Supersymmetric Black Holes in $N=8$ Supergravity & Duality

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Introduction/Motivation:

The extreme ($a=1$) dilaton black hole problem

The extreme ($a=1$) dilaton black hole is a classical solution of the theory

$$S = \int d^4x \sqrt{|g|}\, e^{-2\phi}\left[-R + 4(\partial\phi)^2 + \frac{1}{2}F^2\right]$$

(Gibbons
Gibbons & Maeda
Garfinkle, Horowitz
Strominger)

In the paper by Garfinkle, Horowitz & Strominger it was said that, even though it saturates a Bogomol'nyi-type bound, it is not supersymmetric because

$$\nexists\, \epsilon \;/\; \delta\xi \sim \not{F}\, \epsilon = 0 .$$

(photino supersymmetry transformation law)

The authors were assuming, implicitly that the vector field was a matter vector field (one of the 16 $U(1)$'s of the heterotic string).

Later, it was proven (Kallosh, Linde, O., Peet, van Proeyen) that if the vector field was a supergravity vector field of $N=4, D=4$ supergravity $(g_{\mu\nu}, A^I_\mu, B^I_\mu, \phi)$ $I=1,2,3$

$$\delta\psi^I_\mu \sim \nabla_\mu \epsilon^I + \not{F}^{IJ}\gamma_\mu \epsilon^J = 0$$

(gravitino supersymmetry transformation law)

$\delta\xi = 0$ automatically $\exists\, \epsilon^I$

This result seems a bit paradoxical : the "same" configuration can be considered supersymmetric or not supersymmetric

Where is the difference?

$N=4$, $D=4$ supergravity vector fields V_μ are combinations $V_\mu = A_\mu^{(1)} - A_\mu^{(2)}$

$A_\mu^{(1)} \longrightarrow$ from the $D=10$ metric

$A_\mu^{(2)} \longrightarrow$ from the $D=10$ axion

$N=4$, $D=4$ matter vector fields D_μ are combinations

$D_\mu = A_\mu^{(1)} + A_\mu^{(2)}$;

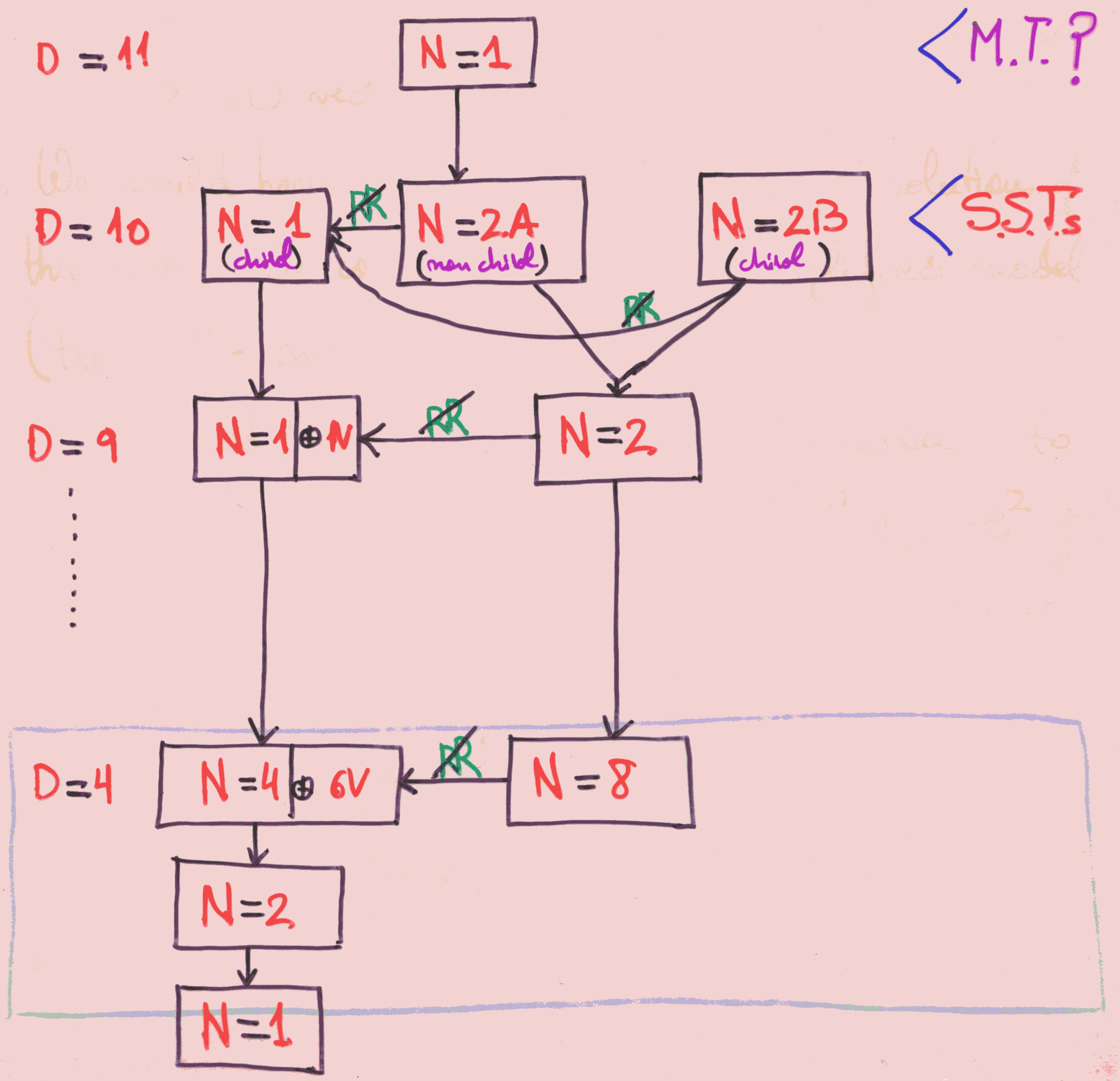
Then, we have two different ten-dimensional configurations (one supersymmetric, one not supersymmetric) that reduce to the same $D=4$ configuration.

- Are there any more of such embeddings?
- What are their supersymmetries?
- What happens for other dilaton black holes?
- Why different embeddings have different supersymmetries?

$\rightsquigarrow \boxed{N=8}$

Supergravities

The low-energy effective field theories of superstring theories are supergravity theories which describe the massless part of the spectrum:



The typical $D=4$ Lagrangean one finds is of the general form

$$S=\int d^4x\sqrt{|g|}\Big[-\tilde{R}-g_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j+\tfrac{1}{2}M_{IJ}(\phi)F^{\mu\nu I}F_{\mu\nu}{}^J$$
$$+N_{IJ}(\phi)\,F^{\mu\nu I}\,{}^*F_{\mu\nu}{}^J\Big]$$

$\phi^i \longrightarrow$ scalars (σ-model with metric $g_{ij}(\phi)$)

$A_\mu{}^I \longrightarrow$ U(1) vector fields

We would have to study black-hole-type solutions of this action. We will do it in a simplified model (the "a"-model). However

i) We need/want to identify the dilaton ϕ to be able to find the "string metric" $g_{\mu\nu}=e^{2\phi}\tilde{g}_{\mu\nu}$

ii) We need to know the supersymmetry transformation rules.

All this is simpler if we derive the action and the supersymmetry transformation rules from $D=10$ in the string metric.

This is our strategy:

1) We get the supergravity theory in $D=10$ from $D=11$
   a) Bosonic Lagrangian
   b) Fermionic supersymmetry rules
2) We get the bosonic Lagrangian in $D=4$ (and the relations between the $D=10$ and $D=4$ fields)
3) We identify the σ-model solutions with solutions of the supergravity theory.
4) We rewrite the solutions in $D=10$ and study the unbroken supersymmetry in $D=10$.
5) In some cases, the study of the unbroken supersymmetry of one chiral half of the $N = \left(\begin{matrix}2A\\8\end{matrix}\right)$ theory will be enough if we use "c-duality".

The "a - model"

$$S = \int d^4x \sqrt{|\tilde{g}|}\left[-\tilde{R} - 2(\partial\varphi)^2 - \frac{1}{2}e^{-2a\varphi}F^2\right]$$

This model has black-hole-type solutions for all "a" and also extreme-multi-black-hole solutions which clearly are the only ones with chances of being supersymmetric:

(Gibbons, Gibbons & Maeda, Garfinkle, Horowitz & Str., Holzhey & Wilczek)

$$\begin{cases} d\tilde{s}^2 = V^{-\frac{2}{1+a^2}}dt^2 - V^{\frac{2}{1+a^2}}d\vec{x}^2\,; \\ e^{\varphi} = V^{-\frac{a}{1+a^2}}\,; \\ F_{t\underline{i}} = \pm\sqrt{\frac{2}{1+a^2}}\,\partial_{\underline{i}}V^{-1}\,; \end{cases}$$

(Shiraishi, T.O., others)

$$\begin{cases} a = 0 \longrightarrow \text{ERN} + \text{constant scalar} \\ a = \begin{Bmatrix}\sqrt{3} \\ 1/\sqrt{3}\end{Bmatrix} \longrightarrow \text{KK-bh / H-monopole} \\ a = 1 \longrightarrow \text{Extreme Dilaton bh in string theory} \\ \qquad \varphi \longrightarrow \phi \quad \text{(the dilaton)} \end{cases}$$

These are the known and expected cases.

Only the $a = \sqrt{3}$ can be BPS. (Shiraishi, Gauntlett, ...)

Reduction from $D=11$ to $D=10$

Huq & Namazie
Bergshoeff, De Wit, D.R.
Bergshoeff, Hull, T.O.
T.O.

The fields of $N=1$, $D=11$ supergravity

$$\left\{\hat{\hat g}_{\hat{\hat\mu}\hat{\hat\nu}},\ \hat{\hat C}_{\hat{\hat\mu}\hat{\hat\nu}\hat{\hat\rho}}\right\} \qquad \left\{\hat{\hat\psi}_{\hat{\hat\mu}}\right\} \qquad \hat{\hat G} = \partial \hat{\hat C}$$

The bosonic part of the action is

$$\hat{\hat S} = \int d^{11}x \sqrt{|\hat{\hat g}|}\left[-\hat{\hat R} + a_1 \hat{\hat G}^2 + a_2 \frac{\hat{\hat\varepsilon}}{\sqrt{|\hat{\hat g}|}}\hat{\hat G}\hat{\hat G}\hat{\hat C}\right]$$

and the supersymmetry transformation rule of $\hat{\hat\psi}_{\hat{\hat\mu}}$ in absence of fermionic fields is

$$\frac{1}{\sqrt2}\delta\hat{\hat\psi}_{\hat{\hat\mu}} = \hat{\hat\nabla}_{\hat{\hat\mu}}\hat{\hat\epsilon} - 6i\frac{a_2}{a_1}\left(\hat{\hat\gamma}^{\hat{\hat\alpha}\hat{\hat\beta}\hat{\hat\gamma}\hat{\hat\delta}}{}_{\hat{\hat\mu}} - 8\,\hat{\hat\gamma}^{\hat{\hat\beta}\hat{\hat\gamma}\hat{\hat\delta}}\delta_{\hat{\hat\mu}}{}^{\hat{\hat\alpha}}\right)\hat{\hat G}_{\hat{\hat\alpha}\hat{\hat\beta}\hat{\hat\gamma}\hat{\hat\delta}}\,\hat{\hat\epsilon}$$

The ansatz that takes us to $N=2A$ in $D=10$ in the string metric is

$$\left(\hat{\hat e}_{\hat{\hat\mu}}{}^{\hat{\hat a}}\right) = \begin{pmatrix} e^{-\frac13\hat\phi}\,\hat e_{\hat\mu}{}^{\hat a} & e^{\frac23\hat\phi}\,\hat A^{(1)}_{\hat\mu} \\ 0 & e^{\frac23\hat\phi} \end{pmatrix}$$

$$\begin{cases} \hat\epsilon = e^{\frac16\hat\phi}\,\hat{\hat\epsilon}\,; \\ \hat\psi_{\hat a} = \dfrac{e^{-\frac16\phi}}{\sqrt2}\left(\hat{\hat\psi}_{\hat a} - \dfrac{i}{2}\hat\gamma_{\hat a}\hat\gamma_{11}\hat{\hat\psi}_{11}\right); \\ \hat\lambda = -\dfrac{3i}{\sqrt2}\,e^{-\frac16\hat\phi}\,\hat\gamma_{11}\,\hat\psi_{\hat a}\,; \end{cases}$$

$$\begin{cases} \hat{\hat C}_{\hat\mu\hat\nu\hat\rho} = \hat C_{\hat\mu\hat\nu\hat\rho}\,; \\ \hat{\hat C}_{\hat\mu\hat\nu\underline{x}} = \frac23\hat B^{(1)}_{\hat\mu\hat\nu}\,; \end{cases}$$

$$\hat{\hat\gamma}^{\hat a} = \hat\gamma^{\hat a}$$

$$\hat{\hat\gamma}^{10} = -i\hat\gamma_{11} \qquad (\hat\gamma_{11})^2 = +1$$

One can define chirality

The result is:

$$\hat{S} = \int d^{10}x \sqrt{|\hat{g}|} \Big\{ \overbrace{e^{-2\hat{\phi}} \Big[-\hat{R} + 4(\partial\hat{\phi})^2 - \tfrac{3}{4}(\hat{H}^{(1)})^2 \Big]}^{\text{NS-NS sector}} + \underbrace{\tfrac{1}{4}(\hat{F}^{(1)})^2 + \tfrac{3}{4}\hat{G}^2 + 2^{-7}\frac{\hat{\varepsilon}}{\sqrt{|\hat{g}|}}\partial\hat{C}\partial\hat{C}\hat{B}^{(1)}}_{\text{RR sector}} \Big\}$$

$$\begin{cases} \delta\hat{\psi}_{\hat{a}} = \partial_{\hat{a}}\hat{\varepsilon} - \tfrac{1}{4}\left(\hat{\omega}_{\hat{a}\hat{b}\hat{c}} - \tfrac{3}{4}\hat{H}_{\hat{a}\hat{b}\hat{c}}\hat{\gamma}_{11}\right)\hat{\gamma}^{\hat{b}\hat{c}}\hat{\varepsilon} - \tfrac{i e^{\hat{\phi}}}{16}\left(\hat{\gamma}_{\hat{a}}{}^{\hat{b}\hat{c}} - 2\delta_{\hat{a}}{}^{\hat{b}}\hat{\gamma}^{\hat{c}}\right)\hat{\gamma}_{11}\hat{F}^{(1)}_{\hat{b}\hat{c}}\hat{\varepsilon} \\ \qquad - \tfrac{i}{32}e^{\hat{\phi}}\left(\hat{\gamma}_{\hat{a}}{}^{\hat{b}\hat{c}\hat{d}\hat{e}} - 4\delta_{\hat{a}}{}^{\hat{b}}\hat{\gamma}^{\hat{c}\hat{d}\hat{e}}\right)\hat{G}_{\hat{b}\hat{c}\hat{d}\hat{e}}\hat{\varepsilon}\,; \\ \delta\hat{\lambda} = \left(\hat{\cancel{\partial}}\hat{\phi} - \tfrac{1}{4}\hat{\cancel{H}}^{(1)}\hat{\gamma}_{11}\right)\hat{\varepsilon} - \tfrac{i}{8}e^{\hat{\phi}}\left(3\hat{\cancel{F}}^{(1)}\hat{\gamma}_{11} + \tfrac{1}{2}\hat{\cancel{G}}\right)\hat{\varepsilon}\,; \end{cases}$$

If we only keep the NS-NS sector (which is a consistent truncation) and we split all spinors into their two chiral halfs $\boxed{\hat{\gamma}_{11}\,\hat{\varepsilon}^{(\pm)} = \pm\hat{\varepsilon}^{(\pm)}}$ etc.

$$\hat{S} = \int d^{10}x \sqrt{|\hat{g}|}\, e^{-2\hat{\phi}}\Big[-\hat{R} + 4(\partial\hat{\phi})^2 - \tfrac{3}{4}(\hat{H}^{(1)})^2\Big]$$

$$\begin{cases} \delta\hat{\psi}^{(\pm)}_{\hat{a}} = \hat{\nabla}^{(\pm)}_{\hat{a}}\hat{\varepsilon}^{(\pm)}\,; \\ \delta\hat{\lambda}^{(\pm)} = \left(\hat{\cancel{\partial}}\hat{\phi} \pm \tfrac{1}{4}\hat{\cancel{H}}^{(1)}\right)\hat{\varepsilon}^{(\pm)}\,; \end{cases} \qquad \begin{aligned} \hat{\nabla}^{(\pm)}_{\hat{a}} &= \partial_{\hat{a}} - \tfrac{1}{4}\hat{\Omega}^{(\pm)}_{\hat{a}\hat{b}\hat{c}}\hat{\gamma}^{\hat{b}\hat{c}}\,; \\ \hat{\Omega}^{(\pm)}_{\hat{a}\hat{b}\hat{c}} &= \hat{\omega}_{\hat{a}\hat{b}\hat{c}} \mp \tfrac{3}{2}\hat{H}^{(1)}_{\hat{a}\hat{b}\hat{c}}\,; \end{aligned}$$

This describes the NS-NS sector of the $N = 2A$ for our purposes. Keeping one chirality we get the $N = 1$ $(\pm)$.

Then:

- Truncating the bosonic RR fields of the $N=2A$ theory one gets the bosonic fields of the $N=1$.
- But the supersymmetry rules of the truncated $N=2A$ theory are equivalent to those of two $N=1$ theories of opposite chirality.
- These two $N=1$ $(\pm)$ theories are related by a "c-duality" transformation.

$$\begin{cases} \pm \longrightarrow \mp & \text{(chiralities of the spinors)} \\ \hat{B}^{(1)}_{\hat\mu\hat\nu} \longrightarrow -\hat{B}^{(1)}_{\hat\mu\hat\nu} & \left(\Rightarrow \begin{array}{l} \hat{\Omega}^{(\pm)} \longrightarrow \hat{\Omega}^{(\mp)} \\ \pm\hat{H}^{(1)} \longrightarrow \mp\hat{H}^{(1)} \end{array} \right) \end{cases}$$

(Bergshoeff Janssen T.O.)

- This c-duality transformation is a symmetry of the $N=2A$ theory but not of the $N=1$ theory. In fact, it is a string/string-type duality between two different $N=1$ theories of opposite chiralities $N=1^{(+)}$, $N=1^{(-)}$

The next step to get closer to the a-model is

Dimensional reduction from $N=1, D=10$ to $N=4, D=4 \oplus 6V$

(Chamseddine ⋮)

The fields that one expects in $D=4$ are

$$\begin{cases} \hat{g}_{\hat{\mu}\hat{\nu}} \longrightarrow g_{\mu\nu}, \; A^{(1)m}_{\mu}, \; G_{mn}; & m = 1, \cdots, 6 \\ \hat{B}^{(1)}_{\hat{\mu}\hat{\nu}} \longrightarrow B_{\mu\nu}, \; A^{(2)}_{\mu m}, \; B_{mn}; \\ \hat{\phi} \longrightarrow \phi; \end{cases}$$

The corresponding action is, in the canonical metric $\tilde{g}$

$$S = \int d^4x \sqrt{|\tilde{g}|} \Big\{ -\tilde{R} - 2(\partial\phi)^2 - \frac{3}{4} e^{-4\phi} H^2 + \frac{1}{4}\left[\partial G_{mn} \partial G^{mn} - G^{mn} G^{pq} \partial B_{mp} \partial B_{nq}\right] - \frac{1}{4} e^{-2\phi}\left[G_{mn} F^{(1)m} F^{(1)n} + G^{mn} \mathcal{F}_m \mathcal{F}_n\right] \Big\} ;$$

$$\mathcal{F}_m = F^{(2)}{}_m + F^{(1)q} B_{qm} ;$$

(Maharana & Schwarz)

$$H = \partial B - \frac{1}{2} A^{(1)m} F^{(2)}{}_m - \frac{1}{2} A^{(2)}{}_m F^{(1)m}$$

$$\Rightarrow \boxed{\partial H \sim \sum_m F^{(1)m} * F^{(2)}{}_m}$$

The effect of C-duality on the $D=4$ fields is

$$\boxed{\begin{array}{l} B_{mn} \longrightarrow -B_{mn} \\ B_{\mu\nu} \longrightarrow -B_{\mu\nu} \\ A^{(2)}_{\mu m} \longrightarrow -A^{(2)}_{\mu m} \end{array} \Rightarrow \begin{cases} H_{\mu\nu\rho} \longrightarrow -H_{\mu\nu\rho} \\ \mathcal{F}_{m\mu\nu} \longrightarrow -\mathcal{F}_{m\mu\nu} \end{cases}}$$

Dimensional reduction of the supersymmetry rules from $D=10$ to $D=4$

We only need to reduce a chiral half and use C-duality. We do not need to reduce the spinor indices for our purposes. We take $N=1^{(+)}$

$$\begin{cases} \hat{\psi}_{\hat{a}}^{(+)} \longrightarrow \psi_a^{(+)},\ \psi_i^{(+)}; \\ \hat{\lambda}^{(+)} \longrightarrow \lambda^{(+)}\left(=\lambda^{(+)}-\hat{\gamma}^i\psi_i^{(+)}\right); \end{cases}$$

$$\left.\begin{array}{l} \psi_a^{(+)} \rightarrow 4 \text{ gravitini} \\ \lambda^{(+)} \rightarrow 4 \text{ dilatini} \end{array}\right] \begin{array}{l} N=4,\ D=4, \\ \text{supergravity} \\ \text{multiplet} \end{array}$$

$$\left.\psi_i^{(+)} \rightarrow 4\times 6 \text{ photini}\right\} \rightarrow 6V \text{ of } N=4$$

$$\begin{cases} \delta\psi_a^{(+)} = \nabla_a^{(+)}\hat{\varepsilon}^{(+)} - \frac{1}{4}\left[\not{F}_i^{(1)} - \not{\tilde{F}}_i\right]\hat{\gamma}^i\tilde{\varepsilon}^{(+)} + \left(\partial_a e_{mj} + \partial_a B_{mn}\, g_j^n\right)e_i^m\,\hat{\gamma}^{ij}\,\hat{\varepsilon}^{(+)}; \\ \delta\lambda^{(+)} = \left\{\not{\partial}\phi + \frac{1}{4}\not{H} - \frac{1}{8}\left[\not{F}_i^{(1)} - \not{\tilde{F}}_i\right]\hat{\gamma}^i\right\}\hat{\varepsilon}^{(+)}; \end{cases}$$

$$\delta\psi_i^{(+)} = \frac{1}{8}\left[\not{F}_i^{(1)} + \not{\tilde{F}}_i\right]\hat{\varepsilon}^{(+)} - \frac{1}{4}\left(\not{\partial} g_{mn} + \not{\partial} B_{mn}\right)e_i^m e_j^n\,\hat{\gamma}^j\,\hat{\varepsilon}^{+};$$

$\psi_a^{(+)}$ and $\lambda^{(+)}$ always transform into $\left(F_i^{(1)} - \tilde{F}_i\right)$.

$\psi_i^{(+)}$ always transforms into $\left(F_i^{(1)} + \tilde{F}_i\right)$.

$$\boxed{\text{Then, C-duality interchanges supergravity vectors with matter vectors.}}$$

Observe:

In the presence of matter vector fields, the Killing spinor equations are difficult to satisfy: $\delta\psi_i^{(+)} = 0$ is the problem.

However, a C-duality transformation takes the matter vector fields onto supergravity vector fields, and, then unbroken supersymmetry is possible in the $N=1^{(-)}$ theory (the C-dual).

Then, if we know that for ($a=1$) dilaton black holes embedded in $N=4^{(-)}$, $D=4$, only when we say that the vector is a supergravity vector we have unbroken supersymmetry, by C-duality we find that when we say that the vector is a matter vector, we have unbroken supersymmetry in the $N=4^{(+)}$ theory.

	$N=4^{(+)}$	$N=4^{(-)}$
supergravity (+)	$\frac{1}{2}$	0
matter (+)	0	$\frac{1}{2}$

To get to the σ-model we need further truncations:

$$\boxed{\begin{aligned} &B_{\mu\nu}=0\,; \quad H=0\,; \\ &B_{mn}=0\,; \quad G_{mn}=-e^{2\rho_m}\delta_{mn}\,; \end{aligned}}$$

The eqs. of mo. of the vanishing fields generate constraints:

$$\begin{aligned} G_{mn}=0 \ \ \forall m\neq n \ &\Rightarrow \quad \boxed{F^{(1)m}F^{(1)n} = e^{-2(\rho_m+\rho_n)}F^{(2)}_{m}F^{(2)}_{n}} \\ B_{mn}=0 \ &\Rightarrow \quad \boxed{F^{(1)m}F^{(2)}_{n} = e^{-2(\rho_m-\rho_n)}F^{(1)n}F^{(2)}_{m}} \end{aligned}$$

Also the consistency of $H_{\mu\nu\rho}=0$, $B_{\mu\nu}=0$ with the Bianchi identity generates the local constraint

$$\boxed{\sum_m F^{(1)m}\,{}^*F^{(2)}_{m}=0}$$

The equations of motion of the remaining fields are:

$$\begin{cases} \tilde G_{\alpha\beta} + 2T^{\phi}_{\alpha\beta} + \sum_m T^{\rho_m}_{\alpha\beta} - \frac{1}{2}e^{-2\phi}\sum_m\left(e^{2\rho_m}T^{(1)m}_{\alpha\beta} + e^{-2\rho_m}T^{(2)}_{\alpha\beta\,m}\right)=0\,; \\ \tilde\nabla^2\phi - \frac{1}{8}e^{-2\phi}\sum_m\left[e^{2\rho_m}\left(F^{(1)m}\right)^2 + e^{-2\rho_m}\left(F^{(2)}_{m}\right)^2\right]=0\,; \\ \tilde\nabla^2\rho_m + \frac{1}{4}e^{-2(\phi-\rho_m)}\left(F^{(1)m}\right)^2 - \frac{1}{4}e^{-2(\phi+\rho_m)}\left(F^{(2)}_{m}\right)^2=0\,; \\ \tilde\nabla_\mu\left(e^{-2(\phi-\rho_m)}F^{(1)m\,\mu\alpha}\right)=0\,; \\ \tilde\nabla_\mu\left(e^{-2(\phi+\rho_m)}F^{(2)\,\mu\alpha}_{m}\right)=0\,; \end{cases}$$

Our objective is to choose $\phi, \rho_m, F^{(1)m}, F^{(2)}{}_m$ in such a way that all constraints are satisfied and the system of eqs. of mo. reduces to that of the a-model:

$$\begin{cases} \tilde{G}_{\alpha\beta} + 2T_{\alpha\beta}{}^{\varphi} - e^{-2a\varphi}T_{\alpha\beta} = 0; \\ \tilde{\nabla}^2\varphi - \frac{a}{4}e^{-2a\varphi}F^2 = 0; \\ \tilde{\nabla}_\mu\left(e^{-2a\varphi}F^{\mu\alpha}\right) = 0; \end{cases}$$

Since this model only has a vector and a scalar, we have to take

$$\boxed{F^*F = 0}$$

$$\boxed{\begin{array}{l|l} \vec{F}^{(1)} = \vec{n}F + \vec{p}\,{}^*F & \rho_m = c_m\varphi \\ \vec{F}^{(2)} = \vec{m}F + \vec{q}\,{}^*F & \phi = b\varphi \end{array}}$$

$c_m, b \to$ constants
$\vec{n}, \vec{p}, \vec{q}, \vec{m}$
functions of φ

This is a non-trivial problem, but all solutions can be found. As expected, there are solutions only for the values of $a = \sqrt{3}, 1, 1/\sqrt{3}, 0$. There is more than one solution for each a, even taking into account heterotic ($N=4$) dualities.

All possible solutions in this framework are, up to heterotic dualities:

a	ϕ	ρ_1	ρ_2	e^{φ}	$F^{(1)1}$	$F^{(2)}_1$	$F^{(1)2}$	$F^{(2)}_1$	(m_+, m_-)	Total	
$a=\sqrt{3}$ (KK BH)	$\frac{1}{\sqrt{3}}\varphi$	$-\frac{2}{\sqrt{3}}\varphi$	0	$V^{-\frac{\sqrt{3}}{4}}$	$\sqrt{2}F$	0	0	0	$(\frac{1}{2}, \frac{1}{2})$	$\frac{1}{2}$	BPS
(H-mono)	$\frac{1}{\sqrt{3}}\varphi$	$+\frac{2}{\sqrt{3}}\varphi$	0		0	$\sqrt{2}F$	0	0	$(\frac{1}{2}, \frac{1}{2})$		T-duals
$a=1$ (DBH)	φ	0	0	$V^{-1/2}$	F	$-F$	0	0	$(\frac{1}{2}, 0)$	$\frac{1}{4}$	
	φ	0	0		F	$+F$	0	0	$(0, \frac{1}{2})$		c-duals
	0	$-\varphi$	φ		F	0	$e^{-2\varphi}{}^*F$	0	$(\frac{1}{4}, \frac{1}{4})$		
$a=1/\sqrt{3}$	$-\frac{1}{\sqrt{3}}\varphi$	$-\frac{2}{\sqrt{3}}\varphi$	0	$V^{-\frac{\sqrt{3}}{4}}$	$\sqrt{\frac{2}{3}}F$	0	$\sqrt{\frac{2}{3}}e^{2\phi}{}^*F$	$-\sqrt{\frac{2}{3}}e^{2\phi}{}^*F$	$(\frac{1}{4}, 0)$	$\frac{1}{8}$	
	$-\frac{1}{\sqrt{3}}\varphi$	$-\frac{2}{\sqrt{3}}\varphi$	0		$\sqrt{\frac{2}{3}}F$	0	$\sqrt{\frac{2}{3}}e^{2\phi}{}^*F$	$+\sqrt{\frac{2}{3}}e^{2\phi}{}^*F$	$(0, \frac{1}{4})$		c-duals
	?	?	?		?	?	?	?	$(\frac{1}{8}, \frac{1}{8})$		
$a=0$ (ERN)	0	0	0	1	$\frac{1}{\sqrt{2}}F$	$-\frac{1}{\sqrt{2}}F$	$\frac{1}{\sqrt{2}}{}^*F$	$-\frac{1}{\sqrt{2}}{}^*F$	$(\frac{1}{4}, 0)$	$\frac{1}{8}$	
	0	0	0		$\frac{1}{\sqrt{2}}F$	$+\frac{1}{\sqrt{2}}F$	$\frac{1}{\sqrt{2}}{}^*F$	$\frac{1}{\sqrt{2}}{}^*F$	$(0, \frac{1}{4})$		c-duals
	0	0	0		?	?	?	?	$(\frac{1}{8}, \frac{1}{8})$		
	0	0	0		$F \pm {}^*F$	0	0	0	$(0, 0)$	0	?

Comments on the results

1) In $N=8$ all the different embeddings of the same solution (same a) have the same supersymmetry (except for the dyonic ERN)

2) When we study only one chirality ($N=4$) different embeddings have different unbroken "susys". This is due to the fact that in some cases the vectors are supergravity and, in others, matter vectors, while in $N=8$ all are supergravity vectors.

3) Since the $\begin{pmatrix} N=4 \\ N=8 \end{pmatrix}$ dualities respect supersymmetry (in general) the different embeddings shown must be related by dualities of $N=8$ which are not dualities of $N=4$ (for instance C-duality which transforms supergravity vectors onto matter vectors in $N=4$.)

4) Where are the "symmetric" RR embeddings $a = \frac{1}{\sqrt{3}}$ $\left(\frac{1}{8}, \frac{1}{8}\right)$ and $a=0$ $\left(\frac{1}{8}, \frac{1}{8}\right)$? $\longrightarrow$ They do not exist?

Conclusion

We have solved the paradox "The extreme dilaton black hole in $N=4$ supergravity" of configurations being supersymmetric or not in $N=4$ depending on the identifications of the vector fields.
Apparently, whenever there is a bound

$M^2 \geqslant$ quadratic combination of charges

saturated, the solution is supersymmetric in the theory with maximal supersymmetry.

Questions:

- What about the dyonic embedding of the ERN?
- What is the situation in the type IIB?
- What happens when RR fields are included?

⋮

The situation in the type IIB theory

The type IIB and IIA theories have the same degrees of freedom organised in two different ways:

Both have in common the NS-NS sector ($\to$ type I)

$$\hat{S}_{NSNS} = \int d^{10}x \sqrt{|g|} \left\{ e^{-2\hat{\phi}} \left[-\hat{R} + 4(\partial\hat{\phi})^2 - \frac{3}{4}\left(\hat{H}^{(1)}\right)^2 \right] \right\}$$

but differ in their RR sectors:

$$\hat{S}^{IIA}_{RR} = \int d^{10}x \sqrt{|g|} \left\{ \frac{1}{4}\left(\hat{F}^{(1)}\right)^2 + \frac{3}{4}\hat{G}^2 + 2^{-7}\frac{\hat{\varepsilon}}{\sqrt{|g|}}\partial\hat{C}\partial\hat{C}\hat{B}^{(1)} \right\}$$

$$\hat{S}^{IIB}_{RR} = \int d^{10}x \sqrt{|g|} \left\{ -\frac{1}{2}(\partial\hat{\ell})^2 - \frac{3}{4}\left(\hat{H}^{(2)} - \hat{\ell}\hat{A}^{(1)}\right)^2 - \frac{5}{6}\hat{F}^2 - \frac{1}{2\cdot 4!}\frac{\hat{\varepsilon}}{\sqrt{|g|}}\hat{D}\hat{H}^{(1)}H^{(2)} \right\}$$

$$\oplus \qquad F = +{}^{*}F$$

$$IIA \longrightarrow \left\{ \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}},\ \hat{A}^{(1)}_{\hat{\mu}} \right\} \qquad \left\{ \begin{aligned} \hat{F}^{(1)} &= \partial\hat{A}^{(1)} \\ \hat{G} &= \partial\hat{C} - 2\hat{H}^{(1)}\hat{A}^{(1)} \end{aligned} \right.$$

$$IIB \longrightarrow \left\{ \hat{D}_{\hat{\mu}_1\cdots\hat{\mu}_4},\ \hat{B}^{(2)}_{\hat{\mu}\hat{\nu}},\ \hat{\ell} \right\} \qquad \left\{ \begin{aligned} \hat{F} &= \partial\hat{D} + \frac{3}{2}\hat{B}^{(1)}H^{(2)} + \frac{3}{2}B^{(2)}\hat{H}^{(1)} \\ \hat{H}^{(2)} &= \partial\hat{B}^{(2)} \end{aligned} \right.$$

Also supersymmetry is organized in a different way:

There are two supersymmetries

	$(+)$	$(-)$
$\mathrm{IIA}^1 \longrightarrow$	$\hat{\psi}^{(+)1}_{\hat{\mu}}$	$\hat{\psi}^{(-)1}_{\hat{\mu}}$
$\mathrm{IIA}^2 \longrightarrow$	$\hat{\psi}^{(+)2}_{\hat{\mu}}$	$\hat{\psi}^{(-)2}_{\hat{\mu}}$
	$\uparrow$	$\uparrow$
	$\mathrm{IIB}^{(+)}$	$\mathrm{IIB}^{(-)}$

When all RR fields vanish, there must be a basis in which

$$\begin{cases} \delta\hat{\psi}^{(+)1}_{\hat{\mu}} = \hat{\nabla}^{(+)}_{\hat{\mu}}\,\hat{\varepsilon}^{(+)1} ; \\ \delta\hat{\psi}^{(+)2}_{\hat{\mu}} = \hat{\nabla}^{(-)}_{\hat{\mu}}\,\hat{\varepsilon}^{(+)2} ; \end{cases} \qquad \begin{cases} \delta\hat{\lambda}^{(+)1} = \left(\not\partial\hat{\phi} + \tfrac{1}{4}\hat{\not{H}}^{(1)}\right)\hat{\varepsilon}^{(+)1} ; \\ \delta\hat{\lambda}^{(+)2} = \left(\not\partial\hat{\phi} - \tfrac{1}{4}\hat{\not{H}}^{(1)}\right)\hat{\varepsilon}^{(+)2} ; \end{cases}$$

and, then C-duality is replaced by 1-2 duality and the results for the IIA theory are valid for the IIB just by replacing the $(+)$ chirality sector by $(+)_1$ and the $(-)$ chirality sector by the $(+)_2$ sector.

The dyonic embedding of the ERN BH

Two possible explanations

1) We have to generalise our concept of supersymmetry and use 2-dimensional criteria.

2) There is a bigger supergravity theory inside which this embedding is supersymmetric (?)

Possible scenario: (Wild speculation)

i) It includes the IIB theory as a consistent chiral truncation ($N=4 \to N=2$) just as $N=1$ is a consistent chiral truncation of the IIA

ii) All the higher spin or exotic fields disappear in the truncation

iii) The truncation includes the (anti)-self-duality condition of $\hat{F}(\hat{D})_{\hat{\mu}_1 \cdots \hat{\mu}_6}$

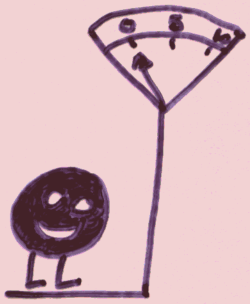
iv) It could be a non-chiral theory derivable from $D=11$ dimensions ($N=2$) 12 ?

$\Longrightarrow$ "M_2 theory"

v) Observe that, without assuming any isometry, the type IIB supergravity theory has the global (duality) symmetry group

$$SL(2,\mathbb{R}) \times \mathbb{R} = GL(2,\mathbb{R})$$

This would be exactly the amount of symmetry that one would get in the dimensional reduction from $D = 12$ to $D = 10$.

Supersymmetric MASSIVE  & massless  "BLACK HOLES"

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T.O.