# DUALITY & SUPERSYMMETRY

E. BERGSHOEFF
R. KALLOSH
T. O.

- DUALITY SYMMETRIES OF THE LOW-ENERGY STRING EFFECTIVE ACTION:
  1.- SL(2,R) (S) - DUALITY
  2.- TARGET-SPACE (R) - DUALITY

- SUPERSYMMETRIC BOSONIC CONFIGURATIONS
  - KILLING SPINORS
  - BOGOMOLNYI BOUNDS - EXTREMALITY

- THE EFFECT OF S- AND R-DUALITY ON THE SUPERSYMMETRY PROPERTIES

# THE LOW-ENERGY STRING EFFECTIVE ACTION

$$S = \frac{1}{2}\int d^D x \sqrt{g}\, e^{-2\phi}\left\{ -R + 4(\partial\phi)^2 - \frac{3}{4}H^2 + O(\alpha')\right\}$$

$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$ $B_{\nu\rho} \to$ axion 2-form

STARTING POINT $D=10$ ($N=1$, $d=10$ SUGRA)

$\downarrow$

$D=4$ (+ 6 Abelian vector fields
$N=4$, $d=4$ SUGRA)

$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$ + Chern-Simons

(Only) in $D=4$ one can perform the change of variables

$H_{\mu\nu\rho} \sim \varepsilon_{\mu\nu\rho}{}^{\sigma}\, \partial_\sigma a$ ← pseudoscalar axion

$\lambda = a - i e^{-2\phi}$ ; $\tilde{F} = \lambda F^{+} - \bar{\lambda} F^{-}$

$$g_{\mu\nu}^{\text{can}} = e^{-2\phi}\, g_{\mu\nu}^{\text{string}}$$

$$S = \frac{1}{2}\int d^4x \sqrt{g}\left\{ -R + 2\,\frac{\partial\lambda\,\partial\bar{\lambda}}{(\operatorname{Im}\lambda)^2} - F\tilde{F}\right\}$$ $N=4$ $d=4$ SUGRA

Eq of motion $\nabla_\mu {}^*\tilde{F}^{\mu\nu} = 0;$

Bianchi identity $\nabla_\mu {}^*F^{\mu\nu} = 0;$

duality

$$\begin{cases} \tilde{F}' = \alpha \tilde{F} + \beta F; \\ F' = \gamma \tilde{F} + \delta F; \end{cases} \qquad \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \in SL(2,R)$$

$$\Rightarrow \quad \lambda' = \frac{\alpha\lambda + \beta}{\gamma\lambda + \delta};$$

SL(2,R) DOES NOT ACT ON THE CANONICAL METRIC

BUT IT DOES ON THE STRING METRIC

IT CAN BE USED TO GENERATE NEW SOLUTIONS:

MANIFESTLY SL(2,R)-INVARIANT FAMILY OF DILATON-AXION TAUB-NUT METRICS

(KASTOR, TORMA, GALT'SOV, GARCÍA MYERS, JOHNSON)

$$ds^2 = f(dt + 2l\cos\theta\, d\varphi)^2 - f^{-1}dr^2 - R^2 d\Omega^2;$$

$$f = \frac{(r-r_+)(r-r_-)}{R^2}; \quad R^2 = r^2 + l^2 - |\Upsilon|^2,$$

$$r_\pm = m \pm r_0; \quad r_0^2 = m^2 + l^2 + |\Upsilon|^2 - 4|\Gamma|^2;$$

$$\lambda = \frac{\lambda_0 (r + il) + \bar{\lambda}_0 \Upsilon}{(r + il) + \Upsilon}; \quad \Upsilon = -2\frac{\bar{\Gamma}^2}{M} \quad \ldots$$

- $SL(2,R)$ IS QUANTUM-MECHANICALLY BROKEN TO $SL(2,Z)$
  - CHARGE QUANTIZATION
  - INSTANTONS
- $SL(2,R)$ INTERCHANGES THE STRONG- AND WEAK-COUPLING REGIMES OF STRING THEORY $\phi \longrightarrow -\phi$
- $SL(2,Z)$ COULD BE AN EXACT SYMMETRY OF STRING THEORY (FONT, IBÁÑEZ, LÜST, QUEVEDO REY SEN )
  - $SL(2,Z)$ -INVARIANT SPECTRUM (SEN, SCHWARZ, KALLOSH ... )
  - STATES = BLACK HOLES? (DUFF)

# TARGET-SPACE DUALITY



FOR ANY D, IF A SOLUTION OF THE EQUATIONS OF MOTION $\{ g_{\mu\nu}, B_{\mu\nu}, \phi \}$ DOES NOT DEPEND ON THE COORDINATE $x$, THERE EXISTS A DUAL CONFIGURATION $\{ \tilde{g}_{\mu\nu}, \tilde{B}_{\mu\nu}, \phi \}$

$$\tilde{g}_{xx} = 1/g_{xx} \; ; \quad \tilde{g}_{x\mu} = B_{x\mu}/g_{xx} \; ;$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - (g_{x\mu} g_{x\nu} - B_{x\mu} B_{x\nu})/g_{xx} \; ;$$

$$\tilde{B}_{x\mu} = g_{x\mu}/g_{xx} \; ; \quad \tilde{B}_{\mu\nu} = B_{\mu\nu} + (g_{x\mu} B_{\nu x} - g_{x\nu} B_{\mu x})/g_{xx}$$

$$\tilde{\phi} = \phi - \frac{1}{2} \ln |g_{xx}| \qquad \text{(BUSCHER)}$$

WHICH IS ALSO A SOLUTION OF THE EQS. OF MOTION.

FROM THE $\sigma$-MODEL POINT OF VIEW BOTH CONFIGURATIONS CORRESPOND TO THE SAME THEORY

EXAMPLE: $\quad R \longrightarrow 1/R$

IT IS THE MOST CHARACTERISTIC SYMMETRY OF STRING THEORY

# SUPERSYMMETRIC BOSONIC CONFIGURATIONS

- THEORY WITH LOCAL SUSY $\{\phi^b, \phi^f\}$
- PARTICULAR SOLUTIONS BREAK SUSY.
- CONSIDER CONFIGURATIONS $\{\phi^b, \phi^f = 0\}$.

  CAN THEY BE INVARIANT UNDER SUSY?

$$\delta_\epsilon \phi^b \propto \phi^f = 0 \quad \checkmark$$

$$\delta_\epsilon \phi^f \propto \phi^b \neq 0 \quad \text{IN GENERAL}$$

$$\delta_\epsilon \phi^f \equiv D_\mu \epsilon = 0 \quad \text{KILLING SPINOR EQUATION}$$

- THE EXISTENCE OF KILLING SPINORS IS ASSOCIATED WITH THE SATURATION OF BOGOMOLNY BOUNDS AND (SOMETIMES) WITH **EXTREMALITY**.
- SUPERSYMMETRIC CONFIGURATIONS ARE FREE OF QUANTUM CORRECTIONS.

IS SUSY PRESERVED BY S- OR R-DUALITY?

# SUSY vs. S-DUALITY

**TH**: IF $\{g_{\mu\nu}, \lambda, F_{\mu\nu}\}$ ADMITS A KILLING SPINOR $\varepsilon$, ANY S-DUAL CONFIGURATION $\{g_{\mu\nu}, \lambda', F'_{\mu\nu}\}$ ALSO ADMITS A KILLING SPINOR

$$\varepsilon' \sim e^{i\,\mathrm{Im}(\gamma\lambda\ \delta)}\ \varepsilon \qquad \begin{pmatrix}\alpha & \beta\\ \gamma & \delta\end{pmatrix} \in SL(2,R)$$

→ THE BOGOMOLNYI BOUND $M^2 + |\Upsilon|^2 - 4|\Gamma|^2 \geq 0$ IS INVARIANT UNDER S-DUALITY (T.O. SEN)

→ THIS PROPERTY IS CONSISTENT WITH THE EXISTENCE OF A SUPERSYMMETRIC $SL(2,Z)$-INVARIANT SPECTRUM OF THE HETEROTIC STRING → **EXTREME BLACK HOLES ?**

# SUSY vs. R-DUALITY

TH: IF $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$ IS A BOSONIC CONFIGURATION WHICH DOES NOT DEPEND ON THE COORDINATE $x$ AND ADMITS AN $N=1$, $d=10$ KILLING SPINOR $\varepsilon$ WHICH DOES NOT DEPEND ON $x$, EITHER, THEN

1) IF $x$ IS SPACE-LIKE, $\varepsilon$ IS ALSO KILLING FOR THE DUAL CONFIGURATION

2) IF $x$ IS TIME-LIKE, $\tilde{\varepsilon} = \gamma_x \varepsilon$ IS ALSO KILLING FOR THE DUAL CONFIGURATION.

$$\mathrm{SSW} \rightleftarrows \mathrm{GFS}$$

$$\mathrm{GFS} \longrightarrow \mathrm{BH} \quad (d=4)$$

BERGSHOEFF
ENTROP
KALLOSH

DUALITY ⊕ SUSY IS GOVERNED BY AN EFFECTIVE $d=9$-DIMENSIONAL THEORY

# FUTURE DIRECTIONS

- INCLUSION OF VECTOR FIELDS ($\alpha'$)
- EXTENSION TO NON-ABELIAN DUALITY
- $N=4$, $d=4$ CASE

  COSMIC CENSORSHIP VS. SUSY
- DUALITY IN SUPERSPACE

⋮