Gödel Spacetimes and *Flacuum* Solutions

Tomás Ortín (I.F.T.-C.S.I.C)

Seminar given on April 29th 2004 at University of Tel Aviv

Based on hep-th/0401005. Work done in collaboration with

Patrick Meessen (C.E.R.N.)

Introduction/Motivation

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  - The BPST instanton configuration is realized in solutions with S^7 subspaces.

We are going to classify the maximally supersymmetric vacua of SUGRAs with 8 $\mathcal{Q}$s and find an interesting example of maximally supersymmetric, topologically non-trivial field configuration of SUGRA that corresponds to a well-known Abelian Yang-Mills instanton configuration.

Plan of the Talk:

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1 – SUGRA Vacua

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- ★ Usually enjoys a high degree of (residual) symmetry. This symmetry determines all the kinematical properties of the QFT (conserved charges, spectrum etc.)
- ★ In (Special-Relativistic) QFT it is *required* that the residual symmetry of the vacuum includes the Poincaré group.

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Clearly, the most important question is

"How should (we or the theory) choose the vacuum?"

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This is a generalization of the concept of isometry, an infinitesimal general coordinate transformation generated by $\xi^\mu(x)$ that leaves the metric $g_{\mu\nu}$ invariant because it satisfies the *Killing (vector) equation*

$$\delta_\xi g_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)} = 0\,. \tag{3}$$

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These will be the superalgebras of the QFTs constructed on these vacua!

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maximally supersymmetric $\not\Rightarrow$ **maximally symmetric**

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2 – 8Q SUGRA Vacua

The smallest spinor in $d \geq 7$ has 16 real components. Then the SUGRAs with 8 supercharges in $d > 3$ are just

Theory	Fields	Bosonic Action
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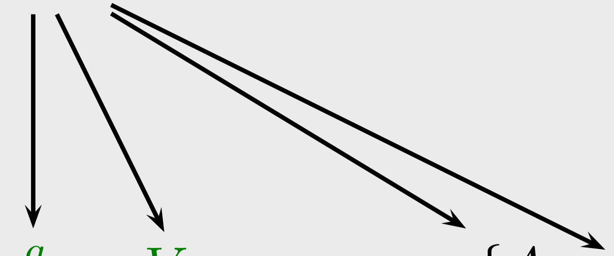

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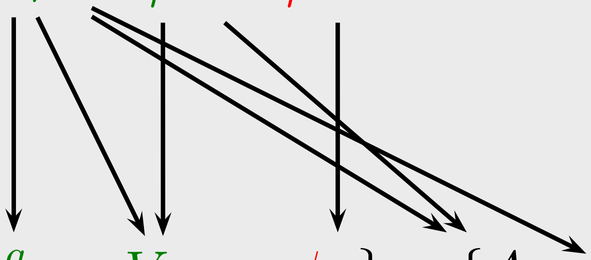

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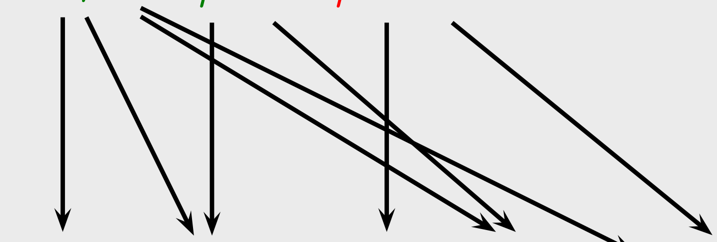

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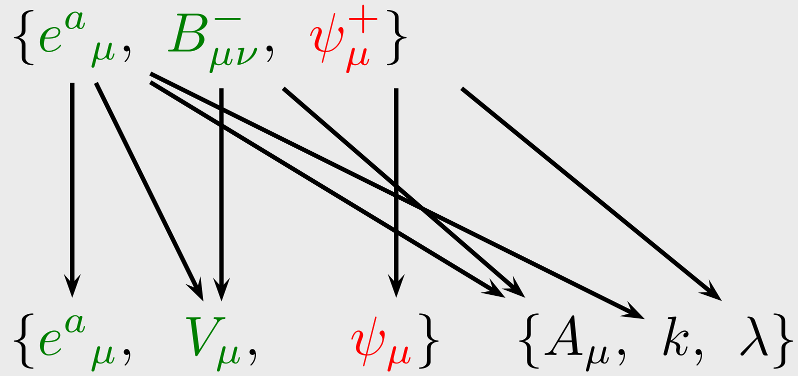

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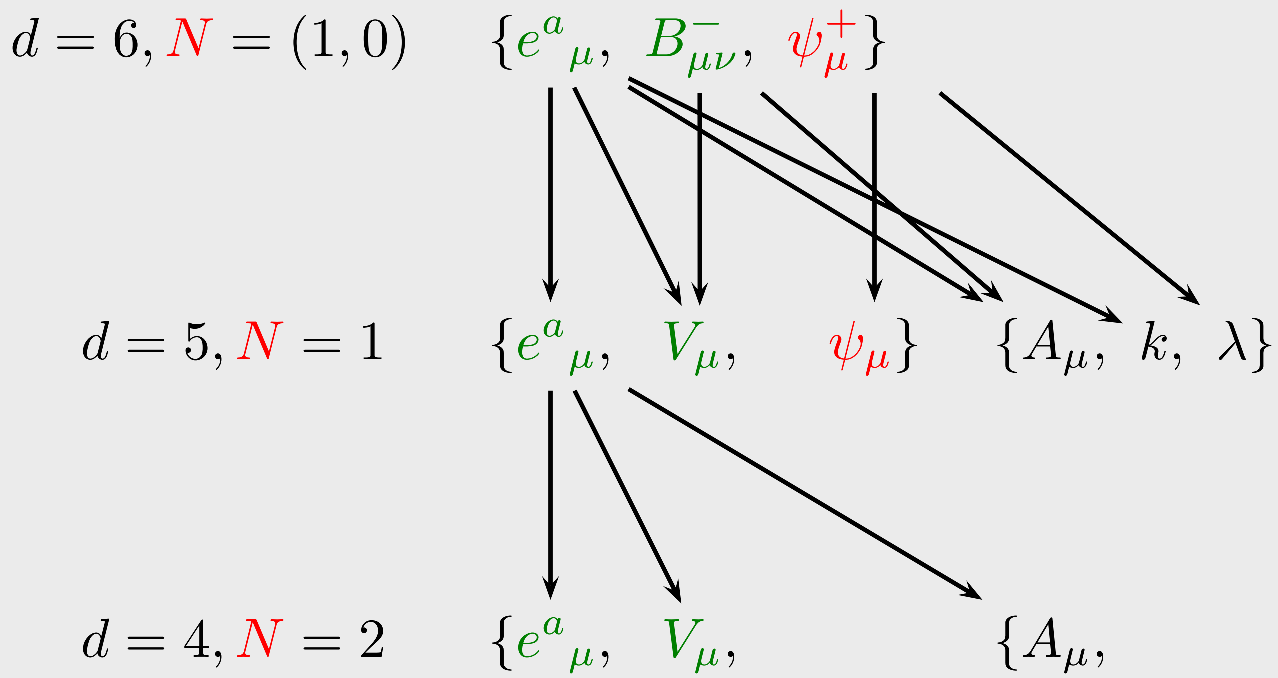

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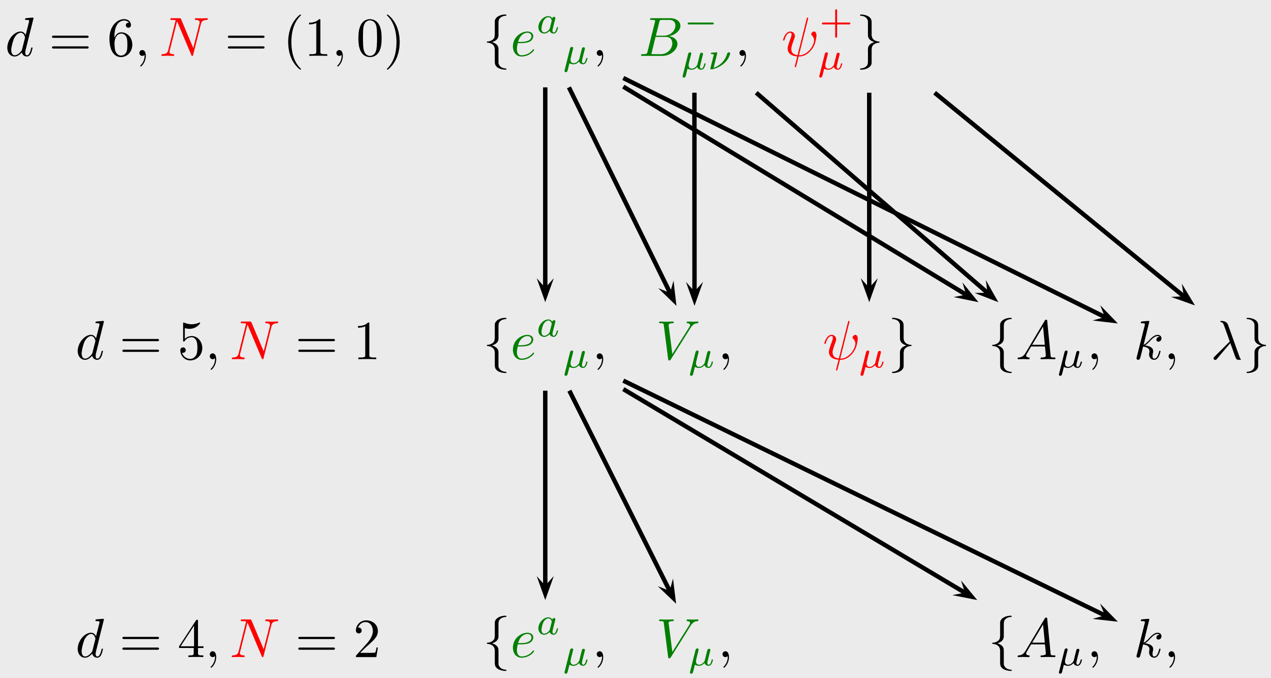

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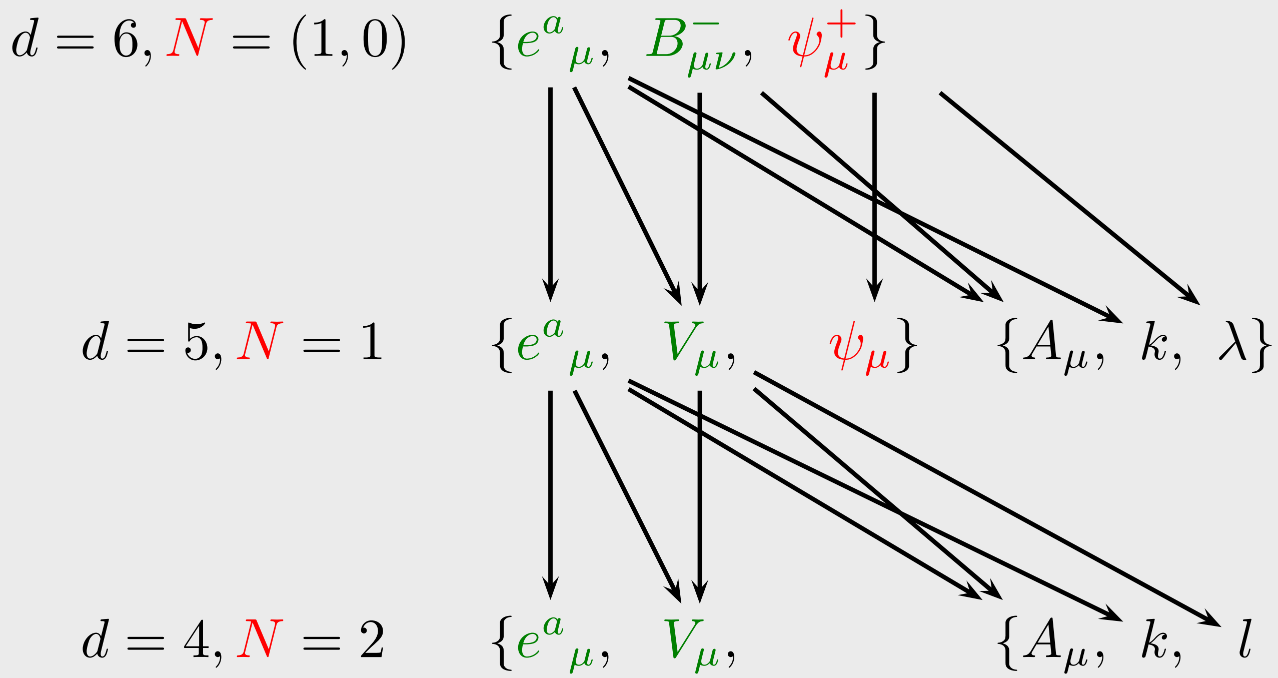

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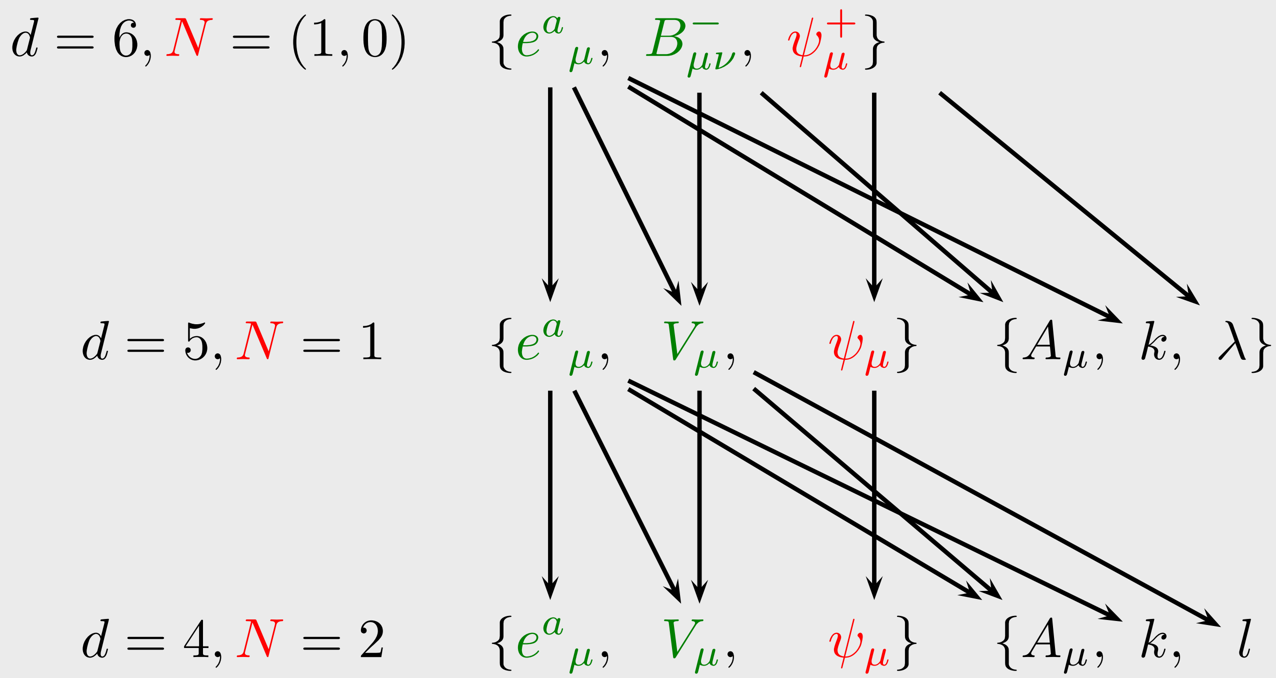

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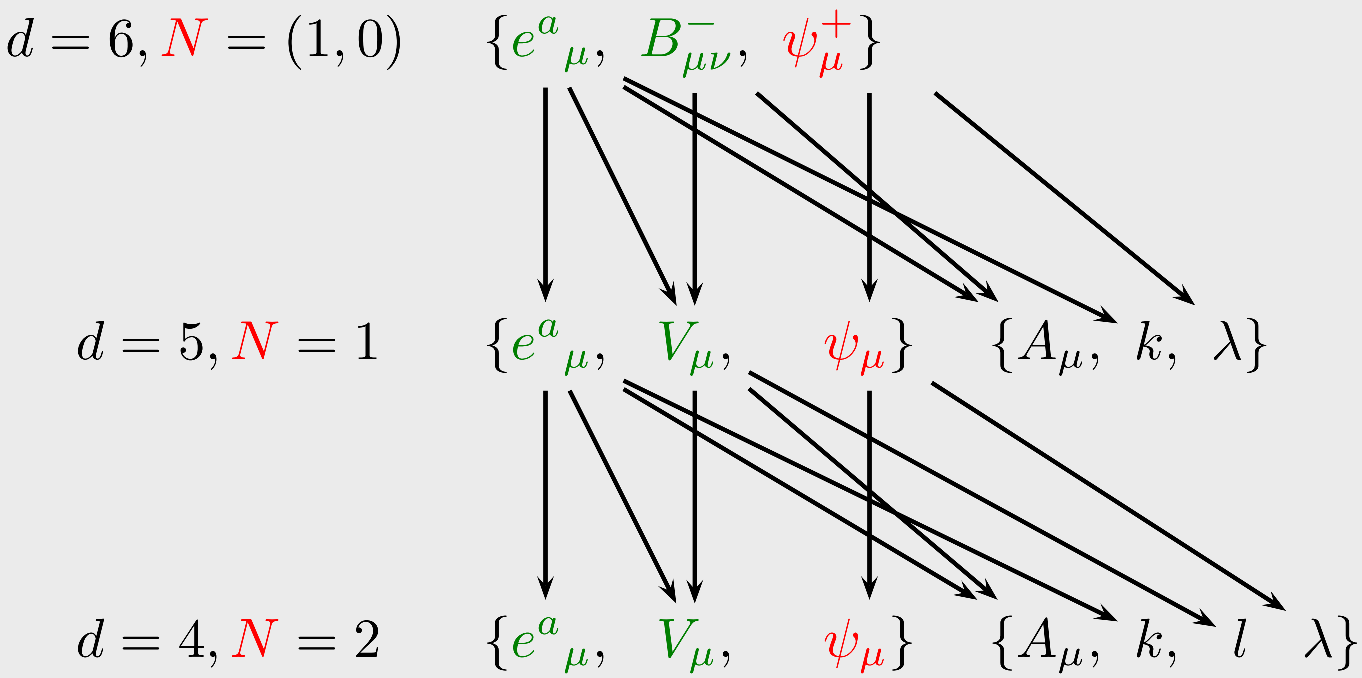

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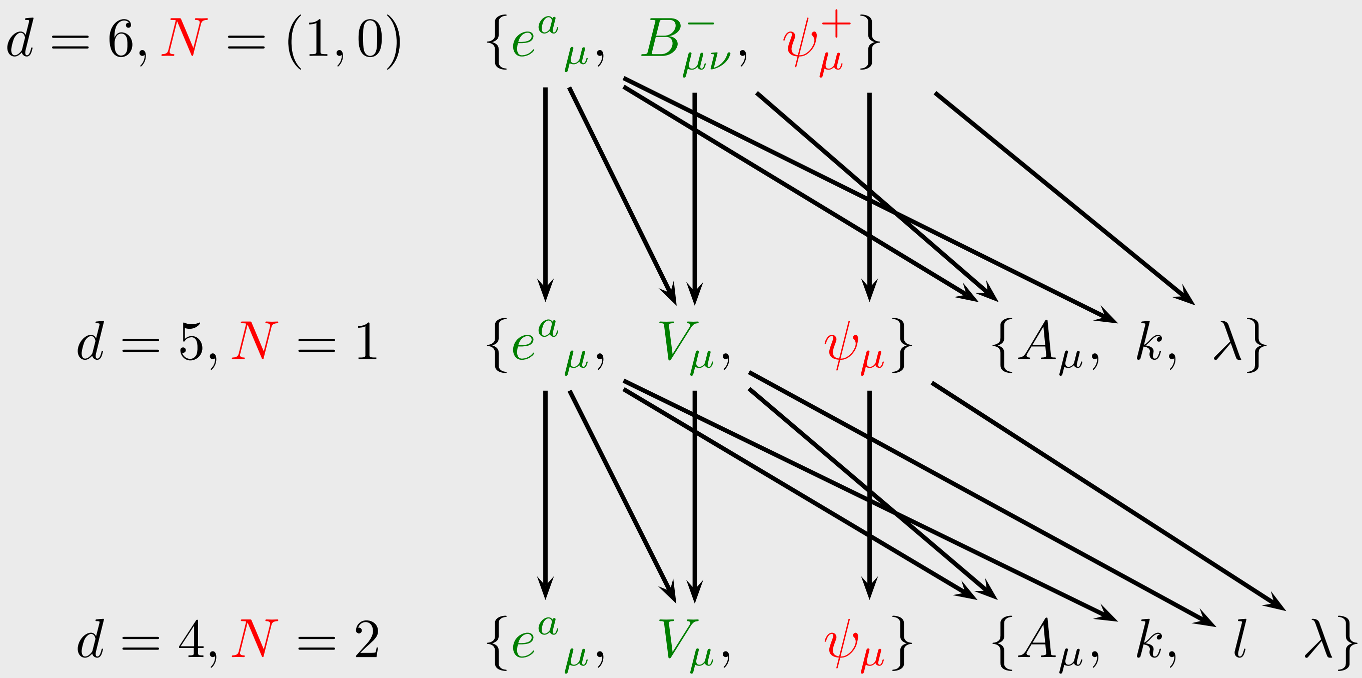

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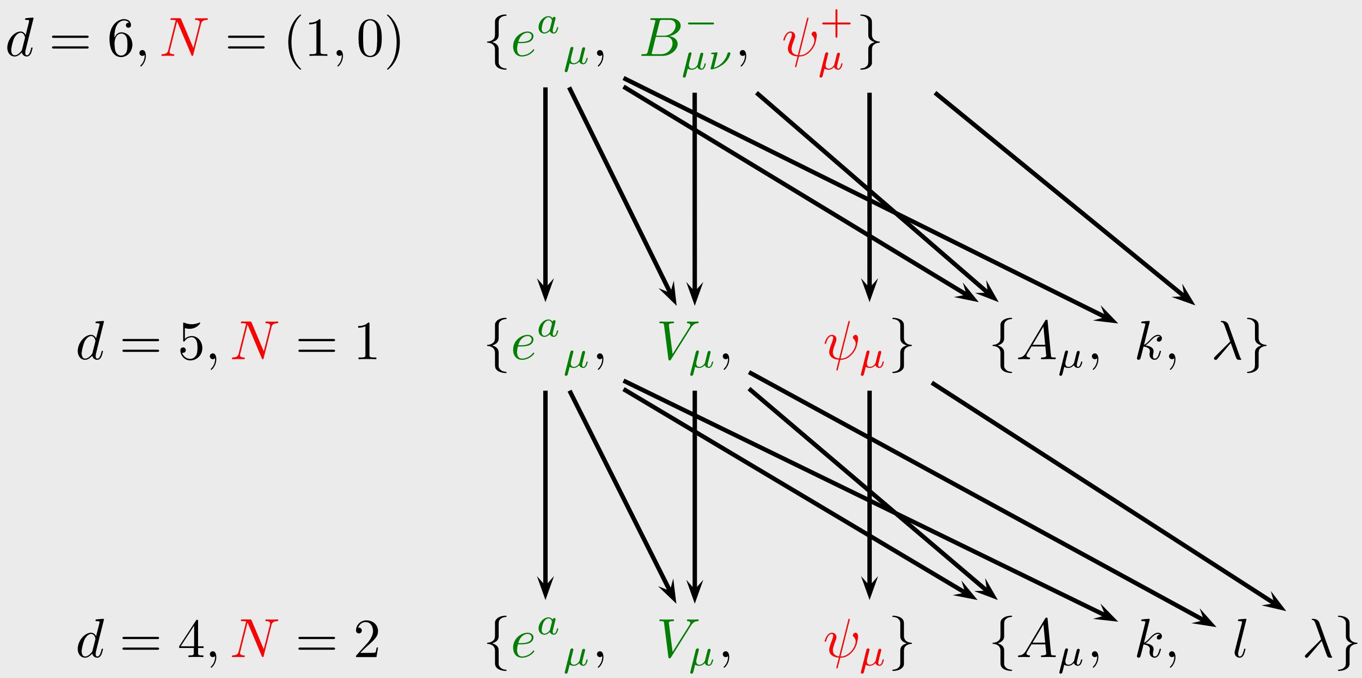

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End of slide

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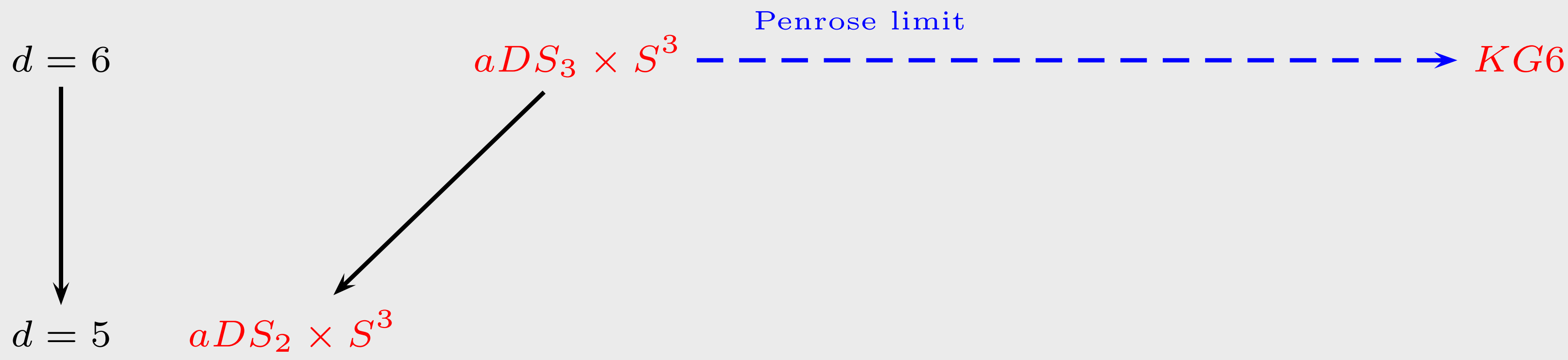

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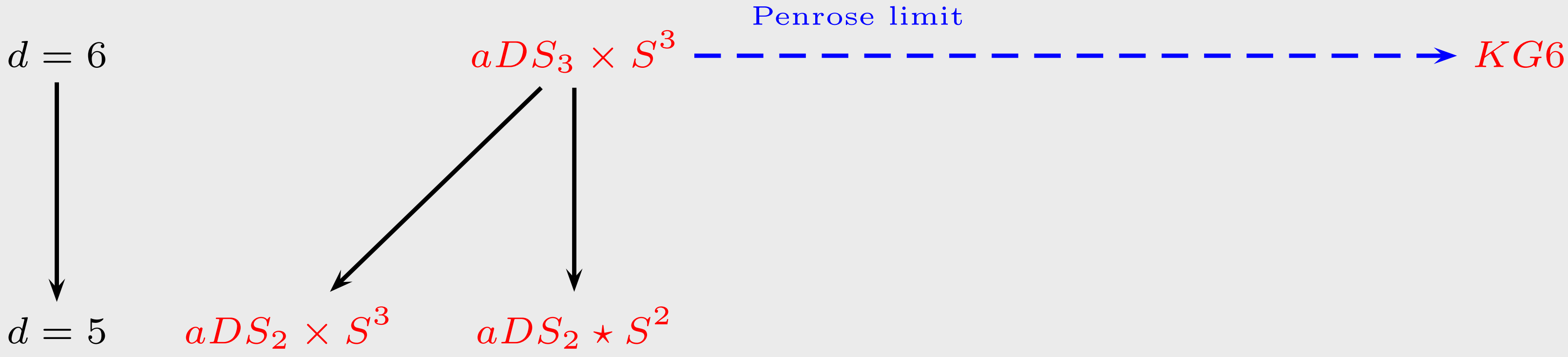

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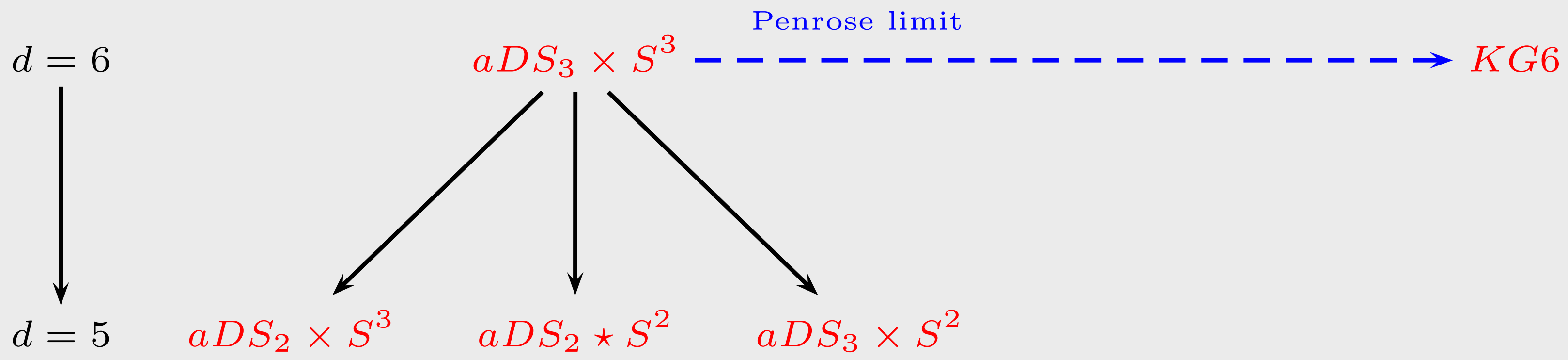

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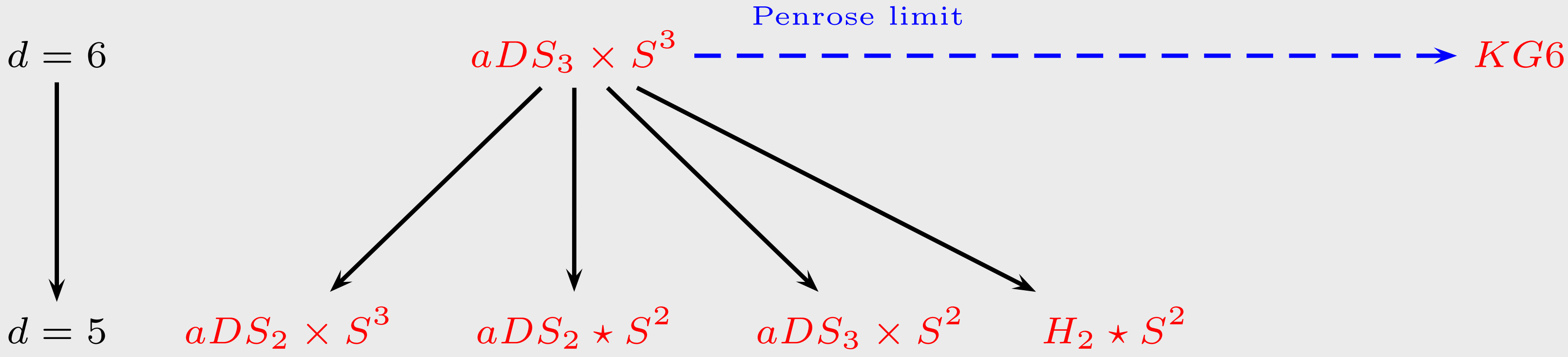

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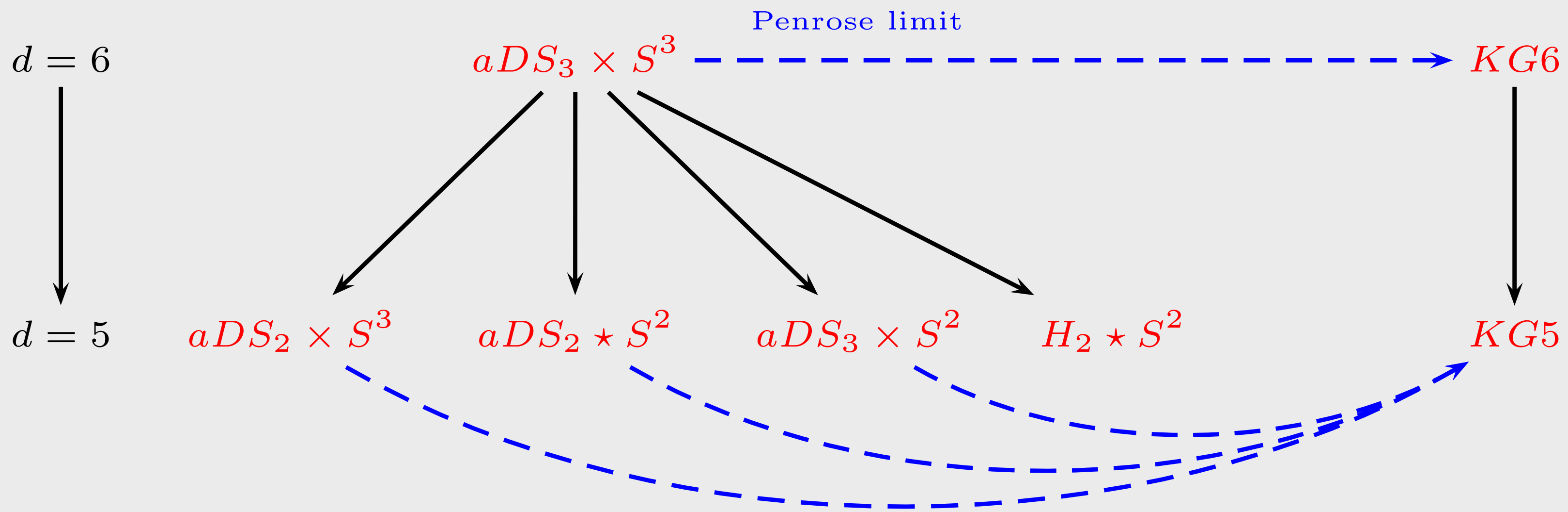

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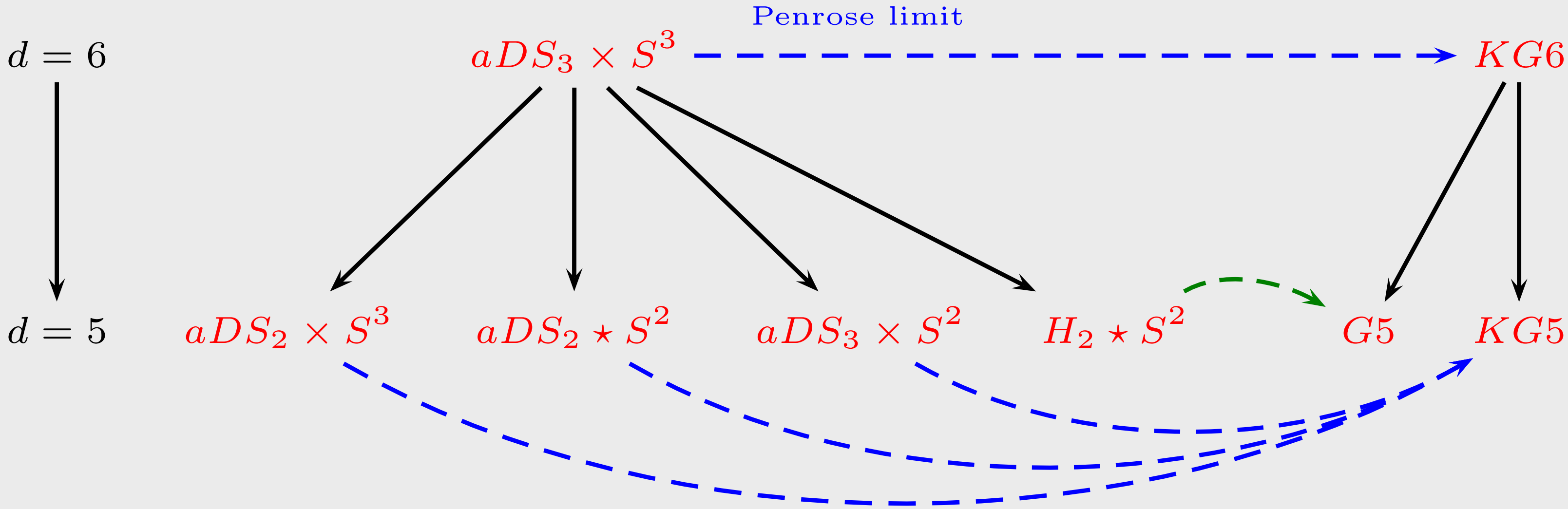

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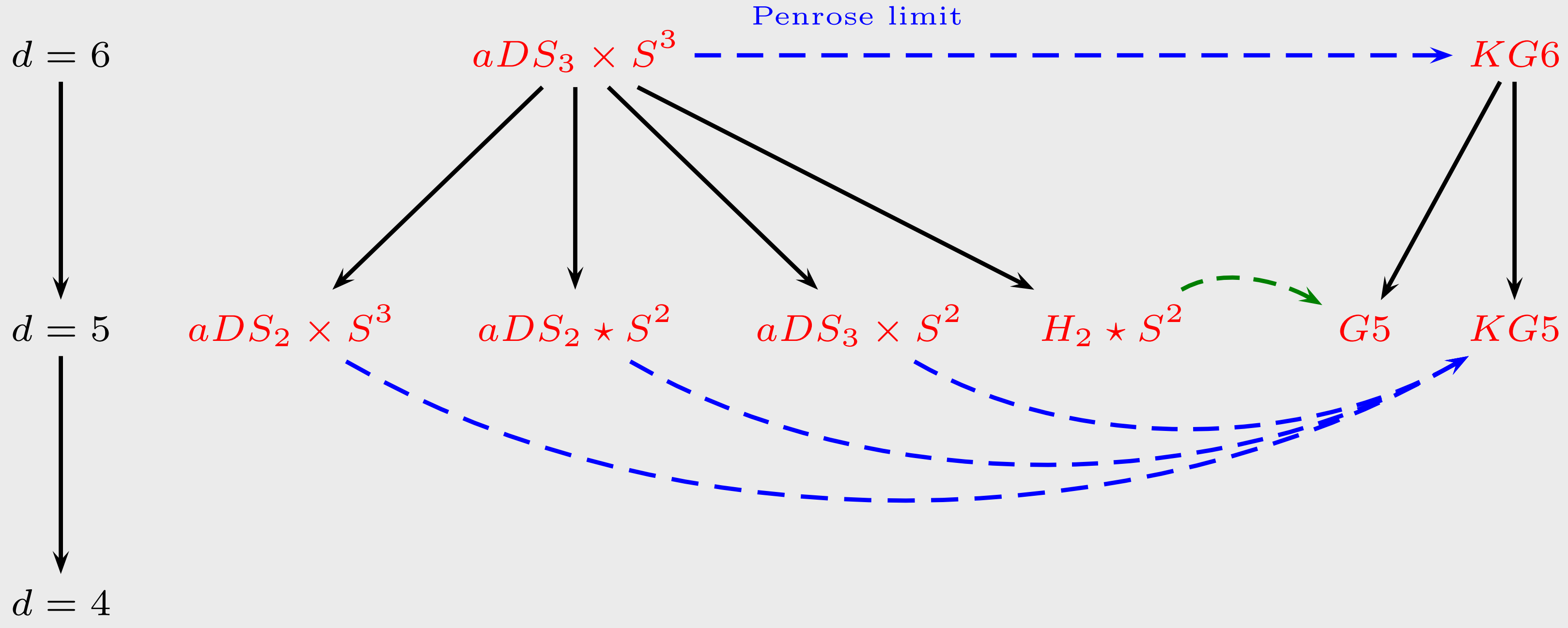

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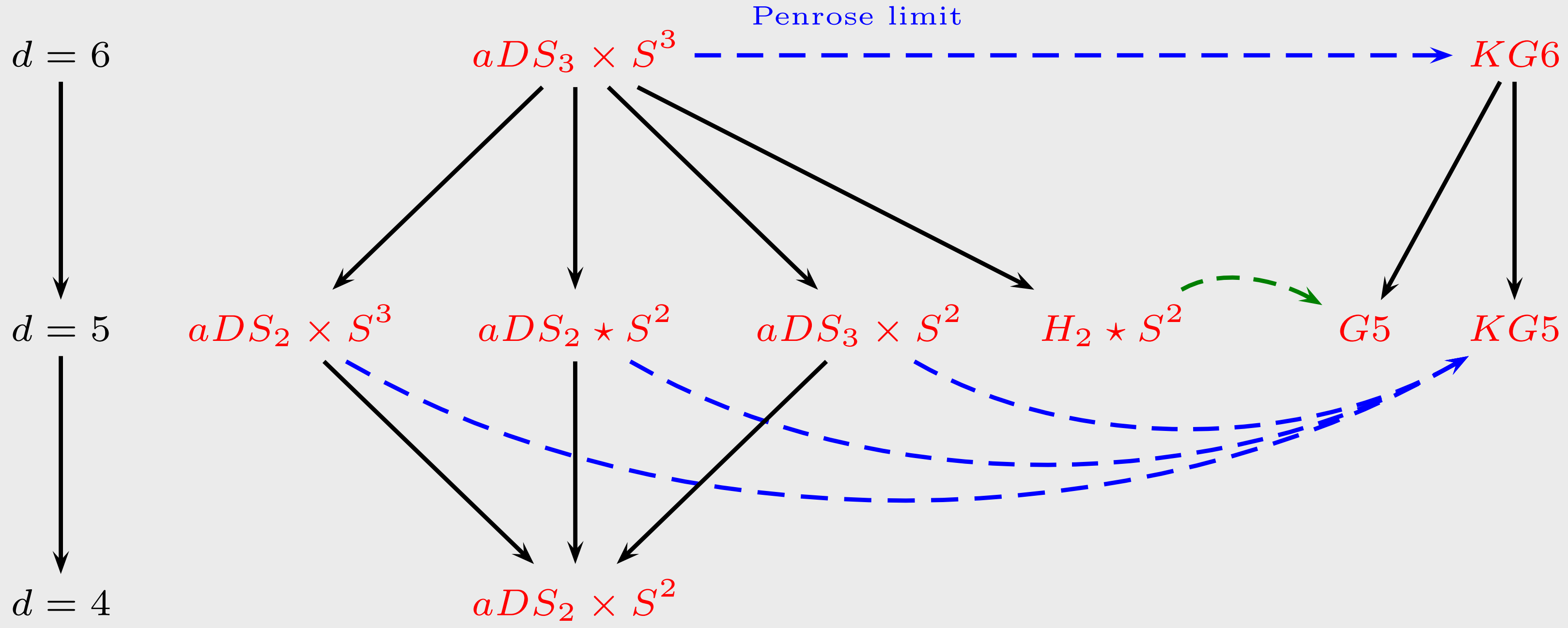

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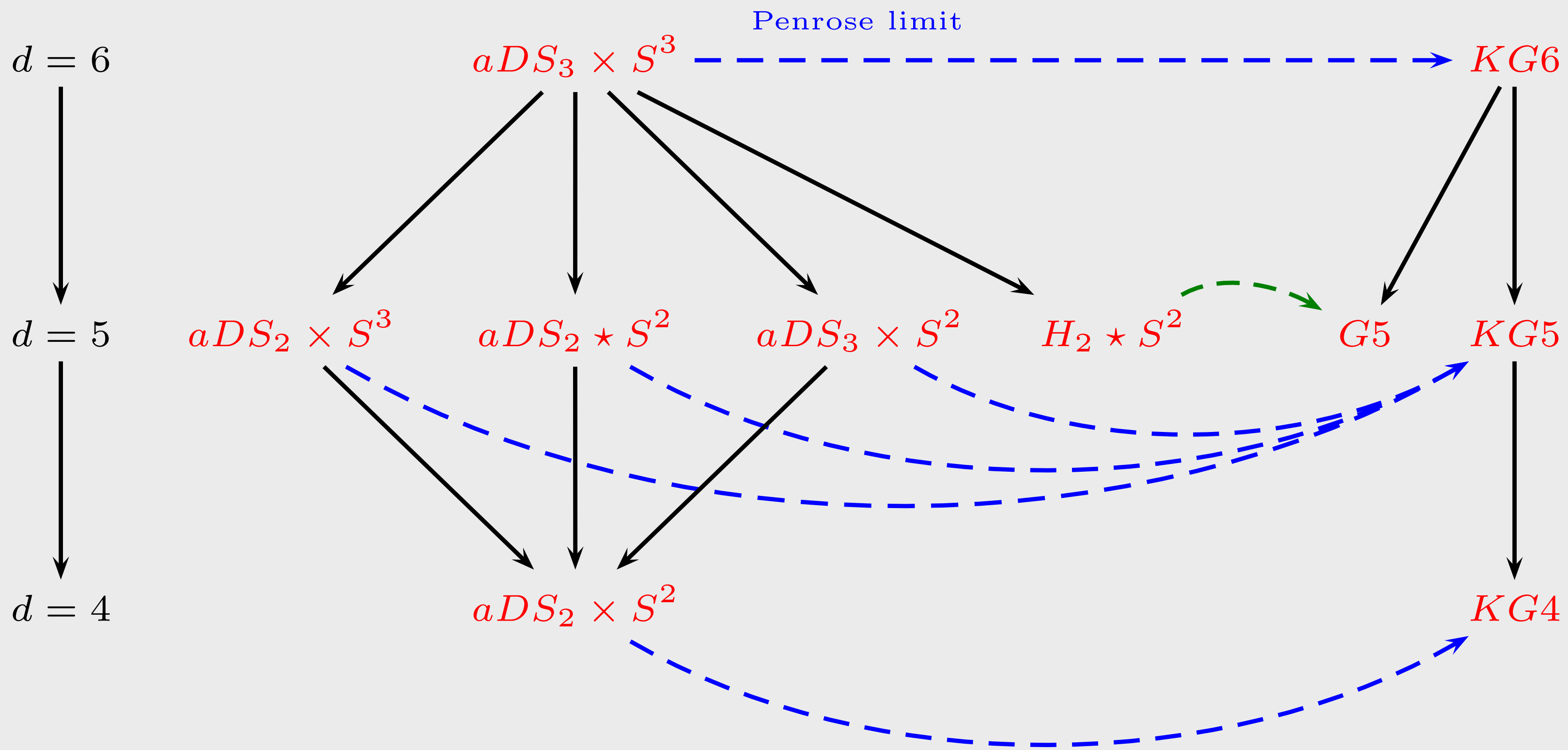

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The spacelike fibrations over base spacetimes are used in standard KK reductions. ω becomes the $d = 4$ Maxwell field.
Can we exploit timelike fibrations over a Euclidean space too?

End of slide

3 – Timelike KK

It is possible to perform Kaluza-Klein dimensional reductions on timelike directions. The original (Lorentzian) theory is reduced to an Euclidean theory and its solutions (with timelike $U(1)$ fibrations) are reduced to Euclidean solutions that may be interpreted as instantons.

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  - Observe the problems one faces in the Wick rotation of a theory a simple as $N = 1, d = 4$ SUGRA whose Euclidean version cannot be found in the literature.
- We will deal only with Dirac fermions, but it is not always clear if we are dealing with vector or pseudovector fields, whose Wick rotations require an extra factor of i.

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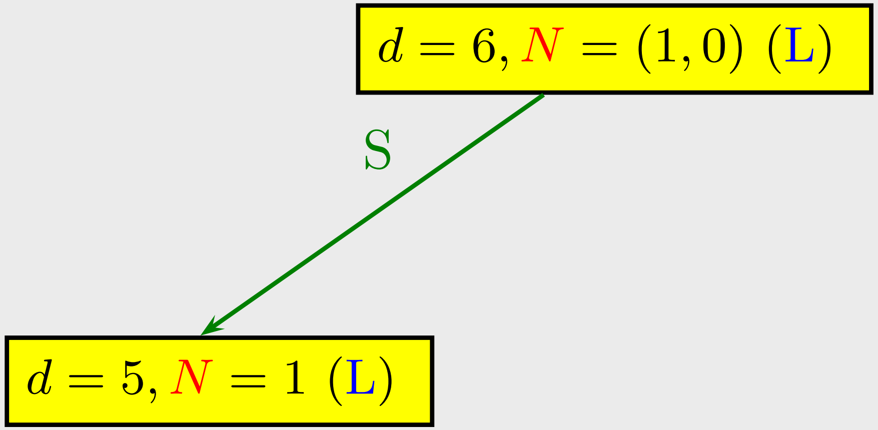

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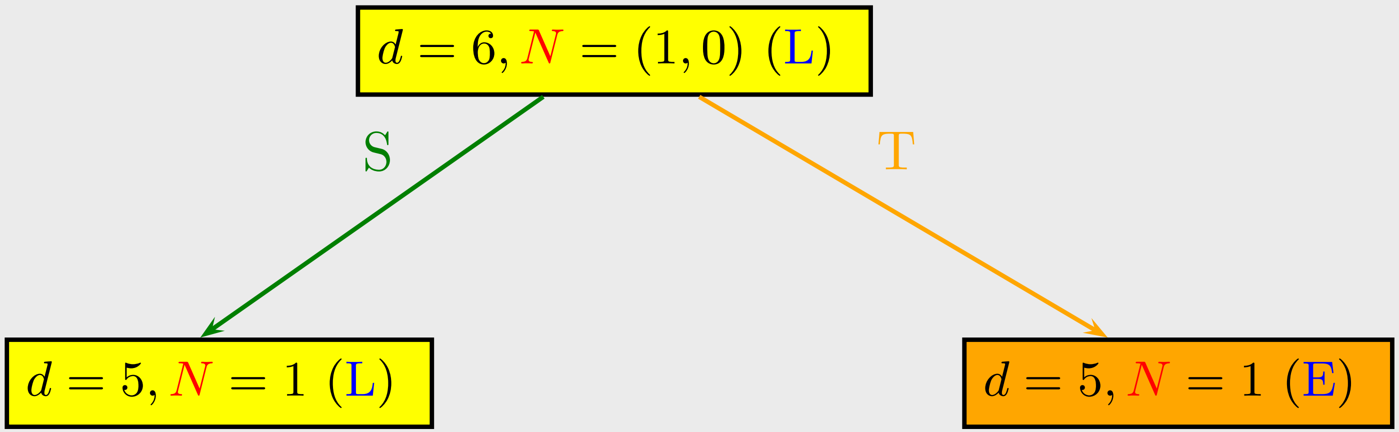

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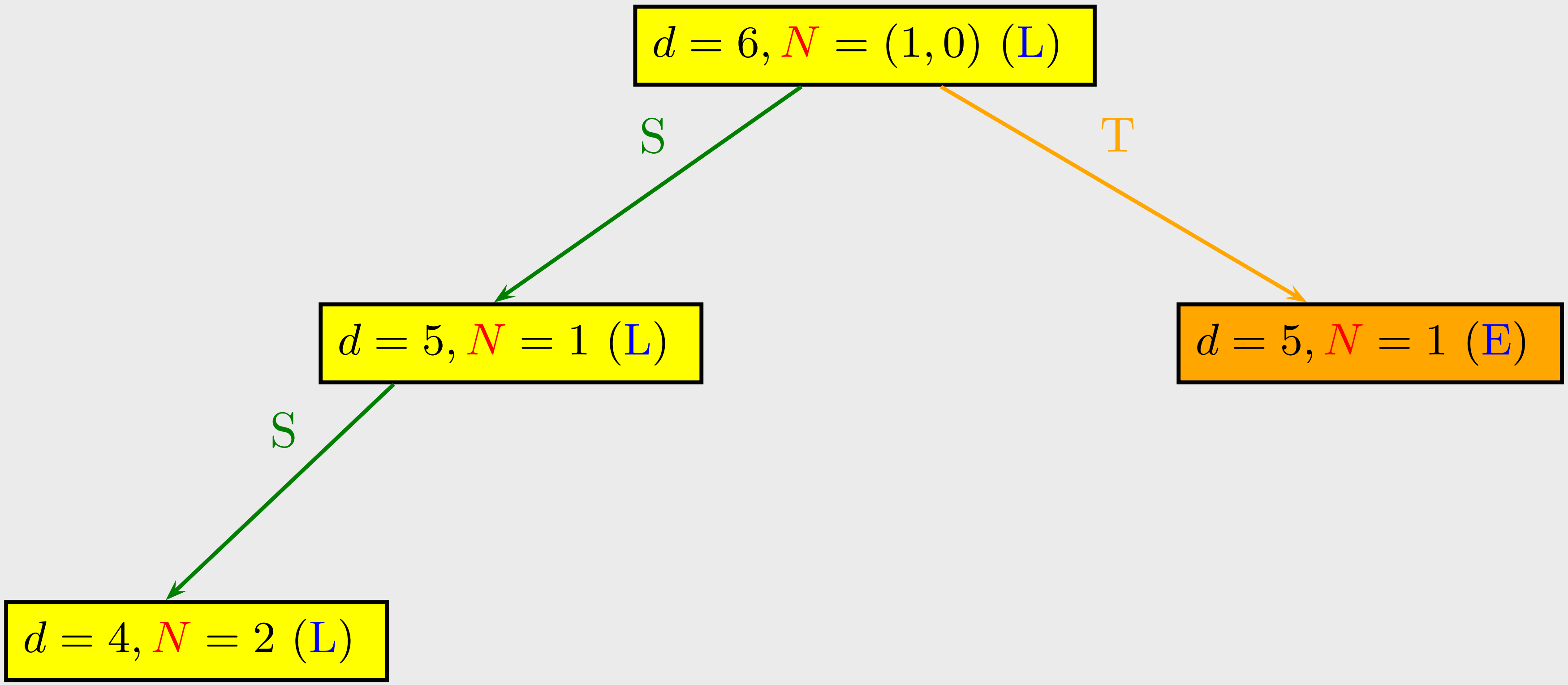

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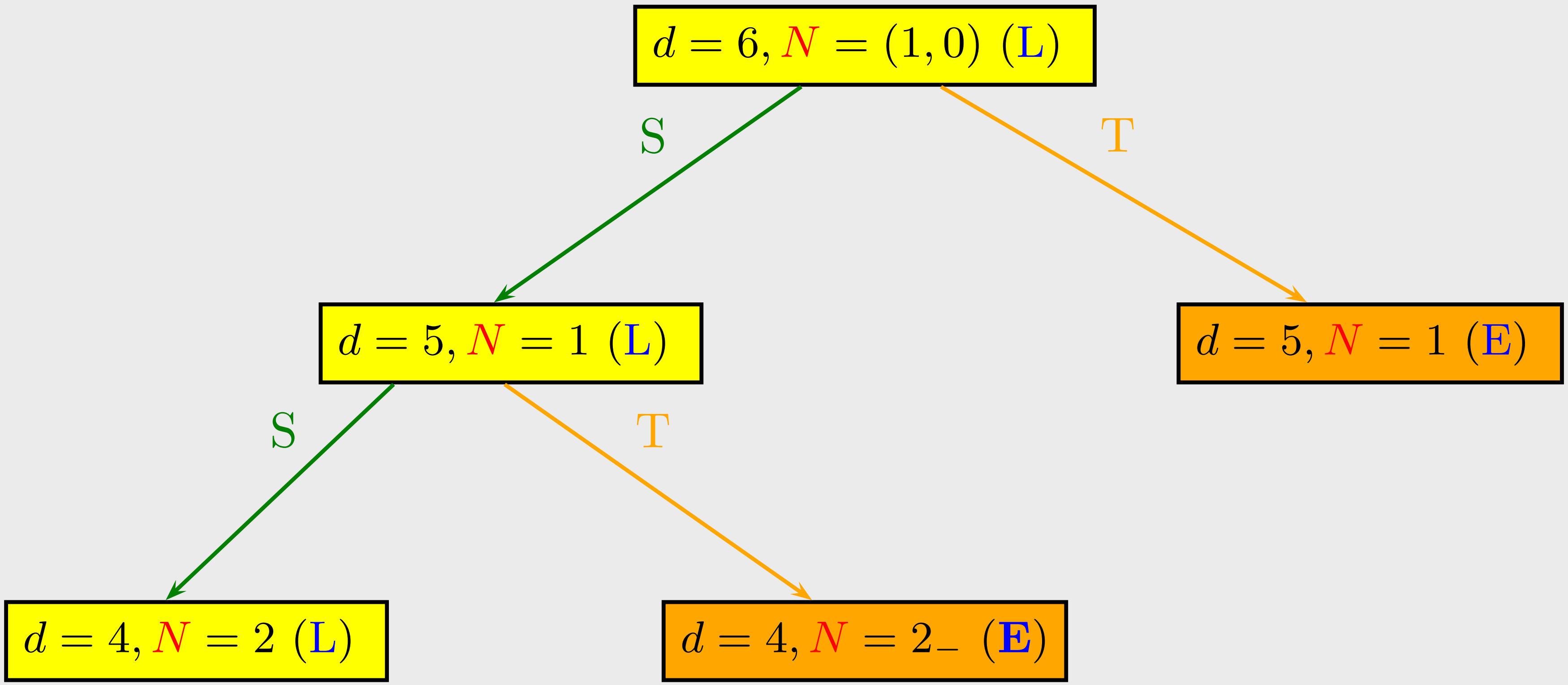

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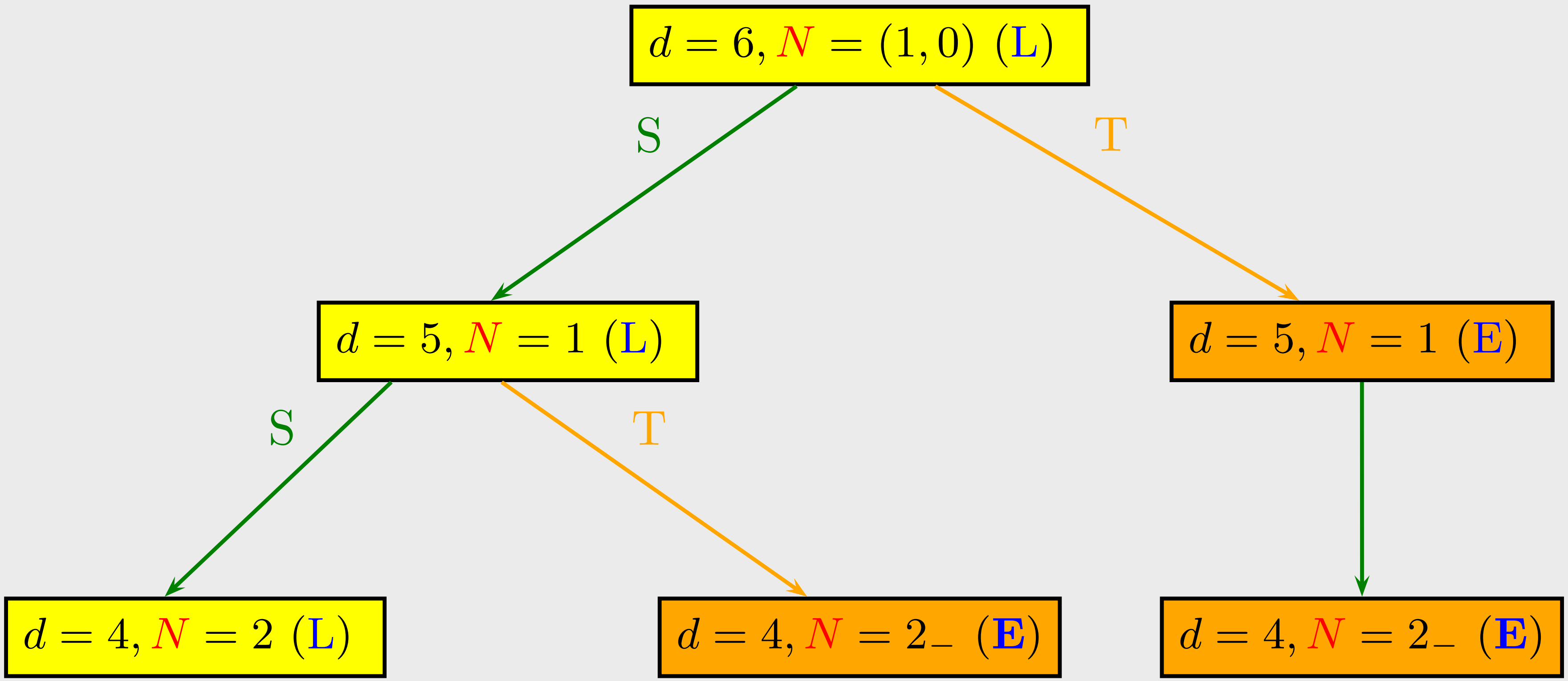

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End of slide

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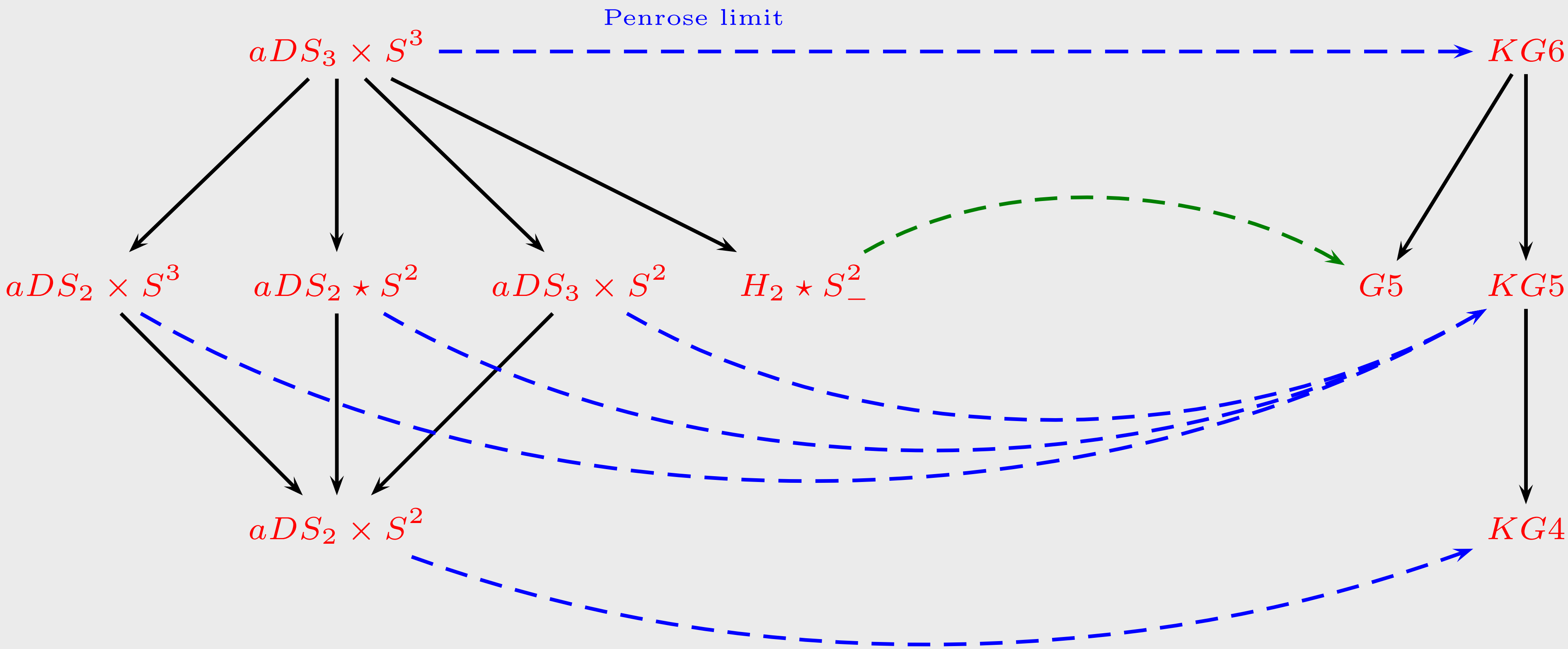

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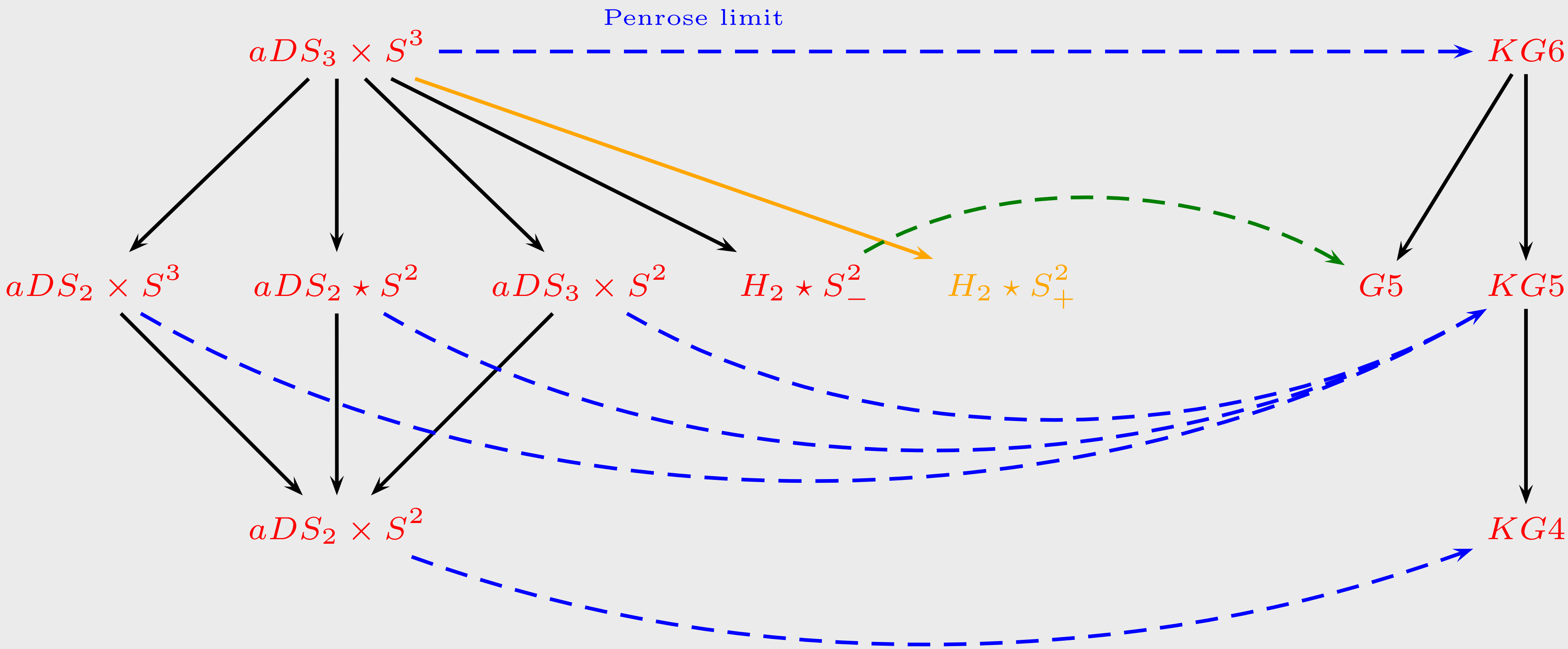

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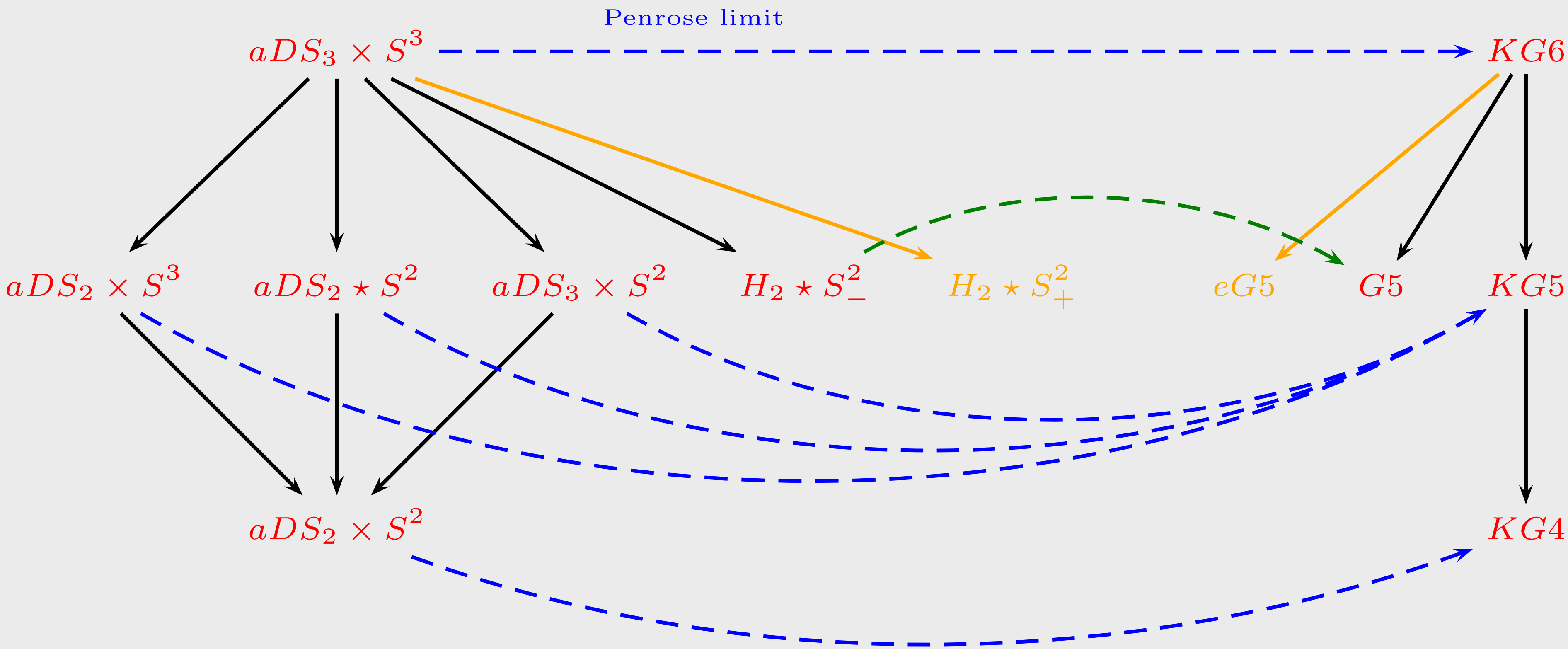

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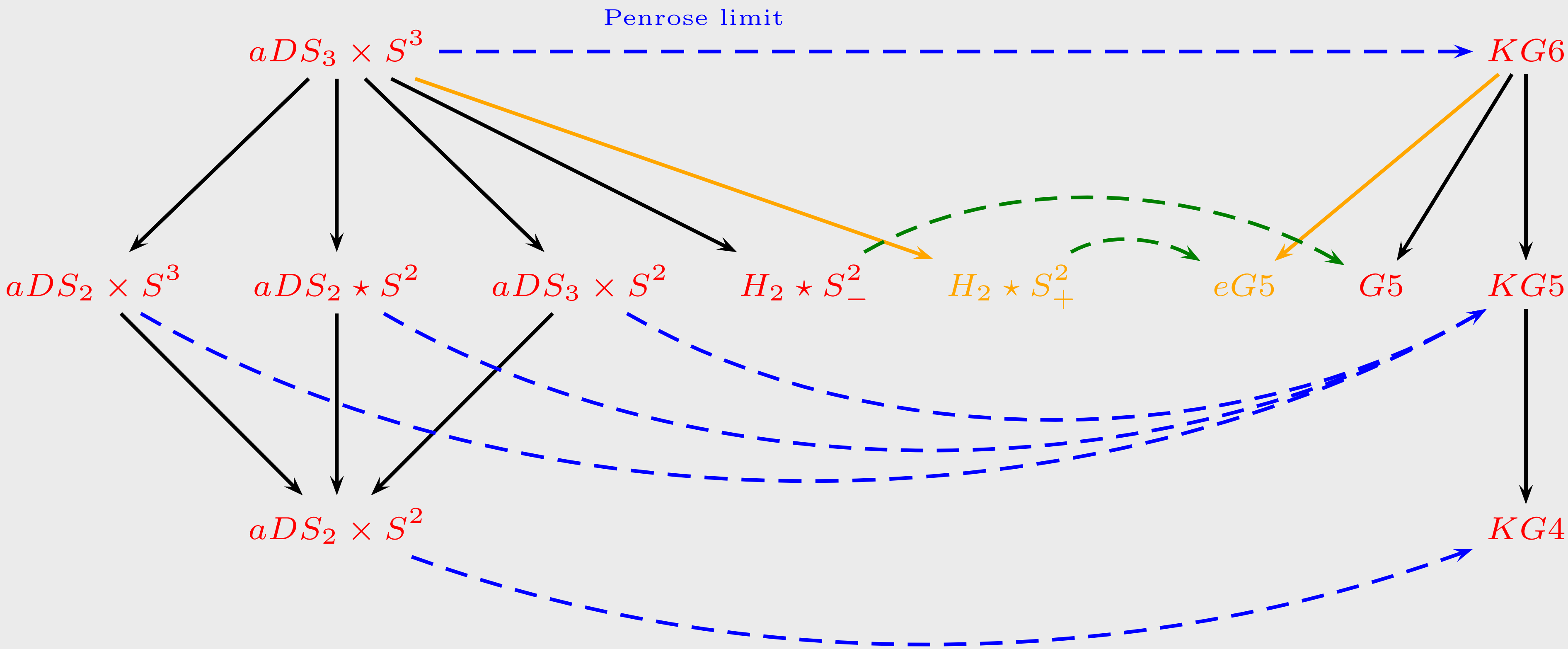

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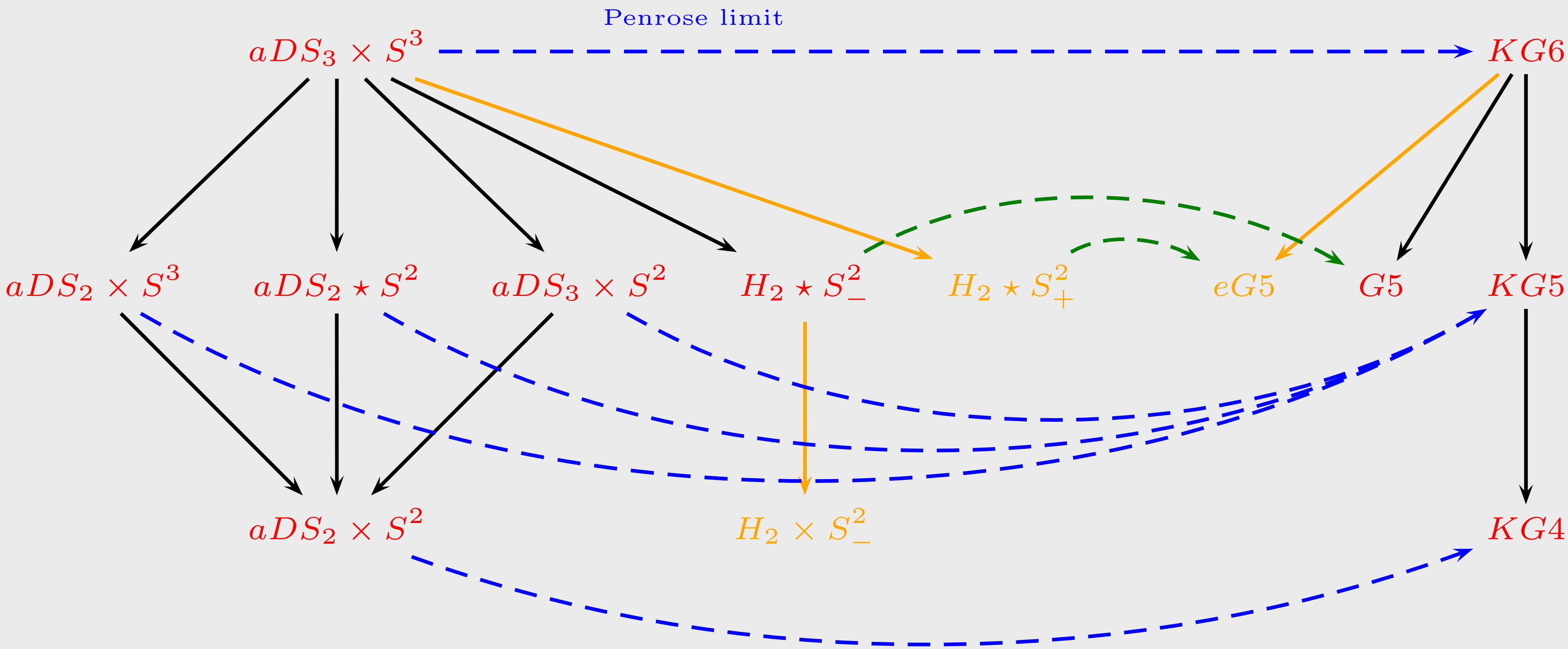

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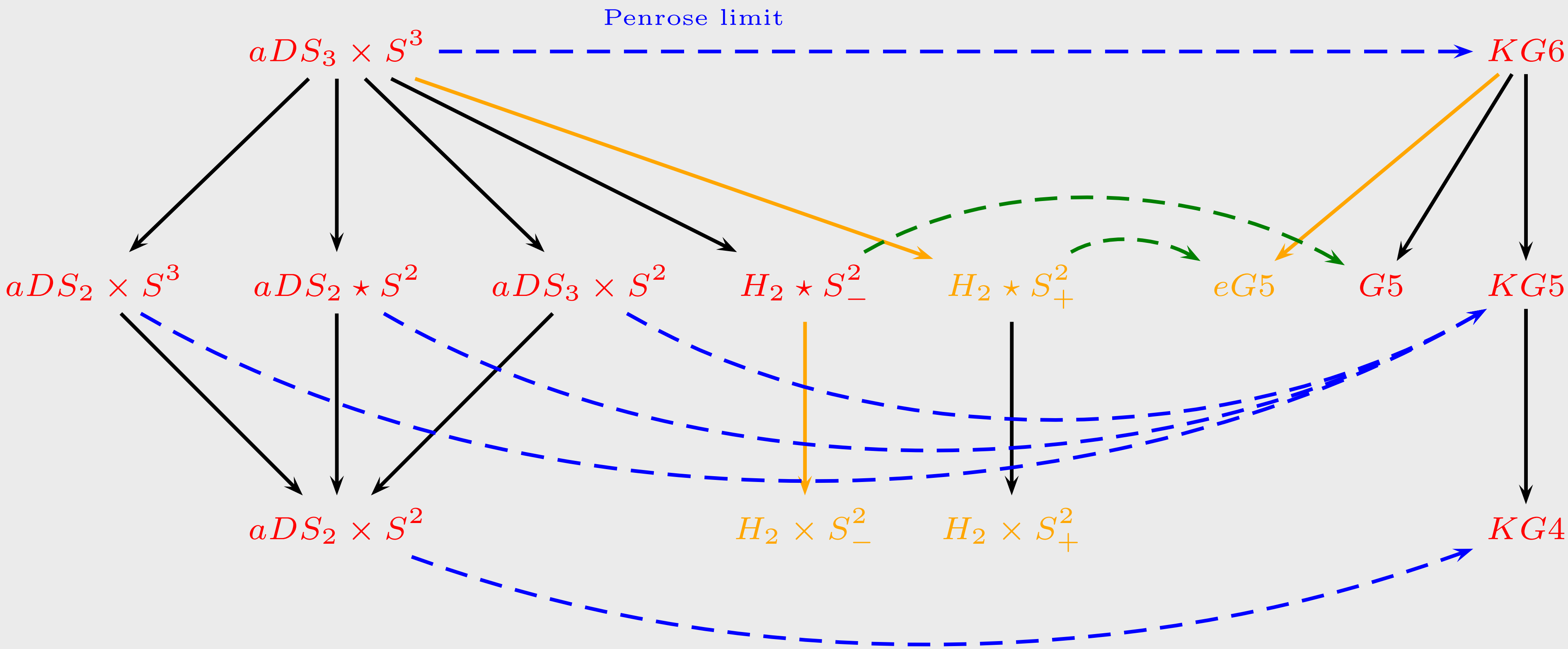

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$H_2 \times S^2_-$ $H_2 \times S^2_+$ are solutions of different theories and are related by analytical continuation.

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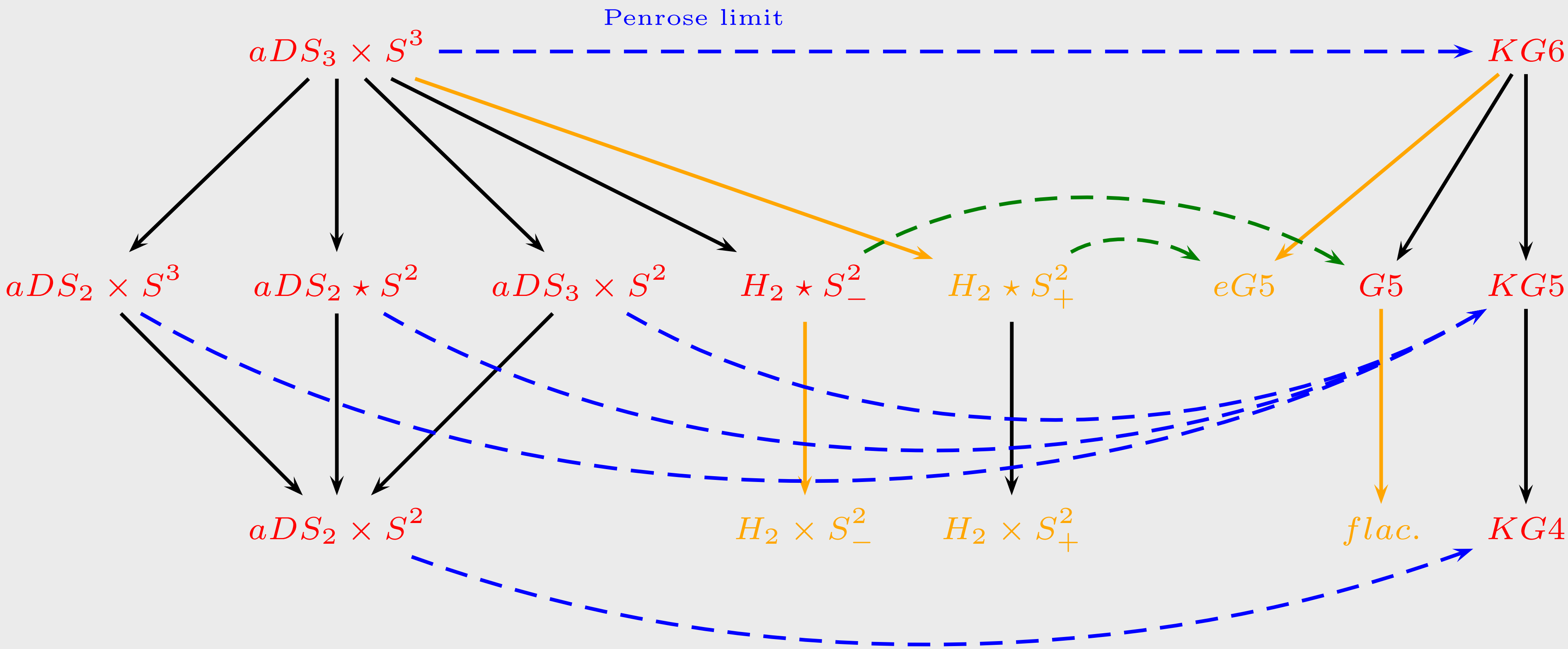

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The ***flacuum*** **solution** is a specially interesting non-trivial solution with flat Euclidean space

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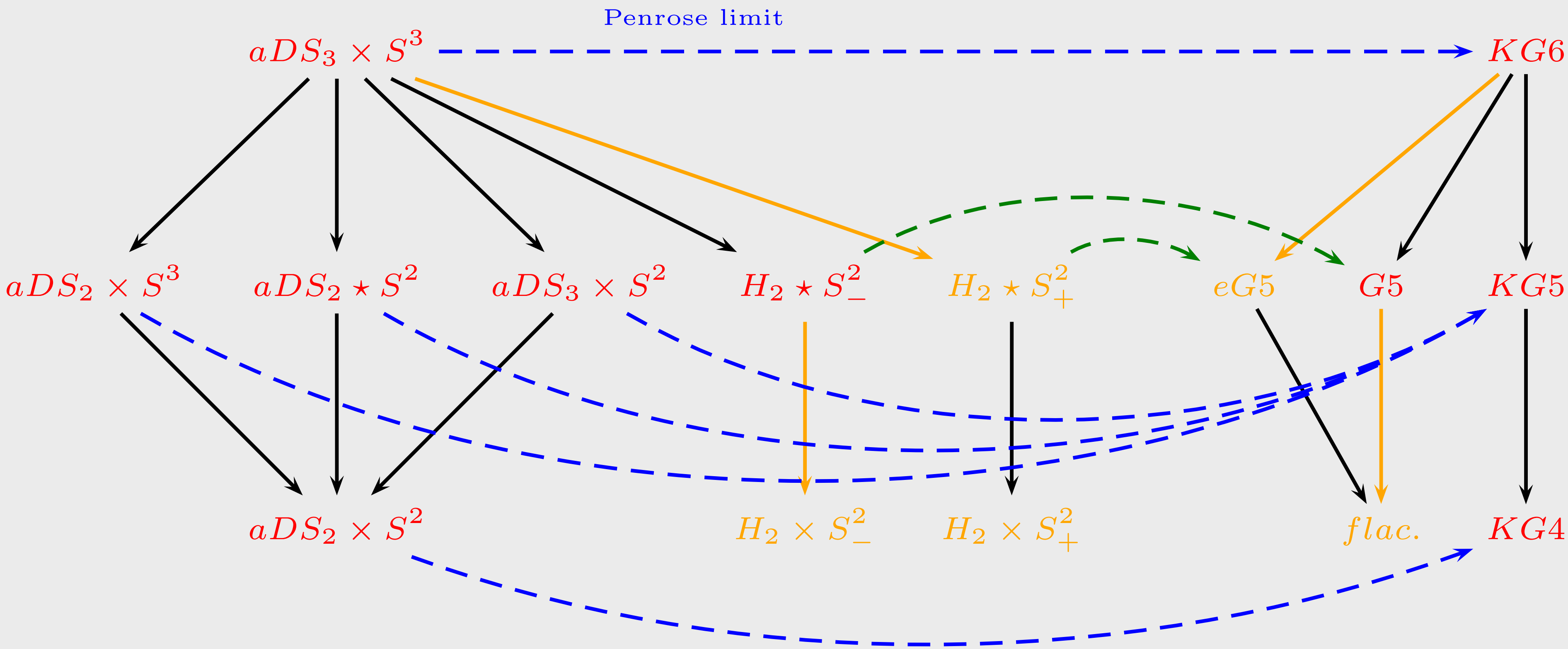

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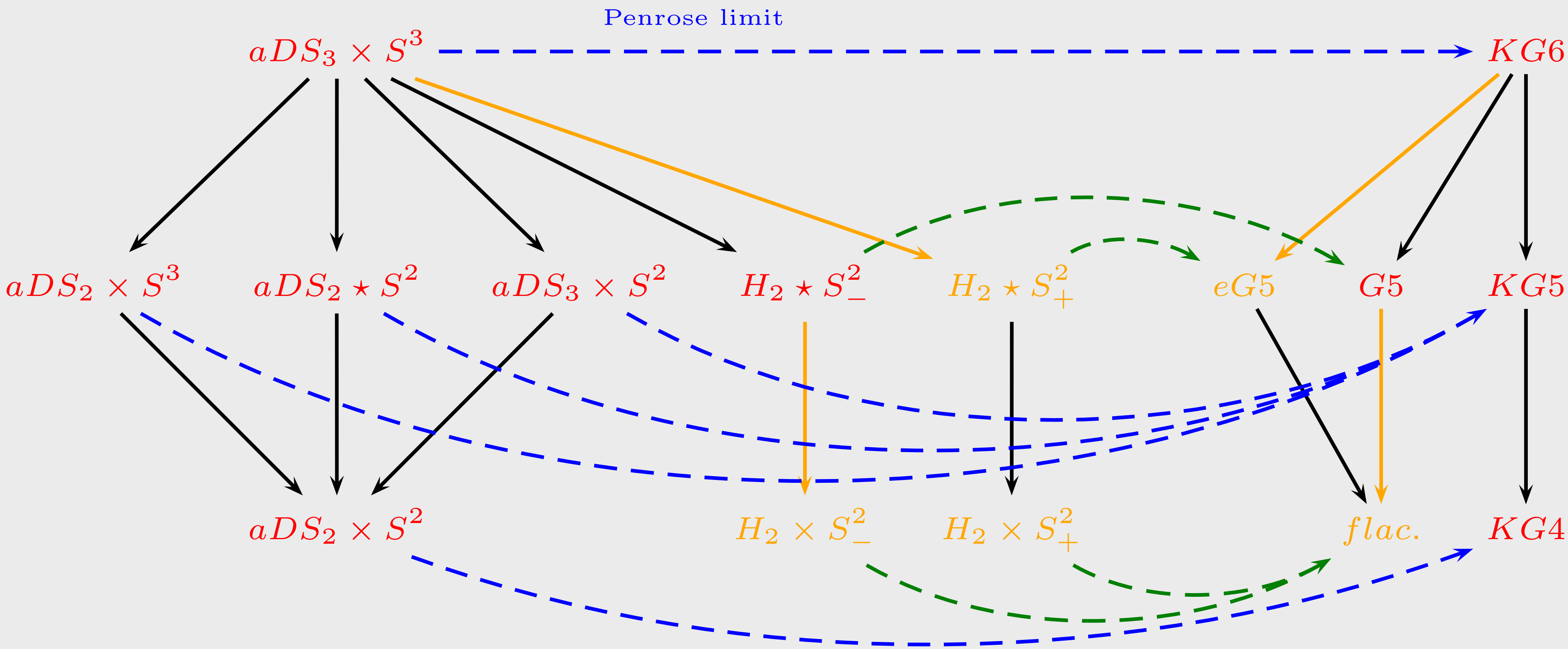

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End of slide

4 – The Flacuum

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As we have seen, the dimensional reduction of the Gödel solution of $d = 5, N = 1$ SUGRA given by

(Gödel) $G5$

$$\begin{aligned} ds^2 &= (dt + \omega)^2 - d\vec{x}_4^{\,2}\,, \\ V &= -\sqrt{3}\omega\,, \\ \omega &= \lambda(x^1 dx^2 - x^3 dx^4)\,. \end{aligned}$$

End of slide

4 – The Flacuum

leads to a non-trivial, maximally supersymmetric Euclidean solution of $d = 4, N = 2$ SUGRA (*i.e.* of the Einstein-Maxwell theory) with flat space and constant **anti-selfdual** field strength $^\star F = -F$ ($F_{12} = -F_{34} = \lambda/2$)

The *flacuum* solution

$$
\begin{aligned}
-ds^2 &= d\vec{x}_4^{\,2}\,, \\
V &= 2\omega\,, \\
\omega &= \lambda(x^1 dx^2 - x^3 dx^4)\,.
\end{aligned}
$$

End of slide

4 – The Flacuum

A constant, anti-selfdual $U(1)$ field strength certainly solves the Maxwell equation in flat space time, but,

how can flat space be a solution in presence of non-trivial matter?

End of slide

The positivity properties of the action and the energy are opposite in Lorentzian and Euclidean signatures:

	Lorentzian	**Euclidean**
Action:	$-F^2 = E^2 - B^2$	$-F^2 = E^2 + B^2 > 0$
$T_{\mu\nu}$:	$F_\mu{}^\rho F_{\nu\rho} + {}^\star F_\mu{}^{\rho\star} F_{\nu\rho} > 0$	$F_\mu{}^\rho F_{\nu\rho} - {}^\star F_\mu{}^{\rho\star} F_{\nu\rho}$

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In particular, selfdual and anti-selfdual Maxwell fields (that can only be defined in Euclidean signature) have a vanishing "*energy-momentum*" tensor. In general, (anti-) selfdual (non-) Abelian Yang-Mills configurations have vanishing *energy-momentum* tensors and almost decouple from the metric.

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The decoupling is not complete because (anti-) selfduality $F_{\rho\sigma} = \pm {}^\star F_{\rho\sigma}$ has to be proven w.r.t. to a given metric:

$$F_{\rho\sigma} = \pm \frac{1}{2\sqrt{|g|}} g_{\rho\mu} g_{\sigma\nu} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \, .$$

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$\Rightarrow$ If $F = \pm^{\star} F$ and $R_{\mu\nu} = \Lambda g_{\mu\nu}$, then $G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{2} T_{\mu\nu}$, and $\nabla_{\mu} F^{\mu\nu} = 0$

End of slide

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The BPST $SU(2)$ instanton

$F = \pm^{\star}F$ with any conformally flat metric. Since $F \to 0$ at ∞ we can take that of the round S^4

$$ds^2 = -\frac{d\vec{x}_4^{\,2}}{(1 + (r/2R)^2)^2}\,, \qquad \Rightarrow \qquad R_{\mu\nu} = \frac{1}{R^2} g_{\mu\nu}\,.$$

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The flacuum $U(1)$ solution

$F = \pm{}^{\star}F$ with any conformally flat metric. However, since F is constant, we have to stay with $\mathbb{R}^4$ which, at most, we can compactify on a torus to have a finite action. $R_{\mu\nu} = 0$ and the Einstein equation is satisfied with zero cosmological constant. Observe that taking the gauge group as $U(1)$ is equivalent to take the time periodic in the Gödel solution.

End of slide

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- $\omega = u^{-1} du + A$ where A is a $U(1)$ connection on $\mathbb{CP}^n$ such that

$$dA = i g_{i\bar{j}} d\xi^i \wedge d\bar{\xi}^j \equiv K\,,$$

the Kähler 2-form K, which is, therefore, closedL $dK = d^2 A = 0$.

End of slide

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Other solutions with vanishing Euclidean energy-momentum tensor can be obtained by time-like compactification of other Gödel solutions.

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$$V = \lambda(x^1 dx^2 - x^2 dx^1 - x^3 dx^4 + x^4 dx^3) \equiv F_{ab} x^a dx^b \,,$$

is not strictly periodic on T^4: when we move around the a-th period from x to $x + \hat{a}$ it changes by a gauge transformation

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The Euclidean action of the SUGRA solutions is

$$S = -4\pi^2 |nm| \,.$$

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Its has been argued that (Duff, Lu, Hull, Papadopoulos, Tsimpis) whant should be considered is the generalized holonomy of the supergravity theory, which is basically that of the gravitino supersymmetry transformation rule (the Killing spinor equation).

In this sense, the above transformations belong to the generalized holonomy group of $N = 2, d = 4$ SUGRA which is $SL(2, \mathbb{H})$ (Batrachenko, Wen `hep-th/0402141`).

End of slide

The symmetry superalgebra of the flacuum solution is particularly interesting because it is a deformation of the supertranslation algebra that preserves the commutativity of momenta but modifies slightly the anticommutator of the supercharges (Berkovits and Seiberg)

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\begin{aligned}
\left\{\mathcal{Q}^{\dagger}_{(\alpha)}, \mathcal{Q}_{(\beta)}\right\} &= (\gamma^1\gamma^a)_{\alpha\beta} P_{(a)} - [\gamma^1 \tfrac{1}{2}(1-\gamma_5)]_{\alpha\beta} M \,, \\
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The quantization of the string on this background leads to a non-commutative Field Theory in which only the fermionic superspace coordinates anticommute anomalously.

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\begin{aligned}
\left\{\mathcal{Q}^{\dagger}_{(\alpha)}, \mathcal{Q}_{(\beta)}\right\} &= (\gamma^1\gamma^a)_{\alpha\beta}P_{(a)} + (\gamma^1\gamma_5)_{\alpha\beta}P_{(0)} - [\gamma^1 \tfrac{1}{2}(1-\gamma_5)]_{\alpha\beta}M\,, \\
\left[\mathcal{Q}_{(\alpha)}, P_{(a)}\right] &= -\mathcal{Q}_{(\beta)}\Gamma_s(P_{(a)})^{\beta}{}_{\alpha}\,, \\
\left[\mathcal{Q}_{(\alpha)}, M\right] &= -\mathcal{Q}_{(\beta)}\Gamma_s(M)^{\beta}{}_{\alpha}\,, \\
\left[P_{(a)}, M\right] &= -P_{(b)}\Gamma_v(M)^{b}{}_{a}\,, \qquad \left[P_{(a)}, P_{(b)}\right] = F_{ab}P_{(0)}\,.
\end{aligned}
$$

The quantization of the string on this background leads to a non-commutative Field Theory in which only the fermionic superspace coordinates anticommute anomalously.

> This superalgebra can be obtained by dimensional reduction of the Gödel superalgebra, in which the momenta $P_{(a)}$ do **not** commute, but give $P_{(0)}$ which should be interpreted as the generator of $U(1)$ gauge transformations on $d=4$. This property is, precisely, what allowed us to relate the periods of the torii.

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- ⋆ We have determined the symmetry superalgebra of the *flacuum* solution. We notice that the symmetry superalgebras of all the maximally supersymmetric vacua are always deformations of the supertranslation (superPoincaré) algebra, which may allow to classify and find all these vacua.

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THE END