Gödel Spacetimes and *Flacuum* Solutions

Tomás Ortín (I.F.T.-C.S.I.C)

Seminar given on February 17th 2004 at IFT-UAM/CSIC

Based on hep-th/0401005. Work done in collaboration with

Patrick Meessen (C.E.R.N.)

Introduction/Motivation

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  - The Dirac monopole configuration is realized in the KK monopole.
  - The BPST instanton configuration is realized in solutions with S^7 subspaces.

In this seminar we are going to study an interesting example of maximally supersymmetric, topologically non-trivial field configuration of supergravity that corresponds to a well-known Abelian Yang-Mills instanton configuration.

Plan of the Talk:

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1 – SUGRA Vacua

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- ★ Usually enjoys a high degree of (residual) symmetry. This symmetry determines all the kinematical properties of the QFT (conserved charges, spectrum etc.)
- ★ In (Special-Relativistic) QFT it is *required* that the residual symmetry of the vacuum includes the Poincaré group.

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Clearly, the most important question is

"How should (we or the theory) choose the vacuum?"

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This is a generalization of the concept of isometry, an infinitesimal general coordinate transformation generated by $\xi^\mu(x)$ that leaves the metric $g_{\mu\nu}$ invariant because it satisfies the *Killing (vector) equation*

$$\delta_\xi g_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)} = 0 \,. \tag{3}$$

End of slide

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These will be the superalgebras of the QFTs constructed on these vacua!

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maximally supersymmetric $\not\Rightarrow$ **maximally symmetric**

End of slide

2 – 8Q SUGRA Vacua

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The smallest spinor in $d \geq 7$ has 16 real components. Then the SUGRAs with 8 supercharges in $d > 3$ are just

Theory	Fields	Bosonic Action
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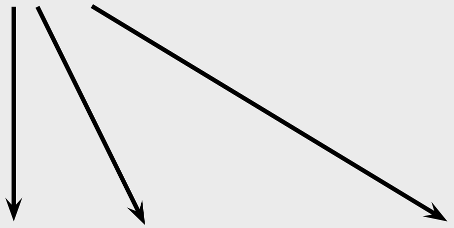

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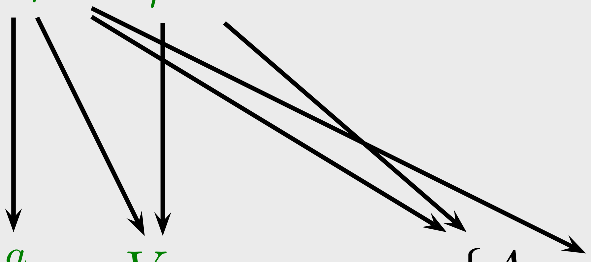

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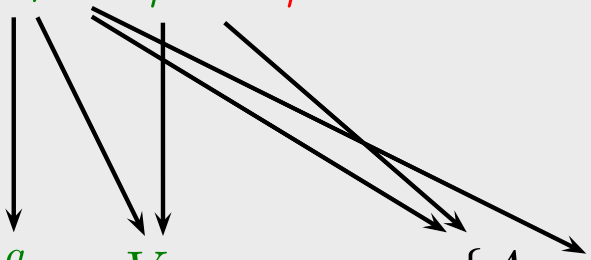

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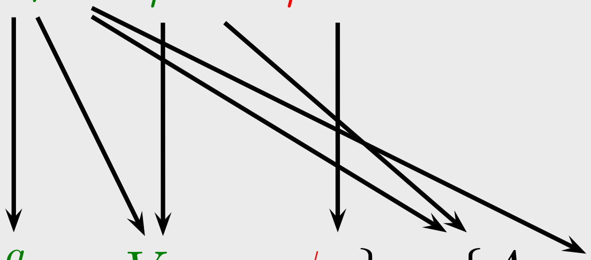

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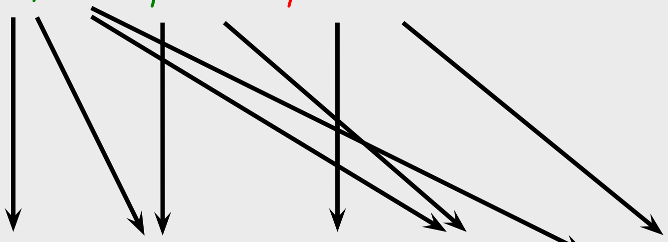

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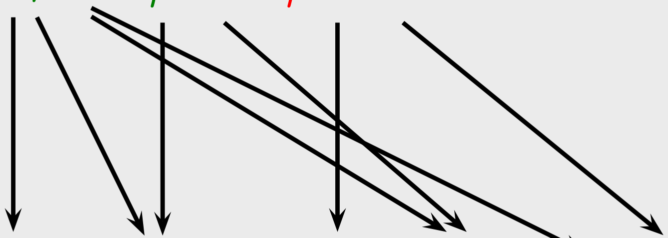

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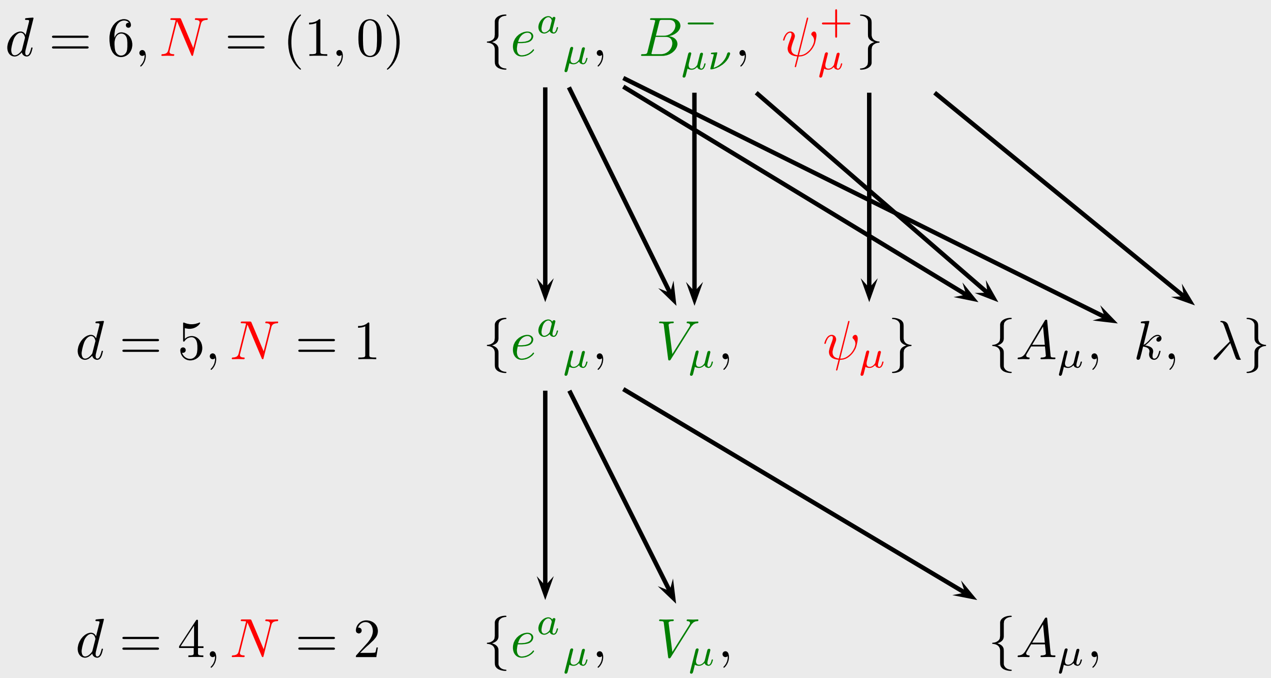

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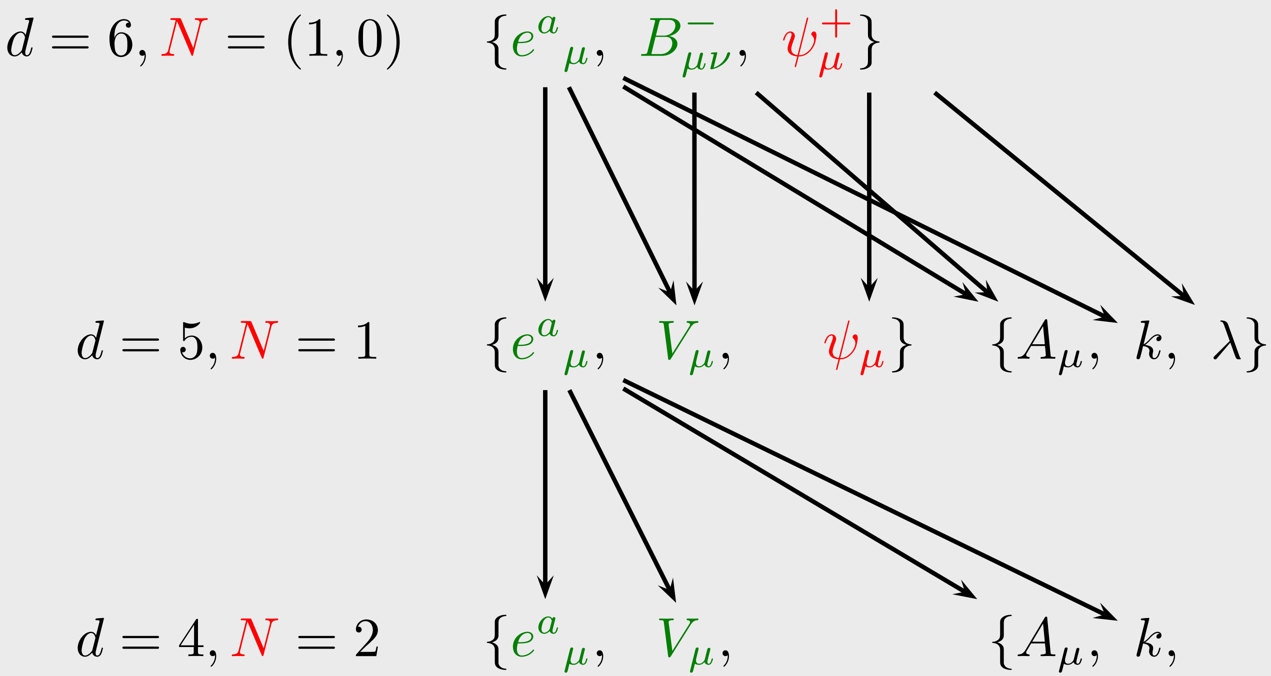

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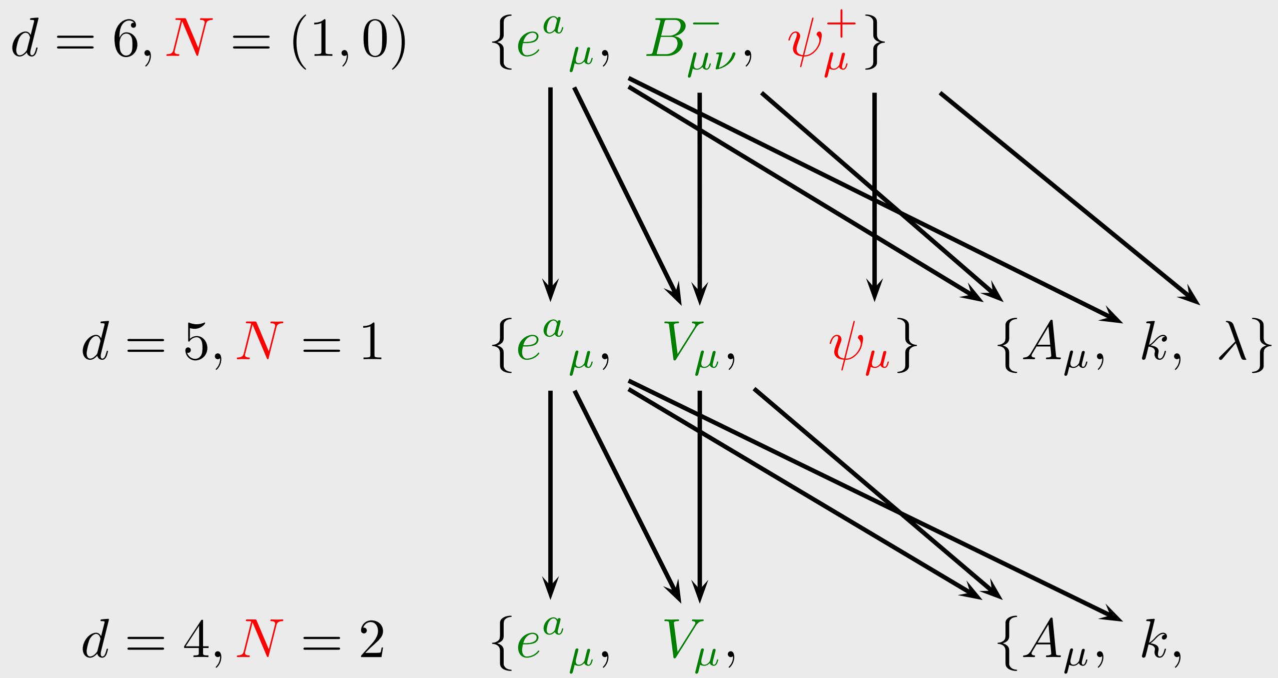

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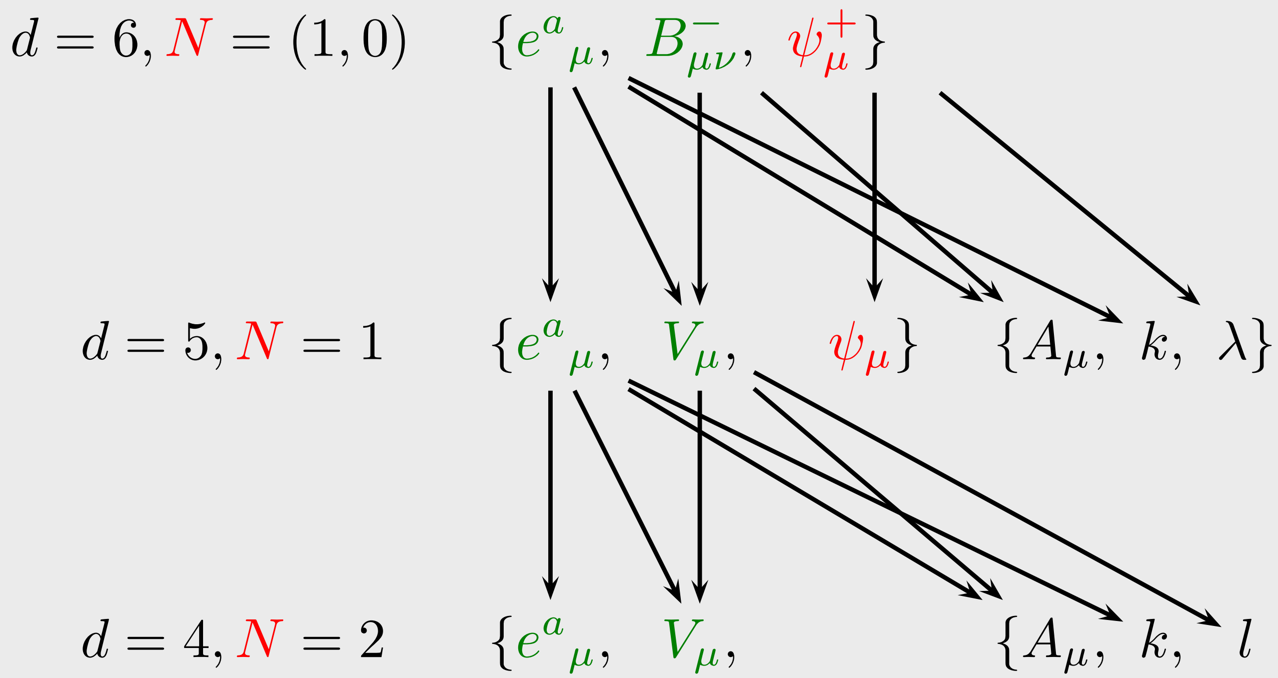

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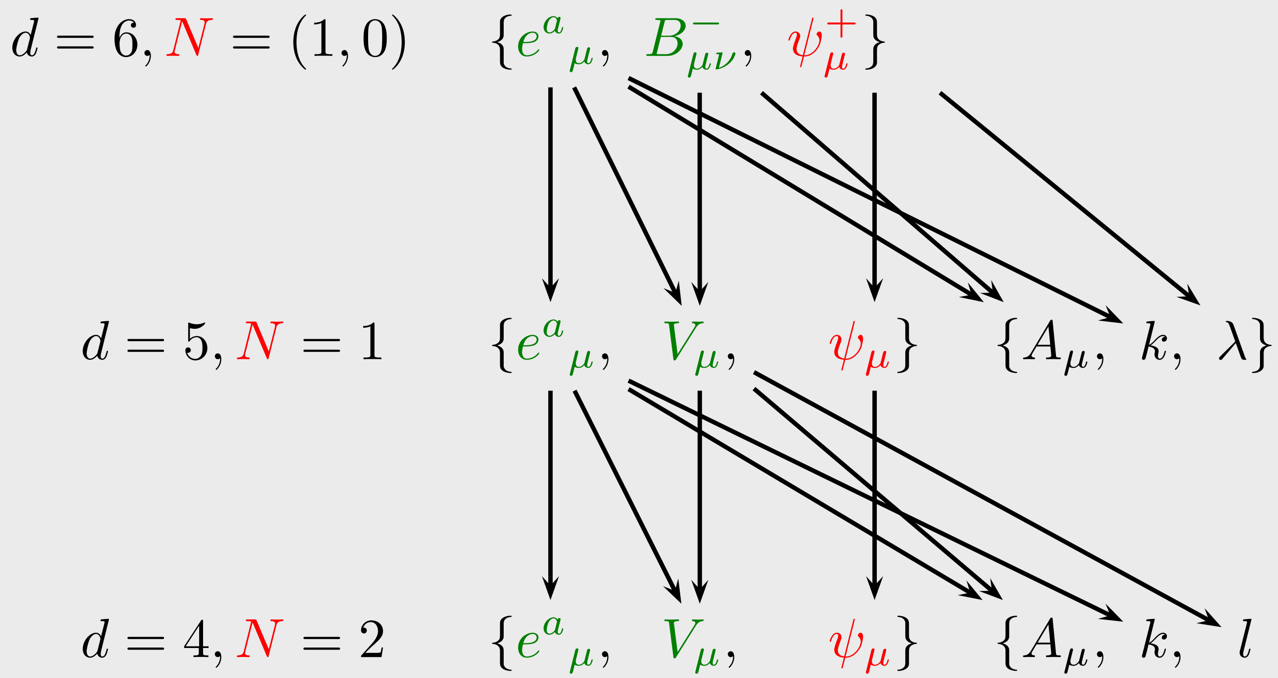

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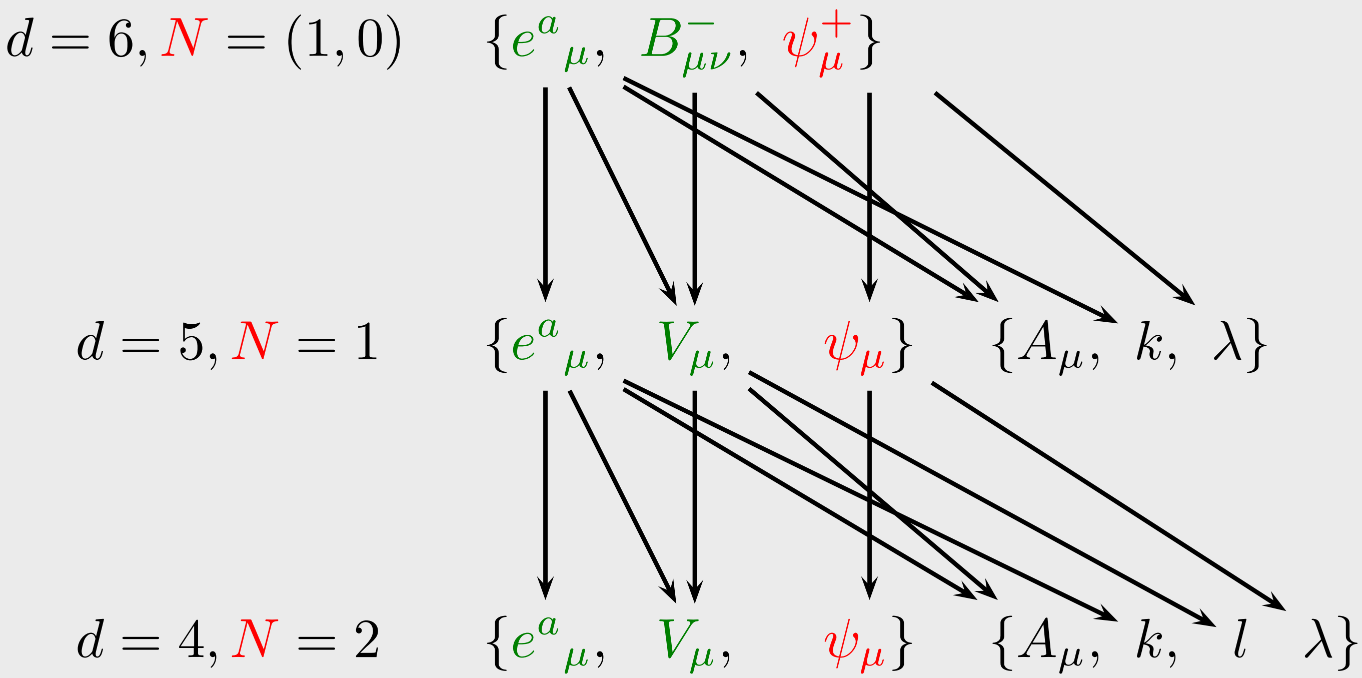

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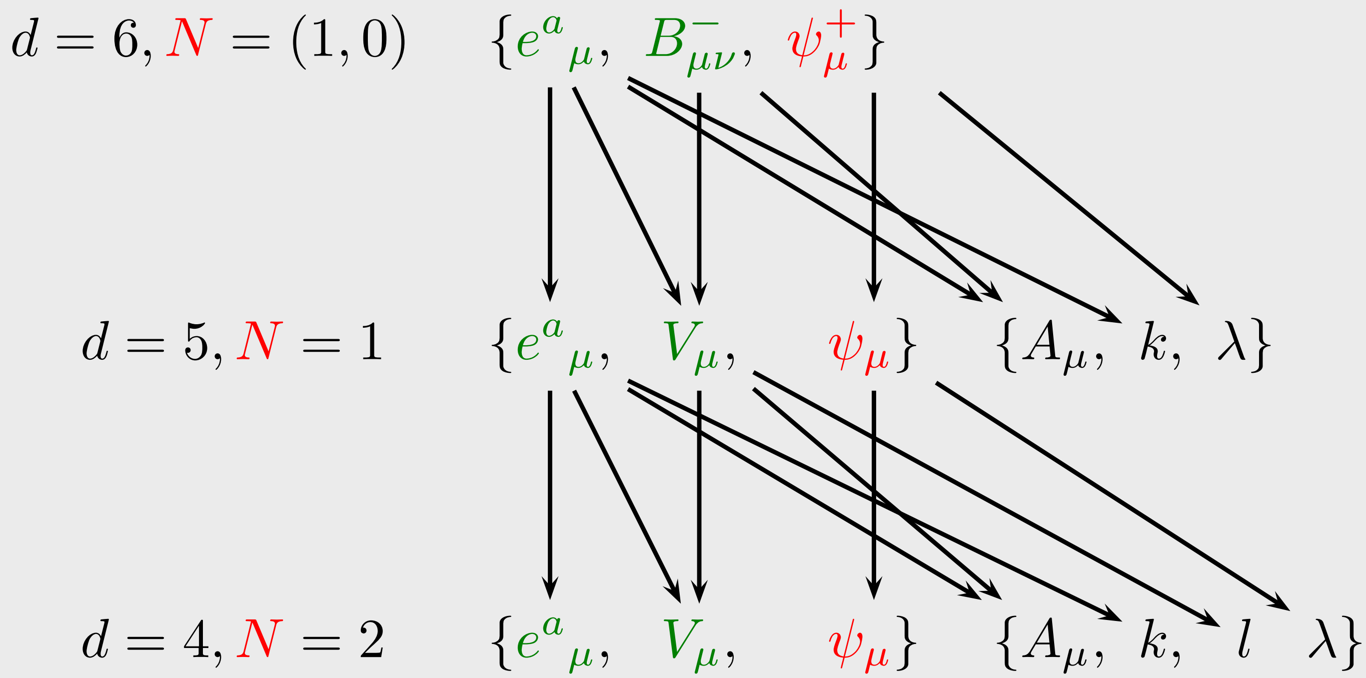

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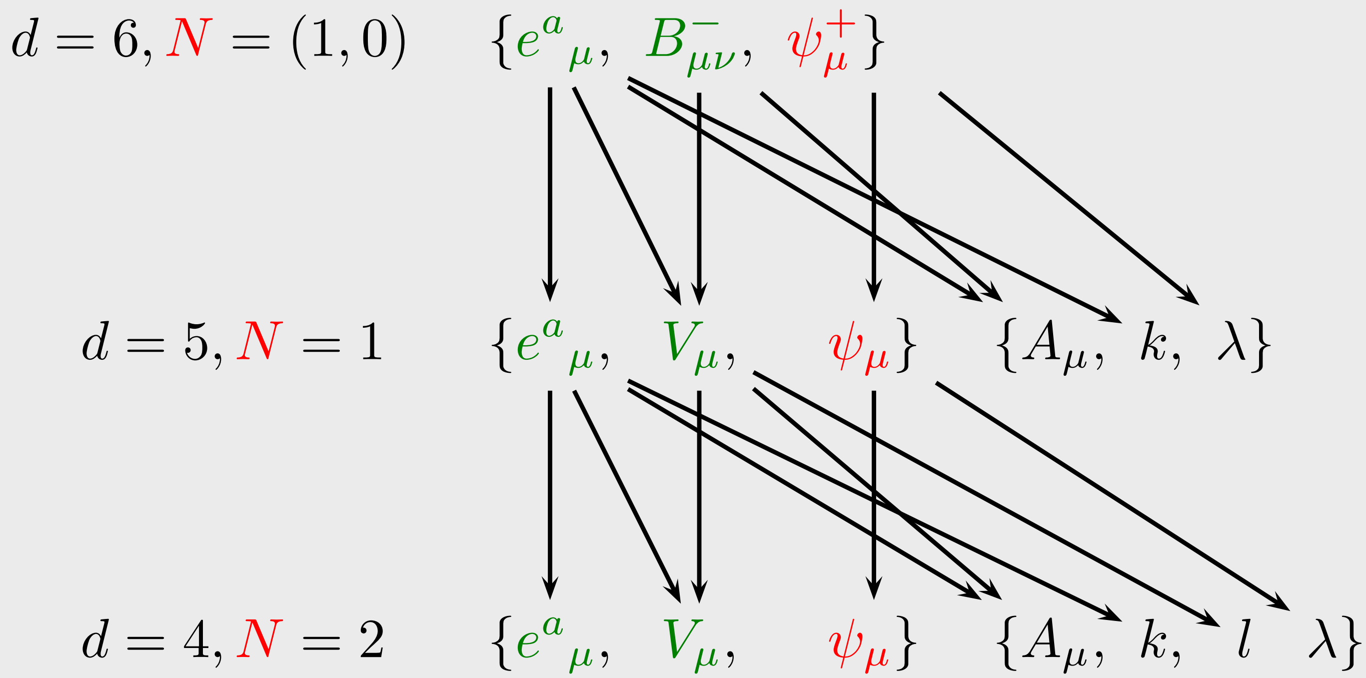

1.- All the solutions of the lower-dimensional theories are also solutions of the higher-dimensional ones with the same unbroken supersymmetries.

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End of slide

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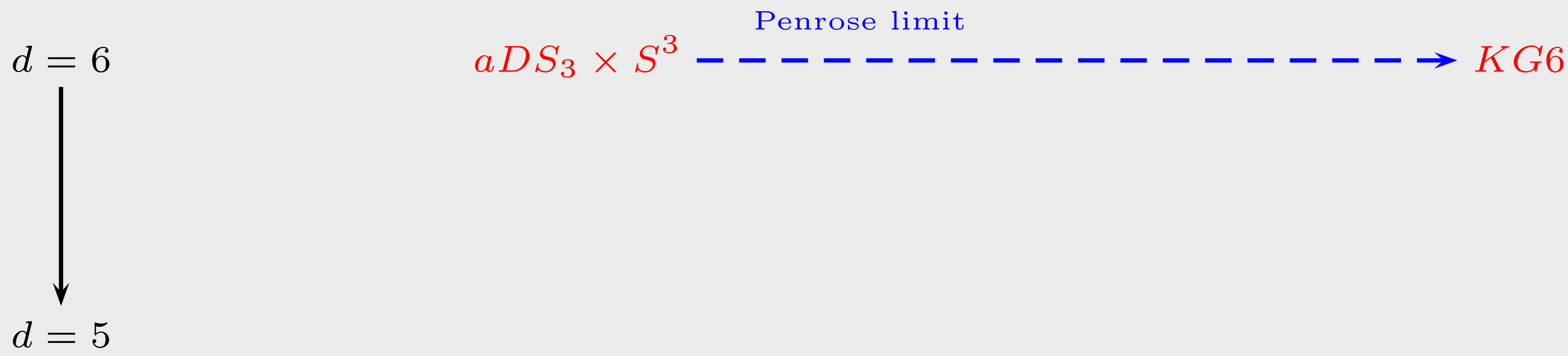

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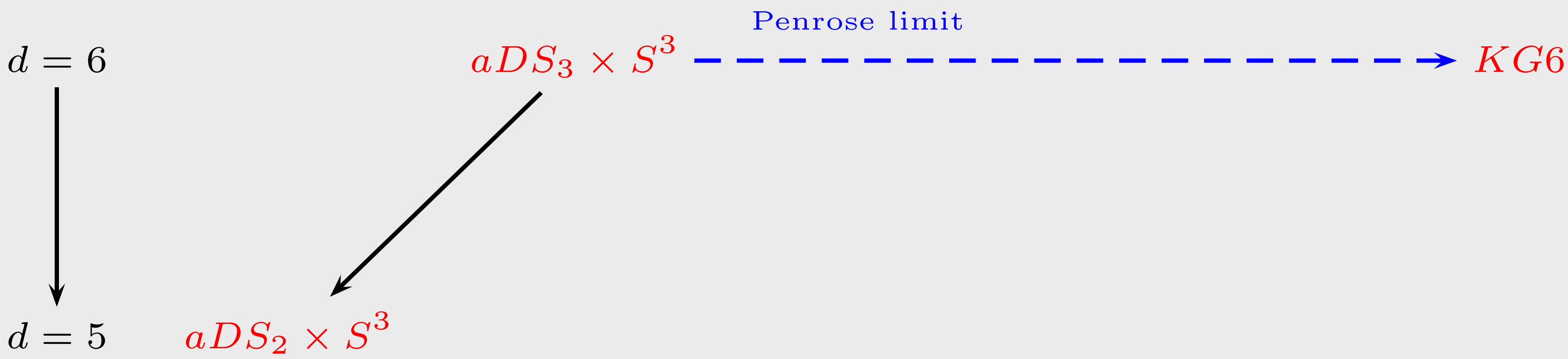

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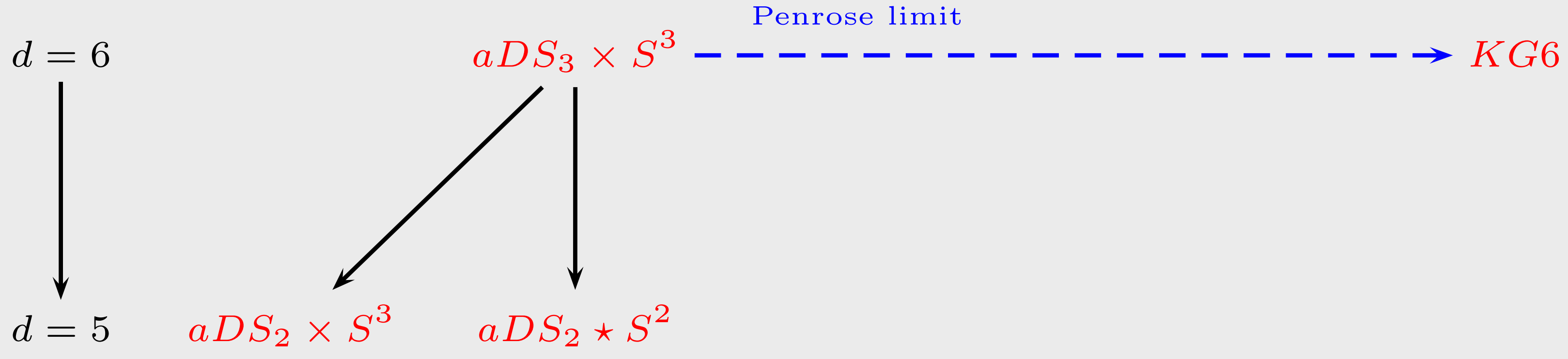

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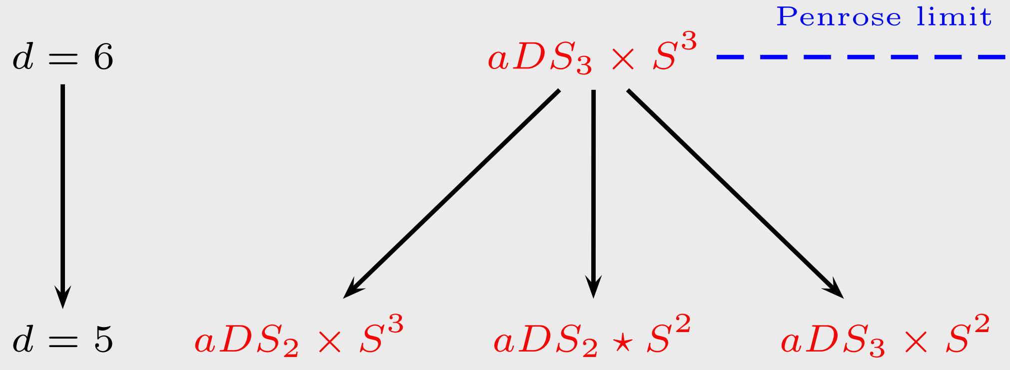

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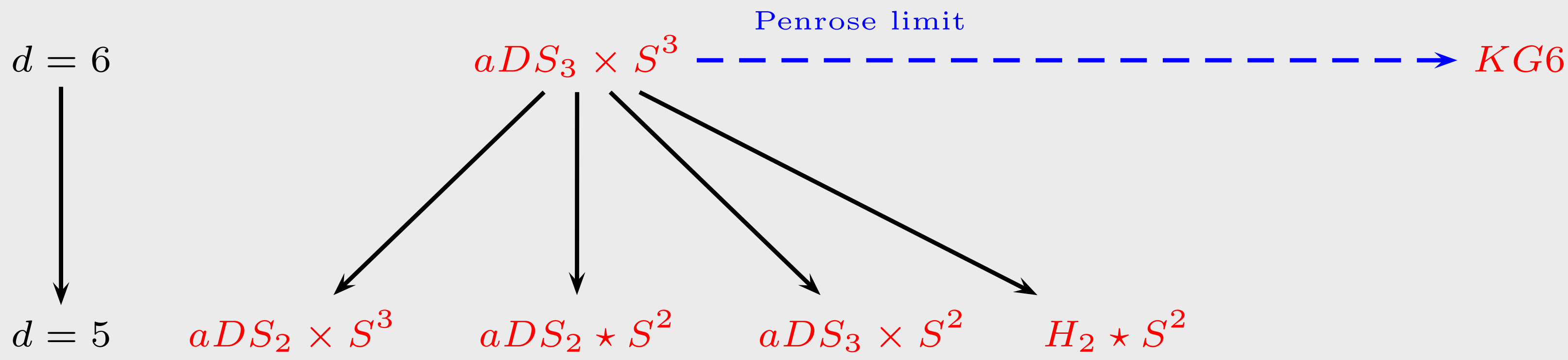

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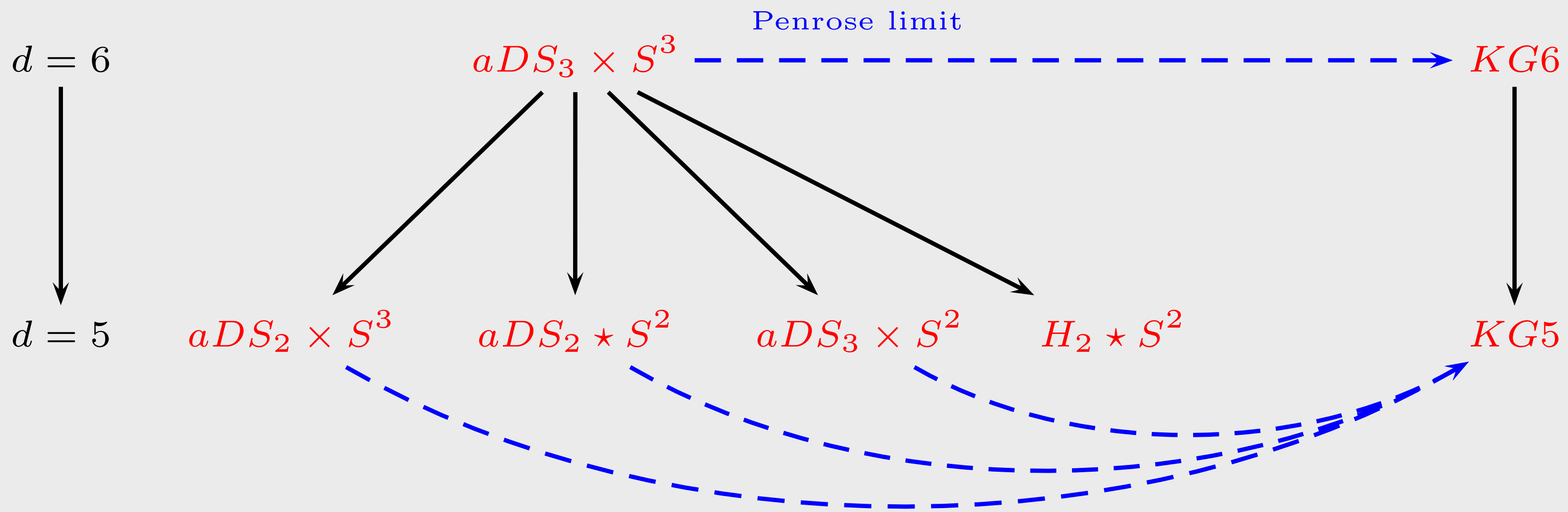

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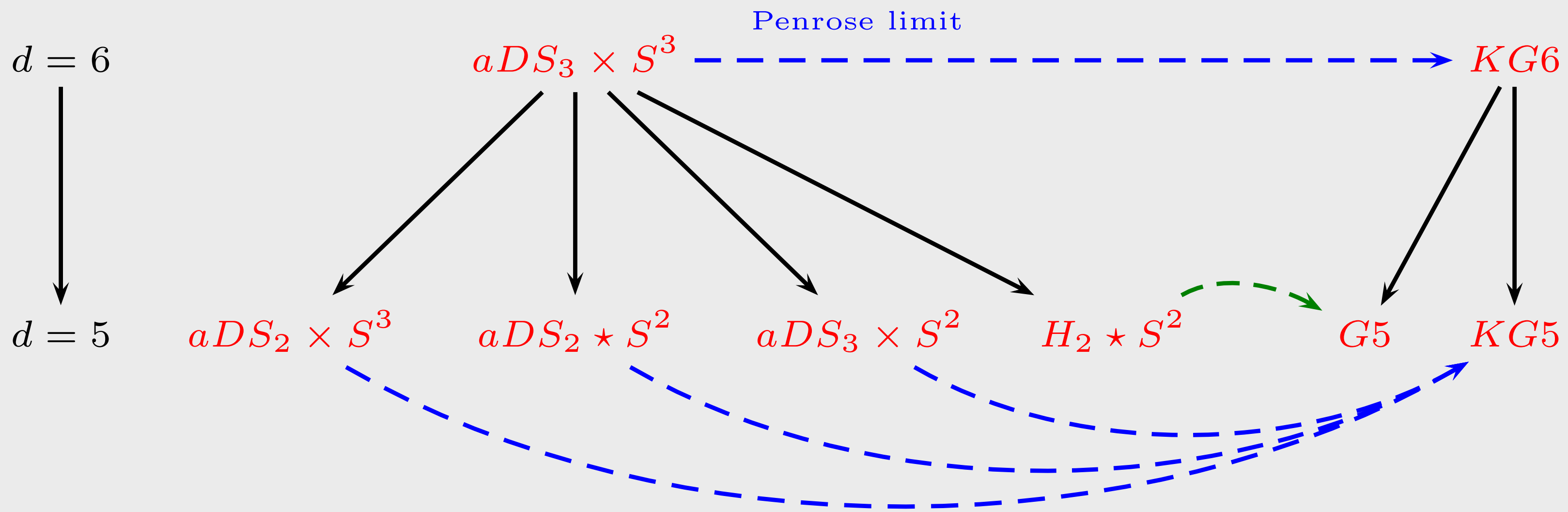

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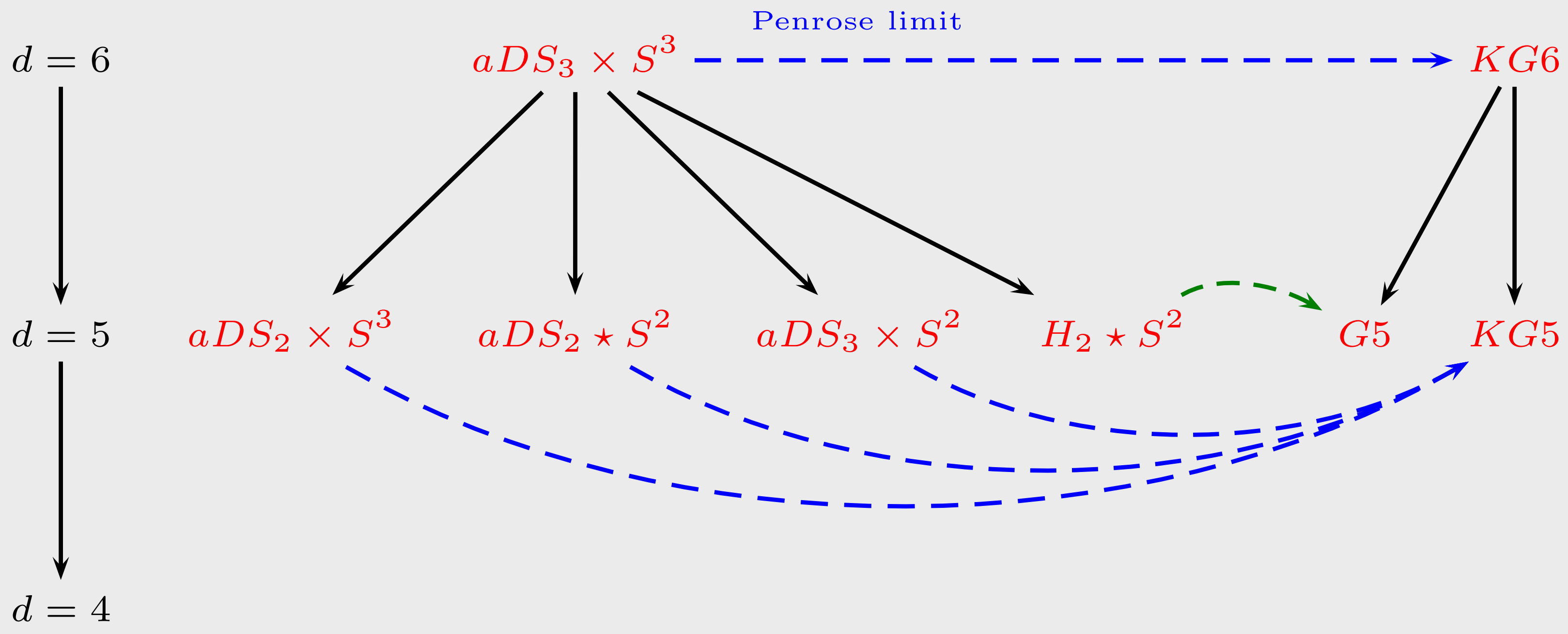

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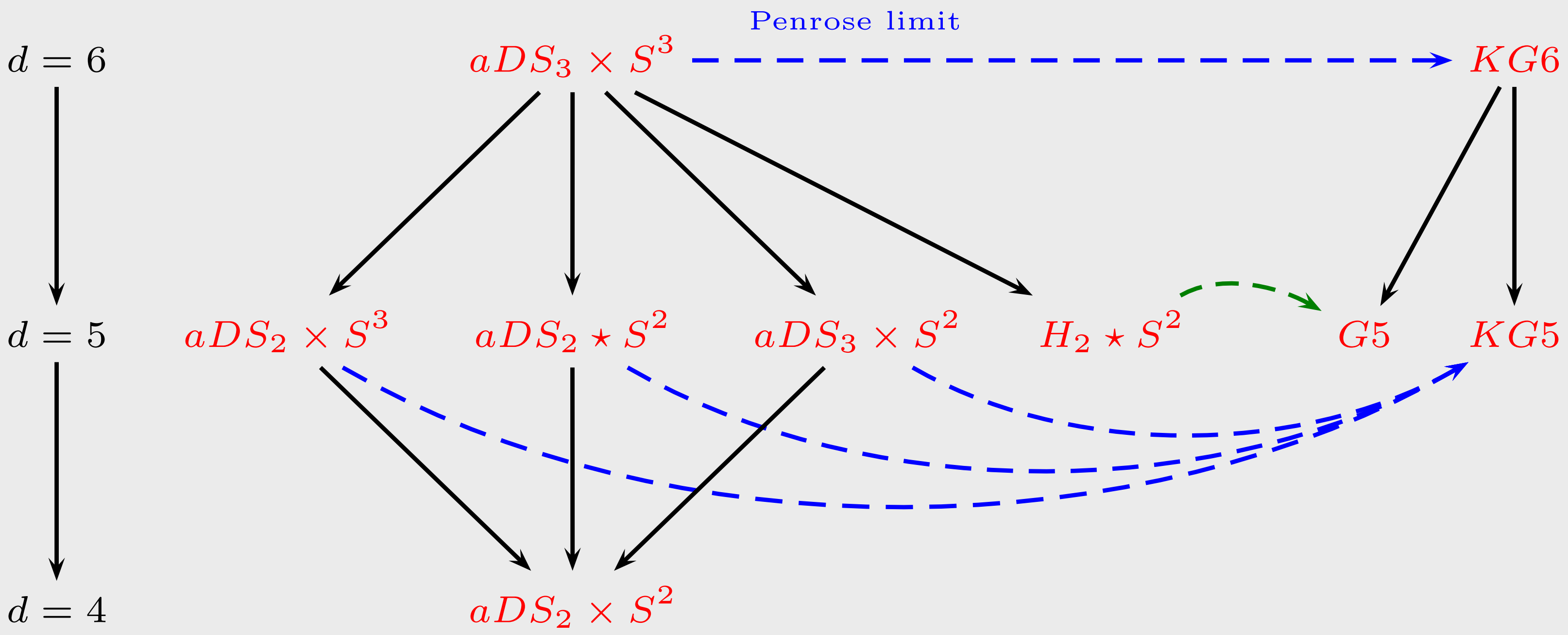

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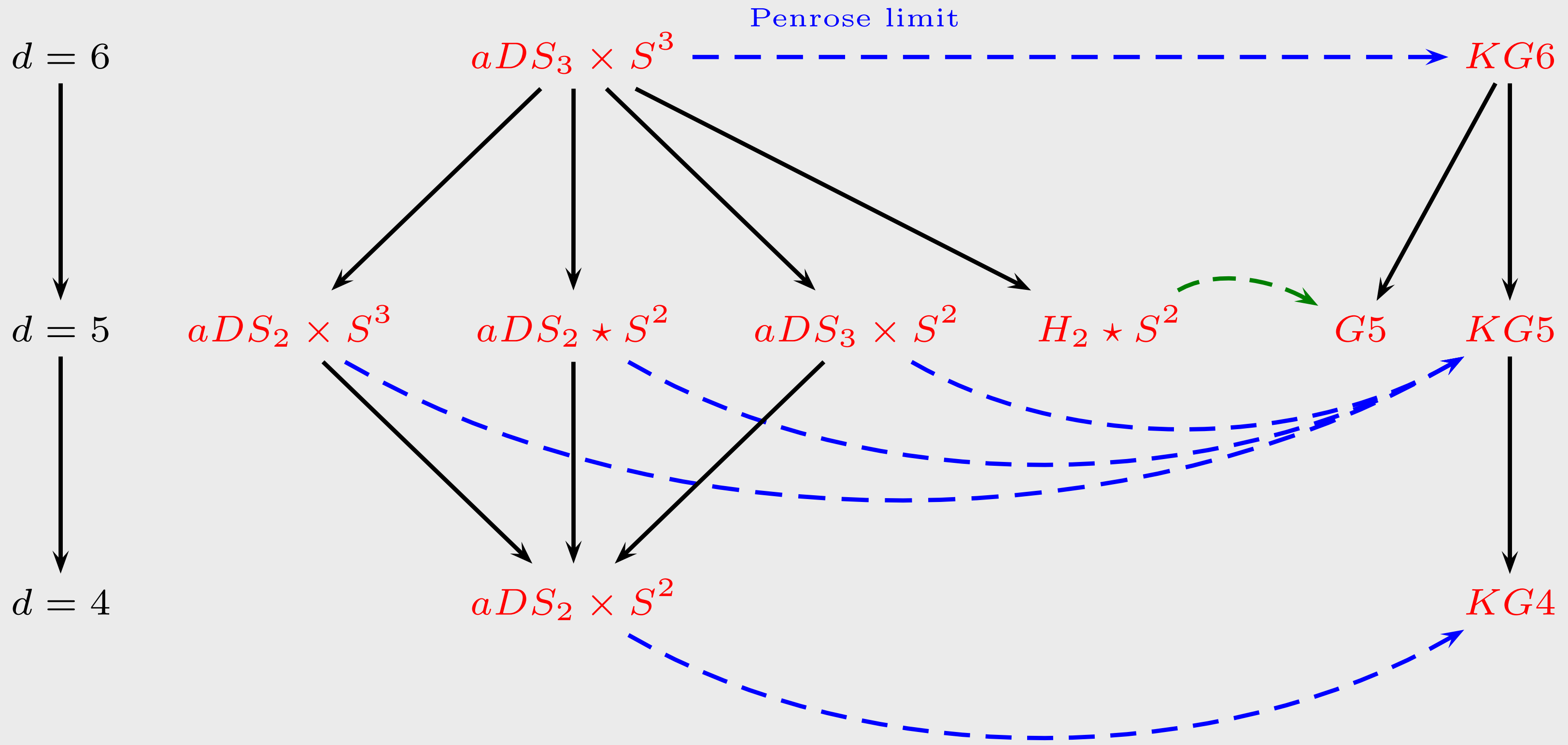

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The spacelike fibrations over base spacetimes are used in standard KK reductions. ω becomes the $d = 4$ Maxwell field.
Can we exploit timelike fibrations over a Euclidean space too?

End of slide

3 – Timelike KK

It is possible to perform Kaluza-Klein dimensional reductions on timelike directions. The original (Lorentzian) theory is reduced to an Euclidean theory and its solutions (with timelike $U(1)$ fibrations) are reduced to Euclidean solutions that may be interpreted as instantons.

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End of slide

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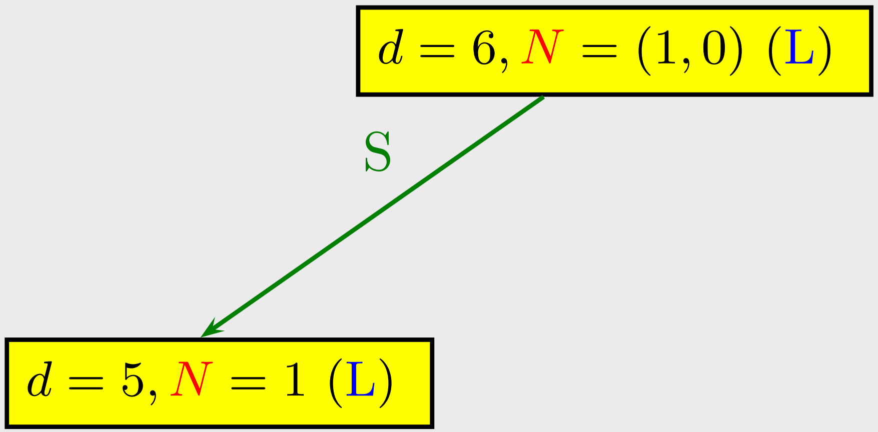

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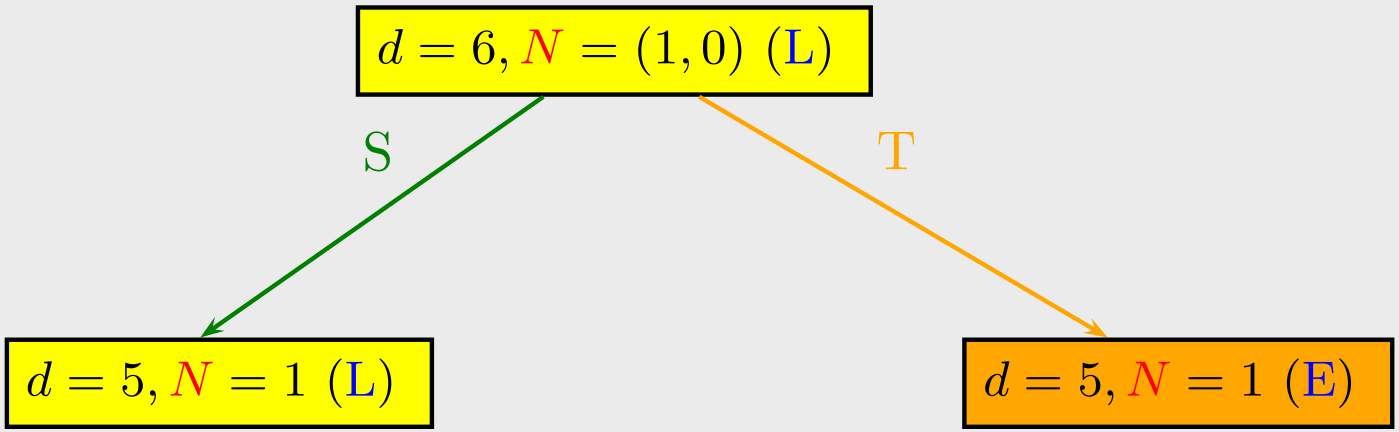

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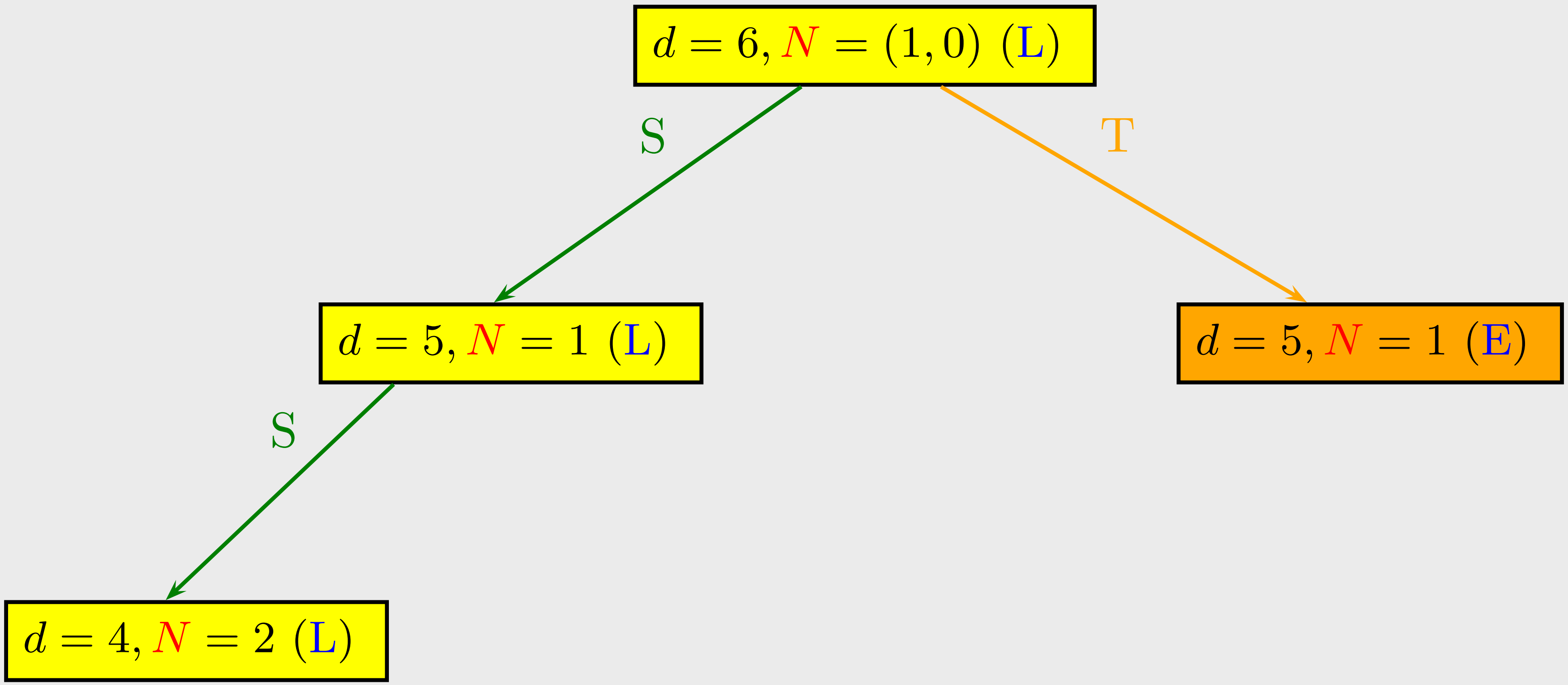

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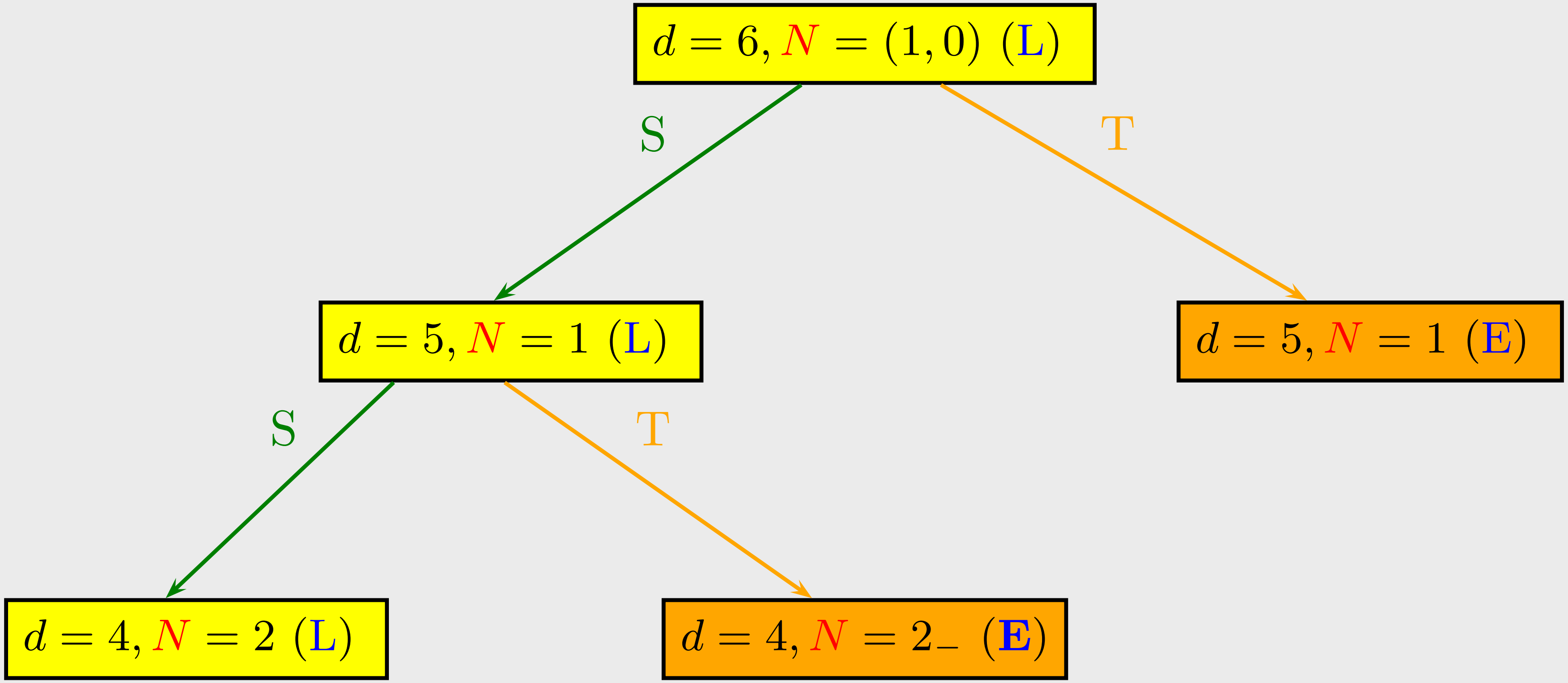

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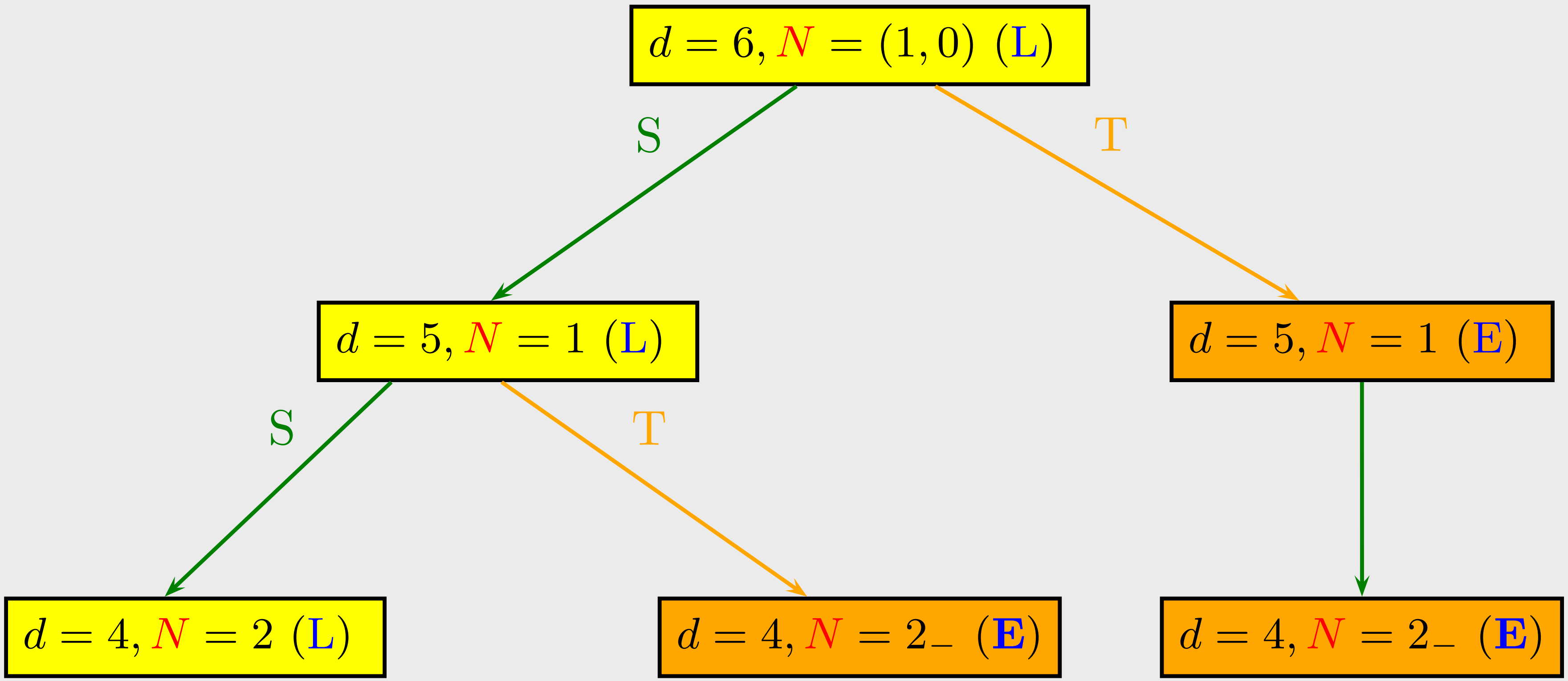

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End of slide

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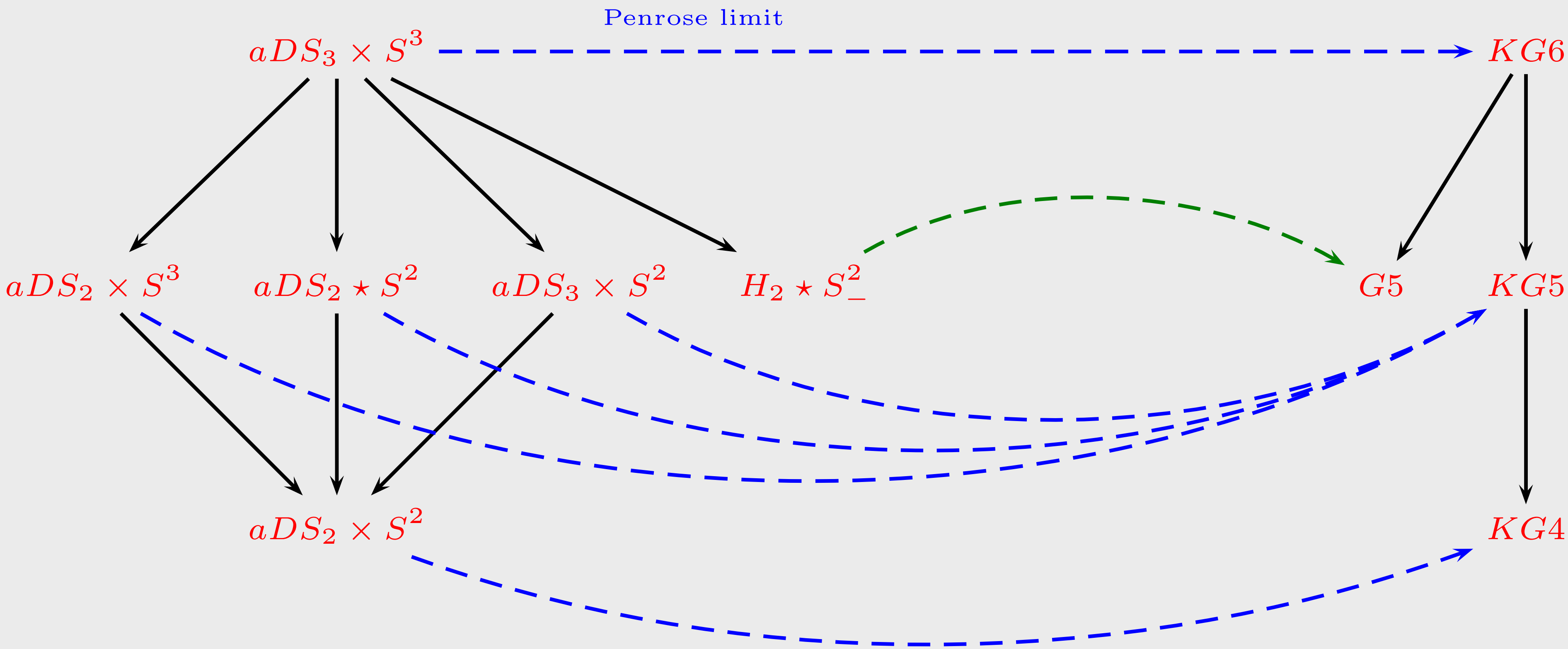

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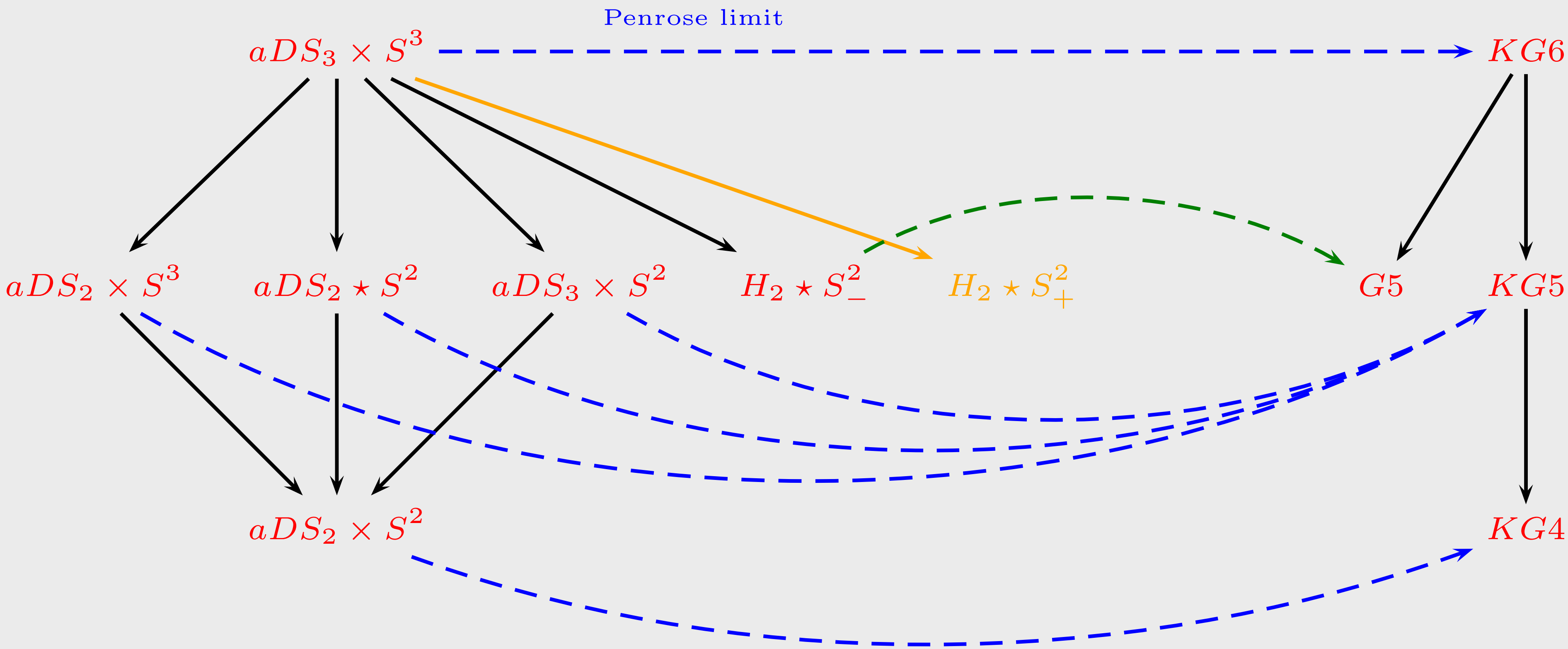

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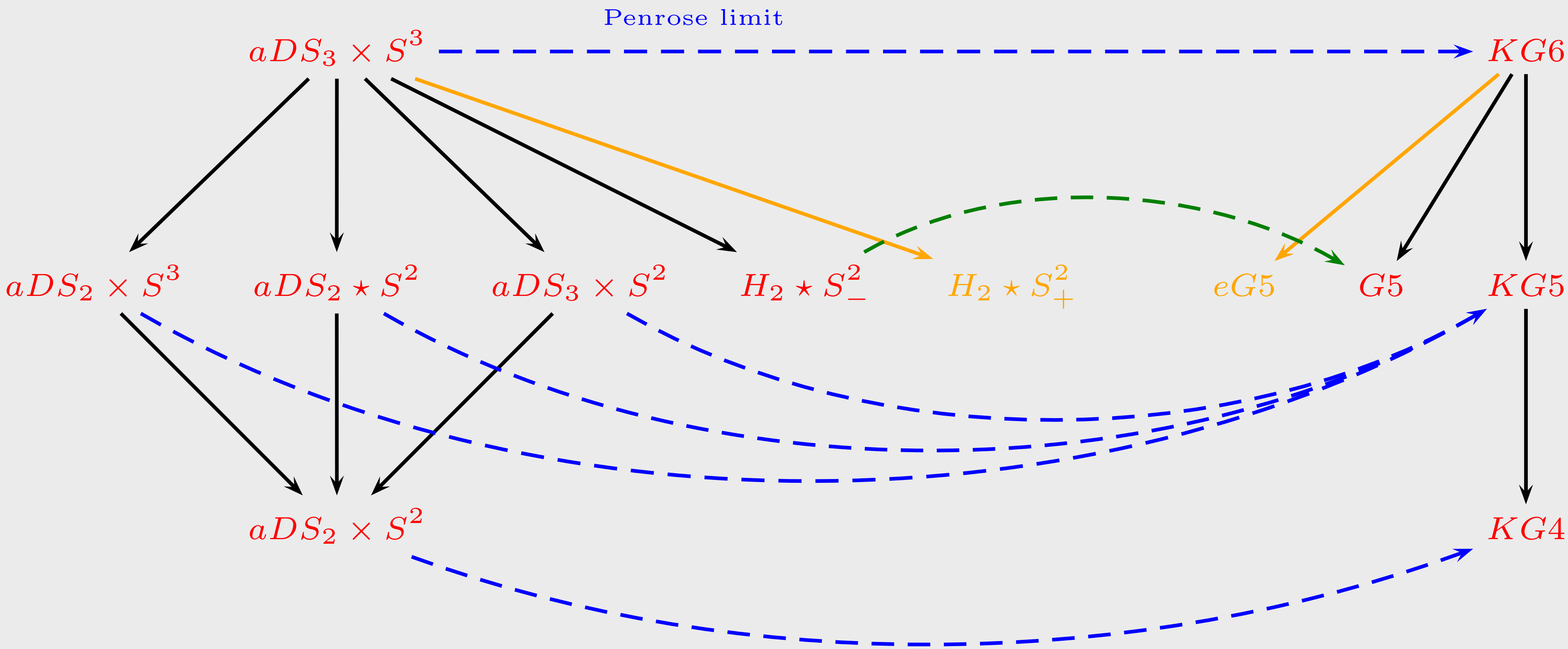

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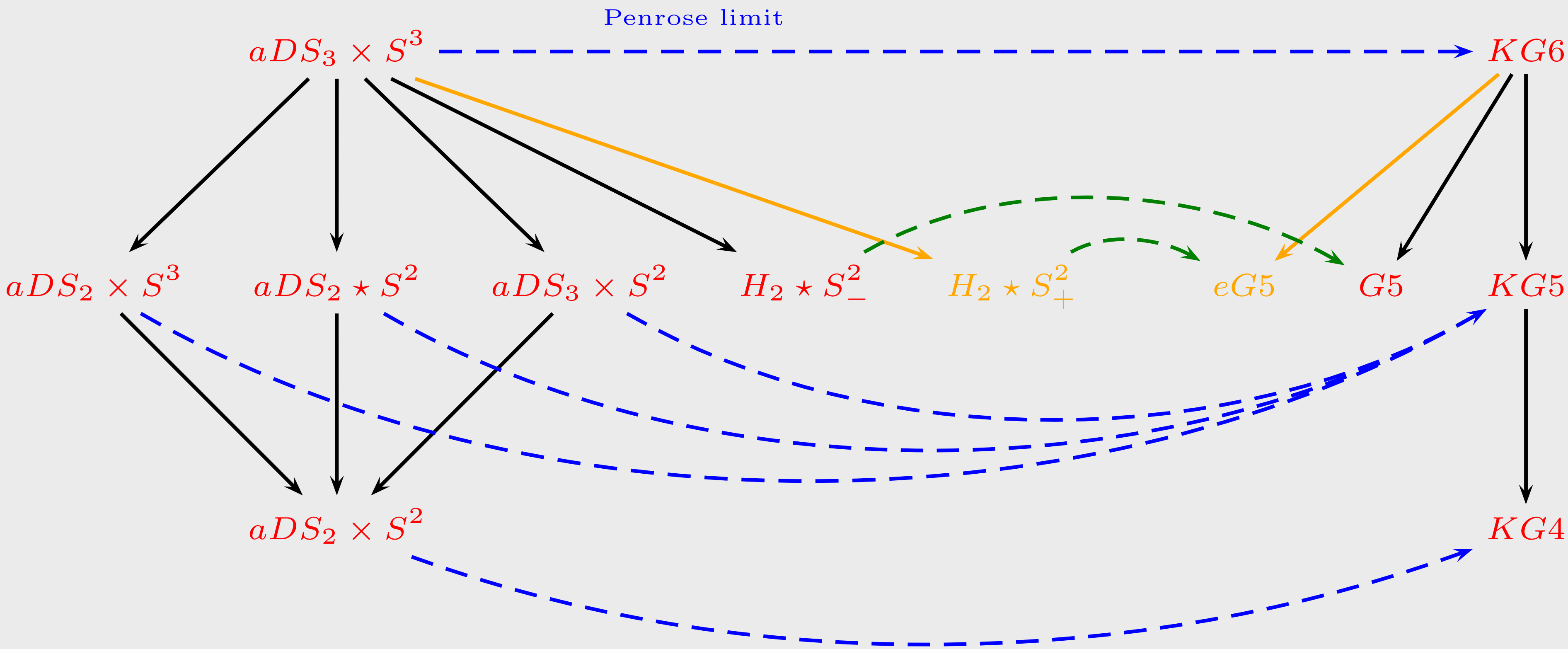

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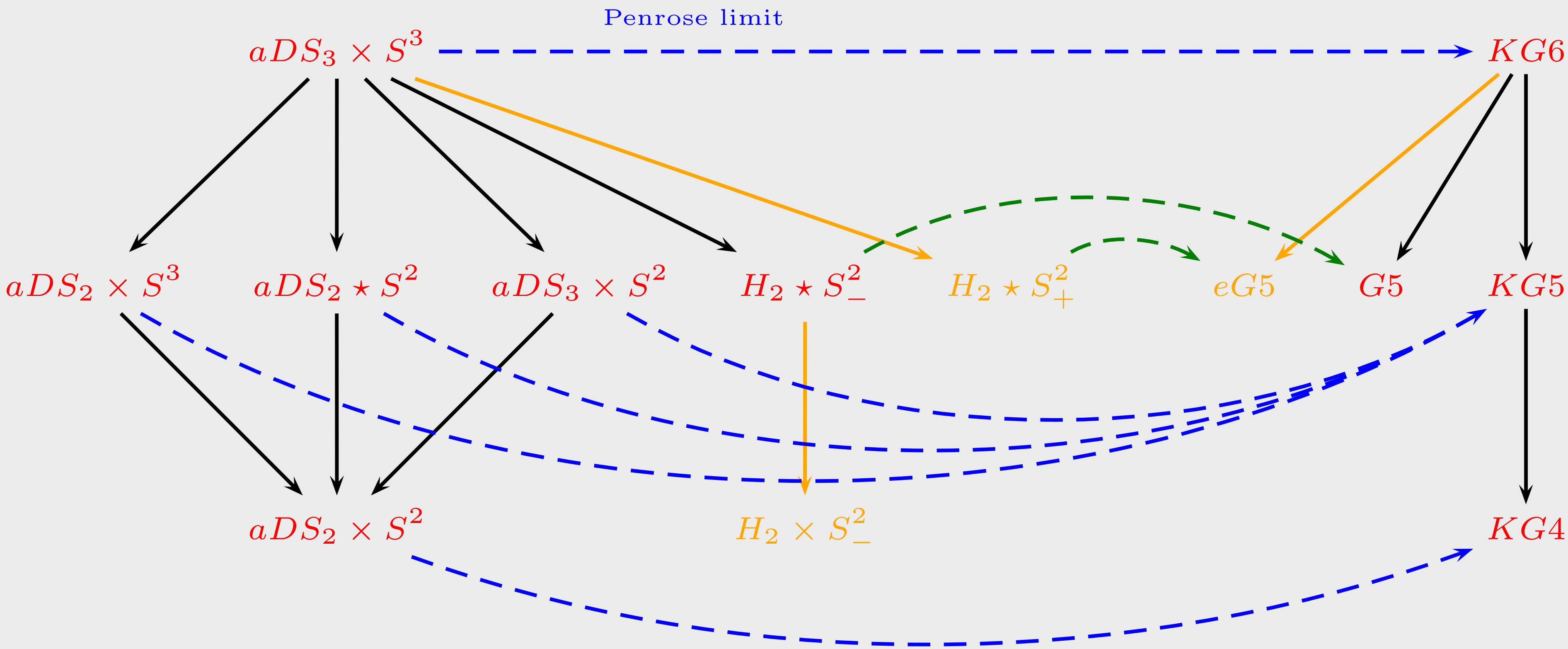

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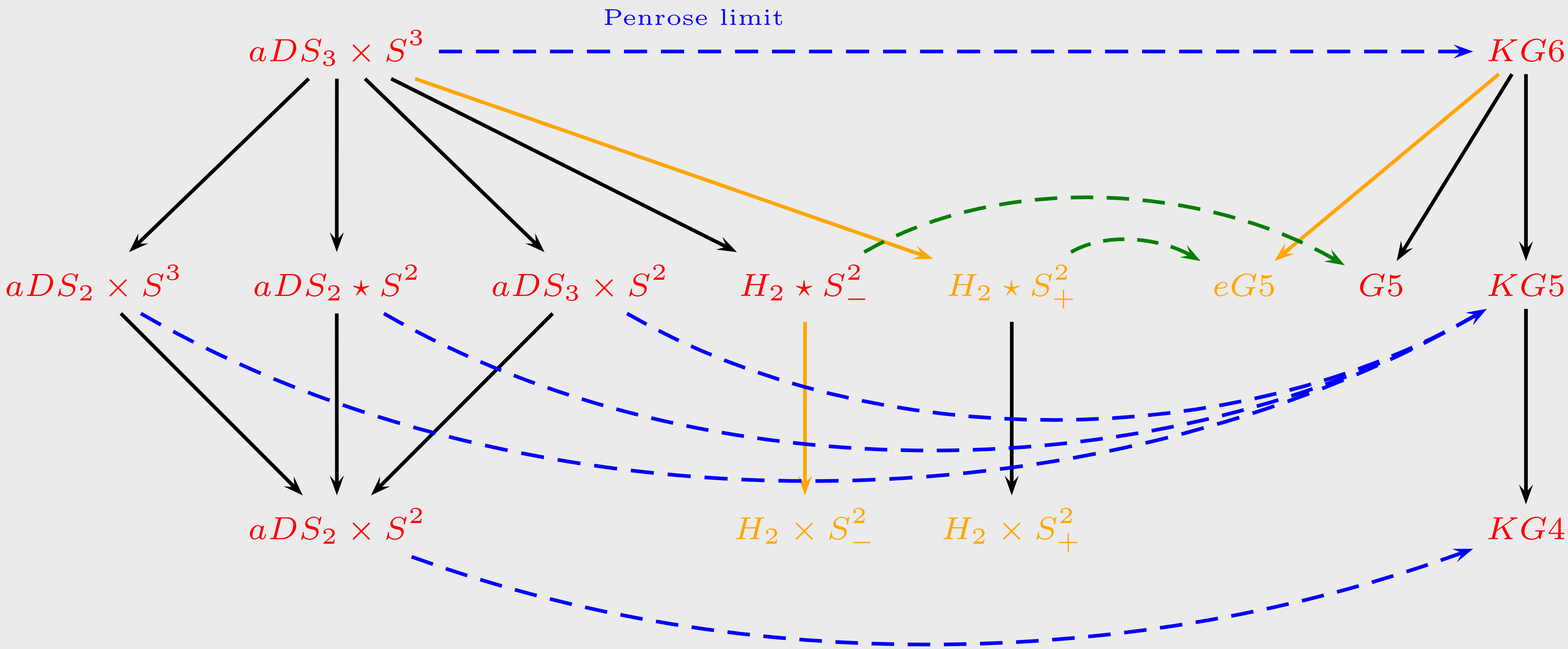

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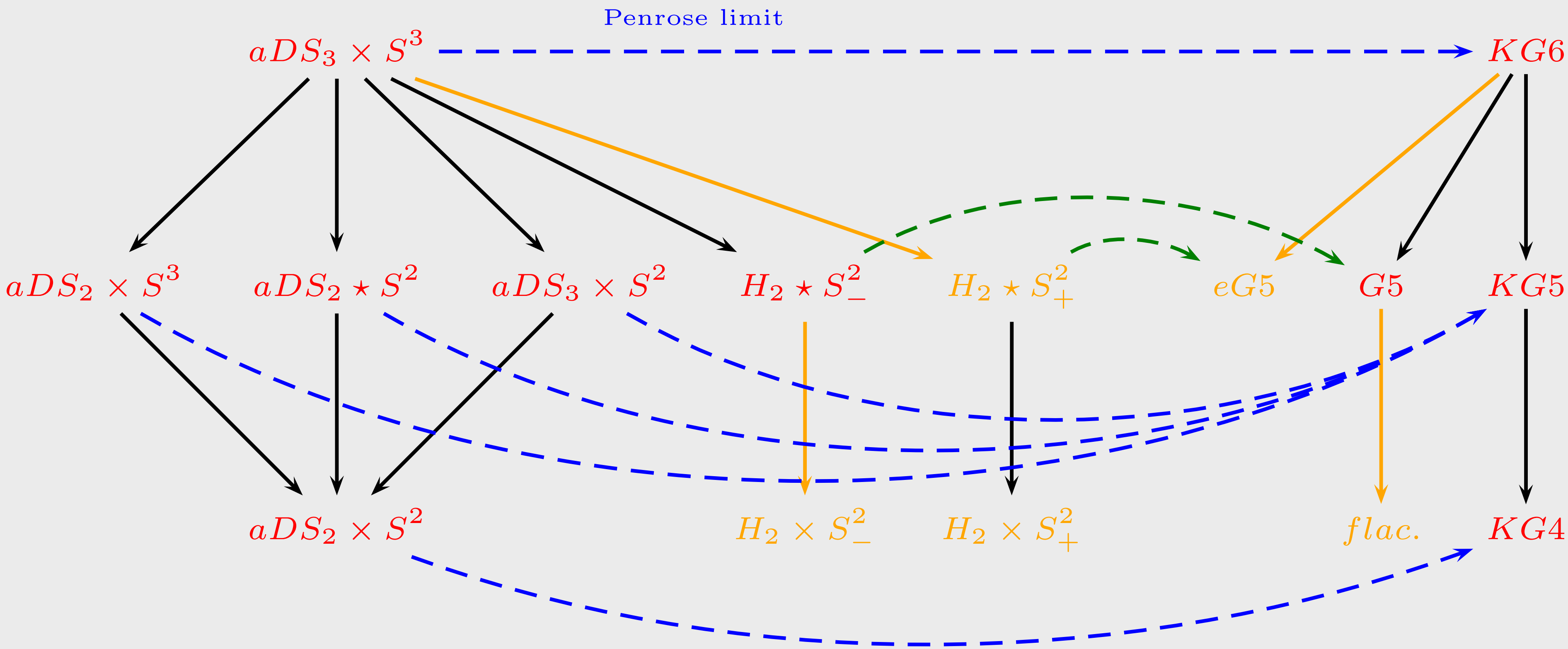

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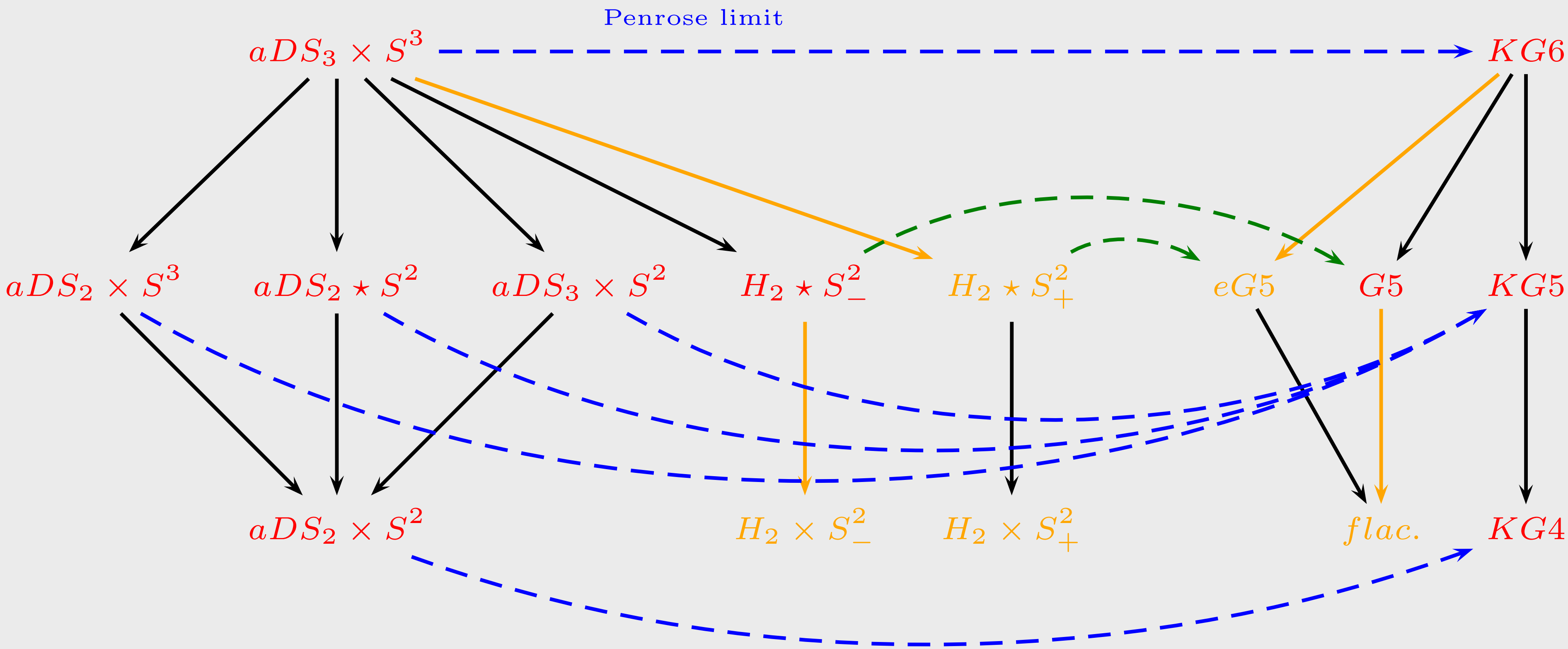

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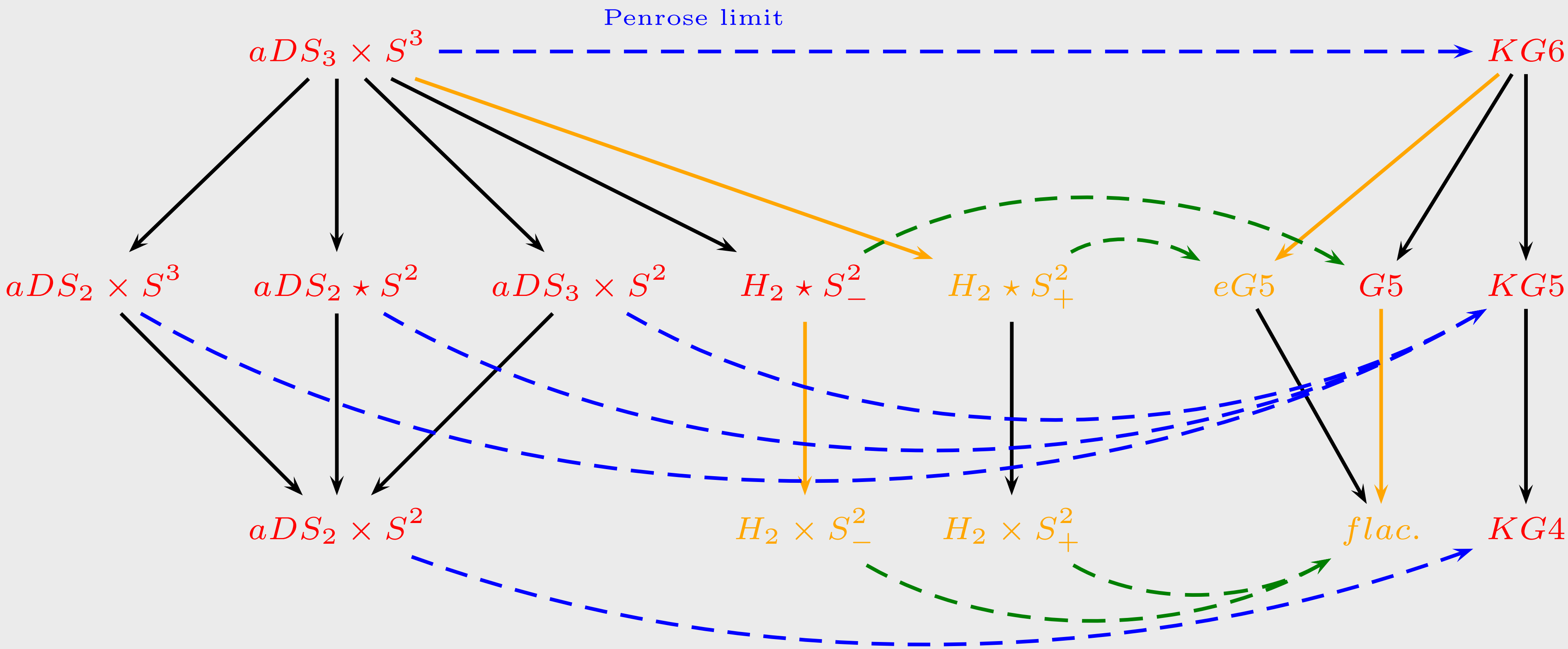

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End of slide

4 – The Flacuum

End of slide

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As we have seen, the dimensional reduction of the Gödel solution of $d = 5, N = 1$ SUGRA given by

(Gödel) $G5$

$$
\begin{aligned}
ds^2 &= (dt + \omega)^2 - d\vec{x}_4^2 \,, \\
V &= -\sqrt{3}\omega \,, \\
\omega &= \lambda(x^1 dx^2 - x^3 dx^4) \,.
\end{aligned}
$$

End of slide

4 – The Flacuum

leads to a non-trivial, maximally supersymmetric Euclidean solution of $d = 4, N = 2$ SUGRA (*i.e.* of the Einstein-Maxwell theory) with flat space and constant **anti-selfdual** field strength ${}^\star F = -F$ ($F_{12} = -F_{34} = \lambda/2$)

The *flacuum* solution

$$-ds^2 = d\vec{x}_4^2\,,$$

$$V = 2\omega\,,$$

$$\omega = \lambda(x^1 dx^2 - x^3 dx^4)\,.$$

End of slide

4 – The Flacuum

A constant, anti-selfdual $U(1)$ field strength certainly solves the Maxwell equation in flat space time, but,

how can flat space be a solution in presence of non-trivial matter?

End of slide

The positivity properties of the action and the energy are opposite in Lorentzian and Euclidean signatures:

	Lorentzian	**Euclidean**
Action:	$-F^2 = E^2 - B^2$	$-F^2 = E^2 + B^2 > 0$
$T_{\mu\nu}$:	$F_\mu{}^\rho F_{\nu\rho} + {}^\star F_\mu{}^{\rho\star} F_{\nu\rho} > 0$	$F_\mu{}^\rho F_{\nu\rho} - {}^\star F_\mu{}^{\rho\star} F_{\nu\rho}$

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In particular, selfdual and anti-selfdual Maxwell fields (that can only be defined in Euclidean signature) have a vanishing "*energy-momentum*" tensor. In general, (anti-) selfdual (non-) Abelian Yang-Mills configurations have vanishing *energy-momentum* tensors and almost decouple from the metric.

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The decoupling is not complete because (anti-) selfduality $F_{\rho\sigma} = \pm{}^\star F_{\rho\sigma}$ has to be proven w.r.t. to a given metric:

$$F_{\rho\sigma} = \pm\frac{1}{2\sqrt{|g|}} g_{\rho\mu} g_{\sigma\nu} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}\,.$$

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$\Rightarrow$ If $F = \pm {}^\star F$ and $R_{\mu\nu} = \Lambda g_{\mu\nu}$, then $\quad G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{2} T_{\mu\nu}\,,$ and $\nabla_\mu F^{\mu\nu} = 0$

End of slide

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The BPST $SU(2)$ instanton

$F = \pm{}^\star F$ with any conformally flat metric. Since $F \to 0$ at ∞ we can take that of the round S^4

$$ds^2 = -\frac{d\vec{x}_4^{\,2}}{(1+(r/2R)^2)^2}\,, \qquad \Rightarrow \qquad R_{\mu\nu} = \frac{1}{R^2} g_{\mu\nu}\,.$$

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The flacuum $U(1)$ solution

$F = \pm{}^{\star}F$ with any conformally flat metric. However, since F is constant, we have to stay with $\mathbb{R}^4$ which, at most, we can compactify on a torus to have a finite action. $R_{\mu\nu} = 0$ and the Einstein equation is satisfied with zero cosmological constant. Observe that taking the gauge group as $U(1)$ is equivalent to take the time periodic in th eGödel solution.

End of slide

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The vector field of our solution (in a new gauge)

$$V = \lambda(x^1 dx^2 - x^2 dx^1 - x^3 dx^4 + x^4 dx^3) \equiv F_{ab} x^a dx^b \,,$$

is not strictly periodic on T^4: when we move around the a-th period from x to $x + \hat{a}$ it changes by a gauge transformation

$$V(x + \hat{a}) = V(x) + d\Lambda_a(x) \,, \qquad \Lambda_a(x) = l^{(a)} F_{(a)b} x^b \,,$$

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Consistency requires that $V(x+\hat{a}+\hat{b}) = V(x+\hat{b}+\hat{a})$, that is

$$\Lambda_a(x+\hat{b}) + \Lambda_b(x) = \Lambda_b(x+\hat{a}) + \Lambda_a(x) \mod(2\pi)\,,$$

which in our case implies

$$\lambda l^1 l^2 = \pi n\,, \qquad \lambda l^3 l^4 = \pi m\,,$$

for two integers n, m that label the possible non-trivial bundles.

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The Euclidean action of the SUGRA solutions is

$$S = -4\pi^2 |nm| \,.$$

End of slide

The symmetry superalgebra of the flacuum solution is particularly interesting because it is a deformation of the supertranslation algebra that preserves the commutativity of momenta but modifies slightly the anticommutator of the supercharges (Berkovits and Seiberg)

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The quantization of the string on this background leads to a non-commutative Field Theory in which only the fermionic superspace coordinates anticommute anomalously.

> This superalgebra can be obtained by dimensional reduction of the Gödel superalgebra, in which the momenta $P_{(a)}$ do **not** commute, but give $P_{(0)}$ which should be interpreted as the generator of $U(1)$ gauge transformations on $d=4$. This property is, precisely, what allowed us to relate the periods of the torii.

End of slide

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- ★ We have determined the symmetry superalgebra of the *flacuum* solution. We notice that the symmetry superalgebras of all the maximally supersymmetric vacua are always deformations of the supertranslation (superPoincaré) algebra, which may allow to classify and find all these vacua.

End of slide

This is

THE END