SUPER GRAvity VACUA TODAY

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(An overview of some recent results on VACUA of SUGRA theories.)

TOMÁS ORTÍN

EREs 2002

VACUUM = the fundamental state of a QFT. $\left\{\begin{array}{l}\text{lowest energy}\\ \text{max. symmetry}\end{array}\right\}$

ITS SYMMETRIES DETERMINE THE KINEMATICS

→ CONSERVED QUANTITIES → Q. NUMBERS → SPECTRUM

(GRAVITY → ENERGY ZERO POINT)

Some theories (in particular, candidates to TOE) admit more than one vacuum and the problem of the vacuum selection (→ moduli problem) is the most pressing and interesting one.

GR_d + matter ⟶ SUGRA $\xleftarrow[\alpha' \to 0]{\text{low energy}}$ SUPERSTRINGS

SUGRA vacua will be candidates to String Theory vacua, at least to some approximation.

PLAN OF THE TALK

I SUGRA THEORIES

SUSY → SUGRA

SPACETIME SUPERALGEBRAS → SUGRA THEORIES (EXTENDED GAUGED/MASSIVE)

SPACETIME SUPERALGEBRAS ↓ (EXTENDED) POINCARÉ, (EXTENDED) AdS, Hpp

SUGRA THEORIES ↓ VACUA ↓ SYMMETRY SUPERALGEBRAS → (EXTENDED) POINCARÉ, (EXTENDED) AdS, Hpp

II MAXIMALLY SUSY SOLUTIONS IN DIVERS DIMENSIONS

- LIST
- COSET CONSTRUCTIONS:
  - KILLING SPINORS
  - KILLING VECTORS
  - SUPERALGEBRAS

III SOME GENERAL RESULTS ON SUPERSYMMETRIC SOLUTIONS

- SUSY SOLUTIONS OF $N=\overset{1}{\underset{4}{2}}$, $d=4$ SUGRA (EINSTEIN-MAXWELL)
- SUSY SOLUTIONS OF $N=1$, $d=5$ SUGRA

I SUGRA THEORIES

SUPER SYMMETRY

SUSY is the ultimate symmetry (allowed by the S matrix)

$$\text{MATTER (Fermions)} \rightleftarrows \text{RADIATION (Bosons)}$$

(carriers of interactions)

In QFT it interchanges Bosonic fields (tensors) with Fermionic (anticommuting) fields (spinors).

$\Rightarrow$ it's generated by spinorial, anticommuting SUPERCHRGES Q^α

and the infinitesimal parameters are anticommuting SPINORS ε^α

$$\begin{cases} \delta_\varepsilon B \sim \bar{\varepsilon} F \\ \delta_\varepsilon F \sim \partial\varepsilon + B\varepsilon \end{cases}$$

The simplest SUPERALGEBRA includes the Poincaré algebra

$$\{Q^\alpha, Q^\beta\} = i(\gamma^a C^{-1})^{\alpha\beta} P_a$$

P_a ← generators of translations

$$[Q^\alpha, M_{ab}] = \frac{1}{2}(\gamma_{ab})^\alpha{}_\beta Q^\beta$$

← Lorentz spinors

etc. is a SPACETIME ALGEBRA ($\rightsquigarrow$ SUPERSPACE)

SUSY ⇒ SUPERGRAVITY

General Relativity can be viewed as the gauge theory of the Poincaré group.

$$\{M_{ab}, P_a\} \xrightarrow[\text{"gauging"}]{} A_\mu \equiv \frac{1}{2}\omega_\mu{}^{ab} M_{ab} + e^a{}_\mu P_a$$

(gauge potential)

(Gauge field strength)

Curvature: $R_{\mu\nu} = 2\partial_{[\mu}A_{\nu]} + [A_\mu, A_\nu] =$

$$= \underbrace{\frac{1}{2} R_{\mu\nu}{}^{ab}}_{\text{(Lorentz curvature)}} M_{ab} + \underbrace{R_{\mu\nu}{}^a}_{\text{(Torsion)}} P_a$$

Action: $S \sim \int d^4x\, e R(e,\omega)$

$$\rightarrow \begin{cases} R_{\mu\nu}{}^a = 0 \\ G_{\mu\nu} = 0 \quad \text{(Einstein equation)} \end{cases}$$

This is equivalent to Einstein's GR, but allows for the coupling of fermions to gravity (Cartan-Sciama-Kibble theory)

SUperGRAvity can be seen as the gauge theory of the Poincaré SUPER ALGEBRA

$$\{M_{ab}, P_a, Q^\alpha\} \xrightarrow[\text{"gauging"}]{} A_\mu = \frac{1}{2}\omega_\mu^{ab} M_{ab} + e^a_\mu P_a + \bar{\psi}_{\mu\alpha} Q^\alpha$$

ψ_μ^α compensates the local SUSY transformations

$\rightarrow$ Rarita-Schwinger field $\rightarrow$ gravitino

Curvature: $R_{\mu\nu} \equiv 2\partial_{[\mu} A_{\nu]} + [A_\mu, A_\nu]$

$$= \frac{1}{2} R_{\mu\nu}{}^{ab} M_{ab} + R_{\mu\nu}{}^a P_a + \bar{R}_{\mu\nu\alpha} Q^\alpha$$

Action $S \sim \int d^4x\, e \left\{ R(e,\omega) + \varepsilon^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_5 \gamma_\nu D_\rho(\omega) \psi_\sigma \right\}$

This is simply the 1^{st} order form of the CSK theory for a Rarita-Schwinger field ψ_μ

$\Rightarrow$ $\omega_\mu{}^{ab}$ is a composite field $\omega_\mu{}^{ab}(e,\psi)$

$\Rightarrow$ there is non-dynamical torsion $T_{\mu\nu}{}^a \sim \bar{\psi}_\mu \gamma^a \psi_\nu$

$e^a{}_\mu \rightarrow$ massless spin 2 quanta $\rightarrow$ $\boxed{\text{gravitons}}$

$\psi_\mu^\alpha \rightarrow$ massless spin 3/2 quanta $\rightarrow$ $\boxed{\text{gravitini}}$

#BOSONS = #FERMIONS

SUPERGRAVITY MULTIPLET

MORE GENERAL SPACETIME SUPERALGEBRAS

$\Downarrow$

MORE SUGRAS

1) $\to N > 1$
2) $\to$ AdS
3) $\to d > 4$

3 Ways to generalize the $(N=1, d=4)$ Poincaré superalgebra

1) Add more supercharges $\to Q^{i,\alpha}$, $i = 1, \dots, N$

$\Rightarrow$ N-EXTENDED $d=4$ Poincaré superalgebras

These admit central charges $Q^{ij} = -Q^{ji}$, $P^{ij} = -P^{ji}$

$$\{Q^{i\alpha}, Q^{j\beta}\} = i\,\delta^{ij}(\gamma^a C^{-1})^{\alpha\beta} P_a - i (C^{-1})^{\alpha\beta} Q^{ij} - (\gamma_5 C^{-1})^{\alpha\beta} P^{ij}$$

$$[Q^{ij}, \text{anything}] = 0 \quad ; \quad [P^{ij}, \text{anything}] = 0 \; ;$$

Gauging them, we obtain N-EXTENDED SUGRAS

$$A_\mu = \frac{1}{2}\omega_\mu{}^{ab} M_{ab} + e^a{}_\mu P_a + \frac{1}{2} A^{ij}{}_\mu Q^{ij} + \bar{\psi}^i_{\mu\alpha} Q^{i\alpha}$$

$\frac{N(N-1)}{2}$ Abelian vectors   N gravitini

$$S \sim \int d^4x\, e \left\{ R(e,\omega) + \varepsilon^{\mu\nu\rho\sigma} \bar{\psi}^i_\mu \gamma_5 \gamma_\nu D_\rho(\omega) \psi^i_\sigma - \frac{1}{8} F^{ij}_{\mu\nu} F^{ij\,\mu\nu} \right\}$$

+ $\underset{V=0}{\text{scalars}}$ and extra fermions

$$F^{ij}{}_{\mu\nu} = 2\partial_{[\mu} A^{ij}{}_{\nu]} \; ;$$

N=8 is the maximum allowed (Nahm)

2) Using a different spacetime bosonic subalgebra

- ~~Anti de Sitter~~ Anti de Sitter — $SO(2,3)$
- ~~de Sitter~~ — $SO(1,4)$
- Hpp-wave — $\{P_+ P_-, P_i, P_{*i}, M_{ij}\}$
- ~~Conformal~~ — $SO(3,4)$

These give inconsistent quantum theories

To be discussed later (never used to construct a SUGRA)

N-EXTENDED $d=4$ AdS superalgebras:

$$\{Q^{i\alpha}, Q^{j\beta}\} = i\delta^{ij}\, m^{\hat a \hat b\, \alpha\beta}\, \hat M_{\hat a \hat b} - i (C^{-1})^{\alpha\beta}\, T^{ij};$$

$$[T^{ij}, T^{kl}] = \delta^{ik} T^{jl} \dots;$$

$$[Q^{i\alpha}, T^{jl}] = 2\eta^{i[j} Q^{l]\alpha}; \qquad \dots$$

$SO(N)$ generators

$$A_\mu = \tfrac{1}{2}\hat\omega_\mu{}^{\hat a \hat b}\, \hat M_{\hat a \hat b} + \tfrac{1}{2} A_\mu{}^{ij}\, T^{ij} + \bar\psi^i_{\mu\,\alpha}\, Q^{i\alpha};$$

$SO(N)$ non-Abelian vector potential

$SO(N)$-charged gravitini

$\Rightarrow$ $SO(N)$ gauged SUGRA

The action contains a negative cosmological constant $-g^2$ $g \to SO(N)$ coupling constant.

3) Using $d > 4$ spacetime bosonic subalgebras

Poincaré up to $d = 11$ (Nahm)

AdS up to $d = 7$

The main feature of these superalgebras is that they admit QUASI-CENTRAL EXTENSIONS $Z_{[a_1 \dots a_p]}$

EXAMPLE: $d=11, N=1$ Poincaré superalgebra

$$\{Q^\alpha, Q^\beta\} = i(\gamma^a C^{-1})^{\alpha\beta} P_a + \tfrac{1}{2}(\gamma^{ab} C^{-1})^{\alpha\beta} Z_{ab} + \frac{i}{5!}(\gamma^{a_1 \dots a_5} C^{-1})^{\alpha\beta} Z_{a_1 \dots a_5};$$

$$\rightarrow A_\mu = \tfrac{1}{2}\omega_\mu{}^{ab} M_{ab} + e^a{}_\mu P_a + \tfrac{1}{2} C_\mu{}^{ab} Z_{ab} + \Psi_{\mu\alpha} Q^\alpha$$

$C_{\mu\nu\rho}$: 3-form potential

$$S \sim \int d^{11}x \, e \left\{ R(e,\omega) + \varepsilon^{\mu\nu\rho\sigma} \bar{\Psi}_\mu \gamma_{11} \gamma_\nu D_\rho \Psi_\sigma + - \frac{1}{48} G^2 + \dots \right.$$

$$G_{\mu\nu\rho\sigma} = 4\, \partial_{[\mu} C_{\nu\rho\sigma]};$$

These SUGRAS generically contain p-form fields

$\longrightarrow$ BRANES!

SUPERALGEBRAS FROM SUGRA VACUA

Most SUGRAS are NOT obtained from superalgebras but we can associate to them

VACUUM SUSY ALGEBRA

VACUUM

GR $\rightsquigarrow$ { maximally symmetric solution: Mink AdS dS

SUGRA $\rightsquigarrow$ { maximally SUPERSYMMETRIC BOSONIC solution

A SUGRA solution is SUPERSYMMETRIC if there is a (KILLING) spinor K such that $\delta_K = 0$

$$\Rightarrow \begin{cases} \delta_K B = \bar{K} F = 0 \\ \delta_K F = \partial K + B K = 0 \end{cases}$$

$\boxed{\text{Killing spinor equation}}$

"superisometry"

GR → the Killing vectors $k_{(I)}$ satisfy a LIE ALGEBRA $[k_{(I)}, k_{(J)}] = f_{IJ}{}^{K} k_{(K)}$ and are associated to generators of the ISOMETRY GROUP $k_{(I)} \longrightarrow P_{(I)}$

WHAT HAPPENS IN SUGRA?

SUGRA solutions will be invariant under some SUPERGROUP generated by infinitesimal isomorfisms (Killing vectors) and infinitesimal supersymmetries (Killing spinors) that

MUST SATISFY A SUPERALGEBRA

How can the superalgebra be found?

RECIPE

(Figueroa O'Farrill 1999)

1. Associate

$$
\begin{array}{lcl}
K_{(A)}{}^{\alpha} \longrightarrow & Q_{(A)} \to & \text{ODD} \\
k_{(I)}{}^{\mu} \longrightarrow & P_{(I)} \to & \text{EVEN}
\end{array}
\Bigg\} \text{ SUPERALGEBRA GENERATORS}
$$

The superalgebra is determined by the structure constants

$$
f_{IJ}{}^{K}\,; \qquad f_{AB}{}^{I}\,; \qquad f_{AI}{}^{B}\,;
$$

$$
[P,P] \sim P \qquad \{Q,Q\} \sim P \qquad [Q,P] \sim Q
$$

2. The structure constants $f_{IJ}{}^{K}$ of the even subalgebra are those of the isometry Lie algebra

$$
[k_{(I)}, k_{(J)}] = f_{IJ}{}^{K} k_{(K)}\,;
$$

3. The structure constants $f_{AB}{}^{I}$ are given by the decomposition of the bilinears

$$
-i\,\bar{K}_{(A)}\gamma^{a}K_{(B)}\,e_{a} = f_{AB}{}^{I}\,k_{(I)}
$$

4. The structure constants $f_{AI}{}^{B}$ are given by the spinorial Lie derivatives

$$
\mathbb{L}_{k_{(I)}} K_{(A)} = f_{AI}{}^{B} K_{(B)}
$$

COMMENTS:

③ If κ_1 and κ_2 are Killing spinors

$$\nabla_\mu \kappa = 0 \quad \text{in } N=1,\ d=4 \text{ Poincaré SUGRA}$$

$$\left(\nabla_\mu - i\frac{g}{2}\gamma_\mu\right)\kappa = 0 \quad \text{in } N=1,\ d=4 \text{ AdS SUGRA}$$

$$\left(\nabla_\mu + \frac{1}{8}\not{F}\gamma_\mu\right)\kappa = 0 \quad \text{in } N=2,\ d=4 \text{ Poincaré SUGRA}$$

$\vdots$

→ then $-i\bar{\kappa}_1\gamma^\mu\kappa_2$ is a Killing vector $\sim c^I k_{(I)}{}^\mu$

④ The spinorial Lie derivative is a particular case of a GENERALIZED G-REDUCTIVE LIE DERIVATIVE (see e.g. Godina & Matteucci math.DG/0201235)

In pedestrian/physicist terms, it is just a gauge-covariant Lie derivative

$$\mathbb{L}_v = \underbrace{\mathcal{L}_v}_{\text{standard Lie derivative}} + \underbrace{W(v)}_{\text{compensator}}$$

Spinors are defined up to local (gauge) Lorentz transformations and $\mathbb{L}_v \psi = v^\mu \nabla_\mu \psi + \frac{1}{4}\nabla_{[\mu}v_{\nu]}\gamma^{\mu\nu}\psi$

interesting properties! (Kosmann 1972) (T. O. 2002)

II MAXIMALLY SUSY SOLUTIONS

(And their supersymmetry algebras)

a) Poincaré **SUGRAS** $d = 2, \dots, 11$

Minkowski is always a solution, maximally symmetric and also maximally supersymmetric and invariant under the Poincaré superalgebra

(d=4)

① Killing vectors: $k_{(a)} = \partial_a \longrightarrow P_a$
$k_{(ab)} = 2x_{[a}\partial_{b]} \longrightarrow M_{ab}$

Killing spinor equation $\longrightarrow \partial_a \kappa = 0$

4 solutions: $\kappa_{(\alpha)}{}^{\beta} = \delta_{(\alpha)}{}^{\beta} \rightarrow Q_{(\alpha)}$

② $\{P_a, M_{ab}\}$ satisfy the Poincaré algebra

③ $-i\,\overline{\kappa_{(\alpha)}}\gamma^a \kappa_{(\beta)}\partial_a = -i(C\gamma^a)_{\alpha\beta}\, k_{(a)}$

$$\Rightarrow \boxed{\{Q_{(\alpha)}, Q_{(\beta)}\} = -i\,(C\gamma^a)_{\alpha\beta}\, P_{(a)}}$$

④ $\nabla_{[\mu} k_{(a)\,\nu]} = 0 \Rightarrow \mathbb{L}_{k_{(a)}} \kappa_{(\alpha)}{}^{\beta} = 0 \Rightarrow \boxed{[Q_{(\alpha)}, P_{(a)}] = 0}$

$\nabla_{[\mu} k_{(ab)\,\nu]} = 2\, e_{[a|\mu}\, e_{|b]\nu} \Rightarrow \mathbb{L}_{k_{(ab)}} \kappa_{(\alpha)}{}^{\beta} = \frac{1}{2}(\gamma_{ab})^{\beta}{}_{\gamma}\, \kappa^{\gamma}{}_{(\alpha)}$

$$\Rightarrow \boxed{[Q_{(\alpha)}, M_{ab}] = Q_{(\beta)}\, \tfrac{1}{2}(\gamma_{ab})^{\beta}{}_{\alpha}}$$

b) AdS SUGRAS $d = 3, \ldots, 7$

AdS is always a maximally symmetric and supersymmetric solution, invariant under the AdS superalgebra. (see later)

c) In ~~some!~~ extended Poincaré SUGRAS there are additional maximally supersymmetric (but not maximally symmetric) solutions:

$AdS_n \times S^m$ & KG Hpp-waves

↳ spontaneous compactification ↳ ?

$N = 1$, $d = 11$: $\left\{ \begin{array}{l} AdS_7 \times S^4 \\ AdS_4 \times S^7 \end{array} \right\} \xrightarrow[\text{limit}]{\text{Penrose}} KG11$

(Freund–Rubin 1980) (Kowalski-Glikman 1984)

$N = 2B$, $d = 10$: $\left\{ AdS_5 \times S^5 \right\} \xrightarrow[\text{limit}]{\text{Penrose}} KG10$

(Blau, Figueroa-O'Farrill, Hull & Papadopoulos hep-th/0110242)

$N = \binom{2,0}{4,0}$, $d = 6$: $\left\{ AdS_3 \times S^3 \right\} \xrightarrow[\text{limit}]{\text{Penrose}} KG6$

(Chamseddine, Ferrara, Gibbons, Kallosh 1996; Gibbons, Horowitz, Townsend 1994) (P. Meessen hep-th/0111031)

$N = 2$, $d = 5$: $\left\{ \begin{array}{l} AdS_3 \times S^2 \\ AdS_2 \times S^3 \\ AdS_2 \times S^2 \\ \oplus \text{ Gödel} \end{array} \right\} \xrightarrow[\text{limit}]{\text{Penrose}} KG5$

new

(Gauntlett, Gutowski, Hull, Pakis & Reall hep-th/0209114)

$N = 2$, $d = 4$: $\left\{ AdS_2 \times S^2 \right\} \xrightarrow[\text{limit}]{\text{Penrose}} KG4$

Bertotti–Robinson 1959 (Kowalski-Glikman 1985) known!

All SUGRA vacua are HOMOGENEOUS SPACES

Minkowski $\sim$ $ISO(1,d-1)/SO(1,d-1)$

AdS_d $\sim$ $SO(2,d-1)/SO(1,d-1)$

$AdS_n \times S^m$ $\sim$ $\dfrac{SO(2,n-1)}{SO(1,n-1)} \times \dfrac{SO(m+1)}{SO(m)}$

$AdS_2 \times S^2$ $\sim$ $[SO(2,1) \times SO(3)]/SO(2)_\alpha$

(Alonso-Alberca, Lozano-Tellechea, T.O. 2002)

Hpp $\sim$ $\dfrac{H(d-2)^*}{T_{d-2}}$ (Cahen-Wallach 1970)

Gödel

This makes easy to find the Killing spinors and SUSY algebras. It can be shown that (Alonso-Alberca, Lozano-Tellechea, T.O. 2002)

1) The Killing spinors are (in an appropriate basis!)

$$\kappa_{(\alpha)}{}^{\beta} = u^{\beta}{}_{\alpha}\ ; \quad u = e^{x^a \Gamma_s(P_a)}$$

: the coset representative in a spinorial rep.

2) In most cases, the bilinears

$$-i\bar{\kappa}_{(\alpha)}\gamma^a \kappa_{(\beta)} e_a = -i\left(\tilde{C}\ \Gamma_s(P^{(I)})\right)_{\alpha\beta} k_{(I)}$$

$f_{\alpha\beta}{}^{I}$

3)

$$\mathbb{L}_{k_{(I)}} \kappa_{(\alpha)}{}^{\beta} = \kappa_{(\gamma)}{}^{\beta}\ \Gamma_s(P_{(I)})^{\gamma}{}_{\alpha}$$

$f_{\alpha I}{}^{\gamma}$

III SOME GENERAL RESULTS ON SUPERSYMMETRIC SOLUTIONS

Solutions with less SUSY are also interesting

1) Some provide vacua with less SUSY on which to define more realistic **FTs**

→ **SPONTANEOUS SUSY BREAKING** ←

(But, in general, we do not know the dynamical mechanism behind the choice of vacuum)

2) Some describe the long-range fields associated to **SOLITONIC OBJECTS** / **STATES** of the **QFT** (**BPS**)

They are stable classically and QMlly ("non-renormalisation theorems")

In some cases, it is possible to solve completely the integrability conditions of the Killing spinor equations and find all the SUSY solutions.

Let us review the known results

$N=1, d=4$ Poincaré SUGRA

The Killing spinor equation is

$$\nabla_\mu \kappa = 0 \Rightarrow -\frac{1}{4} R_{\mu\nu}{}^{ab} \gamma_{ab} \kappa = 0$$

Only two solutions known

1) Minkowski spacetime → maximally supersymmetric
2) pp-waves (spacetimes admitting a covariantly constant null Killing vector ℓ^μ $\left\{ \begin{array}{l} \nabla_\mu \ell^\nu = 0 \\ \ell^2 = 0 \end{array} \right.$)

The Killing spinors satisfy $\ell_\mu \gamma^\mu \kappa = 0 \Rightarrow 2 \Rightarrow \frac{1}{2}$

Euclidean signature

3) Gibbons-Hawking (Instantons of SU(2) holonomy closely related to the BPST instanton) | KK monopole, Eguchi-Hanson metric, ⋮

Also $\frac{1}{2}$ supersymmetric

$$\boxed{\begin{array}{l} ds^2 = H^{-1}(d\tau + A_i dx^i) + H\, dx^i dx^i \,; \\ \epsilon_{ijk}\partial_j A_k = \partial_i H \Rightarrow \partial_i \partial_i H = 0 \,; \end{array}}$$

N=2 d=4 Poincaré SUGRA

This theory is just Einstein-Maxwell coupled to $\psi_\mu{}^i$ and all solutions of E-M are solutions of N=2, d=4 P. SUGRA

The Killing spinor equation is

$$\left[\delta^{ij}\nabla_\mu + \tfrac{1}{4}\not{F}(\sigma^2)^{ij}\right]\kappa^j = 0\,;$$

$$\Rightarrow -\tfrac{1}{4}\left\{C_{\mu\nu}{}^{ab}\gamma_{ab} + 2i\,\not{\nabla}(F_{\mu\nu} + i\,{}^*F_{\mu\nu}\gamma_5)\,i\sigma^2\right\}\kappa = 0\,.$$

These integrability conditions were completely solved by Tod (PL 121B (1981) 241) who found two kinds of solutions that preserve 4→½ SUSIES generically:

1) Israel-Wilson-Perjés solutions

$$\boxed{\begin{aligned} & ds^2 = |\mathcal{H}|^{-2}(dt + A)^2 - |\mathcal{H}|^2 dx^i dx^i \\ & A_t = 2\,\mathrm{Re}\,\mathcal{H}\;; \quad \tilde{A}_t = -2\,\mathrm{Re}(i\bar{\mathcal{H}}) \\ & A = A_i\,dx^i\;; \quad \epsilon_{ijk}\partial_j A_k = \pm\,\mathrm{Im}(\bar{\mathcal{H}}\partial_k\mathcal{H})\;; \quad \partial_i\partial_i\mathcal{H} = 0 \end{aligned}}$$

These include the extreme RN BH $\xrightarrow[\text{limit}]{\text{near horizon}}$ Bertotti-Robinson $AdS_2\times S^2$

$M^2 = Q^2$

extreme Taub-NUT

$M^2 + N^2 = Q^2$

K-N with $M^2 = Q^2$

and multicenter solutions (Papapetrou-Majumdar)

2) Gravito-electromagnetic pp-waves

$$\boxed{\begin{aligned} &ds^2 = 2\,du(dv + K\,du) - 2\,d\xi\,d\bar{\xi}\,; \\ &F_{\xi u} = \partial_\xi C\,; \qquad K = \mathrm{Re}\,f + \tfrac{1}{4}|C|^2\,; \\ &\partial_{\bar{\xi}} f = \partial_{\bar{\xi}} C = 0\,; \end{aligned}}$$

→ Hpp-waves as a particular case

$$\boxed{K = A_{ij}\, x^i x^j}$$

→ KG4 as a particular case

$$\boxed{A_{ij} \sim \delta_{ij}}$$

Kowalski-Glikman proved that the Robinson-Bertotti and the KG4 solutions are the only non-trivial vacua of $N=2$ SUGRA.

$N=4$ $d=4$ SUGRA

This theory consists of

$$\left\{\begin{array}{l} B\left\{\begin{array}{l} g_{\mu\nu} \\ A_\mu{}^i \to 6 \text{ vectors } N(N-1)/2 \\ \tau \to \text{complex scalar "axidilaton"} \end{array}\right. \\ F\left\{\begin{array}{l} \psi_\mu{}^i \to 4 \text{ gravitini} \\ \chi^i \to 4 \text{ dilatini} \end{array}\right. \end{array}\right.$$

It is closely related to the Heterotic String and has very interesting duality properties.

The most general families of SUSY solutions were obtained by Tod (CQG 12 (1995) 1801) who identified several families

1) "SUPER-Israel-Wilson-Perjés" solutions (Bergshoeff, Kallosh, T.O (1996))

Include all the IWP metrics of $N=2$, $d=4$, but now NONE of them is maximally supersymmetric.

2) Waves (pp and more)

Again, none of them is maximally supersymmetric

In fact, the only known vacuum is Minkowski and this could turn out to be an advantage.

N=2, d=5 SUGRA

This theory is an interesting generalisation of Einstein-Maxwell's

$$S = \int d^5x \sqrt{|g|} \left\{ R - \frac{1}{4}F^2 - \underbrace{\frac{1}{\sqrt{|g|}} \varepsilon^{\mu\nu\rho\sigma\delta} F_{\mu\nu} F_{\rho\sigma} A_\delta}_{\text{new "Chern-Simons" topological term}} \right\}$$

This new term changes the Maxwell equation $\nabla_\mu F^{\mu\nu} \sim \frac{\varepsilon^{\nu\rho\sigma\delta\kappa}}{\sqrt{|g|}} F_{\rho\sigma} F_{\delta\kappa}$ but not the Einstein equation, and has very interesting solutions like BHs with horizons of non-trivial topology (the "rotating black ring" of Emparan & Reall PRL 88 (2002) 101101).

Recently (past week!) Gauntlett, Gutowski, Hull, Pakis and Reall have shown how to construct all the SUSY solutions of this theory, although not all of them can be written explicitly.

Their method exploits the identities satisfied by the Killing spinor bilinears $-i\bar{\kappa}\gamma^a\kappa$.

CONCLUSION

In spite of the many solutions found so far, the study and classification of SUSY solutions in SUGRA theories is still in its infancy.

Although we have focused on only one of many aspects of Superstrings/SUGRA theories, it should be clear that SUSY can be a very useful tool to explore the world of GR solutions.