The Supersymmetric Vistas of the Supergravity Landscape

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Seminar given on September 7th 2005 at the Spanish Relativity Meeting '05

Based on hep-th/0506056 and on work in preparation. Work done in collaboration with

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Plan of the Talk:

1 – Unification & Landscape

or

"How We Got Into This Mess"

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There have been many instances of unification:

1 Electricity $\bigoplus$ Magnetism $\stackrel{\text{Faraday,Maxwell}}{\Longrightarrow}$ Electromagnetism

$$\vec{E}, \vec{B} \longrightarrow (F_{\mu\nu}) \equiv \left(\begin{array}{c|c} 0 & -\vec{E}^T \\ \hline \vec{E} & {}^\star\vec{B} \end{array}\right)$$

Required by the Special Theory of Relativity just as Newtonian gravity and gravitomagnetism are combined in General Relativity.

2 Space $\bigoplus$ Time $\overset{\text{Einstein,Minkowski}}{\Longrightarrow}$ Spacetime

$$t, \vec{x} \longrightarrow (x^{\mu}) \equiv (ct, \vec{x})\,.$$

Strongly related to the former, is associated to an enhancement of symmetry from the Galileo to the Poincaré group which is not apparent at low speeds, but is never broken.

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➠ There is enhancement of <u>local</u> symmetry from g.c.t.'s in $d = 4$ to g.c.t.'s in $d = 5$, but this symmetry is spontaneously broken (in modern parlance) to g.c.t.'s in $d = 4$ and $U(1)$ due to the (completely arbitrary) choice of vacuum. The rule is always:

global symmetry of the vacuum $\sim$ local symmetry of the reduced theory.

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> The extraordinary success of this model has made of it the paradigm of unification schemes.

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- It can also be combined with g.c.t.'s, making it local (supergravity theories). We can have supergravity theories with Yang-Mills fields etc. etc. But in most of these theories gravity is not unified with the other interactions.

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This theory could satisfy all our desires for unification, but we have to find in it our Universe's vacuum and explain why and how it is selected.

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But this is only a (computationally necessary) simplification of the genuine problem in which all possible compactifications should be considered.

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- An application to $N = 4, d = 4$ supergravity.

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This is a generalization of the concept of isometry, an infinitesimal general coordinate transformation generated by $\xi^\mu(x)$ that leaves the metric $g_{\mu\nu}$ invariant because it satisfies the *Killing (vector) equation*

$$\delta_\xi g_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)} = 0 \,. \tag{3}$$

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When the supersymmetric vacuum solution has a clear (possibly warped) product structure we distinguish internal and spacetime symmetries
$\longrightarrow$ spontaneous compactification.

3 – Tod's problem

This is the problem of finding **all** the bosonic field configurations ϕ^b for which a SUGRA's Killing spinor equations

$$\delta_\epsilon \phi^f \Big|_{\phi^f=0} \sim \partial\epsilon + \phi^b \epsilon = 0\,,$$

have a solution ϵ, (i.e. all the possible supersymmetric bosonic field configurations ϕ^b), which includes all the possible supersymmetric vacua and compactifications.

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N.B. Not all supersymmetric bosonic field configurations satisfy the classical bosonic equations of motion $\frac{\delta S}{\delta \phi^b}\Big|_{\phi^f=0} \equiv S_{,b}|_{\phi^f=0} \equiv \mathcal{E}(\phi^b)$.

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The supersymmetry invariance of the action implies

$$\delta_\epsilon S = \int d^dx\,(S_{,b}\,\delta_\epsilon\phi^b + S_{,f}\,\delta_\epsilon\phi^f) = 0\,,$$

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These non-trivial identities are linear relations between the bosonic equations of motion and can be used to solve Tod's problem, obtain BPS bounds etc. Let's see some examples.

$N = 1, d = 4$ supergravity

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Its field content is $\{e^{a}{}_{\mu}, \psi_{\mu}\}$. The bosonic action is just the Einstein-Hilbert action

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We will see later how to obtain more information from these identities.

$N = 2, d = 4$ supergravity

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Its field content is $\{e^{a}{}_{\mu}, A_{\mu}, \psi_{\mu}\}$. The bosonic action is just the Einstein-Maxwell action

$$S|_{\psi_\mu=0} = \int d^4x\sqrt{|g|}\,\left[R - \tfrac{1}{4}F^2\right]\,,\;\Rightarrow\;\left\{\begin{array}{rcl}\mathcal{E}_a{}^{\mu}(e) & = & -2\{G_a{}^{\mu} - \tfrac{1}{2}T_a{}^{\mu}\}\,,\\ \mathcal{E}^{\mu}(A) & = & \nabla_\alpha F^{\alpha\mu}\,,\end{array}\right.$$

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$$S|_{\psi_\mu=0} = \int d^4x\sqrt{|g|}\,\left[R - \tfrac{1}{4}F^2\right]\,, \quad \Rightarrow \quad \left\{ \begin{array}{rcl} \mathcal{E}_a{}^\mu(e) & = & -2\{G_a{}^\mu - \tfrac{1}{2}T_a{}^\mu\}\,, \\ \mathcal{E}^\mu(A) & = & \nabla_\alpha F^{\alpha\mu}\,, \end{array} \right.$$

and the supersymmetry transformations are

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$$[\tilde{\mathcal{D}}_\mu, \tilde{\mathcal{D}}_\nu]\epsilon = -\tfrac{1}{4}\left\{\left[R_{\mu\nu}{}^{ab} - e^a{}_{[\mu}T_{\nu]}{}^b\right]\gamma_{ab} + \nabla^a\left(F_{\mu\nu} + {}^\star F_{\mu\nu}\gamma_5\right)\gamma_a\right\}\epsilon = 0\,,$$

$$\Rightarrow \quad \{\mathcal{E}_a{}^\mu(e)\gamma^a + 2[\mathcal{E}^\mu(A) + \mathcal{B}^\mu(A)\gamma_5]\}\epsilon = 0\,.$$

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Let us see some examples.

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All the metrics with covariantly constant null vectors are Brinkmann pp-waves and have the form

$$ds^2 = 2du(dv + Kdu + A_{\underline{i}} dx^i) + \tilde{g}_{\underline{i}\underline{j}} dx^i dx^j\,,$$

where all the components are independent of v $V^\mu \partial_\mu \equiv \partial / \partial v$.

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These metrics are the supersymmetric field configurations of $N = 1, d = 4$ SUGRA, but only those with $R_{\mu\nu} = 0$ are supersymmetric solutions.

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With two Weyl spinors ϵ^I one can construct the following independent bilinears

- A complex scalar $\bar{\epsilon}^I \epsilon^J \equiv M \varepsilon^{IJ}$
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SUSY $\Rightarrow d\omega = i|M|^{-2\star}[M dM^* - \text{c.c.}]$,
Solutions$\Rightarrow \vec{\nabla}^2 M^{-1} = 0$. (Israel-Wilson-Perjes)

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5 − N=4, d=4 SUGRA

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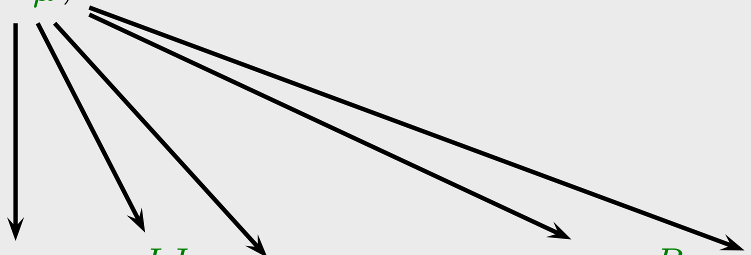

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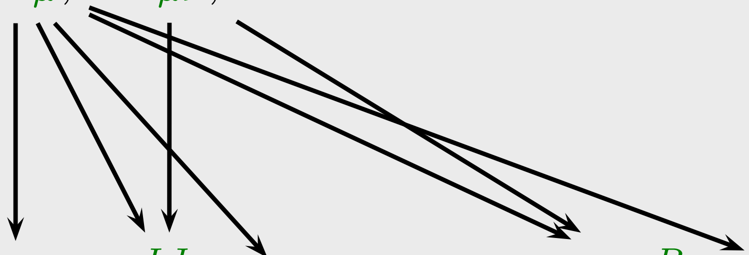

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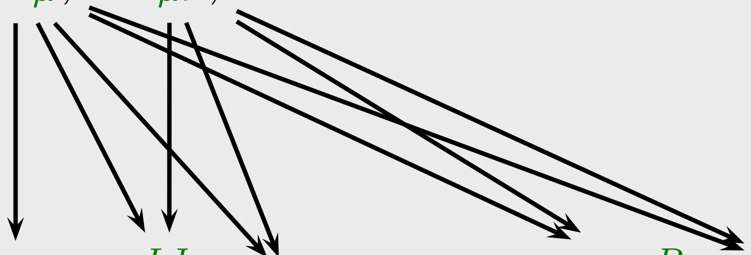

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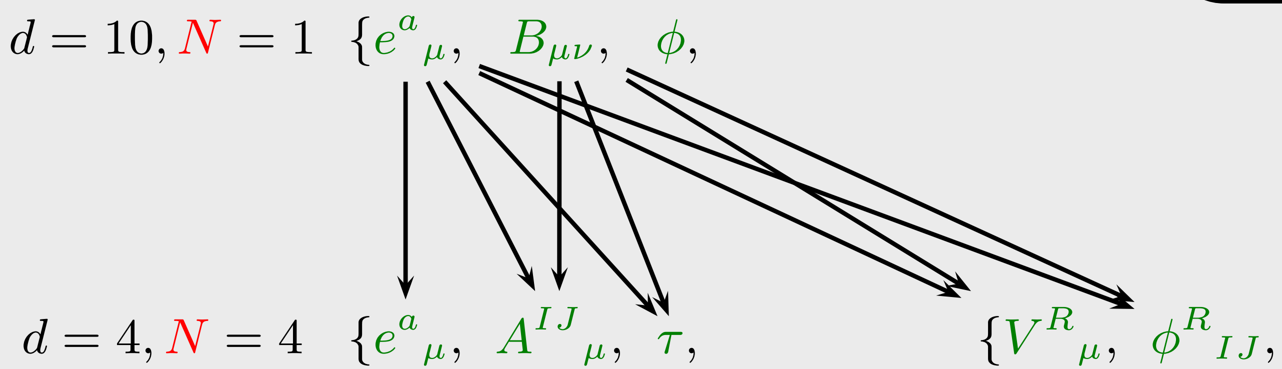

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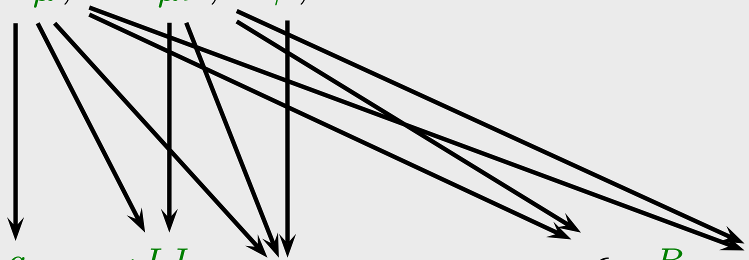

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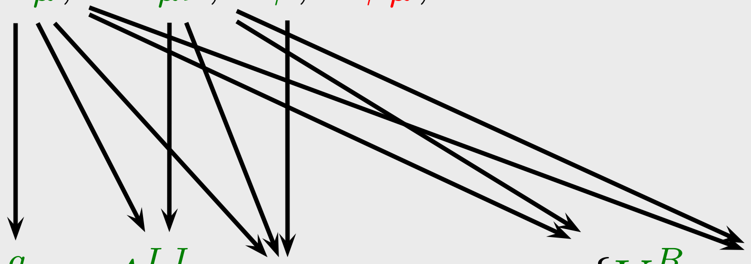

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5 – N=4, d=4 SUGRA

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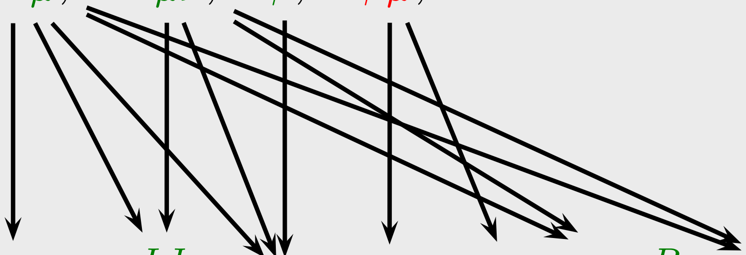

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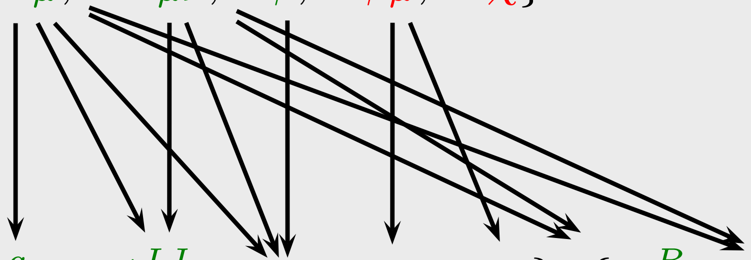

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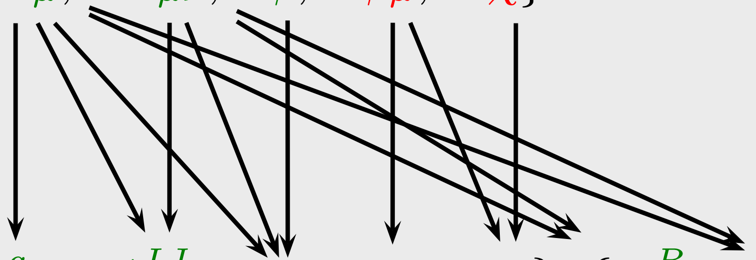

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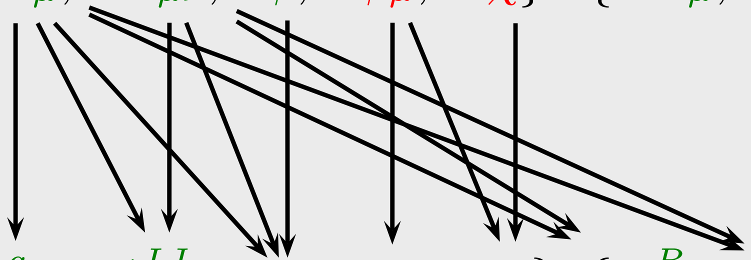

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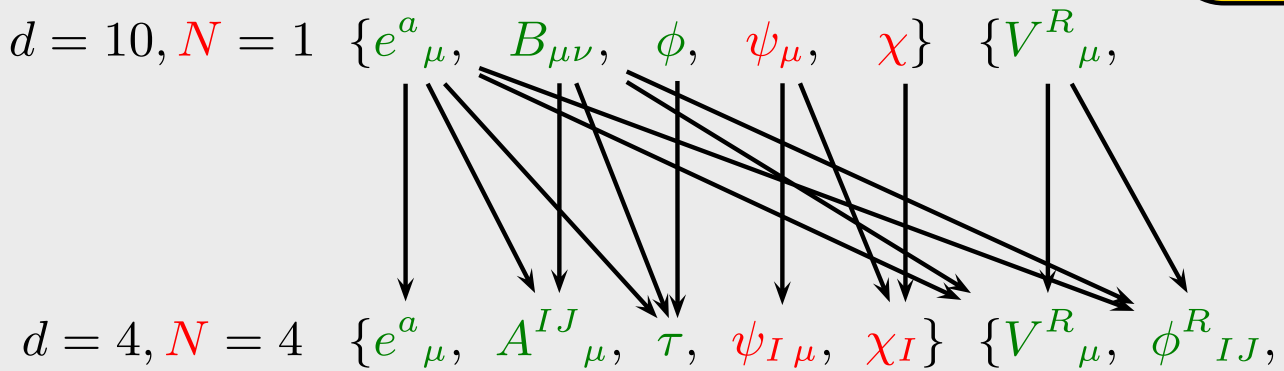

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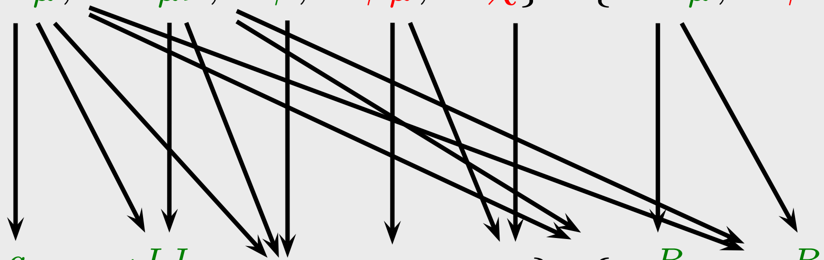

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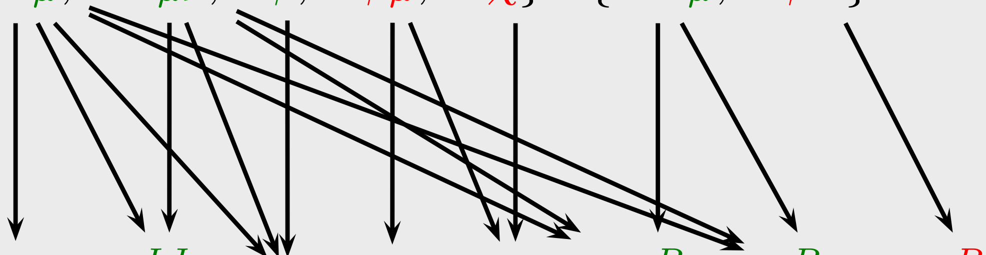

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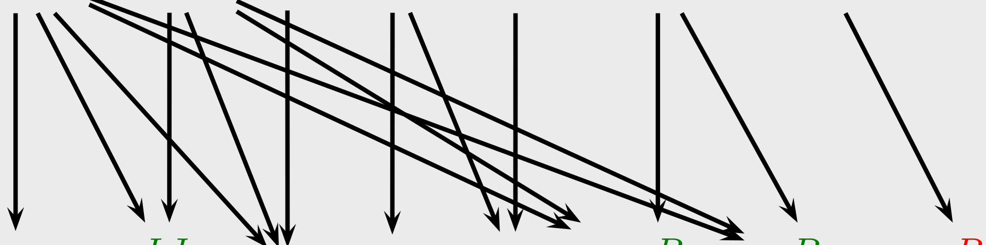

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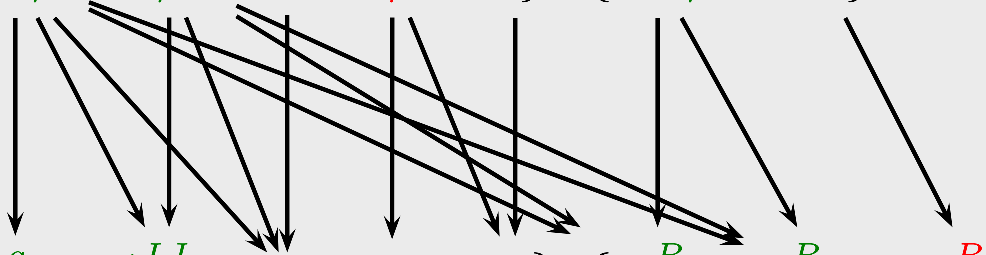

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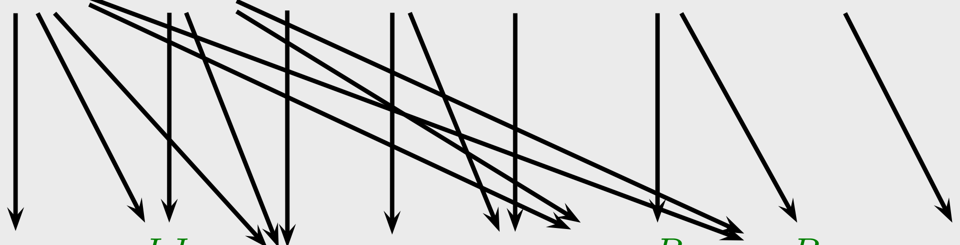

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There is also a global $SL(2,\mathbb{R})$ invariance $\tau' = \dfrac{\alpha\tau + \beta}{\gamma\tau + \delta}$, (S duality).

It is convenient to start by studying the *pure* supergravity theory (without the vector supermultiplets).

This theory still has interesting $SU(4) \sim SO(6)$ and $SL(2,\mathbb{R})$ invariances and very interesting solutions. The $N = 2$ and $N = 1$ are included as truncations.

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The bosonic action is

$$S = \int d^4x \sqrt{|g|} \left[R + \tfrac{1}{2} \frac{\partial_\mu \tau \, \partial^\mu \tau^*}{(\Im\mathrm{m}\,\tau)^2} - \tfrac{1}{16} \Im\mathrm{m}\,\tau F^{IJ\,\mu\nu} F_{IJ\,\mu\nu} - \tfrac{1}{16} \Re\mathrm{e}\,\tau F^{IJ\,\mu\nu\star} F_{IJ\,\mu\nu} \right].$$

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The equations of motion (plus Bianchi identities) are

$$\begin{aligned}
\mathcal{E}_{\mu\nu} &= G_{\mu\nu} + \tfrac{1}{2}(\Im\mathrm{m}\, \tau)^{-2}[\partial_{(\mu}\tau\partial_{\nu)}\tau^* - \tfrac{1}{2} g_{\mu\nu}\partial_\rho\tau\partial^\rho\tau^*] - \tfrac{1}{4}\Im\mathrm{m}\, \tau F_{IJ}{}^{+}{}_{\mu}{}^{\rho} F^{IJ-}{}_{\nu\rho} \, , \\
\mathcal{E} &= \mathcal{D}_\mu \left(\frac{\partial^\mu \tau^*}{\Im\mathrm{m}\, \tau} \right) - \tfrac{i}{8} \Im\mathrm{m}\, \tau F^{IJ\,+\,\rho\sigma} F_{IJ}{}^{+}{}_{\rho\sigma} \, , \\
\mathcal{E}^{IJ\,\mu} &= \nabla_\nu {}^\star \tilde{F}^{IJ\,\nu\mu} \, , \\
\mathcal{B}^{IJ\,\mu} &= \nabla_\nu {}^\star F^{IJ\,\nu\mu} \, .
\end{aligned}$$

The equations of motion are $SL(2,\mathbb{R})$-covariant buth the action is not. This symmetry rotates $\mathcal{E}^{IJ\,\mu}$ and $\mathcal{B}^{IJ\,\mu}$ and, therefore, $\tilde{F}^{IJ} = \tau F_{IJ}{}^{+} + \tau^* F_{IJ}{}^{-}$ and F^{IJ}.

For vanishing fermions, the supersymmetry transformation rules of the gravitini and dilatini, generated by 4 spinors ϵ_I are

$$\delta_\epsilon \psi_{I\,\mu} = \mathcal{D}_\mu \epsilon_I - \tfrac{i}{2\sqrt{2}} (\Im\mathrm{m}\,\tau)^{1/2} F_{IJ}{}^{+}{}_{\mu\nu} \gamma^\nu \epsilon^J \,,$$

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To follow the recipe we first construct the independent bilinears

- An antisymmetric complex matrix of scalars $M^{IJ} \equiv \bar{\epsilon}^I \epsilon^J$.
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A technical problem: at each point only two Weyl spinors can be linearly independent, but we have to work with the 4 ϵ^I to keep $SU(4)$ covariance. Then M^{IJ} is singular:

$$\varepsilon^{IJKL} M_{IJ} M_{KL} = 0 \,.$$

The Killing spinor equations become the following equations for bilinears:

$$
\begin{aligned}
\mathcal{D}_\mu M_{IJ} &= \tfrac{1}{\sqrt{2}}(\Im\mathrm{m}\,\tau)^{1/2} F_{K[I|}{}^{+}{}_{\mu\nu} V^{K}{}_{|J]}{}^{\nu}\,, \\
\mathcal{D}_\mu V^{I}{}_{J\,\nu} &= -\tfrac{1}{2\sqrt{2}}(\Im\mathrm{m}\,\tau)^{1/2}\Big[M_{KJ} F^{KI\,-}{}_{\mu\nu} + M^{IK} F_{JK}{}^{+}{}_{\mu\nu} \\
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We assume that they can indeed be solved and we assume the existence of $\tau, M_{IJ}, V^{I}{}_{J\,\mu}$ and try to determine $g_{\mu\nu}$ and $F^{IJ}{}_{\mu\nu}$ using these equations.

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In the timelike case this equation determines completely F_{IJ}:

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and the metric can be written in the form

$$ds^2 = |M|^2(dt+\omega)^2 - |M|^{-2}\gamma_{\underline{ij}}dx^i dx^j\,, \qquad i,j = 1,2,3\,,$$

where

$$d\omega = \tfrac{i}{2\sqrt{2}}|M|^{-4}\,{}^\star\Big[(M^{IJ}\mathcal{D}M_{IJ} - M_{IJ}\mathcal{D}M^{IJ})\wedge\hat{V}\Big]\,.$$

We substitute into the equations of motion, not to solve them, but to check the K.S.I.s which are necessary conditions to solve the Killing spinor equations and have supersymmetry.

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In the timelike case the K.S.I.s imply the following useful relations involving bilinears:

$$
\begin{aligned}
\mathcal{E}^{ab} - \tfrac{1}{2}\Im\mathrm{m}\,\mathcal{E}V^{a}V^{b} - \frac{1}{\sqrt{2}}(\Im\mathrm{m}\,\tau)^{1/2}\Im\mathrm{m}\,(M^{IJ}\mathcal{B}_{IJ}{}^{a})V^{b} &= 0\,,\\
\mathcal{E}^{*}V^{a} - \frac{i}{\sqrt{2}(\Im\mathrm{m}\,\tau)^{1/2}}M^{IJ}(\mathcal{E}_{IJ}{}^{a} - \tau\mathcal{B}_{IJ}{}^{a}) &= 0\,,\\
\Im\mathrm{m}[M^{IJ}(\mathcal{E}_{IJ}{}^{a} - \tau^{*}\mathcal{B}_{IJ}{}^{a})] &= 0\,.
\end{aligned}
$$

We substitute into the equations of motion, not to solve them, but to check the K.S.I.s which are necessary conditions to solve the Killing spinor equations and have supersymmetry.

In the timelike case the K.S.I.s imply the following useful relations involving bilinears:

$$\begin{aligned}
\mathcal{E}^{ab} - \tfrac{1}{2}\Im\mathrm{m}\,\mathcal{E}V^a V^b - \frac{1}{\sqrt{2}}(\Im\mathrm{m}\,\tau)^{1/2}\Im\mathrm{m}\,(M^{IJ}\mathcal{B}_{IJ}{}^a)V^b &= 0\,,\\
\mathcal{E}^* V^a - \frac{i}{\sqrt{2}(\Im\mathrm{m}\,\tau)^{1/2}} M^{IJ}(\mathcal{E}_{IJ}{}^a - \tau\mathcal{B}_{IJ}{}^a) &= 0\,,\\
\Im\mathrm{m}[M^{IJ}(\mathcal{E}_{IJ}{}^a - \tau^*\mathcal{B}_{IJ}{}^a)] &= 0\,.
\end{aligned}$$

We find two important results:

1. All the equations of motion are combinations of two simple 3-dimensional equations involving only τ, M^{IJ}, $\gamma_{\underline{i}\underline{j}}$, namely

$$n^{IJ}_{(3)} \equiv (\nabla_{\underline{i}} + 4i\xi_{\underline{i}})\left(\frac{\partial^{\underline{i}} N^{IJ}}{|N|^2}\right)\,, \qquad N^{IJ} \equiv (\Im\mathrm{m}\tau)^{1/2} M^{IJ}$$

$$e^*_{(3)} \equiv (\nabla_{\underline{i}} + 4i\xi_{\underline{i}})\left(\frac{\partial^{\underline{i}}\tau}{|N|^2}\right)\,, \qquad \xi \equiv \tfrac{i}{4}|M|^{-2}(M_{IJ}dM^{IJ} - M^{IJ}dM_{IJ})\,.$$

2. These field configurations still have to satisfy two complicated conditions in order to be supersymmetric.

In the end, in a straightforward way, a complete classification of supersymmetric field configurations of *pure* $N = 4, d = 4$ SUGRA can be achieved [a]

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There are also new types of string-like solutions, with metrics of the form

$$ds^2 = |k|^2(dt + \omega_{\underline{x}} dx) - |k|^{-2} dx^2 - 2dzdz^*\,,$$

where $|k|^2 = k_{IJ}(z)k^{IJ}(z^*)$ and $\omega_{\underline{x}}$ satisfies

$$\partial_{\underline{z}}\omega_{\underline{x}} = \partial_{\underline{z}^*}|k|^{-2}\,, \qquad \partial_{\underline{z}^*}\omega_{\underline{x}} = \partial_{\underline{z}}|k|^{-2}\,.$$

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Work on the last two topics is in progress.

This is

THE END