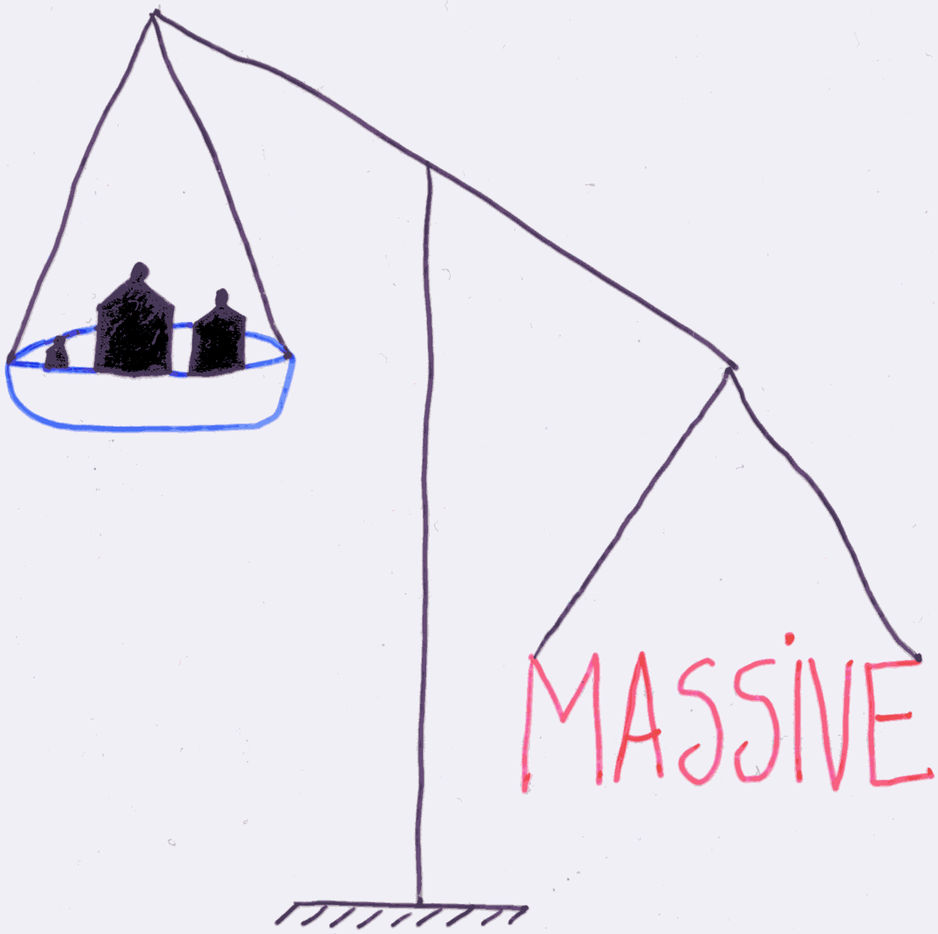

MASSIVE SUPERGRAVITIES

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PLAN OF THE TALK

1.- What are AdS/gauged/massive SUGRAS?

- Description
- Domain-wall interpretation $\left(\frac{\text{Domain wall}}{\text{CFT}}\right)$

2.- Higher-dimensional origin

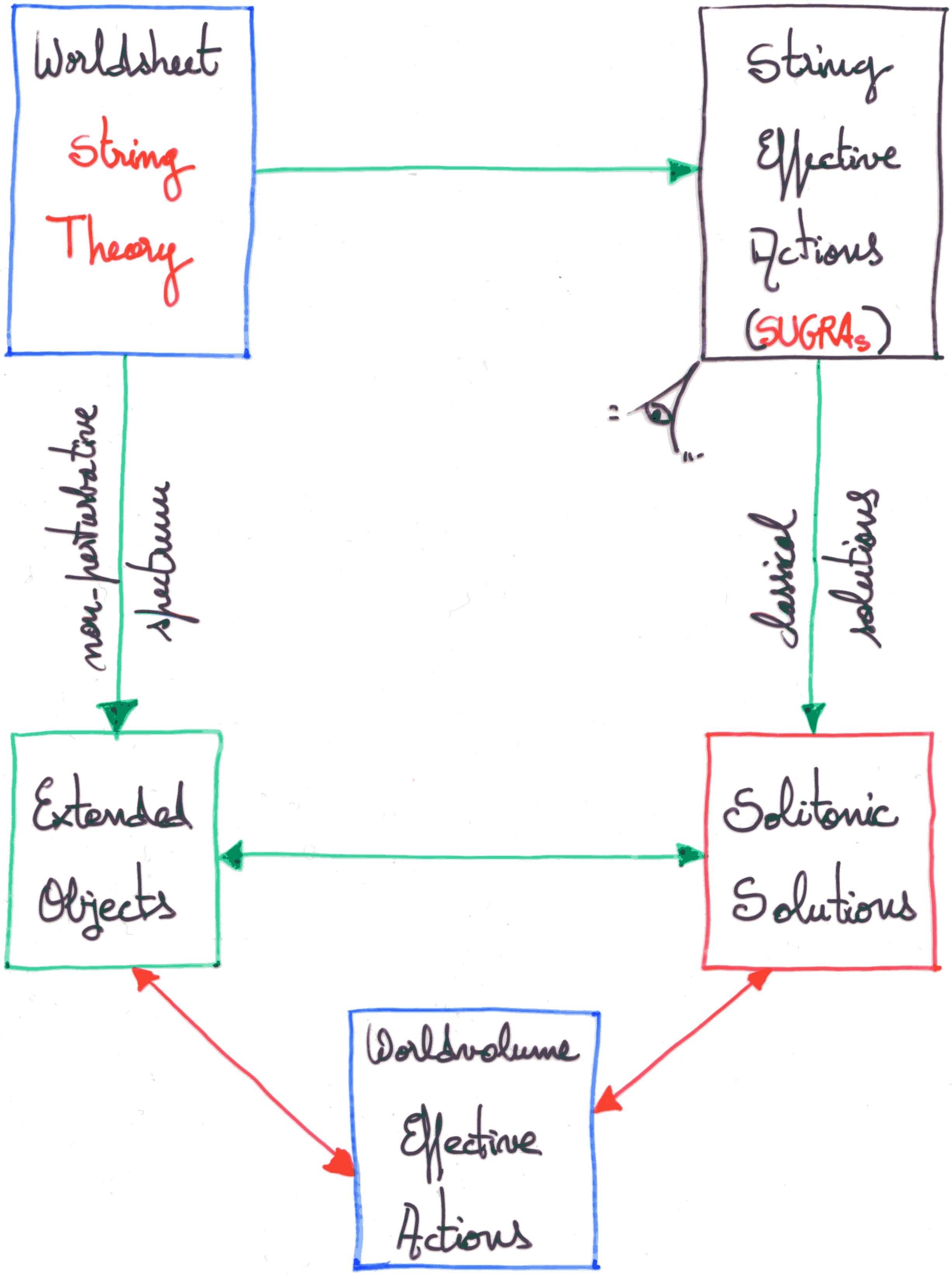
- GDR
- Fluxes
- Spheres
- ....

3.- M-theoretical origin of Romans' massive IIA

- Problems
- "Solution" BLO

4.- Conclusions

THE FRAMEWORK



AdS / GAUGED SU(E)GRAS

Standard SUEGRAS can be constructed by gauging the Extended Poincaré Superalgebra

$$\{Q^{i\alpha}, Q^{j\beta}\} = i\delta^{ij}(\gamma^a C^{-1})^{\alpha\beta} P_a - i(C^{-1})^{\alpha\beta} Q^{ij} - (\gamma_5 C^{-1})^{\alpha\beta} P^{ij}$$

$(d=4;\ i,j = 1,\dots,N)$

central charges

$$A_\mu \sim e^a{}_\mu P_a + \tfrac{1}{2}\omega_\mu{}^{ab} M_{ab} + \tfrac{1}{2} A^{ij}{}_\mu Q^{ij} \quad \bar{\Psi}^i_{\mu\alpha} Q^{i\alpha}$$

$\frac{N(N-1)}{2}$ Abelian vectors

$$S \sim \int d^4x\, e\Big\{ R(e,\omega) + \varepsilon^{\mu\nu\rho\sigma} \bar{\Psi}^i_\mu \gamma_5 \gamma_\nu D_\rho(\omega) \psi^i_\sigma - \frac{1}{8} F^{ij}{}_{\mu\nu} F^{ij\,\mu\nu} + \dots \Big\}$$

$$F^{ij}{}_{\mu\nu} = 2\partial_{[\mu} A_{\nu]}{}^{ij}$$

There is no cosmological constant.

The vacuum is Minkowski spacetime

$\Rightarrow$ maximally symmetric $(\{P_a, M_{ab}\})$

$\Rightarrow$ maximally supersymmetric $(\{Q^{i\alpha}\})$

AdS SUEGRAS can be constructed by gauging Extended AdS Superalgebras

$$\{Q^{i\alpha}, Q^{j\beta}\} = i\delta^{ij}\left[\Gamma_5(\hat{M}^{\hat{a}\hat{b}})C^{-1}\right]^{\alpha\beta}\hat{M}_{\hat{a}\hat{b}} - i(C^{-1})^{\alpha\beta}T^{ij}$$

$$[T^{ij}, T^{kl}] = \delta^{ik}T^{jl}\ldots \rightsquigarrow SO(N) \text{ generators}$$

$$[Q^{i\alpha}, T^{jl}] = \Gamma_5 (T^{jl})^i{}_k Q^{k\alpha}$$

$$A_\mu \sim \hat{\omega}_\mu{}^{\hat{a}\hat{b}}\hat{M}_{\hat{a}\hat{b}} + \frac{1}{2}A_\mu{}^{ij}T^{ij} + \Psi_{\mu\alpha}^i Q^{i\alpha}$$

$A_\mu{}^{ij}$: SO(N) gauge vector field → coupling constant g

$\Psi_{\mu\alpha}^i$: SO(N)-charged gravitini

$g \to 0$ { Abelian limit ⊕ Wigner-Inönü contraction

$$S \sim \int d^4x\, e\left\{R(e,\omega) + \varepsilon^{\mu\nu\rho\sigma}\bar{\Psi}_\mu^i \gamma_5\gamma_\nu\left(D_\rho(\hat{\omega})\psi_\sigma^i + gA_\rho^{ij}\psi_\sigma^j\right) - \frac{1}{8}F^{ij}{}_{\mu\nu}F^{ij\,\mu\nu} + g^2\ldots\right\}$$

g^2: cosmological constant

$$F^{ij}{}_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}{}^{ij} + 2gA_{[\mu}{}^{ik}A_{\nu]}{}^{kj}$$

The vacuum is AdS

→ Maximally symmetric $(\{\hat{M}_{\hat{a}\hat{b}}\}\ SO(2,3))$

→ Maximally supersymmetric $(\{Q^{i\alpha}\})$

MASSIVE SUGRAS

Archetype: Romans' massive $N=2A$, $d=10$ SUGRA

$\{ g_{\mu\nu},\ B_{\mu\nu},\ \phi,\ C^{(1)}_{\mu},\ C^{(3)}_{\mu\nu\rho},\ \psi_\mu \}$ ← $\underline{N=2}$

$$S_{\text{bosonic}} \sim \int d^{10}x \sqrt{|g|} \left\{ e^{-2\phi}\left[R - 4(\partial\phi)^2 + \frac{1}{2\cdot 3!} H^2 \right] - \left[\frac{1}{2} m^2 + \frac{1}{2\cdot 2!} G^{(2)\,2} \ \ldots \right. \right\}$$

$$G^{(2)}_{\mu\nu} = 2\partial_{[\mu} C^{(1)}_{\nu]} + m B_{\mu\nu} \ ;$$

$$\delta C^{(1)}_{\mu} = -m\Lambda_\mu \ ; \quad \delta B_{\mu\nu} = 2\partial_{[\mu}\Lambda_{\nu]} \ ;$$ ← massive gauge transformations

m → mass parameter ∉ cosmological constant?

Einstein frame → $-\frac{1}{2} m^2 e^{5/2\,\phi}$ ⇒ "potential" for ϕ ?

The vacuum is not Minkowski or AdS but the D8

$$\text{D8:}\ \boxed{\begin{cases} ds^2 = H^{1/8}(dt^2 - d\vec{y}^{\,2}_{8}) - H^{9/8}\, dx^2 \ ; \\ e^{\phi} = H^{-5/4} \ ; \quad H = \alpha + m x \ ; \end{cases}}$$

not maximally symmetric ($ISO(1,8)$)

not maximally supersymmetric ($(1-\Gamma_x)\varepsilon = 0$)

1/2

RELATION BETWEEN MASSIVE & AdS SUGRA

→ Both theories have "mass parameters" g, m

$$\hat{D}_\mu(\hat{\omega})\,\psi_\nu^i = \left(\partial_\mu - \tfrac{1}{4}\omega_\mu{}^{ab}\gamma_{ab} - ig\,\gamma_\mu\right)\psi_\nu^i$$

$$G^{(3)}_{\mu\nu} = 2\partial_{[\mu}C^{(1)}_{\nu]} + mB_{\mu\nu} \Rightarrow m^2B^2$$

Their presence is, however, necessary for masslessness in the vacua of these theories (AdS, D8)

→ The vacua of these theories can be seen as domain walls (1 transverse dimension x):

AdS: $ds^2 = H^2(dt^2 - d\vec{y}^{\,2}_{d-2}) - H^{-2}dx^2$; $H = a + gx$; (Poincaré coordinates)

D8: $\begin{cases} ds^2 = H^{1/8}(dt^2 - d\vec{y}^{\,2}_8) - H^{9/8}dx^2 ; \\ e^\phi = H^{-5/4} ; \end{cases}$ $H = \alpha + mx$;

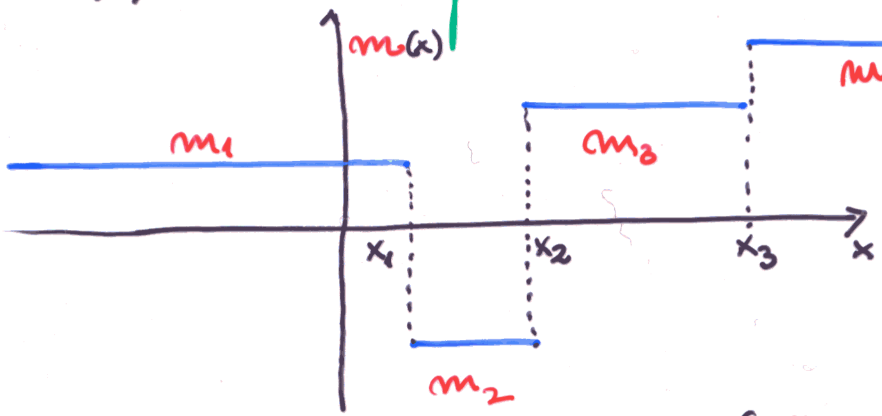
AdS → "non-dilatonic domain-wall solution"

→ The mass parameters can be interpreted as field strengths and dualised into d-forms:

$m \to m(x)$ + Lagrange multiplier : $\varepsilon^{\mu_1 \dots \mu_d} A_{(d-1)\,\mu_1\dots\mu_{d-1}}\,\partial_{\mu_d} m(x)$

$\Rightarrow \quad F_{(d)}^2 \quad ; \quad F_{(d)} = dA_{(d-1)}$

$m(x)$ can be piecewise constant



The discontinuities are the $\left\{ \begin{array}{c} (d-2)\text{-branes} \\ \text{domain walls} \end{array} \right\}$. In between, the spacetime is AdS or D8.

At the discontinuities the e.o.m. are not satisfied and we have to add $(d-2)$-brane sources:

($e^{-\phi}$ for the D8)

$$S = -\frac{T}{2}\int d^{d-1}\xi \sqrt{|\gamma|}\left[\gamma^{ij}\partial_i X^\mu \partial_j X^\nu g_{\mu\nu}(x) - (d-3)\right]$$

$$+ \frac{(-1)^{d-1}T}{(d-1)!}\int d^{d-1}\xi \, A_{(d-1)\,\mu_1\cdots\mu_{d-1}} \partial_{i_1}X^{\mu_1}\cdots\partial_{i_{d-1}}X^{\mu_{d-1}} \varepsilon^{i_1\cdots i_{d-1}}$$

(the RR 9-form for the D8)

now $H = \alpha + T|x - x_0|$

position of the domain wall

$$\boxed{\text{massive \& AdS/gauged SUEGRAS are cousins}}$$

WHY ARE THEY INTERESTING?

Because of the AdS/CFT correspondence (Maldacena)
and its generalisations (Boonstra, Skenderis, Townsend)

$\hat{d}$ $\hat{N}$ SUEGRA admitting a domain-wall $\times \mathcal{M}$ solution (AdS)
preserving N susies

KK on $\mathcal{M}$ ($\mathcal{M}$ with isometry group G)

d N massive/gauge SUEGRA with gauge group G (admits a domain-wall solution (AdS))

AdS/CFT correspondence

Same superalgebra

(d-1) N SQFT on the "boundary" with G as R-symmetry group (describes the "internal" dynamics of the d-w)

EXAMPLES →

1. $\hat{d}=10$, $\hat{N}=2B$ SUGRA on $AdS_5\times S^5$
(32 charges)
$N=8$ (32 charges in $d=5$)
$G \to SO(6)$

KK on S^5

$d=5$, $N=8$, $SO(6)$-gauged SUGRA $\rightarrow$ AdS_5 is the vacuum solution
(32 charges)

A function of the AdS_5 radius

$d=4$ $N=4$ SYM gauge group $SU(\mathcal{N})$
$\rightarrow$ it is a SCFT $\Rightarrow$ invariant under AdS_5 group $SO(2,4)$
32 charges
$\rightarrow$ R-symmetry group $SU(4) \sim SO(6)$

($d=4$ $N=4$ SYM is the theory on $\mathcal{N}$ D3-branes)

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2. $\hat{d}=11$, $\hat{N}=1$ SUGRA on $AdS_7\times S^4$
(32 charges)
$N=4$ (32 charges in $d=7$)
$G = SO(5)$

KK on S^4

$d=7$, $N=4$, $SO(5)$-gauged SUGRA $\rightarrow$ AdS_7 is the vacuum solution
(32 charges)

$d=6$, $N=(2,0)$ $A_{\mathcal{N}-1}$ theory
$\rightarrow$ it is a SCFT $\Rightarrow$ invariant under AdS_7 group $SO(2,6)$
32 charges
R-symmetry group $USp(4) \sim SO(5)$

($A_{\mathcal{N}-1}$ (2,0) is the theory on $\mathcal{N}$ M5 branes)

(3) $\hat{d} = 11$, $\hat{N} = 1$ SUGRA on $AdS_4 \times S^7$
(32 charges)
$N = 8$ (32 charges in $d=4$)
$G = SO(8)$

KK on S^7

$d = 4$, $N = 8$ $SO(8)$-gauged SUGRA → AdS_4 is the vacuum sol.
(32 charges)

Some $d=3$ $N=8$ SCFT which is the IR of $d=3$ $N=8$ SYM
invariant under AdS_4 group $SO(2,3)$
32 charges
R-symmetry group $SO(8)$
(→ this is the theory on $\mathcal{N}$ M2-branes)

NON-CONFORMAL

(4) $\hat{d} = 10$, $\hat{N} = 2A$ SUGRA on $d\text{-}\omega_8 \times S^2$ (near-horizon limit of D6)
(32 charges)
$N = 2$ (16 charges in $d=8$)
$G = SO(3)$
$1/2\ \hat{N}$

KK on S^2

$d = 8$, $N = 4$ $SU(2)$-gauged/massive SUGRA
32 supercharges
$d\text{-}\omega_8$ is the vacuum (breaking 1/2)
(16 charges)

NO DUAL THEORY
(→ The theory of $\mathcal{N}$ D6-branes does not decouple from the bulk)

5. $\hat{d}=10$, $\hat{N}=2^{A}_{B}$ SUGRA on $d\text{-}W_{p+2} \times S^{8-p}$ (non-horizon limit of D-p-branes)

(32 charges)

$N = \frac{1}{2}\hat{N}$ (16 charges)

$G = SO(9-p)$

$\downarrow$ KK on S^{8-p}

$p=0$, $d=2$, $N=16$ $SO(9)$-gauged SUGRA (32 supercharges) — $d\text{-}W_2$ is the vacuum sol. (1/2)

$p=1$, $d=3$, $N=8$ $SO(8)$-gauged SUGRA

$p=2$, $d=4$, $N=8$ $ISO(7)$-gauged SUGRA (only $SO(7)$ linearly realized)

⋮ ⋮ ⋮

$p=6$, $d=8$, $N=4$ $SO(3)$-gauged SUGRA → not unique

$p=7$, $d=9$, $N=2$ $SO(2)$-gauged SUGRA unknown *

$p=8$ $d=10$ $N=2A$ massive Romans' SUGRA → D8

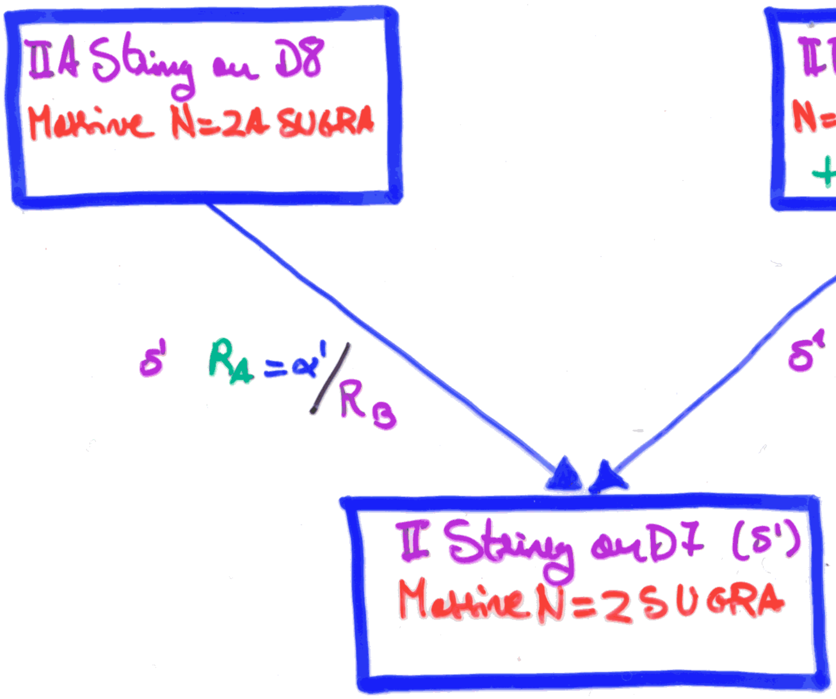
In the non-conformal cases the correspondence with QFT is more complicated (running coupling constants)

We are interested in the relation between

DOMAIN-WALL VS. GAUGED/MASSIVE SUGRA

RELATION BETWEEN MASSIVE N=2A & N=2B

(A.K.A. "MASSIVE T-DUALITY")



D7 flux is introduced via (Scherk-Schwarz) Generalized Dimensional Reduction:

(Bergshoeff, de Roo, Green, Papadopoulos, Townsend)

$$S_{2B} \sim \int d^{10}\hat{x} \left\{ \hat{R} + \tfrac{1}{2}(\partial\hat{\phi})^2 + e^{2\hat{\phi}}(\partial\hat{a})^2 + \ldots\ldots \right.$$

$$\frac{1}{2}\frac{\partial\tau\,\partial\bar{\tau}}{(\mathrm{Im}\,\tau)^2} \qquad \tau = \hat{a} + i e^{\varphi}$$

$SL(2,\mathbb{R})/SO(2)$ σ-model $\rightarrow$ S-duality

Generalized KK Ansatz: $\hat{a}(z,x) = a(x) + m z$

(both periodic) (Romans' mass parameter)

$$\hat{a} = m z \Rightarrow \hat{F}_{(9)} \sim * d\hat{a} \sim m$$

$\longrightarrow$ constant D7 flux

GDR exploits invariance under global symmetries

$\hat{a} \longrightarrow \hat{a} + m$ $\Rightarrow$ KK Ansatz $\hat{a}(z,x) = a(x) + m z$

It is possible to perform GDR exploiting the full $SL(2,\mathbb{R})$ global invariance of N=2B (Lavrinenko, Lü, Pope; Meessen, O.)

$$\hat{\mathcal{M}} = e^{\hat{\varphi}}\begin{pmatrix} |\hat{\tau}|^2 & \hat{a} \\ \hat{a} & 1 \end{pmatrix} \qquad \hat{\mathcal{M}}' = \Lambda \hat{\mathcal{M}} \Lambda^T \qquad \Lambda \in SL(2,\mathbb{R}) \quad e^{m^i T_i}$$

$\Rightarrow$ KK Ansatz
$$\hat{\mathcal{M}}(z,x) = \Lambda(z)\,\mathcal{M}(x)\,\Lambda^T(z)$$
$$\Lambda(z) = e^{z m^i T_i}$$

1) All the z-dependence disappears after reduction

2) We get theories with 3 mass parameters m^i (Romans' + 2) (N=2A origin???)

3) The 9-dimensional theories are gauged SUGRAS (as predicted!) of 3 kinds (Hull) with gauge groups SO(2) (1) and SO(1,1) (2)

4) They are associated to 3 kinds of d=10 7-branes $\Rightarrow$ 3 kinds of domain walls in d=9 (Bergshoeff, Gran, Roest)

ROMANS' MASSIVE N=2A FROM d=11

1) N=1, d=11 SUGRA is unique: it cannot be deformed with mass parameters (or cosmological constant) preserving full d=11 covariance.

2) No mass parameters can be introduced via GDR

3) There are no 8-branes or 9-branes in d=11 N=1 SUGRA that give rise to the D8-brane.

(FORMAL) SOLUTION: $\boxed{\text{"MASSIVE } d=11 \text{ SUGRA"}}$ (Bergshoeff, Lozano, Ortín)

This theory is formally 11-d-covariant but has a Killing vector $\hat{k}^{\hat{\mu}}$ in the Lagrangian

$$\hookrightarrow -\frac{1}{2} m^2 |\hat{k}^2|^2 \xrightarrow[\text{the direction } \hat{k}^{\hat{\mu}}]{\text{Reduction in}} -\frac{1}{2} m^2 \, e^{5/2\phi}$$

This theory looks weird, but it can be generalized to an even weirder theory with 2 K. vectors $\hat{k}_{(m)}{}^{\hat{\mu}}$ and a 2×2 mass matrix Q^{mn}

$$\hookrightarrow \frac{1}{2}\left(\hat{k}_{(m)}{}^{\hat{\mu}} Q^{mn} \hat{k}_{(n)\hat{\mu}}\right)^2 - \left(\hat{k}_{(m)\hat{\mu}} Q^{mn} \hat{k}_{(n)\hat{\nu}}\right)^2$$ (Meessen, Ortín)

The generalised 11-d-massive SUGRA reduced to $d=9$ in the directions $\hat{k}^{\hat{\mu}}_{(1)}$, $\hat{k}^{\hat{\mu}}_{(2)}$ gives the $N=2$ 9-d-massive/gauged SUGRA with 3 mass parameters m^i:

$$m^i T_i{}^{mn} \equiv Q^{mn} \qquad \{T_i\} \ SL(2,\mathbb{R}) \text{ generators}$$

...weird but not totally crazy...

The theory can be generalised to N Killing vectors and an $N\times N$ mass matrix.

Reducing over the N K. vectors we always get a massive/gauged SUGRA in $d=11-N$:

$\underset{SO(N)}{\text{massive/gauged}}$

$N=3 \longrightarrow N=4,\ d=8,\ SO(3)$-gauged SUGRA (2)

(Alonso-Alberca, Meessen, Ortín)

This theory has 6 vector fields:

A^m from $\hat{g}_{\hat{\mu}\hat{\nu}}$ $\rightsquigarrow$ gauged in Salam & Sezgin (GDR from d=11) (1)

B^m from $\hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}$ $\rightsquigarrow$ gauged in A.-A. M. O. (from "massive d=11") (2)

$m=1,2,3$

N=6 $\longrightarrow$ N=8, d=5, SO(6)-gauged SUGRA

This theory has 15+6 vectors & 6 2-forms (Alonso-Alberca, Ortín, in preparation)

15 $V_\mu{}^{mn}$ (SO(6)) from $\hat{C}_{\hat\mu\hat\nu\hat\rho}$ $\qquad$ 6 $B_{\mu\nu\, m}$ from $\hat{C}_{\hat\mu\hat\nu\hat\rho}$

6 $A_\mu{}^m$ from $\hat{g}_{\hat\mu\hat\nu}$ $\rightsquigarrow$ $\partial_{[\mu} A_{\nu]}{}^m - \Phi^{mn} B_{\mu\nu\, n}$

Stückelberg field

6 massive 2-forms

This theory was constructed first by {Gunaydin, Romans, Warner / Pernici, Pilch, van Nieuwenhuizen} dualizing the 6 $A_\mu{}^m$ into 6 $A_{\mu\nu\, m}$

$$\mathcal{B}_{\mu\nu\, m} = B_{\mu\nu\, m} + i A_{\mu\nu\, m}$$

satisfying a selfduality equation (Townsend, Pilch, van Nieuwenhuizen)

$$\mathcal{L} \sim m^2 \mathcal{B}^*_{\mu\nu\, m} \mathcal{B}^{\mu\nu}{}_m - \frac{i}{2} m \varepsilon^{\mu\nu\rho\sigma\lambda} \mathcal{B}^*_{\mu\nu\, m} \partial_\rho \mathcal{B}_{\sigma\lambda\, m}$$

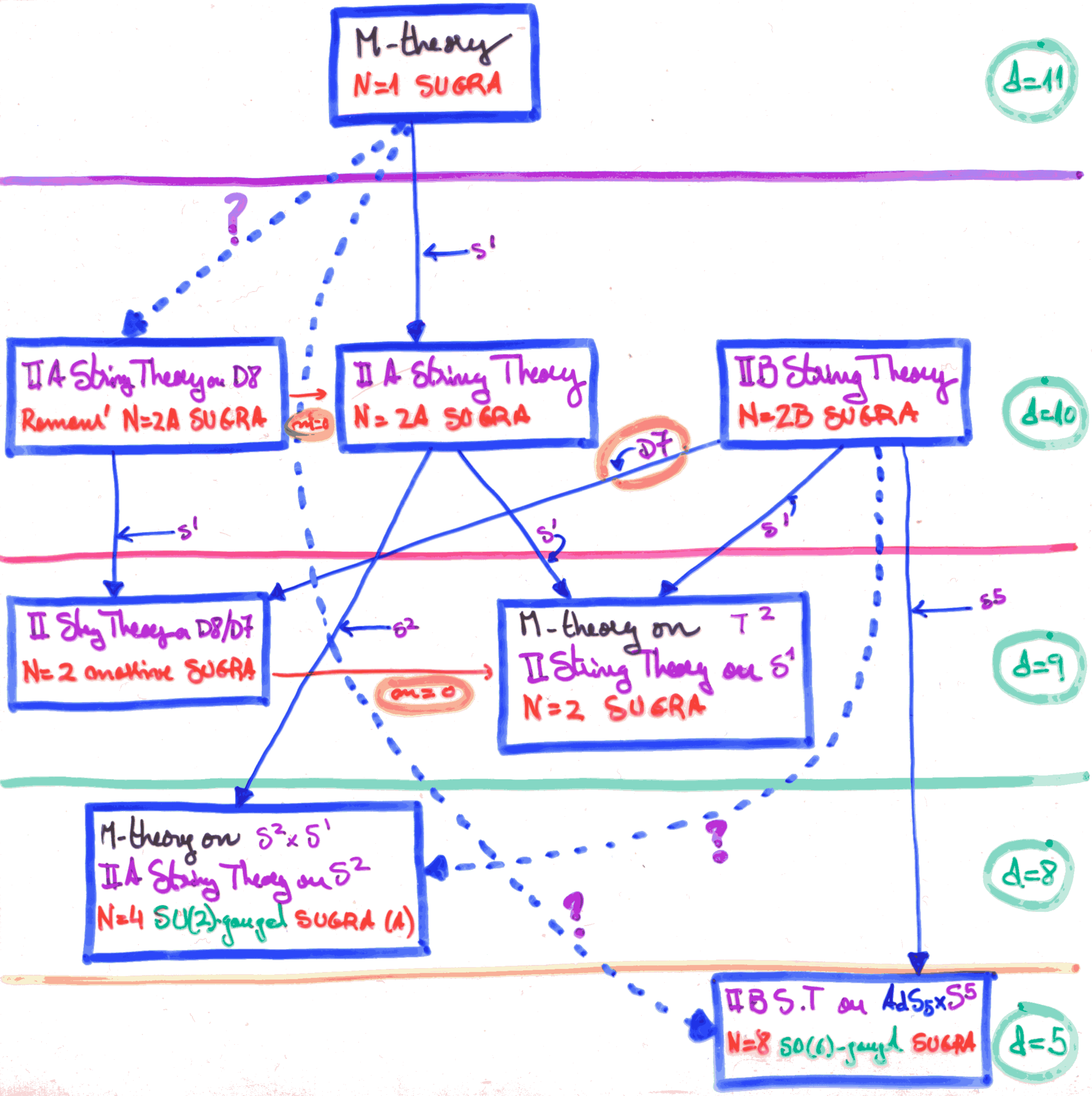
$\Rightarrow$ gives the Proca equation for the 6 $B_{\mu\nu\, m}$'s alone!

1) There is an alternative form of this theory.

2) There is a possible 11-d origin of this theory (that can be obtained from N=2B on $AdS_5 \times S^5$)

OUR PROBLEM

We want to understand the {IIB String-theoretical / M-theoretical} origin of some gauged/massive SUGRAS (and their associated domain-wall vacua)



CONCLUSIONS

- We have proposed a theory (at least a rule to construct theories) that systematically gives massive/gauged SUEGRAS in $d<11$, some new ($N=4, d=8$ (2)) some in a new form ($N=8, d=5$). (more cases: $d=7$ ....)
- p-form fields always get mass by a Stückelberg mechanism.* Giving mass to p-forms is essential to construct gauged/massive SUEGRAS in $d>4$ (the minimal coupling to vector fields would break the p-form gauge invariance ⇝ inconsistency!)
- A physical interpretation of "massive $d=11$ SUGRA" is difficult

  KK9 branes (M9) ? (necessary in Horava-Witten)

  KK-branes in general ?

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* Alternative to selfduality in odd dimensions

$d=5$ SELFDUALITY vs. STÜCKELBERG MECH.

In odd dimensions there are 3 actions for a massive $\frac{d-1}{2}$-form field. Let us take $d=5$ for example:

$B_{\mu\nu}$; $H_{\mu\nu\rho} = 3\,\partial_{[\mu}B_{\nu\rho]}$

1.- Proca action

$$S = \int d^5x \left[\frac{1}{2\cdot 3!} H^2 - \frac{1}{4} m^2 B^2 \right]$$

$$\Rightarrow \quad \partial_\mu H^{\mu\nu\rho} + m^2 B^{\nu\rho} = 0 \qquad \Rightarrow \quad \partial_\nu B^{\nu\rho} = 0$$

$$\Rightarrow \quad (\Box + m^2)\, B^{\mu\nu} = 0$$

2.- Self-dual action

$\tilde{B}_{\mu\nu}$; $\tilde{H}_{\mu\nu\rho} = 3\,\partial_{[\mu}\tilde{B}_{\nu\rho]}$

$$S = \int d^5x \left[-\frac{1}{4} m^2 B^2 - \frac{1}{4} m^2 \tilde{B}^2 + \frac{m}{2\cdot 3!} \varepsilon^{\mu\nu\rho\sigma\delta} \tilde{H}_{\mu\nu\rho} B_{\sigma\delta} \right]$$

$$\Rightarrow \quad \left.\begin{array}{l} * \tilde{H} = m B \\ * H = -m \tilde{B} \end{array}\right\} \quad (\Box + m^2)\, B^{\mu\nu} = 0$$

Each field propagates half of the d.o.f. of a standard massive 2-form. There is no $m \to 0$ limit!

On-shell we can eliminate either 2-form from the action and recover the Proca action for the other one.

3.- Stückelberg action

auxiliary A_μ ; $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]} + m B_{\mu\nu}$

$$S[B,A] = \int d^5x \left[\frac{1}{2\cdot 3!} H^2 - \frac{1}{4} F^2 \right]$$

Invariant under massive gauge transformations

$$\left.\begin{aligned} \delta A_\mu &= -m\Lambda_\mu \\ \delta B_{\mu\nu} &= 2\partial_{[\mu}\Lambda_{\nu]} \end{aligned}\right\} \Rightarrow$$

In the $A_\mu = 0$ gauge we recover the Proca action

Can we relate directly the Stückelberg and self-dual action?

yes: dualise the Stückelberg field A_μ into $\tilde{B}_{\mu\nu}$

$$S[B,\tilde{B},F] = \int d^5x \left[\frac{1}{2\cdot 3!} H^2 - \frac{1}{4} F^2 + \frac{1}{4} \varepsilon \partial \tilde{B} (F - mB) \right]$$

Lagrange-multiplier term → Bianchi for F

$$\Rightarrow \quad F = *\tilde{H}$$

$$\Rightarrow S[B,\tilde{B}] = \int d^5x \left[\frac{1}{2\cdot 3!} H^2 + \frac{1}{2\cdot 3!} \tilde{H}^2 - \frac{m}{12} \varepsilon \tilde{H} B \right]$$

$$\Rightarrow \left.\begin{aligned} & *d\,(*H = -m\tilde{B}) \\ & *d\,(*\tilde{H} = m B) \end{aligned}\right\}$$

These are the e.o.m. of the self-dual action

How about charged 2-forms?