RELATIONS

BETWEEN

D=4, 5, 6

VACUA

WITH

8

SUPERCHARGES

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D=4,5,6 VACUA WITH 8 SUPERCHARGES

SUGRA SOLUTIONS PRESERVING ALL SUSYS

$\wr\wr$

$\boxed{\text{VACUA}}$

KNOWN VACUA IN D=4,5,6 THEORIES WITH 8 Qs

$N=(2,0)$, $d=6$

1.- $KG6(2,0)$ (Hpp-wave) 1 parameter

2.- $AdS_3\times S^3$ (N.H.L. of self dual string) 1 param.

$N=2$, $d=5$

1.- $KG5$ (Hpp wave) 1 parameter

2.- $AdS_3\times S^2$ (N.H.L. of string) 1 parameter

3.- $AdS_2\times S^3$ (N.H.L. of B.H.) 1 parameter

$j=0$

4.- N.H.L. of rotating B.H. 2 parameter

$N=2$, $d=4$

1.- $KG4$ (Hpp wave) 1 parameter

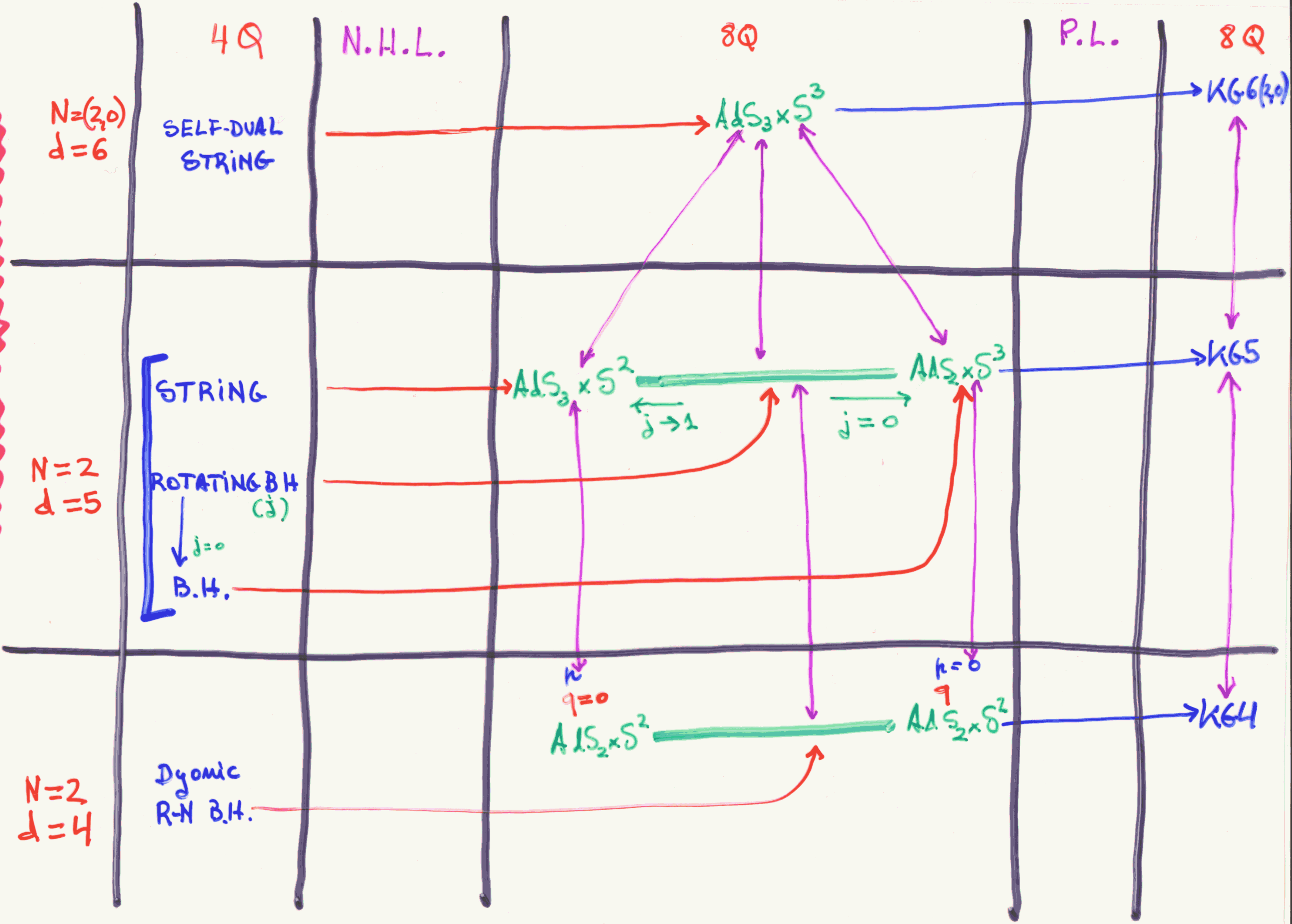
2.- $AdS_2\times S^2$ (N.H.L. of R.N. B.H.) 2 parameter

(ROBINSON-BERTOTTI)

E/M DUALITY

E/M DUALITY × CHANGE OF POLARIZATION

THE RESULTS


4Q
N.H.L.
8Q
P.L.
8Q
N=(2,0)
d=6
SELF-DUAL
STRING
$AdS_3 \times S^3$
KG6(2,0)
N=2
d=5
STRING
ROTATING BH
(j)
j=0
B.H.
$AdS_3 \times S^2$
j→1
j=0
$AdS_2 \times S^3$
KG5
N=2
d=4
Dyonic
R-N B.H.
q=0
$AdS_2 \times S^2$
p=0
$AdS_2 \times S^2$
KG4

SUSY vs. REDUCTION & OXIDATION

(STANDARD LORE)

REDUCTION: The Killing spinors of all maximally supersymmetric spacetimes (except Mink.) depend on all spacelike coordinates and have to be projected out of the theory upon dimensional reduction

⇒ (SOME) SUSY BROKEN IN DIMENSIONAL REDUCTION & T-DUALITY

(Bakas '95; Bergshoeff, Kallosh, O. '95; Duff, Lu, Pope '97)

OXIDATION: Supersymmetric solutions should come from supersymmetric but one has to be careful:

{Any solution of $N=2$, $d=4$ (Einstein-Maxwell) with $F^2=0$} ⟶ {A purely gravitational solution in $d=5$ with vanishing KK scalar}

(This can always be achieved with E/M duality rotations)

In general all supersymmetry will be broken in this oxidation

$N=2$, $d=5$ includes a vector field whose truncation imposes constraints

pp- & Hpp-WAVES

pp-waves: $\exists\ l_\mu$; i) $\nabla_\mu l_\nu = 0$; ii) $l^2 = 0$

$$\Rightarrow \quad ds^2 = 2du\left[dv + K(u,x^i)\,du + A_i(u,x)\,dx^i\right] + \tilde{g}_{ij}(u,x)\,dx^i dx^j$$

(Brinkman) $\qquad$ Sagnac connection $\to 0$

General SUGRA pp-wave solutions

$$S = \int d^dx \sqrt{|g|}\left\{R + \tfrac{1}{2}(\partial\varphi)^2 + \frac{(-1)^{p+1}}{2(p+2)!}\, e^{-2a\varphi} F^2_{(p+2)}\right\}$$

$$+ T\int d^p\xi \sqrt{|\gamma|}\ \gamma^{mn} g_{\mu\nu}\,\partial_m X^\mu \partial_n X^\nu \ + \mu \int A_{(p+1)}$$

$$\boxed{\begin{aligned} & ds^2 = 2du\,(dv + K(u,x)\,du) - \delta_{ij}\,dx^i dx^j; \\ & F_{(p+2)} = du \wedge C_{(p+1)}(u); \qquad \varphi = \varphi(u); \\ & K = H + A; \qquad \partial_i\partial_i H \sim T\,\delta(u)\,\delta^{(d-2)}(x) \\ & A \equiv A_{ij}(u)\,x^i x^j = -\tfrac{1}{4}\left[(\varphi')^2 + \frac{(-1)^{p+1}}{(p+1)!}\, e^{-2a\varphi} C^2_p\right]\frac{M_{ij}\,x^i x^j}{\mathrm{Tr}(M)} \end{aligned}}$$

$H = 0 \Rightarrow$ Hpp-wave (Cahen-Wallach homogeneous spacetime)

$A = 0 \Rightarrow$ Shock pp-wave $\qquad$ (MW $\to$ D0)

All of them preserve at least 1/2 SUSY

Some Hpp have maximal SUSY $\to$ Kowalski-Glikman (KG)

REDUCTION OF KG5 (1)

N=2, d=5

$$\begin{cases} S = \int d^5x \sqrt{|g|}\left[R - \frac{1}{4}F^2 + \frac{\varepsilon}{12\sqrt{3}\sqrt{|g|}} F F V\right]; \\ F_{\mu\nu} = 2\partial_{[\mu}V_{\nu]}; \\ \delta_\varepsilon \psi_a = \left\{\nabla_a - \frac{1}{8\sqrt{3}}\left(\gamma^{bc}{}_a + 2\gamma^b g^c{}_a\right)F_{bc}\right\}\varepsilon; \end{cases}$$

KG5 (Meessen' 02)

$$\begin{cases} ds^2 = 2du\left[dv + \frac{\lambda_5^2}{24}(x^2+y^2+4z^2)\,du\right] - dx^2 - dy^2 - dz^2; \\ F = \lambda_5\, du \wedge dz; \\ \varepsilon = \left(1 + x^i \Omega_i\right)\exp\left(\Omega^- u\right)\varepsilon_0 \quad \Rightarrow \delta_\varepsilon \psi_a = 0 \end{cases}$$

$i = 1, 2, 3$

The Killing spinor depends on all spacelike coordinates

However:

$$\begin{cases} x = \cos\left(\frac{\lambda_5}{2\sqrt{3}}u\right)x' + \sin\left(\frac{\lambda_5}{2\sqrt{3}}u\right)y'; \\ y = -\sin\left(\frac{\lambda_5}{2\sqrt{3}}u\right)x' + \cos\left(\frac{\lambda_5}{2\sqrt{3}}u\right)y'; \\ v = v' - \frac{\lambda_5}{2\sqrt{3}}x'y'; \end{cases}$$

$$\Rightarrow \text{KG5} \quad \begin{cases} ds^2 = 2du\left[dv' + \frac{\lambda_5}{6}z^2 du + \frac{\lambda_5}{\sqrt{3}}x'dy'\right] - \cdots \\ F = \lambda_5\, du \wedge dz; \end{cases}$$

non-vanishing Sagnac connection

The metric is now independent of y'.
How about the Killing spinor?

REDUCTION OF KG5 (2)

We only need to analyse $\delta_\varepsilon \psi_{y'} \sim \partial_{y'} \varepsilon + \cdots$

$$\delta_\varepsilon \psi_{y'} \sim \left\{ \partial_{y'} + \frac{i}{8\sqrt{3}} \left(\not{\mathcal{F}} - \sqrt{3} * \not{F} \right) \right\} \varepsilon$$

KK vector ⟵ Sagnac connection

$$A = \frac{\lambda_5}{\sqrt{3}} x^1 du \ ; \qquad F = -\frac{\lambda_5}{\sqrt{3}} du \wedge dx^1 ; \qquad *F = \frac{\lambda_5}{\sqrt{3}} du \wedge dz$$

$$\boxed{\sqrt{3} * F = \mathcal{F}} \qquad \boxed{\delta_\varepsilon \psi_{y'} \sim \partial_{y'} \varepsilon = 0}$$

The Killing spinor is now independent of the new coordinate y' and the reduction of KG5 will give a maximally supersymmetric solution in $d=4$

→ KG4

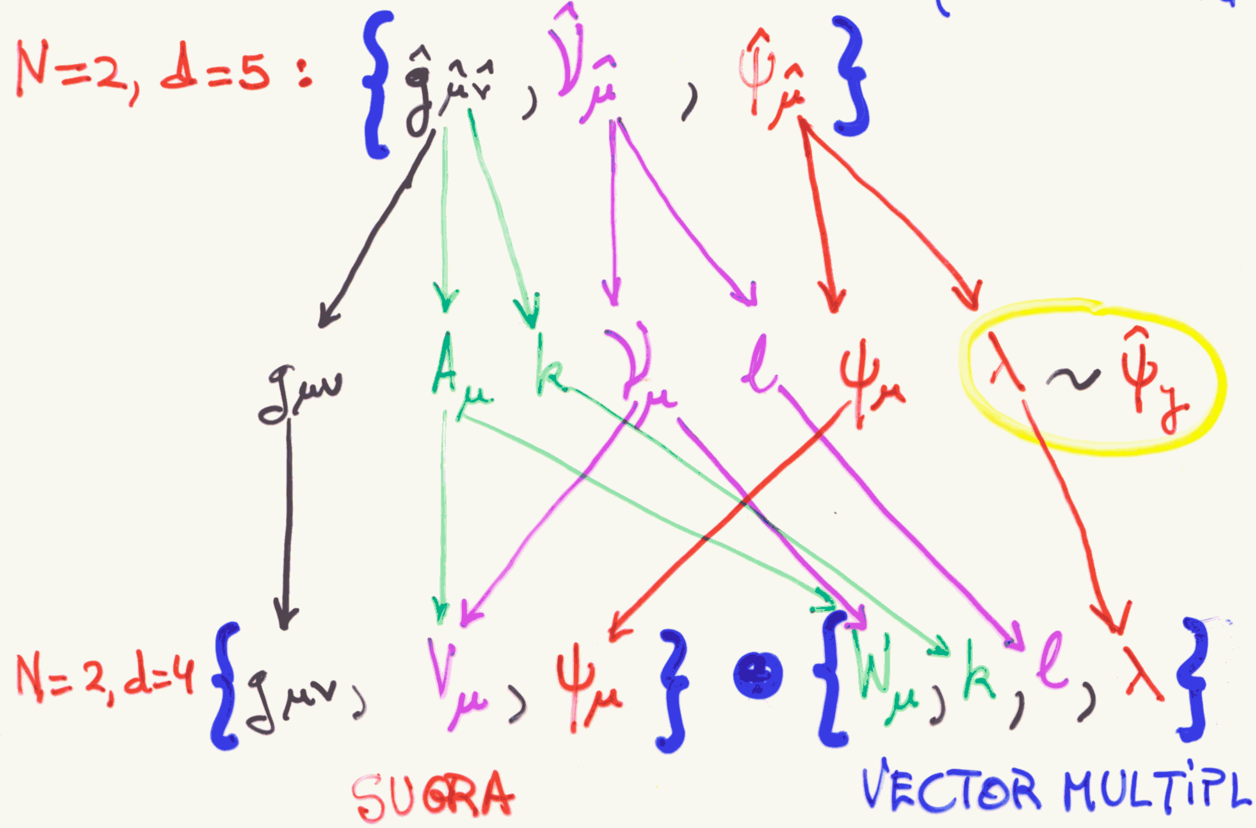
(Kowalski-Glikman '84)

$$\begin{cases} ds^2 = 2du \left[dv + \frac{\lambda_4}{8} (x^2 + z^2) du \right] - dx^2 - dz^2 \\ F = \lambda_4 \, du \wedge dz \ ; \qquad \lambda_4 = \frac{2}{\sqrt{3}} \lambda_5 \ ; \end{cases}$$

The cancellation between the KK vector field (Sagnac connection) and the $d=5$ vector field is no coincidence, but it is the condition for the $\boxed{\text{truncation}}$ of the $N=2$ $d=4$ vector multiplet (matter)

REDUCTION AND TRUNCATION OF $N=2$, $d=5$ SUGRA

(Chamseddine & Nicolai '80)



$$\delta_\varepsilon \lambda \sim \delta_\varepsilon \psi_y \sim \left\{ \partial_y{}' + \not\partial k \gamma_5 + k \not\partial \ell + \not F(W)\gamma_5 \right\} \varepsilon$$

To have maximal supersymmetry in $d=4$ we have to set $k=1$, $\ell=0$, $W=0$ which is the consistent truncation of the matter multiplet.

$$N_\mu = 0 \quad \longrightarrow \quad \boxed{\mathcal{F} = \sqrt{3}\,{}^*F}$$

NOW WE CAN APPLY THIS TO ANY SOLUTION

$AdS_2 \times S^2_{\text{electric}} \rightarrow AdS_2 \times S^3$ | $AdS_2 \times S^2_{\text{magnetic}} \rightarrow AdS_3 \times S^2$

REDUCTION & TRUNCATION OF N=(2,0), d=6 SUGRA

SIMILARLY:

$N=(2,0),\ d=6$: $\left\{ \hat{g}_{\hat{\mu}\hat{\nu}},\ \hat{B}^{-}_{\hat{\mu}\hat{\nu}},\ \hat{\psi}^{-}_{\hat{\mu}} \right\}$

$$g_{\mu\nu} \quad A_\mu \quad k \quad B_{\mu\nu} \quad B_\mu \quad \psi_\mu \quad \lambda \sim \hat{\psi}_z$$

$N=2,\ d=5$: $\left\{ g_{\mu\nu},\ V_\mu,\ \psi_\mu \right\} \oplus \left\{ W_\mu,\ k,\ \lambda \right\}$

SUGRA   VECTOR MULTIPLET

Preserving maximal supersymmetry in $d=5$ requires a consistent truncation of the matter vector multiplet

$$\left.\begin{aligned} W_\mu &= 0 \\ k &= 1 \end{aligned}\right\} \longrightarrow \boxed{F(B) = \sqrt{2}\, F(A)}$$

KGG (2,0):
$$\begin{cases} ds^2 = 2du\left[dv + \dfrac{\lambda_6^2}{8}\, \vec{x}_4^{\,2}\, du\right] - d\vec{x}_4^{\,2}\,; \\ B^- = \lambda_6\, du \wedge (y\, dx - z\, dw)\,; \end{cases}$$

$$\left.\begin{aligned} z &= \cos\left(\frac{\lambda_6}{2}u\right) z' + \sin\left(\frac{\lambda_6 u}{2}\right) w'\,; \\ w &= -\sin\left(\frac{\lambda_6}{2}u\right) z' + \cos\left(\frac{\lambda_6}{2}u\right) w'\,; \\ v &= v' - \frac{\lambda_6}{2} z' w'\,; \end{aligned}\right\} \rightarrow \begin{cases} ds^2 = 2du\left[dv + \dfrac{\lambda_6^2}{8}(x^2+y^2)\,du - \lambda_6\, z'\, dw'\right] - d\vec{x}_4^{\prime\,2} \\ B^- = \lambda_6\, du \wedge (y\, dx - z'\, dw') \end{cases}$$

REDUCTION / OXIDATION OF AdS×S SOLUTIONS

We can use these results with the AdS×S solutions preserving all supersymmetries: starting from d=4

DYONIC R-B

$$\begin{cases} ds^2 = R_2^2\, d\Pi^2_{(2)} - R_2^2\, d\Omega^2_{(2)}\,; \\ F = -\frac{2}{R_2}\cos\xi\, dr\wedge dt + 2R_2 \sin\xi \sin\theta\, d\theta\wedge d\varphi\,; \end{cases}$$

ξ

$$\begin{cases} \cos\xi\; t \to t \\ \frac{\psi}{R_2\cos\xi} \to \psi \end{cases}$$

j

$$\begin{cases} ds^2 = \left[\frac{r}{R_2}dt + R_2 \sin\xi\,(d\psi + \cos\theta\, d\varphi)\right]^2 \\ \qquad - \left(\frac{R_2}{r}\right)^2 dr^2 - (2R_2)^2\, d\Omega^2_{(3)}\,; \\ F = -\frac{2}{R_2}\cos\xi\, dr\wedge dt + 2R_2\sin\xi \sin\theta\, d\theta\wedge d\varphi\,; \end{cases}$$

$$\begin{cases} j=0 \to AdS_2\times S^3 \\ j=1 \to AdS_3\times S^2\;! \end{cases}$$

← (Gaiotto & Lowe '98
Gauntlett, Myers, Townsend '99)

$$\omega = \cos\xi\,\eta + R_2\sin\xi\,\psi\,;$$
$$y = -\sin\xi\,\eta + R_2\cos\xi\,\psi\,;$$

$AdS_3\times S^3$

$$\begin{cases} ds^2 = (2R_2)^2\, d\Pi^2_{(3)} - (2R_2)^2\, d\Omega^2_{(3)}\,; \\ B^- = \frac{r}{R_2}\, dy\wedge dt - R_2^2\cos\theta\, d\varphi\wedge d\psi\,; \end{cases}$$