T-Duality and the Type 0 Superstring Effective Action

And the tachyon coupling to the RR fields

Tomás Ortín (I.F.T.-C.S.I.C)

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Work done in collaboration with

Patrick Meessen (K.U. Leuven)

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Introduction/Motivation

Type 0 superstring theories are fashionable these days for a number of reasons:

1. They are very rich theories: they have NSNS and RR fields, as the Type II theories, and allow in principle) for many brane configurations.
2. They do not have spacetime fermions and, thus, they are not spacetime supersymmetric. Thus, one can explore non-supersymmetric AdS/CFT correspondences.
3. These theories are constructed from the worldsheet-supersymmetric NSR model. Thus, in spite of the absence of spacetime supersymmetry, the theory is constrained and we have certain control over it.
4. They have a tachyon that couples to the RR fields. It was argued that this coupling induces a positive shift of the tachyon mass in presence of D-branes, removing the instability. In any case, there may be interesting phenomena to be studied.

The Type 0A/B Superstrings Theories

The Type 0A and 0B theories have no fermions, have a NSNS tachyon and the massless parts of their spectra are[a]

$$\mathbf{0A}\ \left\{ \underbrace{g_{\mu\nu}\,, B_{\mu\nu}\,, \phi}_{\text{NSNS}}, \begin{array}{c} \overbrace{m, C_{(1)}\,, C_{(3)}\,, C_{(5)}\,, C_{(7)}\,, C_{(9)}}^{\text{RR}_{+-}} \\ \underbrace{\overline{m}, \overline{C_{(1)}}\,, \overline{C_{(3)}}\,, \overline{C_{(5)}}\,, \overline{C_{(7)}}\,, \overline{C_{(9)}}}_{\text{RR}_{-+}} \end{array} \right\}$$

$$\mathbf{0B}\ \left\{ \underbrace{\jmath_{\mu\nu}\,, \mathcal{B}_{\mu\nu}\,, \varphi}_{\text{NSNS}}, \begin{array}{c} \overbrace{C_{(0)}\,, C_{(2)}\,, C_{(4)}\,, C_{(6)}\,, C_{(8)}\,, C_{(10)}}^{\text{RR}_{++}} \\ \underbrace{\overline{C_{(0)}}\,, \overline{C_{(2)}}\,, \overline{C_{(4)}}\,, \overline{C_{(6)}}\,, \overline{C_{(8)}}\,, \overline{C_{(10)}}}_{\text{RR}_{--}} \end{array} \right\}$$

Remarks:

- They include the RR sectors of the two type II theories of the same kind.
- Then, in the 0B there is one self-dual and one anti-self-dual RR 5-form field strength for which no proper action can be written down.
- These are closed string theories and are T-dual to each other.

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[a] Here we have added the dual RR forms, and a mass parameter m by analogy with the Type IIA theory.

The Type 0B Superstring Effective Action

I. Klebanov and A. Tseytlin (**hep-th/9811035**) calculated the effective action of the Type 0B theory to lowest order *including the tachyon field* $\hat{\mathcal{T}}$[a]:

$$\hat{S} \sim$$

$$\int d^{10}\hat{x}\,\sqrt{|\hat{\jmath}|}\,\Big\{e^{-2\hat{\varphi}}\left[\hat{R}-4(\partial\hat{\varphi})^2+\tfrac{1}{2\cdot 3!}\hat{\mathcal{H}}^2+\tfrac{1}{2}(\partial\hat{\mathcal{T}})^2-\mathcal{V}(\hat{\mathcal{T}})\right]$$

$$+f_+(\hat{\mathcal{T}})\left[\tfrac{1}{2}\left(\hat{G}^+_{(1)}\right)^2+\tfrac{1}{2\cdot 3!}\left(\hat{G}^+_{(3)}\right)^2+\tfrac{1}{2\cdot 5!}\left(\hat{G}^+_{(5)}\right)^2\right]$$

$$+f_-(\hat{\mathcal{T}})\left[\tfrac{1}{2}\left(\hat{G}^-_{(1)}\right)^2+\tfrac{1}{2\cdot 3!}\left(\hat{G}^-_{(3)}\right)^2\right]\Big\}\,,$$

where

$\hat{C}^\pm_{(n-1)} \equiv \frac{1}{\sqrt{2}}\left(\hat{C}_{(n-1)} \pm \overline{\hat{C}_{(n-1)}}\right)$ is the diagonal basis,

$\hat{\mathcal{H}} = d\hat{\mathcal{B}}$, $\hat{G}^\pm_{(n)} = d\hat{C}^\pm_{(n-1)}$, are the field strengths,

$\mathcal{V}(\hat{\mathcal{T}}) = \frac{1}{2}m^2\hat{\mathcal{T}}^2 - 4c_1\hat{\mathcal{T}}^4 + \ldots$, is the tachyon potential,

$m^2 = -\frac{2}{\ell_s{}^2}$, is the tachyon mass.

$f_\pm(\hat{\mathcal{T}}) = 1 \pm \sqrt{2}\hat{\mathcal{T}} + \hat{\mathcal{T}}^2 + \ldots$,

$G^+_{(5)}$ combines the self- and anti-self-dual $G_{(5)}, \overline{G}_{(5)}$ into one unconstrained field (could have been $G^-_{(5)}$!!)

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[a] We add hats to denote 10-dimensional objects. After dimensional reduction, 9-dimensional objects will be represented with no hats.

The Type 0A/B Superstrings Effective Actions

Our goal is to complete the Type 0B action and find the complete Type 0A action imposing that they have to be T-dual to each other, using our knowledge on T-duality in string effective actions

First,
some examples

T-Duality in the Bosonic String Effective Action

This is the simplest example[a]

The effective action is identical to that of the NSNS sectors of the Type IIA, IIB and 0B:

$$\hat{S} \sim \int d^{\hat{d}}\hat{x}\,\sqrt{|\hat{g}|}\,e^{-2\hat{\phi}}\left[\hat{R} - 4(\partial\hat{\phi})^2 + \tfrac{1}{2\cdot 3!}\hat{H}^2\right] \,.$$

If the coordinate x is compact, reducing with the KK Ansatz

$$\begin{aligned} \hat{g}_{\mu\nu} &= g_{\mu\nu} - k^2 A^{(1)}{}_{\mu} A^{(1)}{}_{\nu}\,, & \hat{B}_{\mu\nu} &= B_{\mu\nu} - A^{(1)}{}_{[\mu} A^{(2)}{}_{\nu]}\,,\\ \hat{g}_{\mu\underline{x}} &= -k^2 A^{(1)}{}_{\mu}\,, & \hat{B}_{\mu\underline{x}} &= A^{(2)}{}_{\mu}\,,\\ \hat{g}_{\underline{xx}} &= -k^2\,, & \hat{\phi} &= \phi + \tfrac{1}{2}\log k\,, \end{aligned}$$

where

- k is the KK scalar that measures the radius of the internal circle.
- $A^{(1)}$ is the KK vector with respect to which, all massive KK modes are electrically charged.
- $A^{(2)}$ is the *winding vector* with respect to which, all massive winding modes are electrically charged.

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[a] E. Bergshoeff, R. Kallosh and T. Ortín, *Phys.Rev.* **D51**, (1995) 3009–3016. (hep-th/9410230).

T-Duality in the Bosonic String Effective Action

We get the $d = (\hat{d} - 1)$ dimensional action:

$$S \sim \int d^d x \sqrt{|g|}\, e^{-2\phi} \Big[R - 4(\partial\phi)^2 + \tfrac{1}{2\cdot 3!} H^2 + (\partial \log k)^2 - \tfrac{1}{4} k^2 \left(F^{(1)}\right)^2 - \tfrac{1}{4} k^{-2} \left(F^{(2)}\right)^2\Big] \,.$$

If we have *another* bosonic string theory with compact coordinate y with radius inverse to that of x and we compactify with the T-dual KK Ansatz

$$\hat{g}_{\mu\nu} = g_{\mu\nu} - k^{-2} A^{(2)}{}_{\mu} A^{(2)}{}_{\nu}\,, \qquad \hat{B}_{\mu\nu} = B_{\mu\nu} - A^{(2)}{}_{[\mu} A^{(1)}{}_{\nu]}$$

$$\hat{g}_{\mu \underline{y}} = -k^{-2} A^{(2)}{}_{\mu}\,, \qquad \hat{B}_{\mu \underline{y}} = A^{(1)}{}_{\mu}\,,$$

$$\hat{g}_{\underline{yy}} = -k^{-2}\,, \qquad \hat{\phi} = \phi - \tfrac{1}{2}\log k\,,$$

in which the names of the KK vector and winding vector have been interchanged and the KK scalar is k^{-1}

WE GET
THE SAME
d-DIMENSIONAL ACTION
and the two theories are

T-DUAL

T-Duality in the Bosonic String Effective Action

Buscher Rules

This gives a non-trivial relation between the $\hat{d}$-dimensional fields of both theories[a]:

$$
\begin{aligned}
\hat{g}'_{\mu\nu} &= \hat{g}_{\mu\nu} - \left(\hat{g}_{\mu \underline{x}}\hat{g}_{\nu \underline{x}} - \hat{B}_{\mu \underline{x}}\hat{B}_{\nu \underline{x}}\right)/\hat{g}_{\underline{x}\underline{x}}\,,\\
\hat{g}'_{\mu \underline{y}} &= \hat{B}_{\mu \underline{x}}/\hat{g}_{\underline{x}\underline{x}}\,,\\
\hat{B}'_{\mu\nu} &= \hat{B}_{\mu\nu} + 2\hat{g}_{[\mu|\underline{x}}\hat{B}_{\nu]\underline{x}}/\hat{g}_{\underline{x}\underline{x}}\,,\\
\hat{B}'_{\mu \underline{y}} &= \hat{g}_{\mu \underline{x}}/\hat{g}_{\underline{x}\underline{x}}\,,\\
\hat{g}'_{\underline{y}\underline{y}} &= 1/\hat{g}_{\underline{x}\underline{x}}\,,\\
\hat{\phi}' &= \hat{\phi} - \tfrac{1}{2}\log|\hat{g}_{\underline{x}\underline{x}}|\,,
\end{aligned}
$$

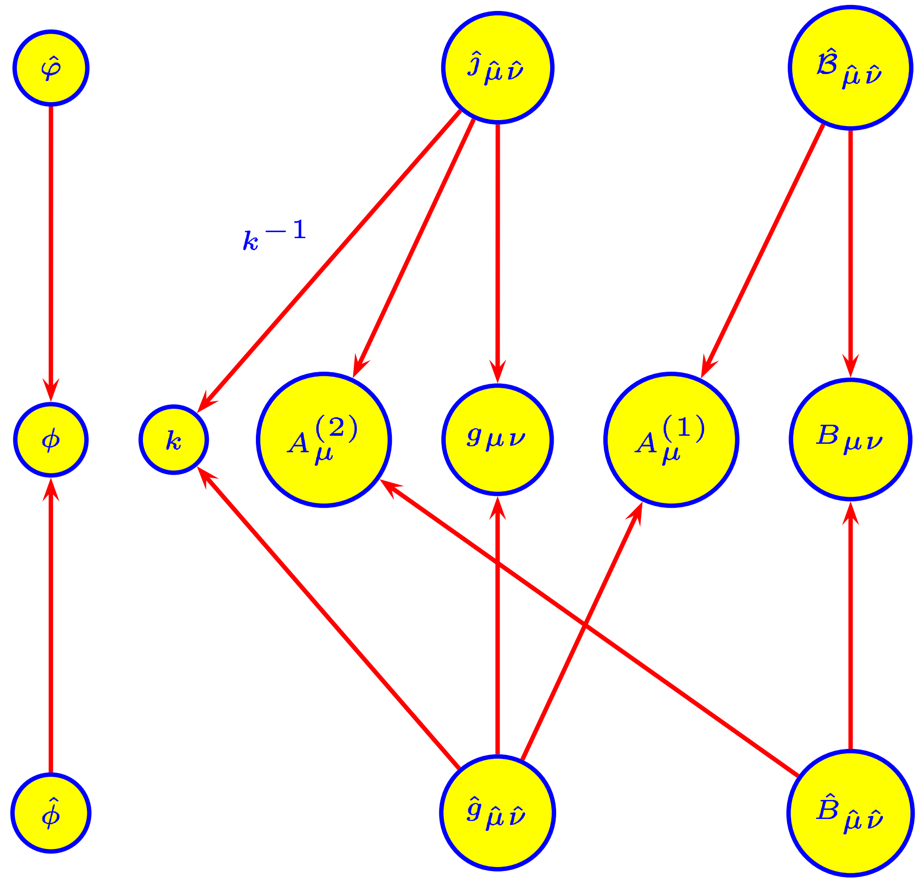
[a] T. Buscher, *Phys. Lett.* **159B** (1985) 127; *ibid* **194B** (1987) 59; *ibid* **201B** (1988) 466.

Type IIA/B T-Duality in the Effective Action

Second example [a]. The Type IIA/B fields are related by T-dual reductions to $d = 9$. The NSNS sector works just as in the bosonic case. Pictorially:

NSNS Sector

Type IIB, $d = 10$

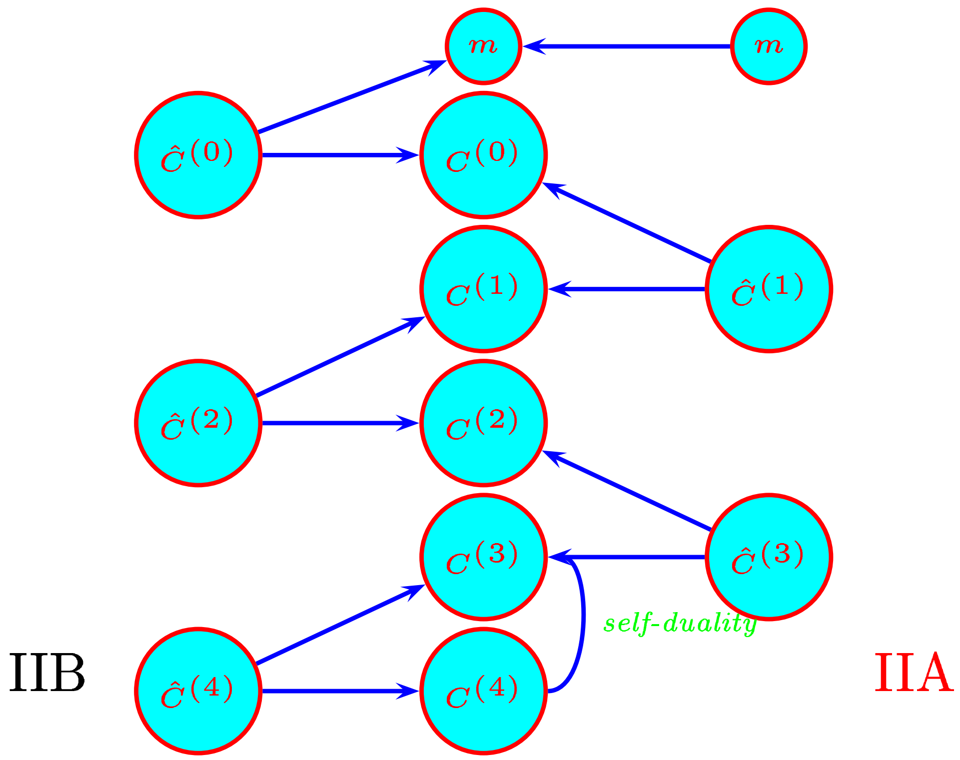


Type IIA, $d = 10$

[a] E. Bergshoeff, C.M. Hull and T. Ortín, *Nucl. Phys.* **B451**(1995), 547 (hep-th/9504081).
E. Bergshoeff, M. de Roo, M.B. Green, G. Papadopoulos and P.K. Townsend, *Nucl. Phys.* **B470**(1996), 113 (hep-th/9601150).
P. Meessen and T. Ortín, *Nucl. Phys.* **B541**(1999), 195 (hep-th/9806120).

Type IIA/B T-Duality in the Effective Action

RR Sector



Remarks:

- The RR and NSNS sectors transform separately (also in the string spectrum).
- In the NSNS sector, the most important feature of T-duality is the interchange of the Kaluza-Klein and the winding vectors (also in the string spectrum).
- In the reduction of the Type IIB effective actions with lower-rank RR forms, it is necessary to Hodge-dualize $C^{(4)}$ into $C^{(3)}$.

Type IIA/B T-Duality in the Effective Action

Type II Buscher Rules

Again, we get a non-trivial relation between the 10-dimensional fields of both theories:

$$
\begin{aligned}
\hat{\jmath}_{\mu\nu} &= \hat{g}_{\mu\nu} - \left(\hat{g}_{\mu \underline{x}}\hat{g}_{\nu \underline{x}} - \hat{B}_{\mu \underline{x}}\hat{B}_{\nu \underline{x}}\right)/\hat{g}_{\underline{xx}}\,,\\
\hat{\jmath}_{\mu \underline{y}} &= \hat{B}_{\mu \underline{x}}/\hat{g}_{\underline{xx}}\,,\\
\hat{\mathcal{B}}_{\mu\nu} &= \hat{B}_{\mu\nu} + 2\hat{g}_{[\mu|\underline{x}}\hat{B}_{\nu]\underline{x}}/\hat{g}_{\underline{xx}}\,,\\
\hat{\mathcal{B}}_{\mu \underline{y}} &= \hat{g}_{\mu \underline{x}}/\hat{g}_{\underline{xx}}\,,\\
\hat{\jmath}_{\underline{yy}} &= 1/\hat{g}_{\underline{xx}}\,,\\
\hat{\varphi} &= \hat{\phi} - \tfrac{1}{2}\log|\hat{g}_{\underline{xx}}|\,,
\end{aligned}
$$

$$
\begin{aligned}
\hat{\mathcal{C}}^{(2n)}{}_{\mu_1\cdots\mu_{2n}} &= \hat{C}^{(2n+1)}{}_{\mu_1\cdots\mu_{2n}\underline{x}}
+2n\hat{B}_{[\mu_1|\underline{x}|}\hat{C}^{(2n-1)}{}_{\mu_2\cdots\mu_{2n}]}\\
&\quad -2n(2n-1)\hat{B}_{[\mu_1|\underline{x}|}\hat{g}_{\mu_2|\underline{x}|}\hat{C}^{(2n-1)}{}_{\mu_3\cdots\mu_{2n}]\underline{x}}/\hat{g}_{\underline{xx}}\,,\\
\hat{\mathcal{C}}^{(2n)}{}_{\mu_1\cdots\mu_{2n-1}\underline{y}} &= -\hat{C}^{(2n-1)}{}_{\mu_1\cdots\mu_{2n-1}}\\
&\quad +(2n-1)\hat{g}_{[\mu_1|\underline{x}|}\hat{C}^{(2n-1)}{}_{\mu_2\cdots\mu_{2n-1}]\underline{x}}/\hat{g}_{\underline{xx}}\,.
\end{aligned}
$$

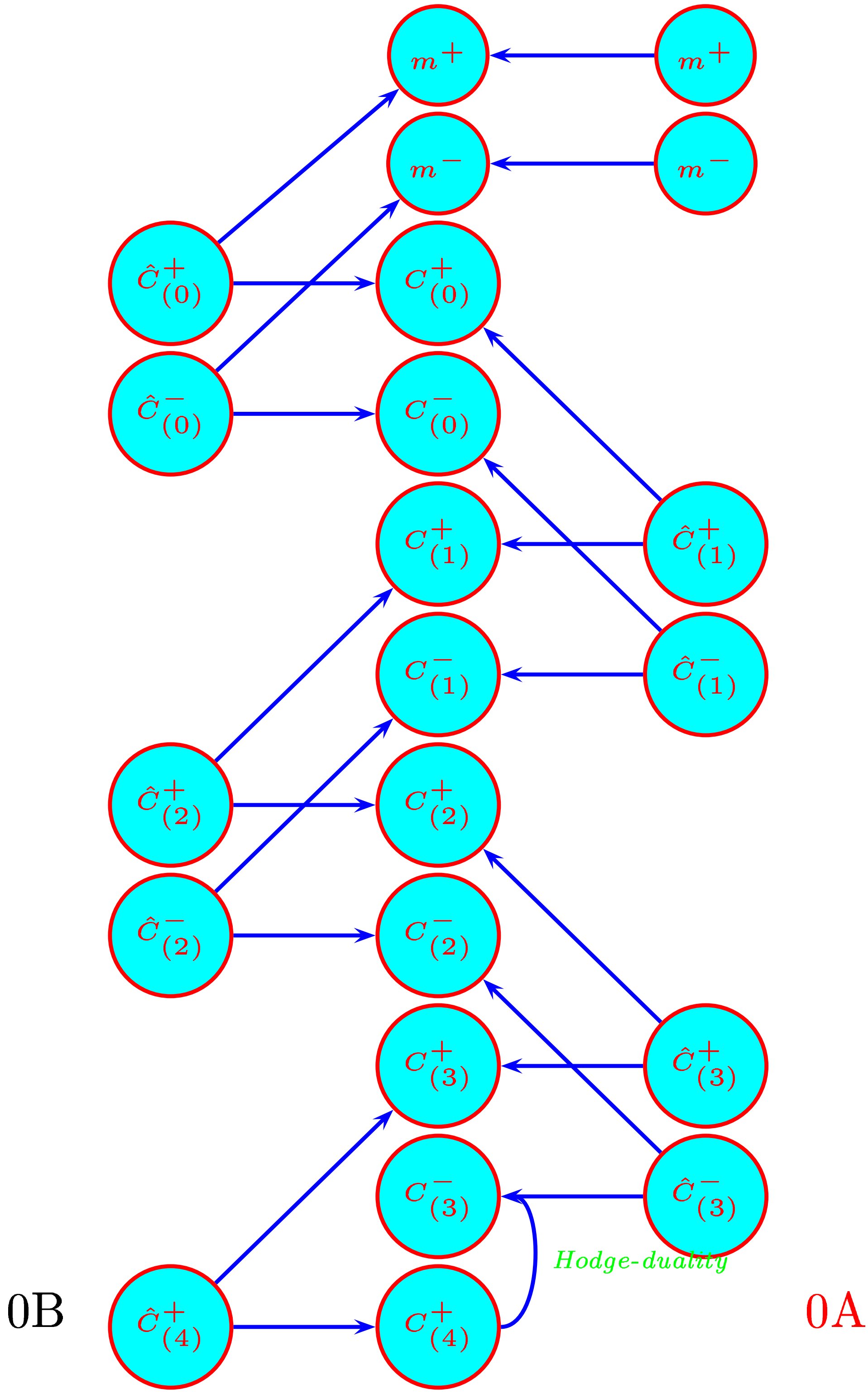
Type 0A/B T-Duality

How do we expect T-duality to work in the Type 0A/B context?

1. We expect the NSNS and RR sectors to transform independently.
2. We expect the NSNS sectors to be related exactly as in the bosonic and Type II cases.
3. The action of the NSNS sector of the Type 0A superstring should, then, be IDENTICAL to that of. the Type 0B theory.
4. The relation between the RR sectors has to be similar to the one in the Type IIA/B case (see next transparence).
5. T-duality requires that $G^{+}_{(5)}$ is the Hodge-dual of $G^{-}_{(4)}$ in $d = 9$, which agrees with the fact that in $d = 10$ $\hat{G}^{+}_{(5)}$ should be the Hodge-dual of $\hat{G}^{-}_{(5)}$ due to the (anti-) self-duality of $\hat{\overline{G}}_{(5)}$ and $\hat{G}_{(5)}$.

Now we are going to try to realize these expectations in the Type 0A/B effective actions.

Type 0A/B T-Duality



Type 0A/B T-Duality

To establish a T-duality relation between the two Type 0A/B effective actions, as in the Type IIA/B case, we need an action for the Type 0A. A reasonable guess based on our expectations is[a]

$$\hat{S} \sim$$

$$\int d^{10}\hat{x}\,\sqrt{|\hat{g}|}\,\Big\{e^{-2\hat{\phi}}\left[\hat{R}-4(\partial\hat{\phi})^2+\tfrac{1}{2\cdot 3!}\hat{H}^2+\tfrac{1}{2}(\partial\hat{T})^2-V(\hat{T})\right]$$

$$-f_+(\hat{T})\left[\tfrac{1}{2\cdot 2!}\left(\hat{G}^+_{(2)}\right)^2+\tfrac{1}{2\cdot 4!}\left(\hat{G}^+_{(4)}\right)^2\right]$$

$$-f_-(\hat{T})\left[\tfrac{1}{2\cdot 2!}\left(\hat{G}^-_{(2)}\right)^2+\tfrac{1}{2\cdot 4!}\left(\hat{G}^-_{(4)}\right)^2\right]\Big\}\,.$$

where $\hat{T}=\hat{\mathcal{T}}$, and the tachyon potentials and functions $f_\pm$ have to be identical, if the Type 0A and 0B actions have to coincide in $d=9$.

[a] We do not include the mass parameter for simplicity yet.

Type 0A/B T-Duality

The NSNS sectors of two Type 0A/B actions are automatically identical in $d = 9$ but we find 3 problems in the RR sector:

1. The $d = 9$ RR field strengths are different. In the reduced Type 0A we get

$$\left\{ \begin{array}{lcl} G^{\pm}_{(2n+1)} & = & dC^{\pm}_{(2n)} \\ G^{\pm}_{(2n)} & = & dC^{\pm}_{(2n-1)} + F^{(1)} C^{\pm}_{(2n-2)} \,, \end{array} \right.$$

   And in the reduced Type 0B we get

$$\left\{ \begin{array}{lcl} G^{\pm}_{(2n+1)} & = & dC^{\pm}_{(2n)} + F^{(2)} C^{\pm}_{(2n-1)} \,, \\ G^{\pm}_{(2n)} & = & dC^{\pm}_{(2n-1)} \end{array} \right.$$

$$+F^{(2)} C^{\pm}_{(2n-1)} \, ,$$

$$+F^{(1)} C^{\pm}_{(2n-2)} \, .$$

Solution: The winding vectors should appear in the $d=9$ field strengths, and so $\hat{B}_{\hat{\mu}\hat{\nu}}$ has to appear in the $d=10$ RR field strengths. Up to field redefinitions, they have to be as in Type II theories:

$$\hat{G}^{\pm}_{(n)} = d\hat{C}^{\pm}_{(n-1)} - \hat{H}\hat{C}^{\pm}_{(n-2)} \, .$$

Type 0A/B T-Duality

2. We get a $G^{+}_{(5)}$ from the 0B side but not from the 0A side and a $G^{-}_{(4)}$ from the 0A side but not from the 0B side.

Solution: We impose

$$f_+(\hat{\mathcal{T}})\hat{G}^+_{(5)} = {}^\star\hat{G}^-_{(5)} \,,$$

in $d = 10$, which leads to

$$f_+(\hat{\mathcal{T}})G^+_{(5)} = {}^\star G^-_{(4)} \,,$$

in $d = 9$, and solves our problem if, and only if

$$f_+ = 1/f_- \,.$$

Remark: A constraint of the form

$$f(\hat{\mathcal{T}})\hat{G}_{(5)} = \pm{}^{\star}\hat{G}_{(5)}\,,$$

cannot be consistently imposed unless $f = 1$.

Type 0A/B T-Duality

3. If we dualize naively $\hat{G}^{+}_{(5)}$ according to $f_{+}(\hat{\mathcal{T}})\hat{G}^{+}_{(5)} = {}^{\star}\hat{G}^{-}_{(5)}$, we get an alternative form of the Type 0B but $\hat{G}^{-}_{(5)}$ is not given by the general formula $\hat{G}^{\pm}_{(n)} = d\hat{C}^{\pm}_{(n-1)} - \hat{H}\hat{C}^{\pm}_{(n-2)}$, because the Chern-Simons term $\hat{H}\hat{C}^{\pm}_{(n-2)}$ is missing. In $d = 9$, then, the $G^{-}_{(4)}$ we get from the 0B side is different than the one we get from the 0A side.

Solution: We have to add a Chern-Simons term to the $d = 10$ Type 0B action. There is only one that allows us to establish T-duality:

$$\int d^{10}\hat{x} - \frac{10}{(5!)^2}\hat{\epsilon}\hat{G}^{+}_{(5)}\hat{G}^{-}_{(3)}\mathcal{B}\,,$$

4. Now we get a $d = 9$ Chern-Simons term from the 0B side, and none from the 0A side

Solution: Again, we add a Chern-Simons term to the $d = 10$ Type 0A action. It is unique:

$$\int d^{10}\hat{x}\,\hat{\epsilon}\;-\;\frac{1}{2\cdot(4!)^2}\hat{G}^{+}_{(4)}\hat{G}^{-}_{(4)}\hat{B}\,.$$

Type 0A/B T-Duality

Result: we get two Type 0 actions which are T-dual with the standard (Type II) Buscher rules:

Type 0B

$$\hat{S} \sim \int d^{10}\hat{x}\, \sqrt{|\hat{\jmath}|}\, \Big\{ e^{-2\hat{\varphi}} \left[\hat{R} - 4(\partial\hat{\varphi})^2 + \tfrac{1}{2\cdot 3!}\hat{\mathcal{H}}^2 + \tfrac{1}{2}(\partial\hat{\mathcal{T}})^2 - \mathcal{V}(\hat{\mathcal{T}}) \right]$$

$$+ f_+(\hat{\mathcal{T}}) \left[\tfrac{1}{2}\hat{G}^{+\,2}_{(1)} + \tfrac{1}{2\cdot 3!}\hat{G}^{+\,2}_{(3)} + \tfrac{1}{2\cdot 5!}\hat{G}^{+\,2}_{(5)} \right]$$

$$+ f_-(\hat{\mathcal{T}}) \left[\tfrac{1}{2}\hat{G}^{-\,2}_{(1)} + \tfrac{1}{2\cdot 3!}\hat{G}^{-\,2}_{(3)} \right] - \tfrac{10}{(5!)^2} \tfrac{\hat{\epsilon}}{\sqrt{|\hat{\jmath}|}} \hat{G}^{+}_{(5)} \hat{G}^{-}_{(3)} \mathcal{B} \Big\} \,,$$

Type 0A

$$\hat{S} \sim \int d^{10}\hat{x}\, \sqrt{|\hat{g}|}\, \Big\{ e^{-2\hat{\phi}} \left[\hat{R} - 4(\partial\hat{\phi})^2 + \tfrac{1}{2\cdot 3!}\hat{H}^2 + \tfrac{1}{2}(\partial\hat{T})^2 - V(\hat{T}) \right]$$

$$- f_+(\hat{T}) \left[\tfrac{1}{2\cdot 2!}\hat{G}^{+\,2}_{(2)} + \tfrac{1}{2\cdot 4!}\hat{G}^{+\,2}_{(4)} \right]$$

$$- f_-(\hat{T}) \left[\tfrac{1}{2\cdot 2!}\hat{G}^{-\,2}_{(2)} + \tfrac{1}{2\cdot 4!}\hat{G}^{-\,2}_{(4)} \right] - \tfrac{1}{2\cdot (4!)^2} \hat{G}^{+}_{(4)} \hat{G}^{-}_{(4)} \hat{B} \Big\} \,.$$

Type 0A/B T-Duality

Remarks:

- The Type 0A theory is (worldsheet) left-right invariant, which implies that the effective action has to be invariant under the interchange of the two $RR_{\pm\mp}$ sectors (i.e. $\hat{C}^{\pm} \rightarrow \pm\hat{C}^{\pm}$) accompanied by sign-reversal of the Kalb-Ramond 2-form $\hat{B} \rightarrow -\hat{B}$. It is easy to see that our action has this symmetry.

- Both Type 0A (as any superstring theory) have to be invariant under the worldsheet $(-)^{f_L}$. This implies that both of their effective actions have to be invariant under the interchange of the + and − RR fields with the simultaneous sign-reversal of the tachyon

$$\hat{C}^{\pm} \rightarrow \hat{C}^{\mp}\,, \quad \hat{T} \rightarrow -\hat{T}$$

In the Type 0A theory this is possible if, and only if

$$f_+(-\hat{T}) = f_-(\hat{T})\,.$$

In the Type 0B we need the same condition, plus

$$f_+(\hat{T})\hat{G}^{+}_{(5)} = {}^{\star}\hat{G}^{-}_{(5)}\,, \quad f_+ = 1/f_-\,.$$

- The two conditions that we have derived for $f_{\pm}(\hat{T})$ can be satisfied if and only if

$$f_{\pm}(\hat{T}) = e^{\pm h(\hat{T})}\,, \quad h(-\hat{T}) = -h(\hat{T})\,,$$

which is consistent with the known terms in their expansions.

Type 0A/B *Democratic* Actions

We can dualize in both actions either all the $\hat{G}^{+}$ or all the $\hat{G}^{-}$ field strengths. The result is an action in which RR fields strengths of all possible ranks appear on equal footing (*democratically*) and has no Chern-Simons term. We can write both actions in a unified way:

$$\hat{S} \sim \int d^{10}\hat{x}\,\sqrt{|\hat{g}|}\,\Big\{e^{-2\hat{\phi}}\Big[\hat{R} - 4(\partial\hat{\phi})^2 + \tfrac{1}{2\cdot 3!}\hat{H}^2 + \tfrac{1}{2}(\partial\hat{T})^2 - V(\hat{T})\Big] - f_{\pm}(\hat{T})\sum_n \tfrac{(-1)^n}{2\cdot n!}\hat{G}^{\pm\,2}_{(n)}\Big\}\,,$$

where $n = 0, 2, 4, 6, 8, 10$ in the Type 0A theory and $n = 1, 3, 5, 7, 9$ in the Type 0B theory.

In this form it is trivial to establish T-duality between both theories.

We have added the mass parameters $m^{+} = \hat{G}^{+}_{(0)}$ and $m^{-} = -\hat{G}^{-}_{(10)}$ in the Type 0A theory.

There is also a *democratic* action for Type II theories that can be supersymmetrized[a].

[a] E. Bergshoeff, R. Kallosh, T. Ortín, D. Roest and A. van Proeyen, *Democratic Formulations of d = 10 Supersymmetry and D8-O8 Domain Walls*, hep-th/0103233

Type 0A/B Branes

Let us now investigate the Type 0A/B brane solutions.

We look for solutions with constant tachyon $\hat{\mathcal{T}}_0$ minimizing the tachyon potential $V'(\hat{\mathcal{T}}_0) = 0$.

The tachyon e.o.m.

$$\nabla^\mu \left(e^{-2\phi}\partial_\mu \mathcal{T}\right) + V'(\hat{\mathcal{T}})$$

$$= h'(\mathcal{T}) \left[e^{h(\mathcal{T})} \sum_n \frac{(-)^n}{2\cdot n!} \hat{G}^{+\ 2}_{(n)} - e^{-h(\mathcal{T})} \sum_n \frac{(-)^n}{2\cdot n!} \hat{G}^{-\ 2}_{(n)}\right] ,$$

becomes a constraint that can be solved by having proportional numbers of $+$ and $-$ D_p branes. The remaining equations are, then, solved, by standard Type II D_p-brane solutions.

Thus

$$\begin{cases} ds^2 = H^{-1/2}\left(dt^2 - d\vec{y}^{\,2}_{(p)}\right) - H^{1/2} d\vec{x}^{\,2}_{(9-p)} \\ e^{2(\phi-\phi_0)} = H^{\frac{p-3}{4}} , \\ C^{+}_{(p+1)\,ty_1\cdots y_p} = \frac{1}{\sqrt{2}} e^{-h(\mathcal{T}_0)} H^{-1} \\ \hat{C}^{-}_{(p+1)\,ty_1\cdots y_p} = \pm\frac{1}{\sqrt{2}} e^{h(\mathcal{T}_0)} H^{-1} . \end{cases}$$

Remark: In this way **every** Type II brane solution can be embedded into a Type 0 theory.

CONCLUSIONS

★ We have determined to lowest order in α' the effective actions of the Type 0A and 0B theories, establishing T-duality between them.

★ In particular we have found the Chern-Simons terms in the field strengths and actions that determine which branes can end on which.

★ We have found a *democratic* form of the action in which RR forms of all ranks appear on equal footing.

★ We have shown how T-duality and other worldsheet symmetries constrain the form of the functions $f_{\pm}(\hat{T})$, but not that of the tachyon potential[a].

[a] The tachyon potential only contains even powers of $\hat{T}$ because all amplitudes involving an odd power vanish due to worldsheet supersymmetry.

Open problems:

★ Do these theories have some kind of M-theoretic origin?

★ What is the true vacuum of these theories?

★ ...