; -i\bar{\epsilon}\gamma^{a}R_a{}^{\mu} = 0\,.$$

$N = 1, d = 4$ supergravity

Its field content is $\{e^a{}_\mu, \psi_\mu\}$. The bosonic action is just the Einstein-Hilbert action

$$S|_{\psi_\mu = 0} = \int d^4x \sqrt{|g|}\, R\,, \;\; \Rightarrow \;\; \mathcal{E}_a{}^\mu(e) \sim G_a{}^\mu\,,$$

and the supersymmetry transformations are

$$\delta_\epsilon e^a{}_\mu = -i\bar{\epsilon}\gamma^a\psi_\mu\,, \qquad\qquad \delta_\epsilon\psi_\mu = \nabla_\mu\epsilon = \partial_\mu\epsilon - \tfrac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\epsilon\,.$$

The K.S.I.s are

$$-i\bar{\epsilon}\gamma^a G_a{}^\mu = 0\,, \;\; \Rightarrow R = 0\,, \;\; -i\bar{\epsilon}\gamma^a R_a{}^\mu = 0\,.$$

The integrability conditions of the Killing spinor equation $\delta_\epsilon\psi_\mu = 0$ are

$$[\nabla_\mu, \nabla_\nu]\epsilon = -\tfrac{1}{4}R_{\mu\nu}{}^{ab}\gamma_{ab}\epsilon = 0\,, \;\; \Rightarrow R^\mu{}_a\gamma^a\epsilon = 0\,.$$

$N = 1, d = 4$ supergravity

Its field content is $\{e^a{}_\mu, \psi_\mu\}$. The bosonic action is just the Einstein-Hilbert action

$$S|_{\psi_\mu=0} = \int d^4x \sqrt{|g|}\, R\,, \;\Rightarrow\; \mathcal{E}_a{}^\mu(e) \sim G_a{}^\mu\,,$$

and the supersymmetry transformations are

$$\delta_\epsilon e^a{}_\mu = -i\bar{\epsilon}\gamma^a\psi_\mu\,, \qquad \delta_\epsilon\psi_\mu = \nabla_\mu\epsilon = \partial_\mu\epsilon - \tfrac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\epsilon\,.$$

The K.S.I.s are

$$-i\bar{\epsilon}\gamma^a G_a{}^\mu = 0\,, \;\Rightarrow R = 0\,, \;\; -i\bar{\epsilon}\gamma^a R_a{}^\mu = 0\,.$$

The integrability conditions of the Killing spinor equation $\delta_\epsilon\psi_\mu = 0$ are

$$[\nabla_\mu, \nabla_\nu]\epsilon = -\tfrac{1}{4}R_{\mu\nu}{}^{ab}\gamma_{ab}\epsilon = 0\,, \;\Rightarrow R^\mu{}_a\gamma^a\epsilon = 0\,.$$

The K.S.I.s are contained in the integrability conditions.

$N = 1, d = 4$ supergravity

Its field content is $\{e^a{}_\mu, \psi_\mu\}$. The bosonic action is just the Einstein-Hilbert action

$$S|_{\psi_\mu=0} = \int d^4x \sqrt{|g|}\, R\,, \;\Rightarrow\; \mathcal{E}_a{}^\mu(e) \sim G_a{}^\mu\,,$$

and the supersymmetry transformations are

$$\delta_\epsilon e^a{}_\mu = -i\bar{\epsilon}\gamma^a \psi_\mu\,, \qquad \delta_\epsilon \psi_\mu = \nabla_\mu \epsilon = \partial_\mu \epsilon - \tfrac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\epsilon\,.$$

The K.S.I.s are

$$-i\bar{\epsilon}\gamma^a G_a{}^\mu = 0\,, \;\Rightarrow R = 0\,, \;\; -i\bar{\epsilon}\gamma^a R_a{}^\mu = 0\,.$$

The integrability conditions of the Killing spinor equation $\delta_\epsilon \psi_\mu = 0$ are

$$[\nabla_\mu, \nabla_\nu]\epsilon = -\tfrac{1}{4} R_{\mu\nu}{}^{ab}\gamma_{ab}\epsilon = 0\,, \;\Rightarrow R^\mu{}_a \gamma^a \epsilon = 0\,.$$

The K.S.I.s are contained in the integrability conditions.
We will see later how to obtain more information from these identities.

$N = 2, d = 4$ supergravity

$N = 2, d = 4$ supergravity

Its field content is $\{e^a{}_\mu, A_\mu, \psi_\mu\}$. The bosonic action is just the Einstein-Maxwell action

$$S|_{\psi_\mu=0} = \int d^4x\sqrt{|g|}\,\left[R - \tfrac{1}{4}F^2\right], \Rightarrow \begin{cases} \mathcal{E}_a{}^\mu(e) = -2\{G_a{}^\mu - \tfrac{1}{2}T_a{}^\mu\}, \\ \mathcal{E}^\mu(A) = \nabla_\alpha F^{\alpha\mu}, \end{cases}$$

$N = 2, d = 4$ supergravity

Its field content is $\{e^a{}_\mu, A_\mu, \psi_\mu\}$. The bosonic action is just the Einstein-Maxwell action

$$S|_{\psi_\mu=0} = \int d^4x\sqrt{|g|}\,\left[R - \tfrac{1}{4}F^2\right]\,, \quad \Rightarrow \left\{ \begin{array}{lcl} \mathcal{E}_a{}^\mu(e) & = & -2\{G_a{}^\mu - \tfrac{1}{2}T_a{}^\mu\}\,, \\ \mathcal{E}^\mu(A) & = & \nabla_\alpha F^{\alpha\mu}\,, \end{array} \right.$$

and the supersymmetry transformations are

$$\delta_\epsilon e^a{}_\mu = -i\bar{\epsilon}\gamma^a\psi_\mu + \text{c.c.}\,, \qquad \delta_\epsilon A_\mu = -2i\bar{\epsilon}\psi_\mu + \text{c.c.}\,. \qquad \delta_\epsilon\psi_\mu = \nabla_\mu\epsilon - \tfrac{1}{8}F^{ab}\gamma_{ab}\epsilon \equiv \tilde{\mathcal{D}}_\mu\epsilon\,.$$

The K.S.I.s are

$$\bar{\epsilon}\{\mathcal{E}_a{}^\mu(e)\gamma^a + 2\mathcal{E}^\mu(A)\} = 0\,.$$

$N = 2, d = 4$ supergravity

Its field content is $\{e^a{}_\mu, A_\mu, \psi_\mu\}$. The bosonic action is just the Einstein-Maxwell action

$$S|_{\psi_\mu=0} = \int d^4x\sqrt{|g|}\,\left[R - \tfrac{1}{4}F^2\right]\,, \quad \Rightarrow \left\{ \begin{array}{rcl} \mathcal{E}_a{}^\mu(e) & = & -2\{G_a{}^\mu - \frac{1}{2}T_a{}^\mu\}\,, \\ \mathcal{E}^\mu(A) & = & \nabla_\alpha F^{\alpha\mu}\,, \end{array} \right.$$

and the supersymmetry transformations are

$$\delta_\epsilon e^a{}_\mu = -i\bar{\epsilon}\gamma^a\psi_\mu + \text{c.c.}\,, \qquad \delta_\epsilon A_\mu = -2i\bar{\epsilon}\psi_\mu + \text{c.c.}\,. \qquad \delta_\epsilon\psi_\mu = \nabla_\mu\epsilon - \tfrac{1}{8}F^{ab}\gamma_{ab}\epsilon \equiv \tilde{\mathcal{D}}_\mu\epsilon\,.$$

The K.S.I.s are

$$\bar{\epsilon}\{\mathcal{E}_a{}^\mu(e)\gamma^a + 2\mathcal{E}^\mu(A)\} = 0\,.$$

The integrability conditions of the Killing spinor equation $\delta_\epsilon\psi_\mu = \tilde{\mathcal{D}}_\mu\epsilon = 0$ are

$$[\tilde{\mathcal{D}}_\mu, \tilde{\mathcal{D}}_\nu]\epsilon = -\tfrac{1}{4}\left\{\left[R_{\mu\nu}{}^{ab} - e^a{}_{[\mu}T_{\nu]}{}^b\right]\gamma_{ab} + \nabla^a\left(F_{\mu\nu} + {}^\star F_{\mu\nu}\gamma_5\right)\gamma_a\right\}\epsilon = 0\,,$$

$$\Rightarrow \quad \{\mathcal{E}_a{}^\mu(e)\gamma^a + 2[\mathcal{E}^\mu(A) + \mathcal{B}^\mu(A)\gamma_5]\}\epsilon = 0\,.$$

4 – Solving it

★ (1983) Tod showed in that in $N = 2, d = 4$ SUGRA the problem could be completely solved using just integrability and consistency conditions.

4 – Solving it

★ (1983) Tod showed in that in $N = 2, d = 4$ SUGRA the problem could be completely solved using just integrability and consistency conditions.

However, he used the Newmann-Penrose formalism, unknown to most particle physicists and suited only for $d = 4$.

4 – Solving it

★ (1983) Tod showed in that in $N = 2, d = 4$ SUGRA the problem could be completely solved using just integrability and consistency conditions.

However, he used the Newmann-Penrose formalism, unknown to most particle physicists and suited only for $d = 4$.

★ (1995) Tod solved partialy the problem in $N = 4, d = 4$ SUGRA.

4 – Solving it

- ★ (1983) Tod showed in that in $N = 2, d = 4$ SUGRA the problem could be completely solved using just integrability and consistency conditions.

However, he used the Newmann-Penrose formalism, unknown to most particle physicists and suited only for $d = 4$.

- ★ (1995) Tod solved partialy the problem in $N = 4, d = 4$ SUGRA.
- ★ (2002) Gauntlett, Gutowski, Hull, Pakis & Reall proposed to translate the Killing spinor equation to tensor language. They solved $N = 1, d = 5$ SUGRA.

4 – Solving it

★ (1983) Tod showed in that in $N = 2, d = 4$ SUGRA the problem could be completely solved using just integrability and consistency conditions.

However, he used the Newmann-Penrose formalism, unknown to most particle physicists and suited only for $d = 4$.

★ (1995) Tod solved partialy the problem in $N = 4, d = 4$ SUGRA.

★ (2002) Gauntlett, Gutowski, Hull, Pakis & Reall proposed to translate the Killing spinor equation to tensor language. They solved $N = 1, d = 5$ SUGRA.

★ (2002) Gauntlett & Gutowski gauged $N = 1, d = 5$ SUGRA.

4 – Solving it

- ★ (1983) Tod showed in that in $N = 2, d = 4$ SUGRA the problem could be completely solved using just integrability and consistency conditions.

However, he used the Newmann-Penrose formalism, unknown to most particle physicists and suited only for $d = 4$.

- ★ (1995) Tod solved partialy the problem in $N = 4, d = 4$ SUGRA.
- ★ (2002) Gauntlett, Gutowski, Hull, Pakis & Reall proposed to translate the Killing spinor equation to tensor language. They solved $N = 1, d = 5$ SUGRA.
- ★ (2002) Gauntlett & Gutowski gauged $N = 1, d = 5$ SUGRA.
- ★ (2003) Gutowski, Martelli & Reall and Chamseddine, J. Figueroa-O'Farrill & Sabra $N = (1, 0), d = 6$ SUGRA.

4 – Solving it

★ (1983) Tod showed in that in $N = 2, d = 4$ SUGRA the problem could be completely solved using just integrability and consistency conditions.

However, he used the Newmann-Penrose formalism, unknown to most particle physicists and suited only for $d = 4$.

★ (1995) Tod solved partialy the problem in $N = 4, d = 4$ SUGRA.

★ (2002) Gauntlett, Gutowski, Hull, Pakis & Reall proposed to translate the Killing spinor equation to tensor language. They solved $N = 1, d = 5$ SUGRA.

★ (2002) Gauntlett & Gutowski gauged $N = 1, d = 5$ SUGRA.

★ (2003) Gutowski, Martelli & Reall and Chamseddine, J. Figueroa-O'Farrill & Sabra $N = (1, 0), d = 6$ SUGRA.

★ (2003) Caldarelli & Klemm gauged $N = 2, d = 4$ SUGRA.

4 – Solving it

★ (1983) Tod showed in that in $N = 2, d = 4$ SUGRA the problem could be completely solved using just integrability and consistency conditions.

However, he used the Newmann-Penrose formalism, unknown to most particle physicists and suited only for $d = 4$.

★ (1995) Tod solved partialy the problem in $N = 4, d = 4$ SUGRA.

★ (2002) Gauntlett, Gutowski, Hull, Pakis & Reall proposed to translate the Killing spinor equation to tensor language. They solved $N = 1, d = 5$ SUGRA.

★ (2002) Gauntlett & Gutowski gauged $N = 1, d = 5$ SUGRA.

★ (2003) Gutowski, Martelli & Reall and Chamseddine, J. Figueroa-O'Farrill & Sabra $N = (1, 0), d = 6$ SUGRA.

★ (2003) Caldarelli & Klemm gauged $N = 2, d = 4$ SUGRA.

★ (2004) Gutowski & Reall and (2005) Gutowski & Sabra gauged $N = 1, d = 5$ SUGRA coupled to Abelian vector multiplets.

4 – Solving it

- ★ (1983) Tod showed in that in $N = 2, d = 4$ SUGRA the problem could be completely solved using just integrability and consistency conditions.

However, he used the Newmann-Penrose formalism, unknown to most particle physicists and suited only for $d = 4$.

- ★ (1995) Tod solved partialy the problem in $N = 4, d = 4$ SUGRA.
- ★ (2002) Gauntlett, Gutowski, Hull, Pakis & Reall proposed to translate the Killing spinor equation to tensor language. They solved $N = 1, d = 5$ SUGRA.
- ★ (2002) Gauntlett & Gutowski gauged $N = 1, d = 5$ SUGRA.
- ★ (2003) Gutowski, Martelli & Reall and Chamseddine, J. Figueroa-O'Farrill & Sabra $N = (1, 0), d = 6$ SUGRA.
- ★ (2003) Caldarelli & Klemm gauged $N = 2, d = 4$ SUGRA.
- ★ (2004) Gutowski & Reall and (2005) Gutowski & Sabra gauged $N = 1, d = 5$ SUGRA coupled to Abelian vector multiplets.
- ★ (2005) Bellorín & T.O. $N = 4, d = 4$ SUGRA.

There is by now a well-defined recipe to attack this problem starting with only one assumption: the existence of one Killing spinor ϵ.

There is by now a well-defined recipe to attack this problem starting with only one assumption: the existence of one Killing spinor ϵ.

- Translate the Killing spinor equations and K.S.I.s into tensorial equations.

There is by now a well-defined recipe to attack this problem starting with only one assumption: the existence of one Killing spinor ϵ.

- Translate the Killing spinor equations and K.S.I.s into tensorial equations. With the Killing spinor ϵ one can construct scalar, vector, and p- form bilinears $M \sim \bar{\epsilon}\epsilon\,, \quad V_\mu \sim \bar{\epsilon}\gamma_\mu\epsilon\,, \cdots$ that are related by Fierz identities and satisfy equivalent equations:

$$\delta_\epsilon\psi_\mu = \tilde{\mathcal{D}}_\mu\epsilon = [\nabla_\mu + \Omega_\mu]\epsilon = 0\,, \;\Rightarrow\; \nabla_\mu M + 2\Omega_\mu M = 0\,, \cdots$$

There is by now a well-defined recipe to attack this problem starting with only one assumption: the existence of one Killing spinor ϵ.

I Translate the Killing spinor equations and K.S.I.s into tensorial equations.
With the Killing spinor ϵ one can construct scalar, vector, and p- form bilinears $M \sim \bar{\epsilon}\epsilon\,, \quad V_\mu \sim \bar{\epsilon}\gamma_\mu\epsilon\,, \cdots$ that are related by Fierz identities and satisfy equivalent equations:

$$\delta_\epsilon \psi_\mu = \tilde{\mathcal{D}}_\mu \epsilon = [\nabla_\mu + \Omega_\mu]\epsilon = 0\,, \;\Rightarrow\; \nabla_\mu M + 2\Omega_\mu M = 0\,, \cdots$$

II One of the vector bilinears (say V_μ) is always a Killing vector which can be timelike or null. These two cases are treated separatelly.

There is by now a well-defined recipe to attack this problem starting with only one assumption: the existence of one Killing spinor ϵ.

I Translate the Killing spinor equations and K.S.I.s into tensorial equations. With the Killing spinor ϵ one can construct scalar, vector, and p- form bilinears $M \sim \bar{\epsilon}\epsilon\,, \quad V_\mu \sim \bar{\epsilon}\gamma_\mu\epsilon\,, \cdots$ that are related by Fierz identities and satisfy equivalent equations:

$$\delta_\epsilon \psi_\mu = \tilde{\mathcal{D}}_\mu \epsilon = [\nabla_\mu + \Omega_\mu]\epsilon = 0\,, \ \Rightarrow \ \nabla_\mu M + 2\Omega_\mu M = 0\,, \cdots$$

II One of the vector bilinears (say V_μ) is always a Killing vector which can be timelike or null. These two cases are treated separatelly.

III One can get an expression of all the gauge field strengths of the theory (the main ingredient of Ω_μ) in terms of the scalar bilinears M and the Killing vector V_μ from tensorial equations.

There is by now a well-defined recipe to attack this problem starting with only one assumption: the existence of one Killing spinor ϵ.

I Translate the Killing spinor equations and K.S.I.s into tensorial equations. With the Killing spinor ϵ one can construct scalar, vector, and p- form bilinears $M \sim \bar{\epsilon}\epsilon\,, \quad V_\mu \sim \bar{\epsilon}\gamma_\mu\epsilon\,, \cdots$ that are related by Fierz identities and satisfy equivalent equations:

$$\delta_\epsilon \psi_\mu = \tilde{\mathcal{D}}_\mu \epsilon = [\nabla_\mu + \Omega_\mu]\epsilon = 0\,, \;\Rightarrow\; \nabla_\mu M + 2\Omega_\mu M = 0\,, \cdots$$

II One of the vector bilinears (say V_μ) is always a Killing vector which can be timelike or null. These two cases are treated separatelly.

III One can get an expression of all the gauge field strengths of the theory (the main ingredient of Ω_μ) in terms of the scalar bilinears M and the Killing vector V_μ from tensorial equations.

IV The Maxwell equations and Bianchi identities are imposed on those field strengths, getting equations for the scalar bilinears.

There is by now a well-defined recipe to attack this problem starting with only one assumption: the existence of one Killing spinor ϵ.

I Translate the Killing spinor equations and K.S.I.s into tensorial equations. With the Killing spinor ϵ one can construct scalar, vector, and p- form bilinears $M \sim \bar{\epsilon}\epsilon\,, \qquad V_\mu \sim \bar{\epsilon}\gamma_\mu\epsilon\,, \cdots$ that are related by Fierz identities and satisfy equivalent equations:

$$\delta_\epsilon \psi_\mu = \tilde{\mathcal{D}}_\mu \epsilon = [\nabla_\mu + \Omega_\mu]\epsilon = 0\,, \;\Rightarrow\; \nabla_\mu M + 2\Omega_\mu M = 0\,, \cdots$$

II One of the vector bilinears (say V_μ) is always a Killing vector which can be timelike or null. These two cases are treated separately.

III One can get an expression of all the gauge field strengths of the theory (the main ingredient of Ω_μ) in terms of the scalar bilinears M and the Killing vector V_μ from tensorial equations.

IV The Maxwell equations and Bianchi identities are imposed on those field strengths, getting equations for the scalar bilinears.

V The Einstein equations are imposed and the K.S.I.s used to find relations between scalar bilinears and metric components.

There is by now a well-defined recipe to attack this problem starting with only one assumption: the existence of one Killing spinor ϵ.

I Translate the Killing spinor equations and K.S.I.s into tensorial equations. With the Killing spinor ϵ one can construct scalar, vector, and p- form bilinears $M \sim \bar{\epsilon}\epsilon\,, \qquad V_\mu \sim \bar{\epsilon}\gamma_\mu\epsilon\,, \cdots$ that are related by Fierz identities and satisfy equivalent equations:

$$\delta_\epsilon \psi_\mu = \tilde{\mathcal{D}}_\mu \epsilon = [\nabla_\mu + \Omega_\mu]\epsilon = 0\,, \;\; \Rightarrow \;\; \nabla_\mu M + 2\Omega_\mu M = 0\,, \cdots$$

II One of the vector bilinears (say V_μ) is always a Killing vector which can be timelike or null. These two cases are treated separatelly.

III One can get an expression of all the gauge field strengths of the theory (the main ingredient of Ω_μ) in terms of the scalar bilinears M and the Killing vector V_μ from tensorial equations.

IV The Maxwell equations and Bianchi identities are imposed on those field strengths, getting equations for the scalar bilinears.

V The Einstein equations are imposed and the K.S.I.s used to find relations between scalar bilinears and metric components.

Let us see some examples.

$N = 1, d = 4$ supergravity

$N = 1, d = 4$ supergravity

With one (Majorana) Killing spinor ϵ one can only construct a real vector bilinear V_μ which is null.

$N = 1, d = 4$ supergravity

With one (Majorana) Killing spinor ϵ one can only construct a real vector bilinear V_μ which is null. V_μ is also covariantly constant:

$$\delta_\epsilon \psi_\mu = \nabla_\mu \epsilon = 0\,, \ \Rightarrow \nabla_\mu V_\nu = 0\,, \ \ R^\mu{}_\nu V^\nu = 0\,, \ \ (\bar{\epsilon} R^\mu{}_a \gamma^a \epsilon = 0)\,.$$

$N = 1, d = 4$ supergravity

With one (Majorana) Killing spinor ϵ one can only construct a real vector bilinear V_μ which is null. V_μ is also covariantly constant:

$$\delta_\epsilon \psi_\mu = \nabla_\mu \epsilon = 0\,, \quad \Rightarrow \nabla_\mu V_\nu = 0\,, \quad R^\mu{}_\nu V^\nu = 0\,, \quad (\bar{\epsilon} R^\mu{}_a \gamma^a \epsilon = 0)\,.$$

All the metrics with covariantly constant null vectors are Brinkmann pp-waves and have the form

$$ds^2 = 2du(dv + K du + A_{\underline{i}} dx^i) + \tilde{g}_{\underline{ij}} dx^i dx^j\,,$$

where all the components are independent of v $V^\mu \partial_\mu \equiv \partial/\partial v$.

$N = 1, d = 4$ supergravity

With one (Majorana) Killing spinor ϵ one can only construct a real vector bilinear V_μ which is null. V_μ is also covariantly constant:

$$\delta_\epsilon \psi_\mu = \nabla_\mu \epsilon = 0\,, \;\Rightarrow \nabla_\mu V_\nu = 0\,, \;\; R^\mu{}_\nu V^\nu = 0\,, \;\; (\bar{\epsilon} R^\mu{}_a \gamma^a \epsilon = 0)\,.$$

All the metrics with covariantly constant null vectors are Brinkmann pp-waves and have the form

$$ds^2 = 2du(dv + K du + A_{\underline{i}} dx^i) + \tilde{g}_{\underline{ij}} dx^i dx^j\,,$$

where all the components are independent of v $V^\mu \partial_\mu \equiv \partial/\partial v$.

These metrics are the supersymmetric field configurations of $N = 1, d = 4$ SUGRA, but only those with $R_{\mu\nu} = 0$ are supersymmetric solutions.

$N = 2, d = 4$ supergravity

$N = 2, d = 4$ supergravity

With two Weyl spinors ϵ^I one can construct the following independent bilinears

- A complex scalar $\bar{\epsilon}^I \epsilon^J \equiv M \varepsilon^{IJ}$
- A Hermitean matrix of null vectors (4) $V^I{}_{J\,\mu} \equiv i \bar{\epsilon}^I \gamma_\mu \epsilon_J$

$N = 2, d = 4$ supergravity

With two Weyl spinors ϵ^I one can construct the following independent bilinears

- A complex scalar $\bar{\epsilon}^I \epsilon^J \equiv M \varepsilon^{IJ}$
- A Hermitean matrix of null vectors (4) $V^I{}_{J\,\mu} \equiv i\bar{\epsilon}^I \gamma_\mu \epsilon_J$

The Killing spinor equations imply the following equations for the bilinears:

$$\nabla_\mu M \sim F^+{}_{\mu\nu} V^I{}_I{}^\nu \,,$$

$$\nabla_\mu V^I{}_{J\,\nu} \sim \delta^I{}_J [M F^+{}_{\mu\nu} + M^* F^-{}_{\mu\nu}] - \Phi_{KJ\,(\mu}{}^\rho \varepsilon^{KI} F^-{}_{\nu)\rho} - \Phi^{IK}{}_{(\mu|}{}^\rho \varepsilon_{KJ} F^+{}_{|\nu)\rho} \,,$$

so $V^\mu \equiv V^I{}_I{}^\mu$ is Killing and the other three are exact forms. $V^\mu V_\mu \sim |M|^2 \geq 0$ can be timelike or null.

$N = 2, d = 4$ supergravity

With two Weyl spinors ϵ^I one can construct the following independent bilinears

- A complex scalar $\bar{\epsilon}^I \epsilon^J \equiv M \varepsilon^{IJ}$
- A Hermitean matrix of null vectors (4) $V^I{}_{J\,\mu} \equiv i\bar{\epsilon}^I \gamma_\mu \epsilon_J$

The Killing spinor equations imply the following equations for the bilinears:

$$\nabla_\mu M \sim F^+{}_{\mu\nu} V^I{}_I{}^\nu \,,$$

$$\nabla_\mu V^I{}_{J\,\nu} \sim \delta^I{}_J [M F^+{}_{\mu\nu} + M^* F^-{}_{\mu\nu}] - \Phi_{KJ\,(\mu}{}^\rho \varepsilon^{KI} F^-{}_{\nu)\rho} - \Phi^{IK}{}_{(\mu|}{}^\rho \varepsilon_{KJ} F^+{}_{|\nu)\rho} \,,$$

so $V^\mu \equiv V^I{}_I{}^\mu$ is Killing and the other three are exact forms. $V^\mu V_\mu \sim |M|^2 \geq 0$ can be timelike or null.
When it is timelike, $V^\mu \partial_\mu \equiv \sqrt{2}\partial/\partial t$ and

$$F^+ \sim |M|^{-2}\{V \wedge dM + i^\star[V \wedge dM]\} \,,$$

$$ds^2 = |M|^2 (dt + \omega)^2 - |M|^{-2} d\vec{x}^{\,2} \,,$$

$N = 2, d = 4$ supergravity

With two Weyl spinors ϵ^I one can construct the following independent bilinears

- A complex scalar $\bar{\epsilon}^I \epsilon^J \equiv M \varepsilon^{IJ}$
- A Hermitean matrix of null vectors (4) $V^I{}_{J\,\mu} \equiv i\bar{\epsilon}^I \gamma_\mu \epsilon_J$

The Killing spinor equations imply the following equations for the bilinears:

$$\nabla_\mu M \sim F^+{}_{\mu\nu} V^I{}_I{}^\nu \,,$$

$$\nabla_\mu V^I{}_{J\,\nu} \sim \delta^I{}_J [M F^+{}_{\mu\nu} + M^* F^-{}_{\mu\nu}] - \Phi_{KJ\,(\mu}{}^\rho \varepsilon^{KI} F^-{}_{\nu)\rho} - \Phi^{IK}{}_{(\mu|}{}^\rho \varepsilon_{KJ} F^+{}_{|\nu)\rho} \,,$$

so $V^\mu \equiv V^I{}_I{}^\mu$ is Killing and the other three are exact forms. $V^\mu V_\mu \sim |M|^2 \geq 0$ can be timelike or null.
When it is timelike, $V^\mu \partial_\mu \equiv \sqrt{2}\partial/\partial t$ and

$$F^+ \sim |M|^{-2}\{V \wedge dM + i^\star[V \wedge dM]\} \,,$$

$$ds^2 = |M|^2 (dt + \omega)^2 - |M|^{-2} d\vec{x}^{\,2} \,,$$

SUSY $\Rightarrow d\omega = i|M|^{-2\star}[M dM^* - \text{c.c.}]$,
Solutions$\Rightarrow \vec{\nabla}^2 M^{-1} = 0$. (Israel-Wilson-Perjes)

5 – Conclusion

Index

Slide 19 / 20

5 – Conclusion

⋆ We have shown how Tod's problem can be solved in a systematic way, at least in $N = 1, 2, d = 4$. (There is a lot of work in $d = 10, 11$ with only partial results so far which should be more relevant to the landscape problem).

5 – Conclusion

- We have shown how Tod's problem can be solved in a systematic way, at least in $N = 1, 2, d = 4$. (There is a lot of work in $d = 10, 11$ with only partial results so far which should be more relevant to the landscape problem).

- We have shown how to obtain and exploit the Killing spinor identities and how they imply the existence of only a few simple independent equations.

5 – Conclusion

- We have shown how Tod's problem can be solved in a systematic way, at least in $N = 1, 2, d = 4$. (There is a lot of work in $d = 10, 11$ with only partial results so far which should be more relevant to the landscape problem).
- We have shown how to obtain and exploit the Killing spinor identities and how they imply the existence of only a few simple independent equations.
- It can be shown that Tod's problem can be also solved pure $N = 4, d = 4$ supergravity using the same systematic techniques.

5 – Conclusion

Index

Slide 19 / 20

- ★ We have shown how Tod's problem can be solved in a systematic way, at least in $N = 1, 2, d = 4$. (There is a lot of work in $d = 10, 11$ with only partial results so far which should be more relevant to the landscape problem).
- ★ We have shown how to obtain and exploit the Killing spinor identities and how they imply the existence of only a few simple independent equations.
- ★ It can be shown that Tod's problem can be also solved pure $N = 4, d = 4$ supergravity using the same systematic techniques.
- ★ Analogous techniques could be used for generic $N = 2, d = 4$ theories and for matter-coupled $N = 4, d = 4$ supergravity.

5 – Conclusion

- We have shown how Tod's problem can be solved in a systematic way, at least in $N = 1, 2, d = 4$. (There is a lot of work in $d = 10, 11$ with only partial results so far which should be more relevant to the landscape problem).
- We have shown how to obtain and exploit the Killing spinor identities and how they imply the existence of only a few simple independent equations.
- It can be shown that Tod's problem can be also solved pure $N = 4, d = 4$ supergravity using the same systematic techniques.
- Analogous techniques could be used for generic $N = 2, d = 4$ theories and for matter-coupled $N = 4, d = 4$ supergravity.

Work on the last topics is in progress.

This is

THE END