The Supersymmetric Vistas of the Supergravity Landscape

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Based on hep-th/0505056 and on work in preparation. Work done in collaboration with

Jorge Bellorín and Mechthild Hübscher (I.F.T., Madrid)

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- However, in theories that include gravity, the energies of different vacua cannot be compared and it is not known how *the* vacuum is chosen, and, therefore, why our Universe is the way it is.
- This is an old and very well known problem. It is also of crucial importance. And it is still **UNSOLVED**.

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But, first, we are going to review briefly how we have come to consider this scenario in our quest for UNIFICATION.

Plan of the Talk:

1	Intro to Unification
9	Susy Solutions
11	Tod's problem
15	Solving it
19	N=4, d=4 SUGRA
26	Conclusion

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"How We Got Into This Mess"

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1 Electricity $\bigoplus$ Magnetism $\stackrel{\text{Faraday,Maxwell}}{\Longrightarrow}$ Electromagnetism

$$\vec{E}, \vec{B} \longrightarrow (F_{\mu\nu}) \equiv \left(\begin{array}{c|c} 0 & -\vec{E}^T \\ \hline \vec{E} & {}^\star\vec{B} \end{array}\right)$$

Required by the Special Theory of Relativity just as Newtonian gravity and gravitomagnetism are combined in General Relativity.

2 Space $\bigoplus$ Time $\overset{\text{Einstein,Minkowski}}{\Longrightarrow}$ Spacetime

$$t, \vec{x} \ \longrightarrow \ (x^\mu) \equiv (ct, \vec{x}) \, .$$

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- There is enhancement of <u>local</u> symmetry from g.c.t.'s in $d = 4$ to g.c.t.'s in $d = 5$, but this symmetry is spontaneously broken (in modern parlance) to g.c.t.'s in $d = 4$ and $U(1)$ due to the (completely arbitrary) choice of vacuum. The rule is always:

> global symmetry of the vacuum $\sim$ local symmetry of the reduced theory.

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The extraordinary success of this model has made of it the paradigm of unification schemes.

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- It can also be combined with g.c.t.'s, making it local (supergravity theories). We can have supergravity theories with Yang-Mills fields etc. etc. But in most of these theories gravity is not unified with the other interactions.

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- These extended supergravities can in general be obtained from compactification of simpler higher-dimensional supergravities. It was also discovered that many $N = 1$ supergravities coupled to Yang-Mills fields could also be obtained in the same way, by a careful choice of compact manifold (i.e. Kaluza-Klein vacuum). This lead to a new brand of unified theories which could describe everything: Theories Of Everything.

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This theory satisfies all our desires for unification, but we have to find in it our Universe's vacuum and explain why and how it is selected.

Since many things seem to work, the vacuum-selection problem (of which the moduli estabilization problem is a sub-problem) becomes more acute.

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One can consider other simplified formulations of the same problem:

- Supergravity landscape (Van Proeyen): the space of all possible supergravities covers all possible low-energy limits of supersymmetric M theory vacua. It is not known if all supergravities can be given an M theory origin, but the problem could be treated in a systematic way.

- Landscape of supersymmetric vacua: the space of all supersymmetric solutions of 11- and 10-dimensional supergravities covers all possible supersymmetric M theory compactification vacua plus the supersymmetric solutions of the corresponding lower-dimensional supergravities. It also covers all the supersymmetric objects (black holes, p-branes...) of M theory.

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- Some useful identities that they always satisfy (Killing spinor identities).
- An application to $N = 4, d = 4$ supergravity.

2 – Susy Solutions

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Supersymmetric solutions (a.k.a. solutions with residual or unbroken or preserved supersymmetry) are classical bosonic solutions of supergravity (SUGRA) theories which are invariant under some supersymmetry transformations.

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Generically, the supersymmetry transformations take the form

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This is a generalization of the concept of isometry, an infinitesimal general coordinate transformation generated by $\xi^\mu(x)$ that leaves the metric $g_{\mu\nu}$ invariant because it satisfies the *Killing (vector) equation*

$$\delta_\xi g_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)} = 0 \,. \tag{3}$$

To each bosonic symmetry we associate a generator

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When the supersymmetric vacuum solution has a clear (possibly warped) product structure we distinguish internal and spacetime symmetries
$\longrightarrow$ spontaneous compactification.

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3 – Tod's problem

This is the problem of finding **all** the bosonic field configurations ϕ^b for which a SUGRA's Killing spinor equations

$$\delta_\epsilon \phi^f \Big|_{\phi^f=0} \sim \partial\epsilon + \phi^b \epsilon = 0\,,$$

have a solution ϵ, (i.e. all the possible supersymmetric bosonic field configurations ϕ^b), which includes all the possible supersymmetric vacua and compactifications.

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N.B. Not all supersymmetric bosonic field configurations satisfy the classical bosonic equations of motion $\frac{\delta S}{\delta \phi^b}\Big|_{\phi^f=0} \equiv S_{,b}\big|_{\phi^f=0} \equiv \mathcal{E}(\phi^b)$.

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The supersymmetry invariance of the action implies

$$\delta_\epsilon S \qquad = \int d^dx\,(S_{,b}\,\delta_\epsilon\phi^b + S_{,f}\,\delta_\epsilon\phi^f) \qquad = 0\,,$$

[a] R. Kallosh & T.O. (1993), J. Bellorín & T.O. (2005)

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The supersymmetry invariance of the action implies after taking the functional derivative w.r.t. fermions and setting them to zero

$$(\delta_\epsilon S\ )_{,f_1}\Big|_{\phi^f=0} = \left\{ \int d^dx \,(S_{,b}\, \delta_\epsilon \phi^b + S_{,f}\, \delta_\epsilon \phi^f) \right\}_{,f_1}\Bigg|_{\phi^f=0} = 0\,,$$

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Many terms vanish automatically because they are odd in fermion fields ϕ^f

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$$\left\{S_{,b}\,(\delta_\epsilon \phi^b)_{,f_1} + S_{,f\,f_1}\,\delta_\epsilon \phi^f\right\}\Big|_{\phi^f=0} = 0\,.$$

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$$\left\{S_{,b}\,(\delta_\epsilon\phi^b)_{,f_1} + S_{,f\,f_1}\,\delta_\epsilon\phi^f\right\}\Big|_{\phi^f=0} = 0\,.$$

This is valid for any fields ϕ^b and any supersymmetry parameter ϵ. For a supersymmetric field configuration ϵ is a Killing spinor $\delta_\epsilon\phi^f\Big|_{\phi^f=0}$ and we obtain the Killing spinor identities

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Many terms vanish automatically because they are odd in fermion fields ϕ^f

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These non-trivial identities are linear relations between the bosonic equations of motion and can be used to solve Tod's problem, obtain BPS bounds etc. Let's see some examples.

$N = 1, d = 4$ supergravity

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Its field content is $\{e^a{}_\mu, \psi_\mu\}$. The bosonic action is just the Einstein-Hilbert action

$$S|_{\psi_\mu=0} = \int d^4x \sqrt{|g|}\, R\,, \;\Rightarrow\; \mathcal{E}_a{}^\mu(e) \sim G_a{}^\mu\,,$$

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$$\delta_\epsilon e^a{}_\mu = -i\bar{\epsilon}\gamma^a\psi_\mu\,, \qquad \delta_\epsilon\psi_\mu = \nabla_\mu\epsilon = \partial_\mu\epsilon - \tfrac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\epsilon\,.$$

The K.S.I.s are

$$-i\bar{\epsilon}\gamma^a G_a{}^\mu = 0\,, \;\Rightarrow R = 0\,, \;\; -i\bar{\epsilon}\gamma^a R_a{}^\mu = 0\,.$$

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$$[\nabla_\mu, \nabla_\nu]\epsilon = -\tfrac{1}{4}R_{\mu\nu}{}^{ab}\gamma_{ab}\epsilon = 0\,, \;\Rightarrow R^\mu{}_a\gamma^a\epsilon = 0\,.$$

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We will see later how to obtain more information from these identities.

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Its field content is $\{e^{a}{}_{\mu}, A_{\mu}, \psi_{\mu}\}$. The bosonic action is just the Einstein-Maxwell action

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$$\Rightarrow \quad \{\mathcal{E}_a{}^\mu(e)\gamma^a + 2[\mathcal{E}^\mu(A) + \mathcal{B}^\mu(A)\gamma_5]\}\epsilon = 0\,.$$

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All the metrics with covariantly constant null vectors are Brinkmann pp-waves and have the form

$$ds^2 = 2du(dv + K du + A_{\underline{i}} dx^i) + \tilde{g}_{\underline{ij}} dx^i dx^j\,,$$

where all the components are independent of v $V^\mu \partial_\mu \equiv \partial/\partial v$.

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These metrics are the supersymmetric field configurations of $N = 1, d = 4$ SUGRA, but only those with $R_{\mu\nu} = 0$ are supersymmetric solutions.

$N = 2, d = 4$ supergravity

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With two Weyl spinors ϵ^I one can construct the following independent bilinears

- A complex scalar $\bar{\epsilon}^I \epsilon^J \equiv M \varepsilon^{IJ}$
- A Hermitean matrix of null vectors (4) $V^I{}_{J\,\mu} \equiv i \bar{\epsilon}^I \gamma_\mu \epsilon_J$

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The Killing spinor equations imply the following equations for the bilinears:

$$\nabla_\mu M \sim F^+{}_{\mu\nu} V^I{}_I{}^\nu \,,$$

$$\nabla_\mu V^I{}_{J\,\nu} \sim \delta^I{}_J [M F^+{}_{\mu\nu} + M^* F^-{}_{\mu\nu}] - \Phi_{KJ\,(\mu}{}^\rho \varepsilon^{KI} F^-{}_{\nu)\rho} - \Phi^{IK}{}_{(\mu|}{}^\rho \varepsilon_{KJ} F^+{}_{|\nu)\rho} \,,$$

so $V^\mu \equiv V^I{}_I{}^\mu$ is Killing and the other three are exact forms. $V^\mu V_\mu \sim |M|^2 \geq 0$ can be timelike or null.

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- A Hermitean matrix of null vectors (4) $V^I{}_{J\,\mu} \equiv i\bar{\epsilon}^I \gamma_\mu \epsilon_J$

The Killing spinor equations imply the following equations for the bilinears:

$$\begin{aligned} \nabla_\mu M &\sim F^+{}_{\mu\nu} V^I{}_I{}^\nu \,, \\ \nabla_\mu V^I{}_{J\,\nu} &\sim \delta^I{}_J [M F^+{}_{\mu\nu} + M^* F^-{}_{\mu\nu}] - \Phi_{KJ\,(\mu}{}^\rho \varepsilon^{KI} F^-{}_{\nu)\rho} - \Phi^{IK}{}_{(\mu|}{}^\rho \varepsilon_{KJ} F^+{}_{|\nu)\rho} \,, \end{aligned}$$

so $V^\mu \equiv V^I{}_I{}^\mu$ is Killing and the other three are exact forms. $V^\mu V_\mu \sim |M|^2 \geq 0$ can be timelike or null.
When it is timelike, $V^\mu \partial_\mu \equiv \sqrt{2}\partial/\partial t$ and

$$\begin{aligned} F^+ &\sim |M|^{-2}\{V \wedge dM + i^\star[V \wedge dM]\} \,, \\ ds^2 &= |M|^2 (dt + \omega)^2 - |M|^{-2} d\vec{x}^{\,2} \,, \end{aligned}$$

$N = 2, d = 4$ supergravity

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SUSY $\Rightarrow d\omega = i|M|^{-2\star}[M dM^* - \text{c.c.}]$,
Solutions$\Rightarrow \vec{\nabla}^2 M^{-1} = 0$. (Israel-Wilson-Perjes)

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5 – N=4, d=4 SUGRA

This theory can be obtained by toroidal compactification on T^6 of $N = 1, d = 10$ SUGRA (the effective field theory of the Heterotic String):

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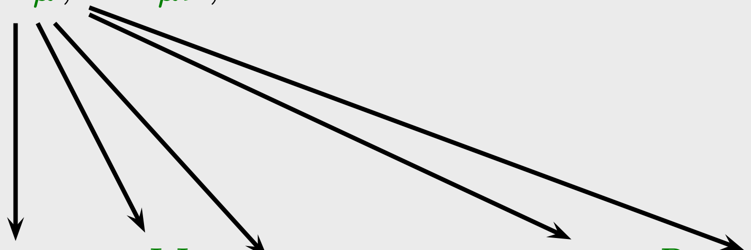

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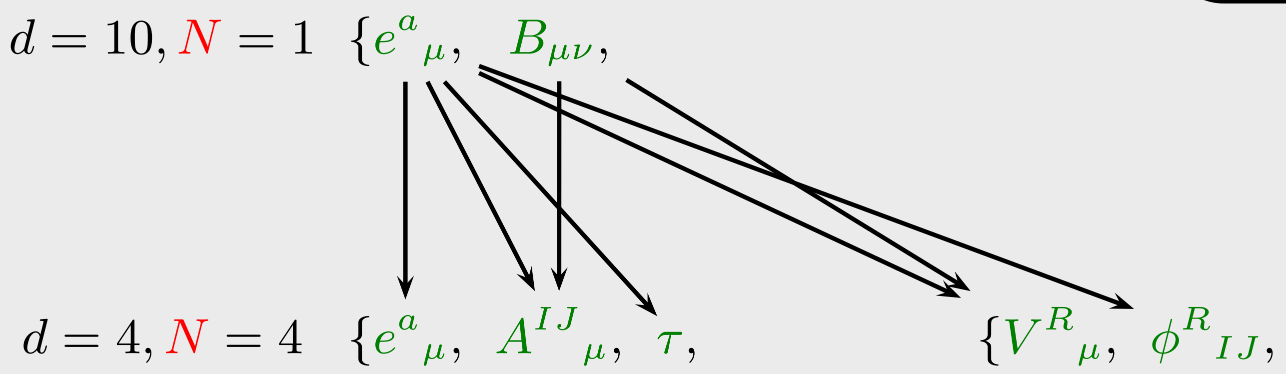

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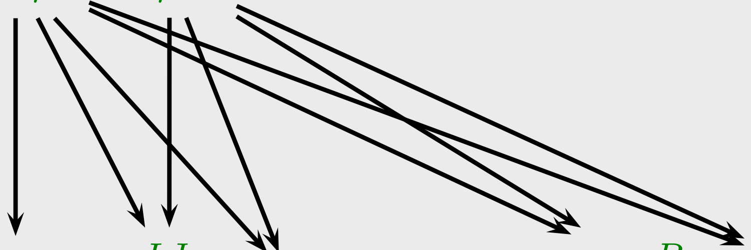

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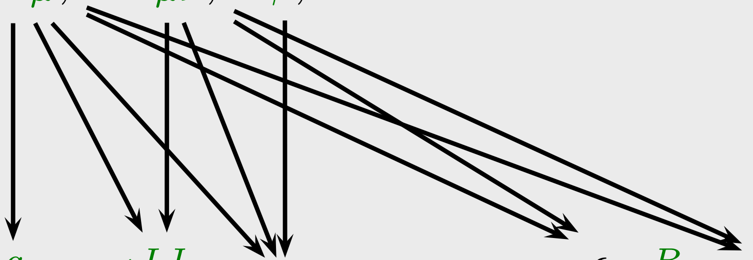

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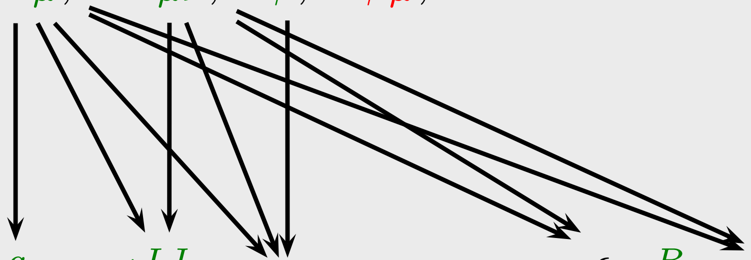

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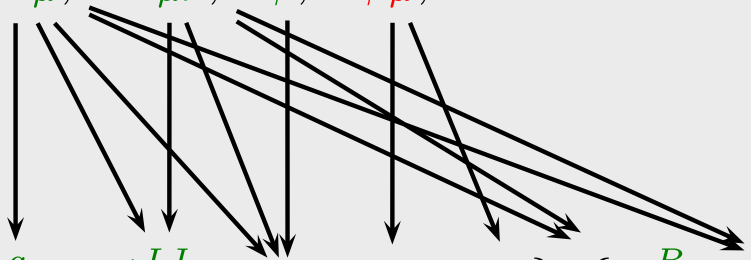

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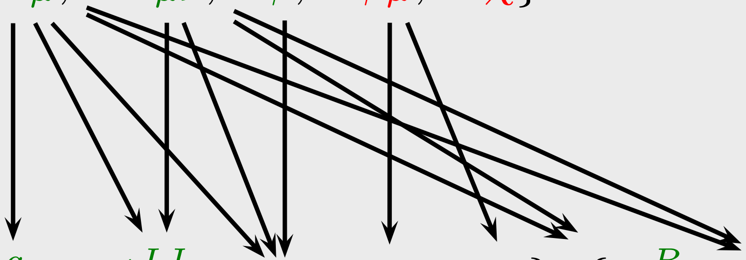

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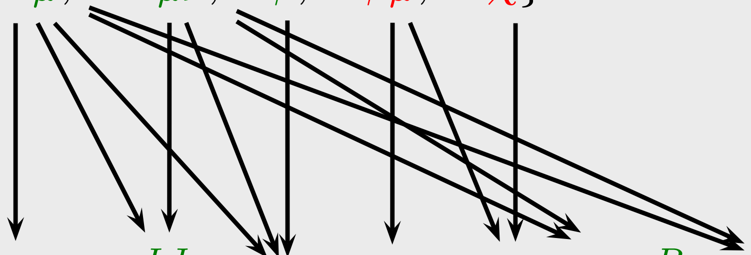

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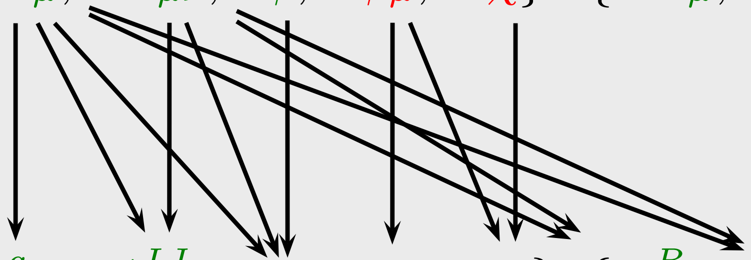

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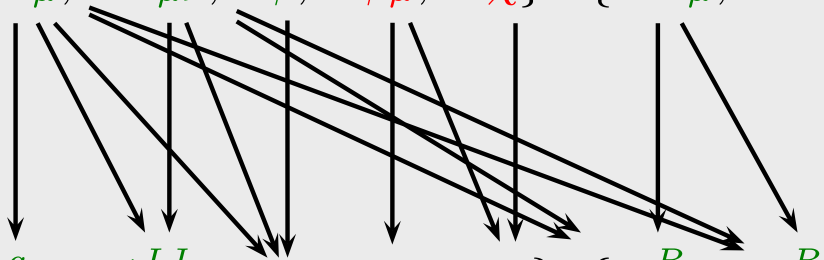

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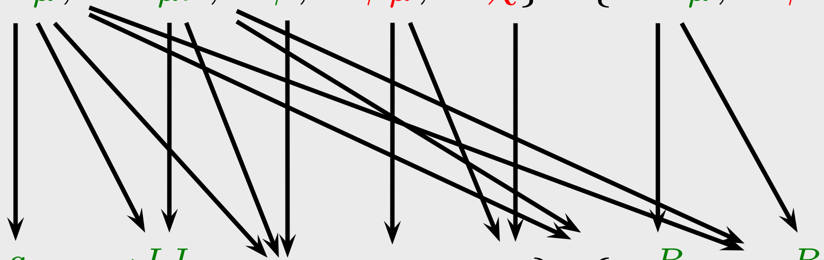

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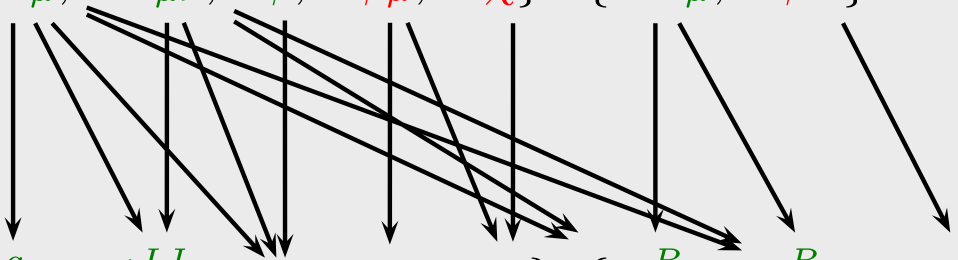

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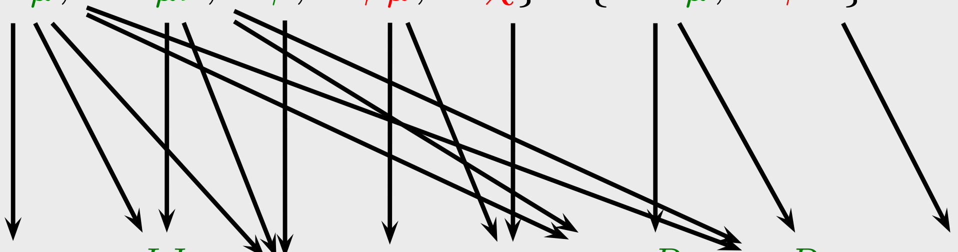

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$$\tau = a + ie^{-\phi}\,, \quad (axidilaton)$$

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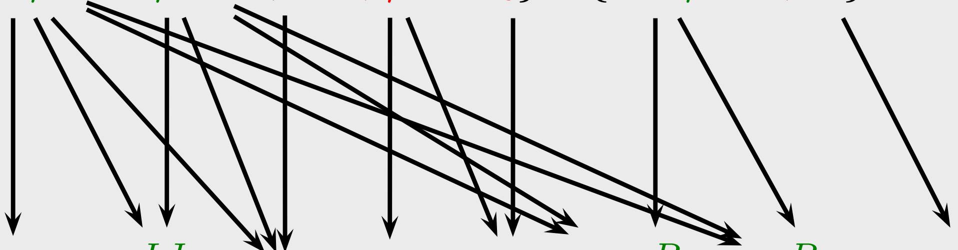

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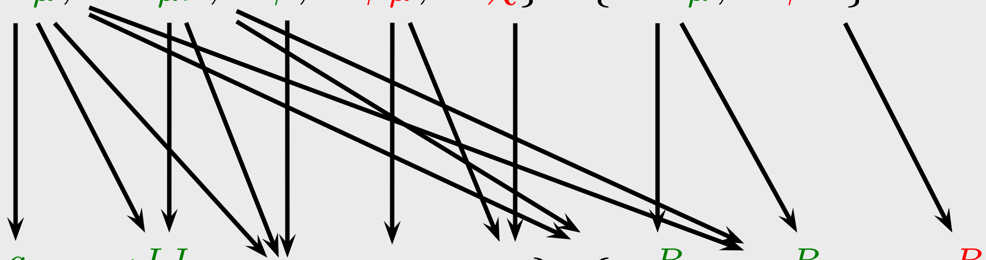

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There is also a global $SL(2,\mathbb{R})$ invariance $\tau' = \dfrac{\alpha\tau + \beta}{\gamma\tau + \delta}$, (S duality).

It is convenient to start by studying the *pure* supergravity theory (without the vector supermultiplets).

This theory still has interesting $SU(4) \sim SO(6)$ and $SL(2, \mathbb{R})$ invariances and very interesting solutions. The $N = 2$ and $N = 1$ are included as truncations.

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The bosonic action is

$$S = \int d^4x \sqrt{|g|} \left[R + \tfrac{1}{2} \frac{\partial_\mu \tau \, \partial^\mu \tau^*}{(\Im\mathrm{m}\, \tau)^2} - \tfrac{1}{16} \Im\mathrm{m}\, \tau F^{IJ\,\mu\nu} F_{IJ\,\mu\nu} - \tfrac{1}{16} \Re\mathrm{e}\, \tau F^{IJ\,\mu\nu\star} F_{IJ\,\mu\nu} \right] .$$

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The equations of motion (plus Bianchi identities) are

$$\begin{aligned}
\mathcal{E}_{\mu\nu} &= G_{\mu\nu} + \tfrac{1}{2} (\Im\mathrm{m}\,\tau)^{-2} [\partial_{(\mu}\tau \partial_{\nu)}\tau^* - \tfrac{1}{2} g_{\mu\nu} \partial_\rho \tau \partial^\rho \tau^*] - \tfrac{1}{4} \Im\mathrm{m}\,\tau F_{IJ}{}^{+}{}_{\mu}{}^{\rho} F^{IJ-}{}_{\nu\rho} \,, \\
\mathcal{E} &= \mathcal{D}_\mu \left(\frac{\partial^\mu \tau^*}{\Im\mathrm{m}\,\tau} \right) - \tfrac{i}{8} \Im\mathrm{m}\,\tau F^{IJ\,+\,\rho\sigma} F_{IJ}{}^{+}{}_{\rho\sigma} \,, \\
\mathcal{E}^{IJ\,\mu} &= \nabla_\nu {}^\star \tilde{F}^{IJ\,\nu\mu} \,, \\
\mathcal{B}^{IJ\,\mu} &= \nabla_\nu {}^\star F^{IJ\,\nu\mu} \,.
\end{aligned}$$

The equations of motion are $SL(2,\mathbb{R})$-covariant buth the action is not. This symmetry rotates $\mathcal{E}^{IJ\,\mu}$ and $\mathcal{B}^{IJ\,\mu}$ and, therefore, $\tilde{F}^{IJ} = \tau F_{IJ}{}^{+} + \tau^* F_{IJ}{}^{-}$ and F^{IJ}.

For vanishing fermions, the supersymmetry transformation rules of the gravitini and dilatini, generated by 4 spinors ϵ_I are

$$\begin{aligned}\delta_\epsilon \psi_{I\,\mu} &= \mathcal{D}_\mu \epsilon_I - \frac{i}{2\sqrt{2}} (\Im\mathrm{m}\, \tau)^{1/2} F_{IJ}{}^{+}{}_{\mu\nu} \gamma^\nu \epsilon^J \,, \\ \delta_\epsilon \chi_I &= \frac{1}{2\sqrt{2}} \frac{\not{\partial}\tau}{\Im\mathrm{m}\, \tau} \epsilon_I - \frac{1}{8} (\Im\mathrm{m}\, \tau)^{1/2} \not{F}_{IJ}{}^{-} \epsilon^J \,.\end{aligned}$$

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Our goal is to find bosonic field configurations such that a Killing spinor (i.e. a set ϵ^I) satisfying $\delta_\epsilon \psi_{I\,\mu} = \delta_\epsilon \chi_I = 0$ exists.

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All the fermions transform with a local $U(1)$ phase under $SL(2,\mathbb{R})$ transformations and, therefore, the Killing spinor equations are $SU(4)$ and $SL(2,\mathbb{R})$-covariant.

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The general supersymmetric configurations must have the same covariance.

To follow the recipe we first construct the independent bilinears

- An antisymmetric complex matrix of scalars $M^{IJ} \equiv \bar{\epsilon}^I \epsilon^J$.
- A Hermitean matrix of null vectors (16) $V^I{}_{J\,\mu} \equiv i\bar{\epsilon}^I \gamma_\mu \epsilon_J$.

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A technical problem: at each point only two Weyl spinors can be linearly independent, but we have to work with the 4 ϵ^I to keep $SU(4)$ covariance. Then M^{IJ} is singular:

$$\varepsilon^{IJKL} M_{IJ} M_{KL} = 0 \,.$$

The Killing spinor equations become the following equations for bilinears:

$$\begin{aligned}
\mathcal{D}_\mu M_{IJ} &= \tfrac{1}{\sqrt{2}}(\Im\mathrm{m}\,\tau)^{1/2} F_{K[I|}{}^{+}{}_{\mu\nu} V^{K}{}_{|J]}{}^{\nu}\,, \\
\mathcal{D}_\mu V^{I}{}_{J\,\nu} &= -\tfrac{1}{2\sqrt{2}}(\Im\mathrm{m}\,\tau)^{1/2}\Big[M_{KJ}F^{KI\,-}{}_{\mu\nu} + M^{IK}F_{JK}{}^{+}{}_{\mu\nu} \\
&\quad -\Phi_{KJ\,(\mu}{}^{\rho}F^{KI\,-}{}_{\nu)\rho} - \Phi^{IK}{}_{(\mu|}{}^{\rho}F_{KI}{}^{+}{}_{|\nu)\rho}\Big]\,,
\end{aligned}$$

$$\begin{aligned}
0 &= V^{K}{}_{I}{}^{\mu}\partial_\mu\tau - \tfrac{i}{2\sqrt{2}}(\Im\mathrm{m}\,\tau)^{3/2}F_{IJ}{}^{-\,\mu\nu}\Phi^{KJ}{}_{\mu\nu}\,, \\
0 &= F_{IJ}{}^{-}{}_{\rho\sigma}V^{J}{}_{K}{}^{\sigma} + \tfrac{i}{\sqrt{2}}(\Im\mathrm{m}\,\tau)^{-3/2}\left(M_{IK}\partial_\rho\tau - \Phi_{IK\,\rho}{}^{\mu}\partial_\mu\tau\right)\,.
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We assume that they can indeed be solved and we assume the existence of $\tau, M_{IJ}, V^{I}{}_{J\,\mu}$ and try to determine $g_{\mu\nu}$ and $F^{IJ}{}_{\mu\nu}$ using these equations.

Working with the equations for the bilinears we find immediately

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$$F_{IJ}{}^-{}_{\mu\nu} V^\nu = -\frac{\sqrt{2}i}{(\Im\mathrm{m}\,\tau)^{3/2}} M_{IJ} \partial_\mu \tau - \frac{\sqrt{2}}{(\Im\mathrm{m}\,\tau)^{1/2}} \varepsilon_{IJKL} \mathcal{D}_\mu M^{KL} \,.$$

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In the timelike case this equation determines completely F_{IJ}:

$$F_{IJ}{}^{-} = -\frac{1}{\sqrt{2}|M|^2 (\Im\mathrm{m}\,\tau)^{1/2}} \left\{ \left[i \frac{M_{IJ}}{(\Im\mathrm{m}\,\tau)} d\tau + \varepsilon_{IJKL} \mathcal{D} M^{KL} \right] \wedge \hat{V} - i\,{}^\star[\cdots] \right\}.$$

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and the metric can be written in the form

$$ds^2 = |M|^2(dt+\omega)^2 - |M|^{-2}\gamma_{\underline{ij}}dx^i dx^j\,, \qquad i,j = 1,2,3\,,$$

where

$$d\omega = \frac{i}{2\sqrt{2}}|M|^{-4}\,{}^{\star}\Big[(M^{IJ}\mathcal{D}M_{IJ} - M_{IJ}\mathcal{D}M^{IJ})\wedge\hat{V}\Big]\,.$$

We substitute into the equations of motion, not to solve them, but to check the K.S.I.s which are necessary conditions to solve the Killing spinor equations and have supersymmetry.

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In the timelike case the K.S.I.s imply the following useful relations involving bilinears:

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\begin{aligned}
\mathcal{E}^{ab} - \tfrac{1}{2}\Im\mathrm{m}\,\mathcal{E}V^a V^b - \frac{1}{\sqrt{2}}(\Im\mathrm{m}\,\tau)^{1/2}\Im\mathrm{m}\,(M^{IJ}\mathcal{B}_{IJ}{}^a)V^b &= 0\,, \\
\mathcal{E}^* V^a - \frac{i}{\sqrt{2}(\Im\mathrm{m}\,\tau)^{1/2}} M^{IJ}(\mathcal{E}_{IJ}{}^a - \tau\mathcal{B}_{IJ}{}^a) &= 0\,, \\
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\end{aligned}$$

We find two important results:

1. All the equations of motion are combinations of two simple 3-dimensional equations involving only $\tau, M^{IJ}, \gamma_{\underline{i}\underline{j}}$, namely

$$\begin{aligned}
n^{IJ}_{(3)} &\equiv (\nabla_{\underline{i}} + 4i\xi_{\underline{i}})\left(\frac{\partial^{\underline{i}} N^{IJ}}{|N|^2}\right)\,, \qquad N^{IJ} \equiv (\Im\mathrm{m}\tau)^{1/2} M^{IJ}\\
e^*_{(3)} &\equiv (\nabla_{\underline{i}} + 4i\xi_{\underline{i}})\left(\frac{\partial^{\underline{i}}\tau}{|N|^2}\right)\,, \qquad \xi \equiv \tfrac{i}{4}|M|^{-2}(M_{IJ}dM^{IJ} - M^{IJ}dM_{IJ})\,.
\end{aligned}$$

2. These field configurations still have to satisfy two complicated conditions in order to be supersymmetric.

In the end, in a straightforward way, a complete classification of supersymmetric field configurations of *pure* $N = 4, d = 4$ SUGRA can be achieved [a]

[a] J. Bellorín & T.O., hep-th/0506056.

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The supersymmetric solutions include black holes, Brinkmann waves and *stringy cosmic strings* of the type found by Greene, Shapere, Vafa & Yau (1989), which can also be seen as Type IIB 7-branes.

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There are also new types of string-like solutions, with metrics of the form

$$ds^2 = |k|^2(dt + \omega_{\underline{x}} dx) - |k|^{-2} dx^2 - 2dzdz^* \,,$$

where $|k|^2 = k_{IJ}(z)k^{IJ}(z^*)$ and $\omega_{\underline{x}}$ satisfies

$$\partial_{\underline{z}}\omega_{\underline{x}} = \partial_{\underline{z}^*}|k|^{-2} \,, \qquad \partial_{\underline{z}^*}\omega_{\underline{x}} = \partial_{\underline{z}}|k|^{-2} \,.$$

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6 – Conclusion

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Work on the last two topics is in progress.

This is

THE END