Non-supersymmetric (but) extreme BLACK HOLES, SCALAR HAIR & other open problems

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Karpacz '97

I Review SUSY BHs + { SWIP solutions / susy extremality

II Scalar charges versus SUSY & Duality

I SUSY BHs

Unbroken SUSY (local version)

A SUGRA theory's solutions are not invariant under SUSY transformations $\begin{cases} \delta_\varepsilon B \sim \varepsilon F \\ \delta_\varepsilon F \sim \partial\varepsilon + \varepsilon B \end{cases}$

Purely bosonic ($F=0$) solutions invariant under some SUSY transformations ($\varepsilon_{Killing}$) are supersymmetric.

$$\begin{cases} \delta_\varepsilon B \sim \varepsilon F = 0 \text{ automatically } \forall \varepsilon \\ \boxed{\delta_\varepsilon F \sim \partial \varepsilon_{Killing} + \varepsilon_{Killing} B = 0} \end{cases}$$ Killing spinor equation

* This is the equivalent of a "super-isometry" in superspace (not rigorously established).

Properties of supersymmetric configurations

– $\exists\ \varepsilon_{Killing} \Rightarrow \exists\ k^\mu_{Killing}$ $\quad k^\mu_{Killing} \sim \bar{\varepsilon}_{Killing}\gamma^\mu \varepsilon_{Killing}$

$k^\mu \begin{cases} \text{lightlike} \\ \text{timelike} \quad d=4 \\ \text{lightlike in } N=1,\ d=10 \\ \text{not a Killing vector in } d=11 \end{cases}$ *

- Supersymmetric configurations saturate B-bounds (only when they are asymptotically flat and massive)
  - Nester construction → GR positivity bounds
  - Associate configuration ⟶ $|\,\rangle$ in quantum theory

N-extended SUSY algebra for massive reps $\Rightarrow$ $M - |Z_i| \geq 0$, $i = 1 \ldots [N/2]$ — Witten Olive; Ferrara Savoy Zumino

$$\delta_\varepsilon \sim \varepsilon Q |\,\rangle$$

$$\Rightarrow \exists\, \varepsilon_{\text{killing}} \Rightarrow \exists\, Q \;/\; Q|\,\rangle = 0$$

$$\exists\, |Z_i| \;/\; M = |Z_i| \geq |Z_j|$$

$N=8$:

$$M = |Z_1| \geq |Z_2|, |Z_3|, |Z_4| \Rightarrow \frac{1}{8}$$

$$M = |Z_1| = |Z_2| \geq |Z_3|, |Z_4| \Rightarrow \frac{1}{4}$$

$$M = |Z_1| = |Z_2| = |Z_3| = |Z_4| \Rightarrow \frac{1}{2}$$ BPS (ground state)

- This is the only duality-invariant one
- Can usually be linearised
- maximal stability

- Supersymmetric configurations obey "no-force conditions" and multi-pole solutions can be found

$$\left.\begin{array}{l} M_1 = \pm Q_1 \\ M_2 = \pm Q_2 \end{array}\right\} \quad F_{12} = -\frac{M_1 M_2}{r_{12}^2} + \frac{Q_1 Q_2}{r_{12}^2} = 0$$

Supersymmetric embedding

A GR solution can be identified with many SUGRA solutions $\Leftrightarrow$ there are many possible "embeddings".

Example Einstein-Maxwell $S = \int R + F^2$

Electric RN BH solution $\begin{cases} ds^2 = H^{-2} W dt^2 - H^2 (W^{-1} d\rho^2 + \rho^2 d\Omega^2) \\ A_t = H^{-1} \end{cases}$

1) Einstein-Maxwell $\approx$ N=2 SUGRA

2) N=4 SUGRA
$$S = \int R + (\partial\phi)^2 + e^{4\phi}(\partial a)^2 + e^{-2\phi} \sum_{i=1}^{6} (F^i)^2 + a \sum_{i=1}^{6} F^{i} {}^*F^i$$

6 possible embeddings $A = A^i$? Wrong

The scalar equations have to be satisfied

$$\nabla^2 \phi + e^{-2\phi} \sum_i F^{i\,2} = 0 ;$$

!!

It is necessary at least 1 electric F
1 magnetic

$F^{(1)} \sim F$
${}^*F^{(2)} \sim F$ give the same RN metric $\Rightarrow$ embedding

Difference between local and global SUSY embeddings:

1) Global

BPS monopole $\subset$ $N=2$ SYM $\quad$ $1/2$ $(N=2)$ SUSY

BPS monopole $\subset$ $N=4$ SYM $\quad$ $1/2$ $(N=4)$ SUSY

2) Local

ERN BH $\subset$ $N=2$ SUGRA $\quad$ $1/2$ $(N=2)$ SUSY

ERN BH $\subset$ $N=4$ SUGRA $\quad$ $1/4$ $(N=4)$ SUSY

ERN BH $\subset$ $N=4+6V$ $\quad$ $\begin{cases} 1/4 & (N=4)\ \text{SUSY} \\ 0 & \end{cases}$

ERN BH $\subset$ $N=8$ $\quad$ $\begin{cases} 1/8 & (N=8)\ \text{SUSY} \\ 0 & !! \end{cases}$

Why?

N = 1 SUGRA $(g_{\mu\nu})$

B-bound: $M \geq 0$

BH-solutions Schwarzschild $(M) \xrightarrow[M \to 0]{}$ Minkowski

$T = \frac{1}{8\pi M} \xrightarrow[M \to 0]{} \infty$? $\Rightarrow 0$

$S = 4\pi M^2 \xrightarrow[M \to 0]{} 0$ ✓

Extremal & SUSY limit

(Kerr $M \to 0 \Rightarrow J \to 0$)

N=2 SUGRA (no matter) $(g_{\mu\nu}, A)$

B-bound: $M^2 \geq Q^2 + P^2$ (SUSY as a cosmic censor?)

BH solutions Reissner-Nordstrom (Papapetrou-Majumdar) $(M, \overset{z}{Q, P}) \xrightarrow[M \to |z|]{}$ ERN

$T = (\qquad) \xrightarrow[M \to |z|]{} 0$

$S = (\qquad) \xrightarrow[M \to |z|]{}$?

Extremal & SUSY limit

Euclidean semiclassical calculation at extremality $\Rightarrow$ $\underset{\sim\sim\sim}{S = 0}$

(Horowitz, Hawking, Kallosh, Gibbons, Teitelboim '94 !!)

Any more SUSY solutions?

$\Rightarrow$ Lots of them!

i) $M^2 = Q^2$ Kerr-Newman $\forall J$

Extremality limit $M^2 - Q^2 - J^2 \geq 0 \Rightarrow$ singular

asymptotically flat

ii) $M^2 + \ell^2 = Q^2$ charged Taub-NUT

NUT charge appears in the B-bound! {Kallosh, Kastor, Ortín}

iii) $M^2 + \ell^2 = Q^2$ Kerr-Newman-Taub-NUT (δ)

iv) IWP solutions

The only BHs in this class are RN (Hartle, Hawking)

$$\boxed{\begin{array}{l} \text{STATIC EXTREME} \approx \text{B-bound-saturating} \approx 1\ d=4\ \text{SUSY} \\ \text{ASYMPT. FLAT} \qquad\qquad\qquad\qquad (4\ \text{SUSY charges}) \\ \approx \text{regular horizon} \approx T=0 \\ \qquad\qquad\qquad\qquad\quad S\ ?? \end{array}}$$

From the $d=10$ heterotic string effective action to (pure) $N=4$, $d=4$ SUGRA

$$d=10 \quad S = \int d^{10}x\, e^{-2\hat\phi}\left[R(\hat g) - 4(\partial\hat\phi)^2 + \frac{1}{12}\hat H^2(\hat B) + \frac{1}{4}\sum_1 \hat F^2(\hat A^i)\right]$$

T^6

$d=4$: $g_{\mu\nu}$, $A_\mu^{(m)}$, g_{mn}, ϕ, $B_{\mu\nu}$, $B_\mu^{(m)}$, B_{mn}

(6) ——— (6) $\longrightarrow$ (12 V)

$N=4$ $d=4$ SUGRA + 6V multiplets

SUSY $\begin{cases} A_\mu^{(m)} \text{ matter} = A_\mu^{(m)} + B_\mu^{(m)} \longrightarrow =0 \Rightarrow g_{mn} = B_{mn} = 0 \\ A_\mu^{(m)} \text{ SUGRA} = A_\mu^{(m)} - B_\mu^{(m)} \end{cases}$

$$\Rightarrow \{g_{\mu\nu},\ A_\mu^{(m)\,\text{SUGRA}},\ B_{\mu\nu},\ \phi\} \quad N=4 \text{ SUGRA multiplet}$$

$$\partial_\mu a \sim \varepsilon_\mu{}^{\nu\rho\sigma}\partial_\nu B_{\rho\sigma}$$

$$S = \int d^4x \sqrt{|g|}\left\{R + \frac{1}{2}\frac{\partial\lambda\partial\bar\lambda}{(\operatorname{Im}\lambda)^2} - i\sum_m F^{(m)} {}^*\tilde F^{(m)}\right\}$$

$$\tilde F^{(m)} = e^{-2\phi}\,{}^*F^{(m)} - i_a F^{(m)} \Rightarrow \boxed{\begin{array}{l} dF^{(m)} = 0 \\ d\tilde F^{(m)} = 0 \end{array}}$$

$\lambda = a + i e^{-2\phi}$ $\qquad B_{\mu\nu}$

N=4 SUGRA $\quad (g_{\mu\nu}, \phi, a, A_\mu^{(n)})$

B-bounds $\left\{ \begin{array}{l} M^2 - |Z_1|^2 \geq 0 \\ M^2 - |Z_2|^2 \geq 0 \end{array} \right\} \quad (M^2 - |Z_1|^2)(M^2 - |Z_2|^2) \geq 0$

Universal, duality-invariant $\rightarrow$ $\boxed{M^2 + \frac{|Z_1|^2 |Z_2|^2}{M^2} - |Z_1|^2 - |Z_2|^2 \geq 0}$

B-bound but two possibilities $\left\{ \begin{array}{l} 1 \text{ susy} \\ 2 \text{ susy} \end{array} \right.$

scalar charges $\qquad$ U(1) charges

SUSY solutions with a time-like Killing vector

$\rightarrow$ SWIP solutions (Super Wilson-Israel-Perjes) $\left\{ \begin{array}{l} \text{Tod} \\ \text{Bergshoeff} \\ \text{Kallosh} \\ \text{O.} \end{array} \right.$

i) Choose any two complex harmonic functions

$$\partial_i \partial_i \begin{pmatrix} \mathcal{H}_1 \\ \mathcal{H}_2 \end{pmatrix} (\vec{x}) = 0$$

ii) Choose a set of complex constants $k^{(n)}$ satisfying

$$\sum_n k^{(n)2} = 0 \, ; \quad \sum_n |k^{(n)}|^2 = \frac{1}{2} \, ;$$

iii) Define $\quad e^{-2U} = 2 \, \mathrm{Im}(\mathcal{H}_1 \overline{\mathcal{H}_2})$

$$\partial_{[i}\omega_{j]} = \epsilon_{ijk} \, \mathrm{Re}(\mathcal{H}_1 \partial_k \overline{\mathcal{H}_2} - \overline{\mathcal{H}_2} \partial_k \mathcal{H}_1)$$

Then:

$$\boxed{\begin{aligned} &ds^2 = e^{2U}(dt+\omega_i dx^i)^2 - e^{-2U}d\vec{x}^2;\\ &\lambda = \mathcal{H}_1/\mathcal{H}_2;\\ &A_t^{(m)} = 2e^{2U}\,\mathrm{Re}\left(k^{(m)}\mathcal{H}_2\right);\\ &\tilde{A}_t^{(m)} = -2e^{2U}\,\mathrm{Re}\left(k^{(m)}\mathcal{H}_1\right); \end{aligned}}$$

* This class is duality invariant

$\begin{pmatrix}\mathcal{H}_1\\ \mathcal{H}_2\end{pmatrix} \to SL(2,\mathbb{R})$ doublet $\qquad k^{(m)} \to SO(6)$ vector

* All terms in the action have a special-geometrical meaning:

in $N=2$, $d=4$ $+1V$ with prepotential $F(X) = X^0X^1$

$X^0 = i\mathcal{H}_1$; $X^1 = \mathcal{H}_2$;

$e^{-K(X,\bar{X})} = e^{-2U}$; $\qquad K$ Kähler potential

$\mathcal{A}_\mu = -\delta_\mu{}^k\,\epsilon_{ijk}\,\partial_i\,\omega_j$; $\qquad \mathcal{A}_\mu$ chiral connection

$\mathcal{A}_\mu = 0 \Rightarrow$ static solution.

* $\mathcal{H}_1 = i\mathcal{H}_2 = \frac{1}{\sqrt{2}}V^{-1}$ $\qquad$ IWP solutions of $N=2$ embedded in $N=4$

* All these solutions are supersymmetric (some $\begin{matrix}1/4\\1/2\end{matrix}$) IWP

* Only these solutions are supersymmetric in $N=4$.

Let us look for BHs in the SWIP class

$$\mathcal{H}_1 = \frac{1}{\sqrt{2}} e^{\phi_0} e^{i\beta} \left(\lambda_0 + \frac{\lambda_0 \mathcal{M} + \bar{\lambda}_0 \mathcal{N}}{r} \right)$$

$$\mathcal{H}_2 = \frac{1}{\sqrt{2}} e^{\phi_0} e^{i\beta} \left(1 + \frac{\mathcal{M} + \mathcal{N}}{r} \right)$$

$$r^2 = x^2 + y^2 + (z - i\alpha)^2$$

After some transformations:

$$ds^2 = \frac{\Delta - \alpha^2 \sin^2\theta}{\Sigma} (dt - \omega\, d\varphi)^2 - \Sigma \left(\frac{d\rho^2}{\Delta} + d\theta^2 + \frac{\Delta \sin^2\theta\, d\varphi^2}{\Delta - \alpha^2 \sin^2\theta} \right)$$

$\alpha = J/M$; $\mathcal{M} = M + i\ell$

angular momentum — NUT charge

Always $\boxed{|\mathcal{M}|^2 + |\mathcal{N}|^2 - 4 \sum_n |\Gamma^{(n)}|^2 = 0}$

Universal B-bound

$$(|\mathcal{M}|^2 - |Z_1|^2)(|\mathcal{M}|^2 - |Z_2|^2) = 0 \qquad Z_i = f(\Gamma^{(n)}) \qquad \Gamma = Q + iP$$

Asymptotically Taub-NUT, rotating, charged "object"

$J \neq 0 \Rightarrow$ naked singularity always

$J = 0 \Rightarrow A = 8\pi \sqrt{(\tilde{q}\cdot\tilde{q})(\tilde{p}\cdot\tilde{p}) - (\tilde{q}\cdot\tilde{p})}$

1/4 (even for $\ell \neq 0$ (?))

1/2 $\Rightarrow A = 0$

General extreme stringy BHs

Is the relation (static + extreme) $\Rightarrow$ SUSY always true?

Stringy BHs can in general be written in this form:

$$A_{(i)t} = \alpha_i H_i^{-1} \; ; \quad \alpha_i = \pm 1 \; ; \quad \partial^2 H_i = 0$$

$$H_i = 1 + \frac{h_i}{\rho^{d-3}} , \quad h_i \geq 0 \text{ for BHs}$$

$q_i = \alpha_i h_i$ can be positive or negative

$$ds^2 = \left(\prod_{i=1}^{m} H_i^{-2r_i}\right) dt^2 - \left(\prod_{i=1}^{m} H_i^{-2r_i}\right)^{-\frac{1}{d-3}} d\vec{x}^2$$

$\sum_{i=1}^{m} r_i = 1$ and, in most cases $r_i = \frac{1}{m} \;\; \forall i$

$$g_{tt} \sim 1 - \frac{2M}{\rho^{d-3}} \Rightarrow \boxed{M = \sum_{i=1}^{i=m} r_i |q_i|}$$

This looks like a B-bound $M = |Z|$. Is it?

Depending on the $2^{(m-1)}$ possible relative signs of the charges

$$\boxed{M = \left|\sum_{q_i>0} r_i q_i - \sum_{q_i<0} r_i q_i\right|}$$

For N-extended SUGRA only $[N/2]$ of these combinations are B-bounds $\Rightarrow$ in many cases they are not B-bounds $\Rightarrow$ the extreme BHs they are not supersymmetric

$N=2$; $n=1$; $2^{n-1}=1$; $[N/2]=1$; SUSY $\Leftrightarrow$ extreme

$N=4$ (pure); $n=2$; $2^{n-1}=2$, $[N/2]=2$; !!

$N=8$; $n=4$; $2^{n-1}=8$, $[N/2]=4$; !!

$N=4+6V$; $n=4$ $2^{n-1}=8$, $[N/2]=2$; !!

What is the meaning of the extremality bounds which are not B-bounds?

- In $N=4+6V$ ($\subset N=8$) 2 of those can be understood as $N=8$ bounds.

- The remaining 4 could be explained in terms of "$N=16$" $\rightarrow$ $\boxed{\text{S-theory?}}$

SCALAR CHARGES versus SUSY & DUALITY

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Consider Einstein + scalar $S = \int R + 2(\partial\phi)^2$

There is no conserved charge associated to ϕ

no-hair "theorem" $\Rightarrow$ there are no static BH solutions with $\phi \neq$ constant (Schwarzschild).

$\Leftrightarrow$ if we define the scalar charge Σ

$$\phi \sim \phi_0 + \frac{\Sigma}{r} \qquad (r \to \infty)$$

there are no static BHs with $\Sigma \neq 0$.

Physical interpretation: Σ should flow to ∞ in the gravitational collapse before the BH reaches the static limit.

In fact, the only known static solutions are

Janis
Newman
Winicour
Agnese
La Camera

$$\boxed{\begin{cases} ds^2 = W^{\frac{M}{r_0}-1} W dt^2 - W^{-\left(\frac{M}{r_0}-1\right)}\left[W^{-1} dr^2 + r^2 d\Omega^2\right]; \\ \phi = \phi_0 + \frac{\Sigma}{2r_0} \ln W; \\ W = 1 - \frac{2r_0}{r}; \qquad r_0^2 = M^2 + \Sigma^2; \end{cases}}$$

and are singular (not BHs) $\forall \Sigma \neq 0$

- These are perfectly good solutions of the string effective action.

non-singular

- We have seen solutions with (allowed) secondary scalar hair $\Sigma \sim \frac{P^2 Q^3}{2M}$

- We do not have a string interpretation for primary scalar hair (just as we do not know what the NUT charge ℓ or the angular momentum J is in this picture).

- Solutions with primary scalar hair exist in all dimensions:

$$\boxed{\begin{cases} ds^2 = W^{-2x} W dt^2 - W^{\frac{2x}{d-3}} \left[W^{-1} dr^2 + r^2 d\Omega^2_{(d-2)} \right] \\ \phi = \phi_0 + \frac{1}{2}\left[\frac{(d-2)}{(d-3)}\left(x - x^2\right)\right]^{1/2} \log W \end{cases}}$$

$\Rightarrow$ So we have a general conceptual problem to be understood.

What does SUSY say about this?

NOTHING!

Primary scalar charges do not appear in B-bounds, just as it happens to the angular momentum J. (Secondary charges formally appear in the "universal B-bound"

$$(M^2-|Z_1|^2)(M^2-|Z_2|^2) \geq 0$$

$$M^2 + \underbrace{\frac{|Z_1|^2|Z_2|^2}{M^2}}_{\sim\, \Sigma \,\sim\, \frac{P^2-Q^2}{2M}} - |Z_1|^2 - |Z_2|^2 \geq 0$$

If we can associate a state to the solution SUSY only tells us that $M \geq 0$. So, perhaps in the massless limit we have unbroken SUSY

$$M \longrightarrow 0 \qquad \begin{cases} ds^2 = dt^2 - dr^2 - W r^2 d\Omega^2 \\ \phi = \phi_0 + \frac{1}{2}\ln W \neq 0 \end{cases}$$

The dilatino SUSY transformation rule is

$$\delta_\epsilon \lambda \sim \not{\partial}\phi\,\epsilon \neq 0 \quad \forall \epsilon \neq 0$$

$\Rightarrow$ NEVER unbroken SUSY (?)

Why?

Observe that this solution is T-dual to Schwarzschild's! $M \rightleftarrows \Sigma$ (F. Quevedo's lectures)

Unbroken SUSY is preserved by T-duality generically.

{ Bakas, Bergshoeff, Kallosh, Ortín, Álvarez, Álvarez-G, [illegible], ⋮ }

$\Rightarrow$ Shouldn't one extend the B-bound to include primary scalar hair? $M^2 + \Sigma^2 \geqslant 0$ (duality invariant)

Remember that $\ell \rightleftarrows \Delta \quad \Rightarrow \quad (M^2 + \ell^2) + (\Sigma^2 + \Delta^2) \geqslant 0$

Idea: perhaps J is related by T duality to a charge appearing in the B-bound and imposing T-duality invariance J should be included and the B-bound would be identical to the extremality bound.

Wrong
(It does not work)

$$\boxed{J \rightleftarrows \Delta'}$$

CONCLUSIONS

There are many points to be understood from the string theory point of view:

- There are BHs with identical metrics and stability properties but with different SUSY properties so the (Dabholkar) D-brane counting seems not to explain the same entropy formula.
- There is no clear string picture for charges like NUT (ℓ), angular momentum (J), and primary scalar charges, which usually cause singularities to appear.

include all of them (primary scalar hair?)

?

STRING BUILDING BLOCKS

exclude all of them (angular momentum?)