New Formulations of Type II Superstring Effective Actions

And their application to the construction of explicitly supersymmetric Randall-Sundrum scenarios

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Seminar given on June 14th 2001 at Universidad de Barcelona

(Based on E. Bergshoeff, R. Kallosh, T.O., A. van Proeyen and D. Roest, hep-th/0103233.)

Plan of the Talk

Standard Type IIA Supergravity

Field content:

NSNS: $g_{\mu\nu}$, $B_{\mu\nu}$, ϕ

NSR: ψ_μ , λ ,

RR: m , $C^{(1)}_\mu$, $C^{(3)}_{\mu\nu\rho}$

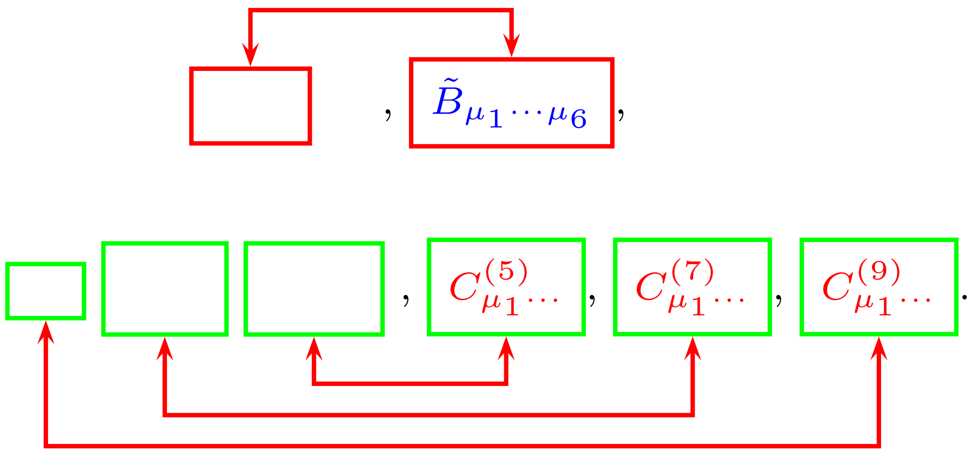
Bosonic field strengths:

$$\begin{aligned} H &= dB \\ G^{(0)} &= m \\ G^{(2)} &= dC^{(1)} + mB \\ G^{(4)} &= dC^{(3)} - HC^{(1)} + \tfrac{1}{2!} mB^2 \end{aligned}$$

Equations of motion and Bianchi identities for the bosonic fields:

$$d\left(e^{-2\phi\,\star}H\right) = +\tfrac{1}{2}\Sigma_{n=1}^{4}\,{}^\star G^{(2n+2)}G^{(2n)}\,, \qquad dH = 0\,,$$

$$dG^{(2n)} - HG^{(2n-2)} = 0\,, \qquad d^\star G^{(2n)} + H^\star G^{(2n+2)} = 0\,.$$



$$
\begin{array}{rcl}
 & \equiv & e^{2\phi}\,{}^\star\tilde{H}\,, \\
 & \equiv & -{}^\star G^{(10)}\,, \\
 & \equiv & +{}^\star G^{(8)}\,, \\
 & \equiv & -{}^\star G^{(6)}\,, \\
G^{(6)} = dC^{(5)} - HC^{(3)} + \frac{1}{3!}mB^3 & = & +{}^\star G^{(4)}\,, \\
G^{(8)} = dC^{(7)} - HC^{(5)} + \frac{1}{4!}mB^4 & = & -{}^\star G^{(2)}\,, \\
G^{(10)} = dC^{(9)} - HC^{(7)} + \frac{1}{5!}mB^5 & = & +{}^\star m\,, \\
\tilde{H} = d\tilde{B}^{(6)} - \frac{1}{2}\Sigma_{n=1}^{4}\,{}^\star G^{(2n+2)}C^{(2n-1)} & = & e^{-2\phi}H\,,
\end{array}
$$

Valid for all n

Standard Type IIA Supergravity

Action for the bosonic fields:

$$S = \frac{g_A{}^2}{16\pi G_{N\,A}^{(10)}} \int d^{10}x \, \sqrt{|g|} \left\{ e^{-2\phi} \left[R - 4 (\partial\phi)^2 + \tfrac{1}{2\cdot 3!} H^2 \right] - \left[\tfrac{1}{2} m^2 + \tfrac{1}{4} \left(G^{(2)} \right)^2 + \tfrac{1}{2\cdot 4!} \left(G^{(4)} \right)^2 \right] - \tfrac{1}{144} \frac{\epsilon}{\sqrt{|g|}} \left[\partial C^{(3)} \partial C^{(3)} B - \tfrac{1}{2} m \partial C^{(3)} B^3 + \tfrac{9}{80} m^2 B^5 \right] \right\}$$

Remarks:

- $g_A =< e^{\phi} >$ is the Type IIA Superstring coupling constant and the Newton constant is $G_{N\,A}^{(10)} = 8\pi^6 g_A{}^2 \ell_s{}^8$.
- It is a functional of the lower rank forms only.
- The higher-rank forms have been defined via *on-shell duality* and the lower-rank forms necessarily appear in their field strengths.
- One can dualize some of the lower RR forms into higher RR forms, but then the lower rank forms disappear.
- m is a given constant but the introduction of $G^{(10)}$ converts it in a (*piecewise constant*) solution to the equation of motion $dm = 0$.
- All the elements of this theory have a known 11-dimensional (“M-theoretical”) origin except for m.

Standard Type IIB Supergravity

Field content:

NSNS: $\jmath_{\mu\nu}$, $\mathcal{B}_{\mu\nu}$, φ

NSR: $\psi_\mu^{1,2}$, $\lambda^{1,2}$, $\frac{1}{2}(1+\Gamma_{10})\lambda^i = \frac{1}{2}(1-\Gamma_{10})\psi_\mu^i = 0\,.$

RR: $C^{(0)}$, $C^{(2)}_{\mu\nu}$, $C^{(4)}_{\mu_1\cdots}$,

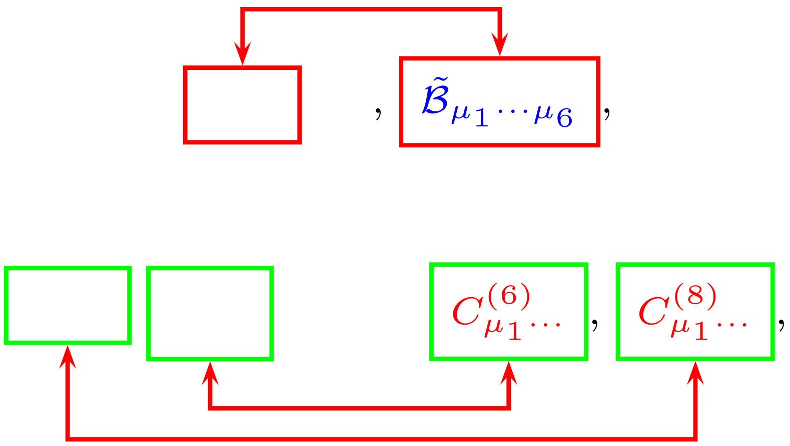
Bosonic field strengths:

$$\begin{aligned} \mathcal{H} &= d\mathcal{B} \\ G^{(1)} &= dC^{(0)} \\ G^{(3)} &= dC^{(2)} - \mathcal{H}C^{(0)} \\ G^{(5)} &= dC^{(4)} - \mathcal{H}C^{(2)} \end{aligned}$$

Equations of motion and Bianchi identities for the bosonic fields:

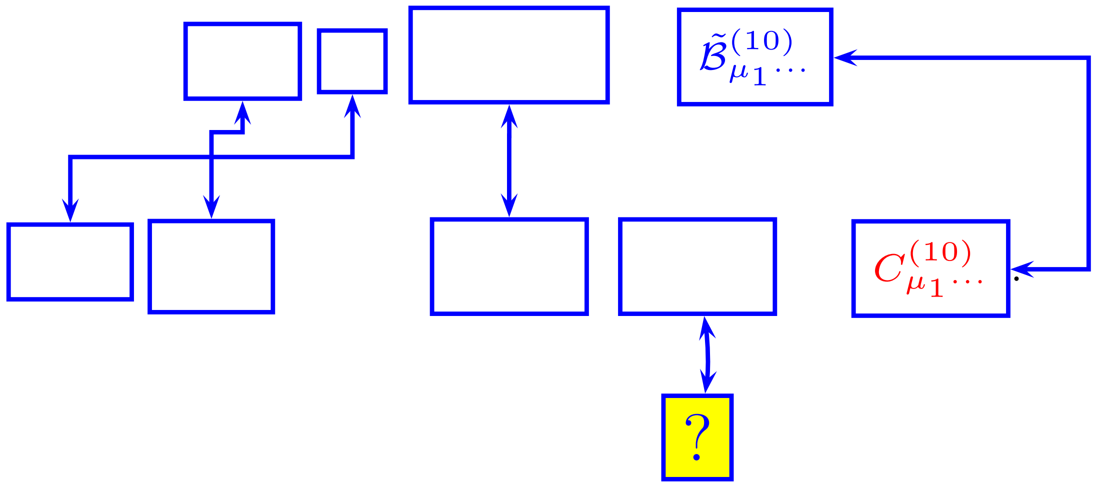
$$d\left(e^{-2\phi\,\star}\mathcal{H}\right) = +\tfrac{1}{2}\Sigma_{n=1}^{4}\,{}^\star G^{(2n+2)}G^{(2n)}\,, \qquad d\mathcal{H} = 0\,,$$

$$dG^{(2n+1)} - \mathcal{H}G^{(2n-1)} = 0\,, \qquad d^\star G^{(2n+1)} + \mathcal{H}^\star G^{(2n+3)} = 0\,.$$



$$
\begin{array}{rcl}
 & \equiv & e^{2\varphi\,\star}\tilde{\mathcal{H}}\,, \\
 & \equiv & +^{\star}G^{(9)}\,, \\
 & \equiv & -^{\star}G^{(7)}\,, \\
 & \equiv & +^{\star}G^{(5)}\,, \\
G^{(7)} = dC^{(6)} - \mathcal{H}C^{(4)} & = & -^{\star}G^{(3)}\,, \\
G^{(9)} = dC^{(8)} - \mathcal{H}C^{(6)} & = & +^{\star}G^{(1)}\,, \\
\tilde{\mathcal{H}} = d\tilde{\mathcal{B}}^{(6)} - \frac{1}{2}\Sigma_{n=0}^{4}\,{}^{\star}G^{(2n+3)}C^{(2n)} & = & e^{-2\phi}\mathcal{H}\,,
\end{array}
$$

Valid for all n


$\tilde{\mathcal{B}}^{(10)}_{\mu_1\cdots}$
$C^{(10)}_{\mu_1\cdots}$
?

Standard Type IIB Supergravity

(Pseudo-) Action for the bosonic fields:

$$S = \frac{{g_B}^2}{16\pi G_{N\ B}^{(10)}} \int d^{10}x \sqrt{|\jmath|} \left\{ e^{-2\varphi} \left[R - 4 (\partial\varphi)^2 + \frac{1}{2\cdot 3!}\mathcal{H}^2 \right] \right.$$
$$+ \left[\frac{1}{2} \left(G^{(1)} \right)^2 + \frac{1}{2\cdot 3!} \left(G^{(3)} \right)^2 + \frac{1}{4\cdot 5!} \left(G^{(5)} \right)^2 \right]$$
$$\left. - \frac{10}{(5!)^2} \frac{\epsilon}{\sqrt{|\jmath|}} G^{(5)} G^{(3)} \mathcal{B} \right\}$$

Remarks:

- $g_B =< e^{\varphi} >$ is the Type IIA Superstring coupling constant and the Newton constant is $G_{N\ B}^{(10)} = 8\pi^6 {g_B}^2 {\ell_s}^8$.
- It is only a *pseudoaction* ($G^{(5)} = {}^\star G^{(5)} \Rightarrow \left(G^{(5)} \right)^2 = 0\,.$)
- It is a functional of the lower rank forms only.
- The higher-rank forms have been defined via *on-shell duality* and the lower-rank forms necessarily appear in their field strengths.
- One can dualize some of the lower RR forms into higher RR forms, but then the lower rank forms disappear.
- There is a $SL(2,\mathbb{R})$ global invariance of the equations of motion (S-duality), only explicit in the Einstein frame.

Type II Branes

One p-brane per $p+1$-form potential

$$\Rightarrow \left\| \begin{array}{rcl} g_{\mu\nu} & \leftrightarrow & \text{Wave (IIA/B)} \\ B_{\mu\nu} & \leftrightarrow & \text{F. String (IIA/B)} \\ \tilde{B}_{\mu_1\cdots\mu_6} & \leftrightarrow & \text{S. 5-brane (IIA/B)} \\ C^{(0)} & \leftrightarrow & \text{D-instanton (IIB)} \\ C^{(1)}_{\mu} & \leftrightarrow & \text{D-particle (IIA)} \\ C^{(2)}_{\mu\nu} & \leftrightarrow & \text{D-string (IIB)} \\ C^{(3)}_{\mu\nu\rho} & \leftrightarrow & \text{D-membrane (IIA)} \\ \vdots & \leftrightarrow & \vdots \end{array} \right.$$

Effective Actions:

$S =$ Kinetic term $+$ Wess-Zumino term

Kinetic term $=$

$$-T_p \int d^{p+1}\xi \, \sqrt{|g_{ij}|} \qquad \text{(F. branes)}$$

$$-T_p g \int d^{p+1}\xi \, e^{-\phi} \sqrt{|g_{ij} + \mathcal{F}_{ij}|} \qquad \text{(D-branes)}$$

$$-T_p g^2 \int d^{p+1}\xi \, e^{-2\phi} \sqrt{|g_{ij}|} \qquad \text{(S. branes)}$$

T_p is the p-brane's tension

$\mathcal{F}_{ij} = 2\partial_{[i}b_{j]} - B_{ij}$ is the Born-Infeld vector field strength.

Type II Branes

Wess-Zumino term: $\sim q_p \int A^{(p+1)}$.

q_p is the p-brane's charge w.r.t. $A^{(p+1)}$.

But they also couple to other (lower rank) potentials (required by gauge-invariance, κ-symmetry and T-duality).

For D-branes:

$$WZ_p = q_p \int \mathbf{C}\ \mathbf{e}^{\mathcal{F}} + m\omega_{p+1}$$

where $\mathbf{C}$ is the formal sum $\mathbf{C} = C^{(0)} + C^{(1)} + C^{(2)} + \ldots$ so

$$\begin{aligned}
WZ_{-1} &= q_{-1} \int C^{(0)}\,, && \mathrm{D-inst.}\\
WZ_0 &= q_0 \int C^{(1)} + mb\,, && \mathrm{D-part.}\\
WZ_1 &= q_1 \int C^{(2)} + C^{(0)}\mathcal{F}\,, && \mathrm{D-string}\\
WZ_2 &= q_2 \int C^{(3)} + C^{(1)}\mathcal{F} + mbdb\,, && \mathrm{D-memb.}\\
WZ_2 &= q_3 \int C^{(4)} + C^{(2)}\mathcal{F} + \tfrac{1}{2}C^{(0)}\mathcal{F}\mathcal{F}\,, && \mathrm{D3-brane}
\end{aligned}$$

For D_p-branes $T_{D_p} \sim 1/g$. and $q_p = T_{D_p} g$.

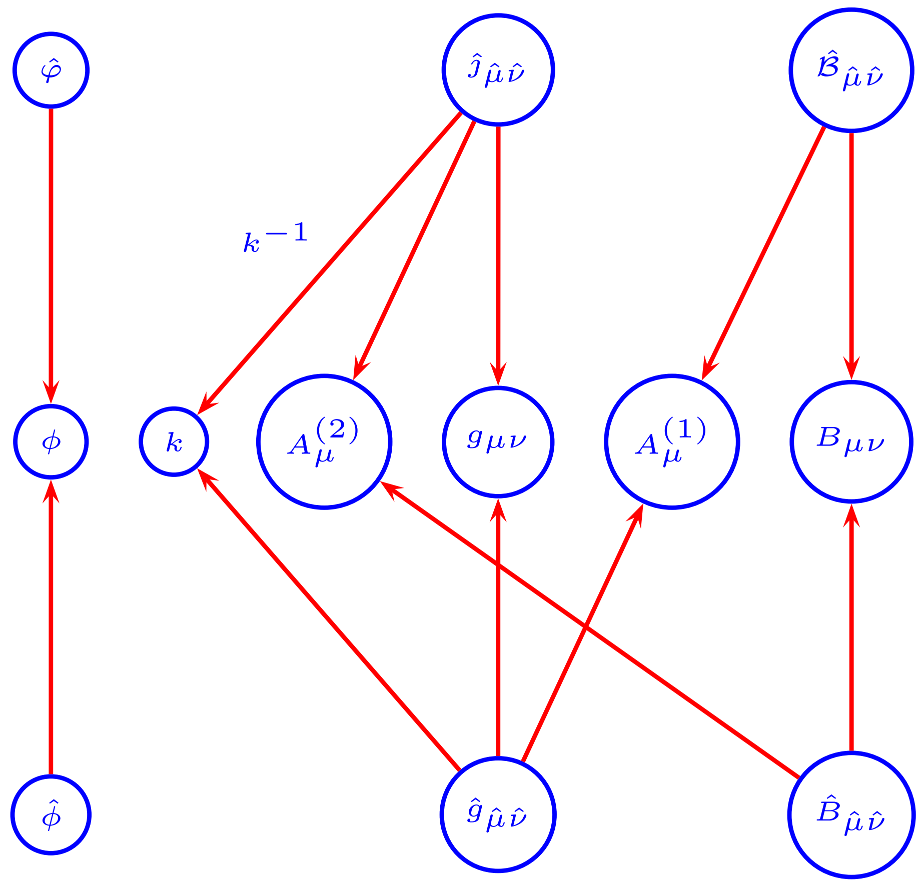
Non-Abelian D-branes also couple to higher rank RR potentials. (Myers)

Type IIA/B T-Duality in the Effective Action

The Type IIA/B fields are related by T-dual reductions to $d = 9$. The NSNS sector works just as in the bosonic case. Pictorially:

NSNS Sector

Type IIB, $d = 10$ NSNS fields

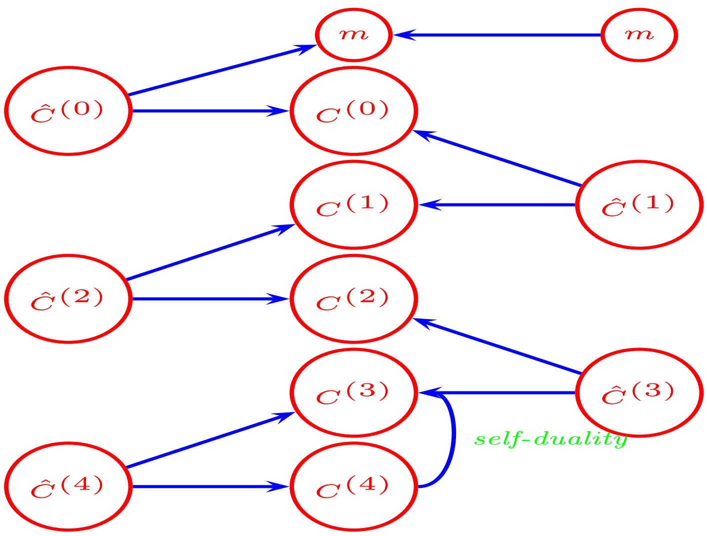


Type IIA, $d = 10$ NSNS fields

Type IIA/B T-Duality in the Effective Action

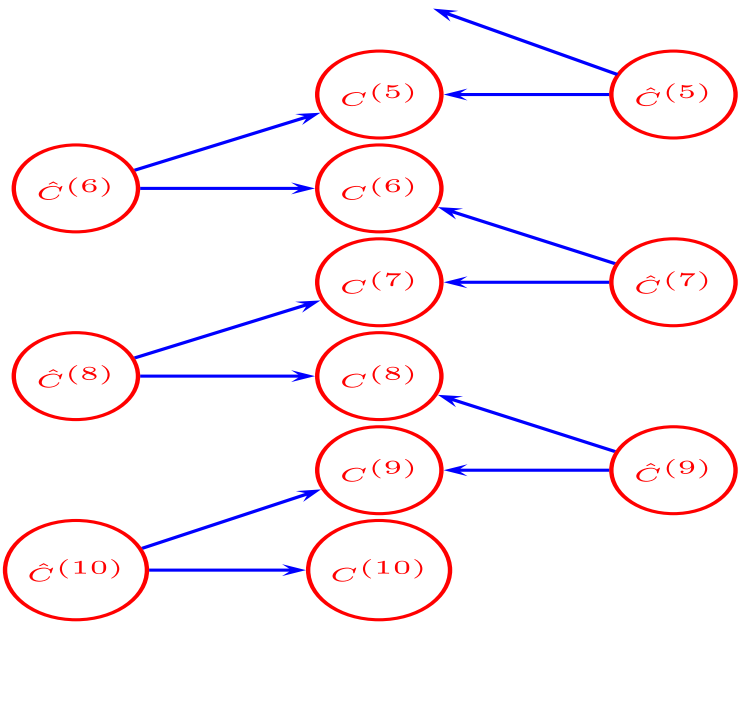
RR Sector

Type IIB, $d = 10$ RR potentials



Type IIA, $d = 10$ RR potentials

Remark: In the reduction of the Type IIB action with lower-rank RR forms, it is necessary to Hodge-dualize $C^{(4)}$ into $C^{(3)}$.



But a simpler picture is obtained introducing the higher-rank RR forms (e.o.m. only!).

Type IIA/B T-Duality in the Effective Action

Type II Buscher Rules

We get a non-trivial relation between the 10-dimensional fields of both theories:

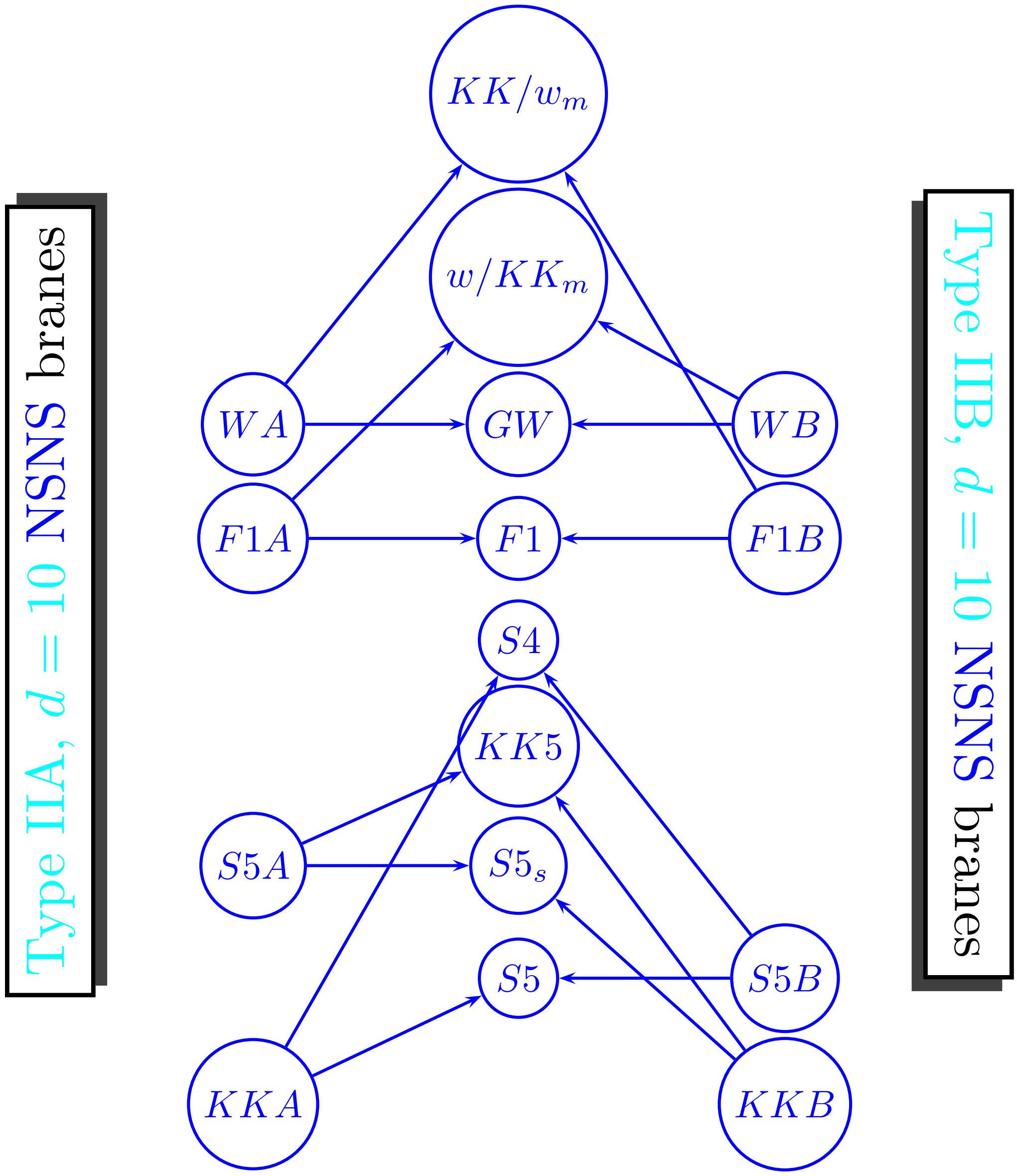
$$
\begin{aligned}
\hat{\jmath}_{\mu\nu} &= \hat{g}_{\mu\nu} - \left(\hat{g}_{\mu\underline{x}}\hat{g}_{\nu\underline{x}} - \hat{B}_{\mu\underline{x}}\hat{B}_{\nu\underline{x}}\right)/\hat{g}_{\underline{xx}}\,, \\
\hat{\jmath}_{\mu\underline{y}} &= \hat{B}_{\mu\underline{x}}/\hat{g}_{\underline{xx}}\,, \\
\hat{\mathcal{B}}_{\mu\nu} &= \hat{B}_{\mu\nu} + 2\hat{g}_{[\mu|\underline{x}}\hat{B}_{\nu]\underline{x}}/\hat{g}_{\underline{xx}}\,, \\
\hat{\mathcal{B}}_{\mu\underline{y}} &= \hat{g}_{\mu\underline{x}}/\hat{g}_{\underline{xx}}\,, \\
\hat{\jmath}_{\underline{yy}} &= 1/\hat{g}_{\underline{xx}}\,, \\
\hat{\varphi} &= \hat{\phi} - \tfrac{1}{2}\log|\hat{g}_{\underline{xx}}|\,,
\end{aligned}
$$

$$
\begin{aligned}
\hat{C}^{(2n)}{}_{\mu_1\cdots\mu_{2n}} &= \hat{C}^{(2n+1)}{}_{\mu_1\cdots\mu_{2n}\underline{x}} \\
&\quad + 2n\hat{B}_{[\mu_1|\underline{x}|}\hat{C}^{(2n-1)}{}_{\mu_2\cdots\mu_{2n}]} \\
&\quad - 2n(2n-1)\hat{B}_{[\mu_1|\underline{x}|}\hat{g}_{\mu_2|\underline{x}|}\hat{C}^{(2n-1)}{}_{\mu_3\cdots\mu_{2n}]\underline{x}}/\hat{g}_{\underline{xx}}\,, \\
\hat{C}^{(2n)}{}_{\mu_1\cdots\mu_{2n-1}\underline{y}} &= -\hat{C}^{(2n-1)}{}_{\mu_1\cdots\mu_{2n-1}} \\
&\quad + (2n-1)\hat{g}_{[\mu_1|\underline{x}|}\hat{C}^{(2n-1)}{}_{\mu_2\cdots\mu_{2n-1}]\underline{x}}/\hat{g}_{\underline{xx}}\,.
\end{aligned}
$$

Type IIA/B T-Duality in the Effective Action

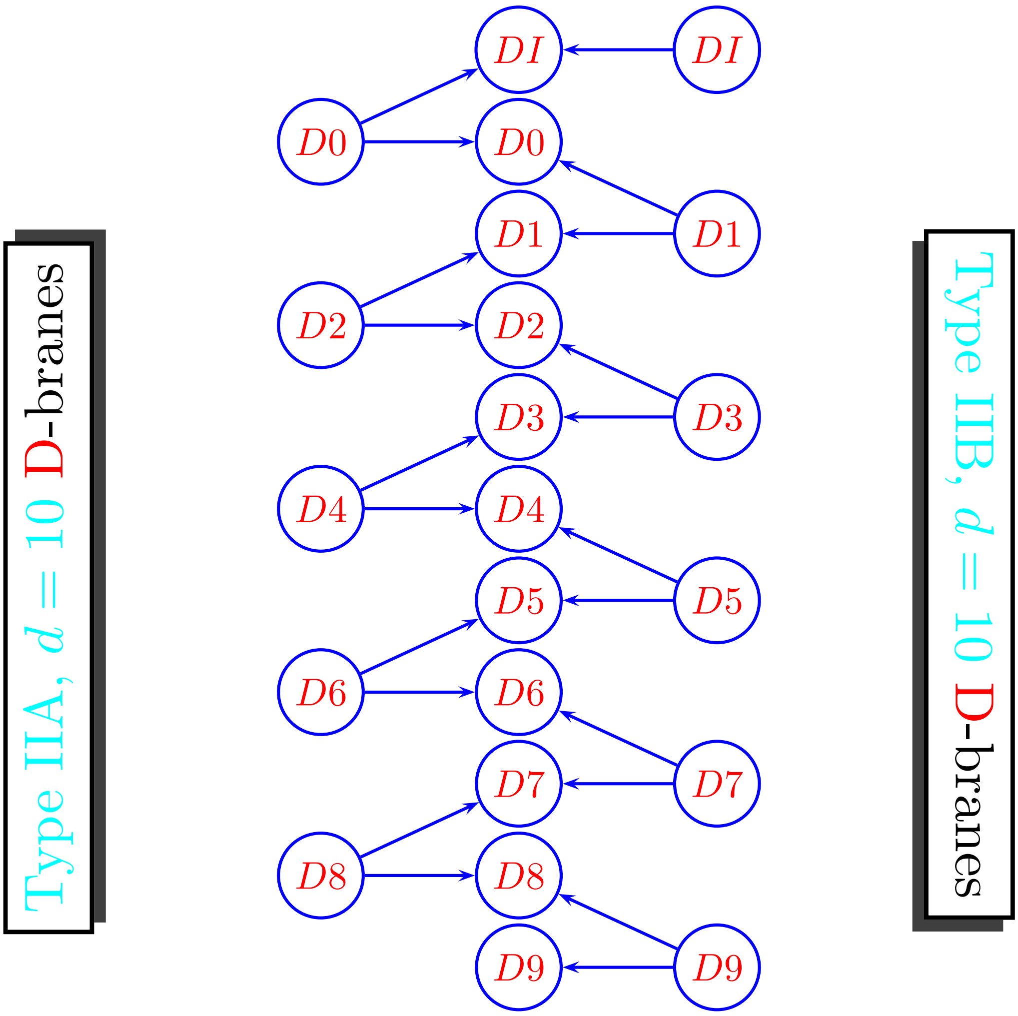
The relation between Type IIA and IIB fields corresponds to a relation between Type IIA and IIB branes (One p-brane per $p+1$-form potential):

NSNS Sector



Type IIA/B T-Duality in the Effective Action

RR Sector



Remark: These T-duality relations between D-branes are at the very basis of the D-brane concept.

Problems to be Solved

We have seen that the essential properties of Type II branes require the simultaneous handling of lower- and higher-rank form potentials. However:

1. The Type IIA SUGRA is only defined for constant m.
2. We do not know how to deal with the higher-rank potentials in a manifestly supersymmetric fashion.
3. We do not know how to deal simultaneously with all (lower and higher-rank) potentials, as required by brane effective actions.
4. We do not know how to construct supersymmetric *bulk & brane* actions. In the standard brane actions κ-symmetry requires the background supergravity fields to be *on-shell* (without sources).

Thus:

We need a generalization of the supergravity setup.

We also need some sort of supersymmetric brane sources.

Democratic Type II (Pseudo-) Actions

It turns out that it is possible to write an (pseudo-) action for all the potentials:

$$S = \frac{g^2}{16\pi G_N^{(10)}} \int d^{10}x \sqrt{|g|} \Big\{ e^{-2\phi} \left[R - 4 (\partial\phi)^2 + \tfrac{1}{2\cdot 3!} H^2 \right] - \sum_n \tfrac{(-1)^{2n}}{4\cdot 2n!} (G^{(2n)})^2 \Big\} \, .$$

where $n = 0, 1, 2, 3, 4, 5$ for IIA and $n = 1/2, 3/2, 5/2, 7/2, 9/2$ for IIB and the field strengths have the standard form.

Further:

- ⋆ It gives the *standard* equations of motion if we impose the duality relations $G^{10-2n} = (-1)^{[n]} {}^\star G^{(2n)}$ after varying the action w.r.t. the independent fields (i.e. all fields) . This is why we call it *pseudo- action*.
- ⋆ It can be supersymmetrized. It is invariant under supersymmetry transformations of *all* fields if at the end we impose the (supersymmetrized) duality relations.
- ⋆ The duality relations cannot be imposed before varying the action (just as in the standard IIB case).
- ⋆ Observe that now there are no Chern-Simons terms in this action. Proving T-duality is now trivial.
- ⋆ This action is only democratic in the RR sector, but not in the NSNS sector. Thus S-duality is broken in the IIB democratic theory.

The Type II Dual Actions

However, this is not the action that we can couple to brane sources because the duality relations plus the Bianchi identities imply *sourceless equations of motion*

For instance, we cannot couple $D8$-brane sources:

$$dG^{(0)} = d\,{}^{\star}G^{(10)} = 0\,,$$

duality

Only for the IIA we can fomulate a proper action for the higher-rank RR potentials which, in certain conditions allows us to couple to it higher-dimensional D-branes just as we can couple lower-dimensional D-branes to the standard action.

$$S = \frac{g^2}{16\pi G_N^{(10)}} \int d^{10}x\, \sqrt{|g|}\, \Big\{ e^{-2\phi} \left[R - 4\,(\partial\phi)^2 + \tfrac{1}{2\cdot 3!} H^2 \right] - \sum_n \tfrac{(-1)^{2n}}{4\cdot 2n!} (G^{(2n)})^2 \Big\}\,.$$

CONCLUSIONS

I.

II.

III.

IV.

V.

VI.

VII.

VIII.

IX. A further generalization is necessary in order to include the dual NSNS potentials $B^{(6)}, \phi$ so the resulting democratic Type IIB theory is manifestly S-duality invariant[a] .

[a] Work in progress.