CHARGE Quantization AND Supersymmetry in Stringy Black Holes

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INTRODUCTION

- String Theory contains the only (known) consistent theory of Quantum Gravity

What is this STQG theory like?

→ Compare low energy String Theory with (General Relativity) gravity coupled to matter fields

- Classically:
  - → Classical solutions (singularities? black holes? ...)
  - Motion of test "objects"
- Semiclassically:
  - → Hawking radiation in black holes (T(emperature), S(entropy), C (specific heat)...)
  - Gravitational collapse etc. (CGHS RST ?)

ETC.

WE ARE GOING TO STUDY

i) Classical solutions

ii) to the low-energy (super-) string effective action ($O(1)$ in α')

iii) dimensionally reduced from $d=10 \to d=4$

$N=4$, $d=4$ ungauged SUGRA

iv) truncated, keeping only the bosonic sector

v) and only a few fields in it:

$$\begin{cases} g_{\mu\nu} \to \text{graviton} \\ \phi \to \text{dilaton} \\ a \to \text{axion} \\ A^{(n)}_{\mu} \to U(1) \text{ vector fields} \end{cases}$$

vi) $g_{\mu\nu}$ will be a static black-hole-type metric

- Geometric structure of the solutions
- Black-hole thermodynamics
- Black-hole evaporation - splitting
- Interactions between black holes:

→ Black holes as elementary particles?

→ Quantisation of black-hole charges?

IDEAS TO IMPLEMENT:

- Strong-weak coupling
  Electric-magnetic } DUALITY SYMMETRY

- Embedding in the full supersymmetric theory.

- DIRAC-SCHWINGER-ZWANZIGER condition of charge quantization.

THE ACTION (I)

$$S = \int d^4x \sqrt{-g} \left\{ -R + 2(\partial\phi)^2 + \frac{1}{2} e^{4\phi}(\partial a)^2 - \right.$$

$$\left. - e^{-2\phi} \sum_n F^{(n)2} + i a \sum_n F^{(n)} {}^*F^{(n)} \right\}$$

$F^{(n)} = dA^{(n)}$

${}^*F^{\mu\nu} = \frac{1}{2}(-g)^{-1/2} \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$

diag $g_{\mu\nu} = (+ - - -)$

$\varepsilon^{0123} = i$

THE EQUATIONS OF MOTION (I)

1) $\nabla_\mu \left(e^{-2\phi} F^{(n)\mu\nu} - i a\, {}^*F^{(n)\mu\nu} \right) = 0;$

2) $\nabla_\mu {}^*F^{(n)\mu\nu} = 0 \iff dF^{(n)} = 0 \underset{\text{locally}}{\iff} F = dA;$

3) $\nabla^2 \phi - \frac{1}{2} e^{4\phi}(\partial a)^2 - \frac{1}{2} e^{-2\phi} \sum_n F^{(n)2} = 0;$

4) $\nabla^2 a + 4\partial_\mu \phi \partial^\mu a - i e^{-4\phi} \sum_n F^{(n)} {}^*F^{(n)} = 0;$

5) $R_{\mu\nu} + 2\partial_\mu \phi \partial_\nu \phi + \frac{1}{2} e^{4\phi} \partial_\mu a \partial_\nu a -$

$\quad - 2e^{-2\phi} \sum_n \left(F^{(n)}_{\mu\rho} F^{(n)\rho}_{\nu} - \frac{1}{4} g_{\mu\nu} F^{(n)2} \right) = 0;$

BLACKOUT TRANSPARENCY

- There exist static spherically symmetric b-h solutions! For them SUSY plays the role of COSMIC CENSOR
- Extreme solutions are supersymmetric.
- There is an electric-magnetic duality symmetry of the equations of motion which produces new solutions and preserves SUSY. It might be a symmetry of the full String Theory. $SL(2,\mathbb{Z}) \subset SL(2,\mathbb{R})$
- q_i ; $i = 1, \dots, 4$ SUSY charges, $\mathcal{S}_i \equiv 2|q_i|^2$

$$\Rightarrow S = \pi\left(\mathcal{S}_1 + \mathcal{S}_2\right)^2, \quad T = \frac{1}{2\pi M}\frac{\mathcal{S}_1\mathcal{S}_2}{(\mathcal{S}_1+\mathcal{S}_2)^2}.$$

→ Black-hole evaporation = SUSY restoration

→ SUSY classification of black holes

→ Non-renormalisation theorems:

B. case $\exists\, \xi$ Killing vector $\Rightarrow \exists\, \eta \,\Big|\, \frac{\partial g_{\mu\nu}}{\partial \eta} = 0 \Rightarrow \int d^4x\, \mathcal{L} \sim \int d\eta \to \infty$

F(SUGRA) case $\exists\, \varepsilon$ Killing spinor $\Rightarrow \exists\, \zeta \,\Big|\, \frac{\partial \mathcal{L}}{\partial \zeta} = 0 \Rightarrow \int d^4\theta\, d^4x\, \mathcal{L} \sim \int d\zeta = 0$

BEREZIN

- The spectrum of quantised charges has at most an $SL(2,\mathbb{Z})$ duality symmetry. (Different from Einstein-Maxwell)

THE CHARGES

INTUITIVE DEFINITIONS GOOD ENOUGH FOR STATIC, SPHERICALLY SYMMETRIC, ASYMPTOTICALLY FLAT SPACES:

r : radial coordinate

MASS: $$g_{tt} \sim 1 - \frac{2M}{r};$$

DILATONIC CHARGE: $$\phi \sim \phi_0 + \frac{\Sigma}{r};$$

AXIONIC CHARGE: $$a \sim a_0 - \frac{2e^{-2\phi_0}\Delta}{r};$$

ELECTRIC CHARGE: $$F^{(n)}_{tr} \sim \frac{e^{+\phi_0} Q^{(n)}}{r^2};$$

"MAGNETIC" CHARGE: $${}^{*}F^{(n)}_{tr} \sim \frac{i\, e^{+\phi_0} P^{(n)}}{r^2};$$

LOOKING AT THE EQUATION OF MOTION:

$$\Sigma \sim -\sum_n Q^{(n)2} + \sum_n P^{(n)2};$$

$$\Delta \sim \sum_n Q^{(n)} P^{(n)};$$

THEY ARE NOT INDEPENDENT QUANTITIES

SIMPLEST SOLUTIONS

- $A_\mu^{(n)} = \phi = a = 0 \; ; \quad \forall n$

Schwarzschild $\quad ds^2 = \left(1-\frac{2M}{r}\right)dt^2 - \left(1-\frac{2M}{r}\right)^{-1}dr^2 - r^2 d\Omega^2$

- $\phi = a = 0 \; ; \quad A_\mu^{(i)} \neq 0 \; ;$ (only one)

$\Rightarrow F^2 = F^*F = 0 \Rightarrow \quad Q^{(i)} = P^{(i)} = 0 \; ;$

Reissner-Nordström is not a solution without ϕ or a

- $a = 0 \; ; \quad A_\mu^{(i)} \neq 0 \; ; \quad \phi \neq 0 \; ;$

Garfinkle, Horowitz & Strominger
Gibbons & Maeda
Gibbons

$$e^{2\phi} = e^{2\phi_0}\frac{r+\Sigma}{r-\Sigma} \; ; \quad \Sigma = -\frac{Q^2}{2M} \; ;$$

$$F_{tr} = e^{\phi} Q \, \frac{e^{-2(\phi-\phi_0)}}{R^2} \; ; \quad R^2 = r^2 - \Sigma^2 \; ;$$

$$ds^2 = \left[\frac{(r-r_+)(r-r_-)}{R^2}\right]dt^2 - \left[\;\right]^{-1}dr^2 - R^2 d\Omega^2$$

$$r_\pm = M \pm \underbrace{\sqrt{M^2+\Sigma^2-Q^2}}_{r_0}$$

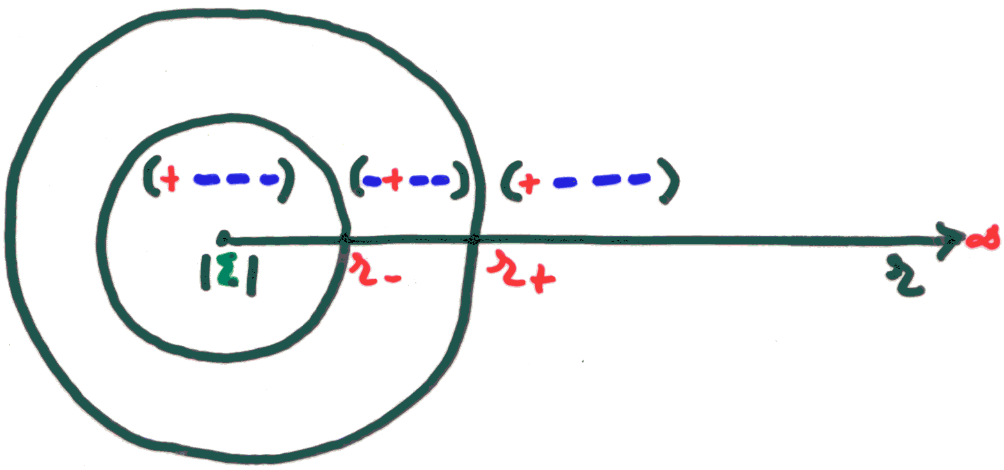
- If we want a R-N-like metric, we need two fields $A_\mu^{(1)} \neq 0 \; ; \; A_\mu^{(2)} \neq 0$, such that

$$\underbrace{F^{(1)\,2} + F^{(2)\,2}}_{\Sigma = 0} = \underbrace{F^{(1)*}F^{(1)} + F^{(2)*}F^{(2)}}_{\Delta = 0} = 0 \; ;$$

$$\boxed{Q^{(i)} = P^{(i)}}$$

GHS-GMG solution:

$r_0^2 > 0$



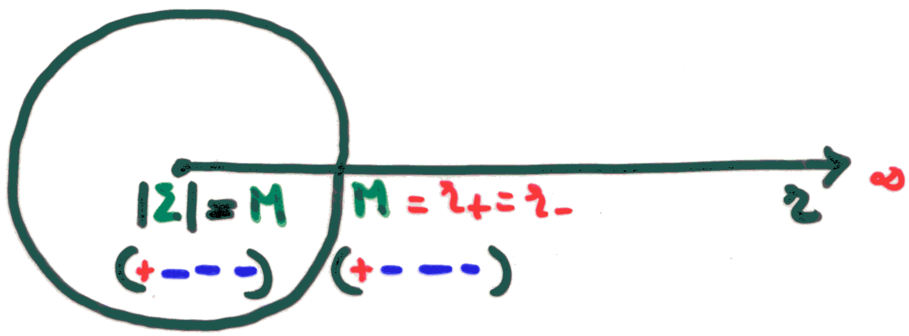
$r_0^2 = 0 \Rightarrow M^2 + \Sigma^2 - Q^2 = 0 \Rightarrow r_+ = r_- = M$

$\Downarrow$

$M^2 = \frac{1}{2} Q^2$

$\Downarrow$

$M = |\Sigma|$



Point-like object

Zero area (singular) horizon

HOLON

$r_0^2 < 0$ ⇝ naked singularity

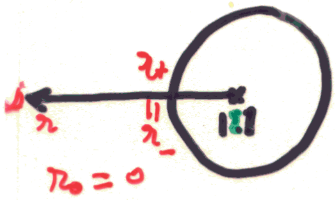
− $a = 0$; $A_\mu^{(1)} \neq 0$; $A_\mu^{(2)} \neq 0$; $\phi \neq 0$;

Gibbons
Gibbons & Maeda
K.L.O.P.R.

$$F_{tr}^{(1)} = e^{\phi_0} Q^{(1)} \frac{e^{2(\phi - \ )}}{R^2} ; \quad e^{2\phi} = e^{2\phi_0} \frac{r+\Sigma}{r-\Sigma} ;$$

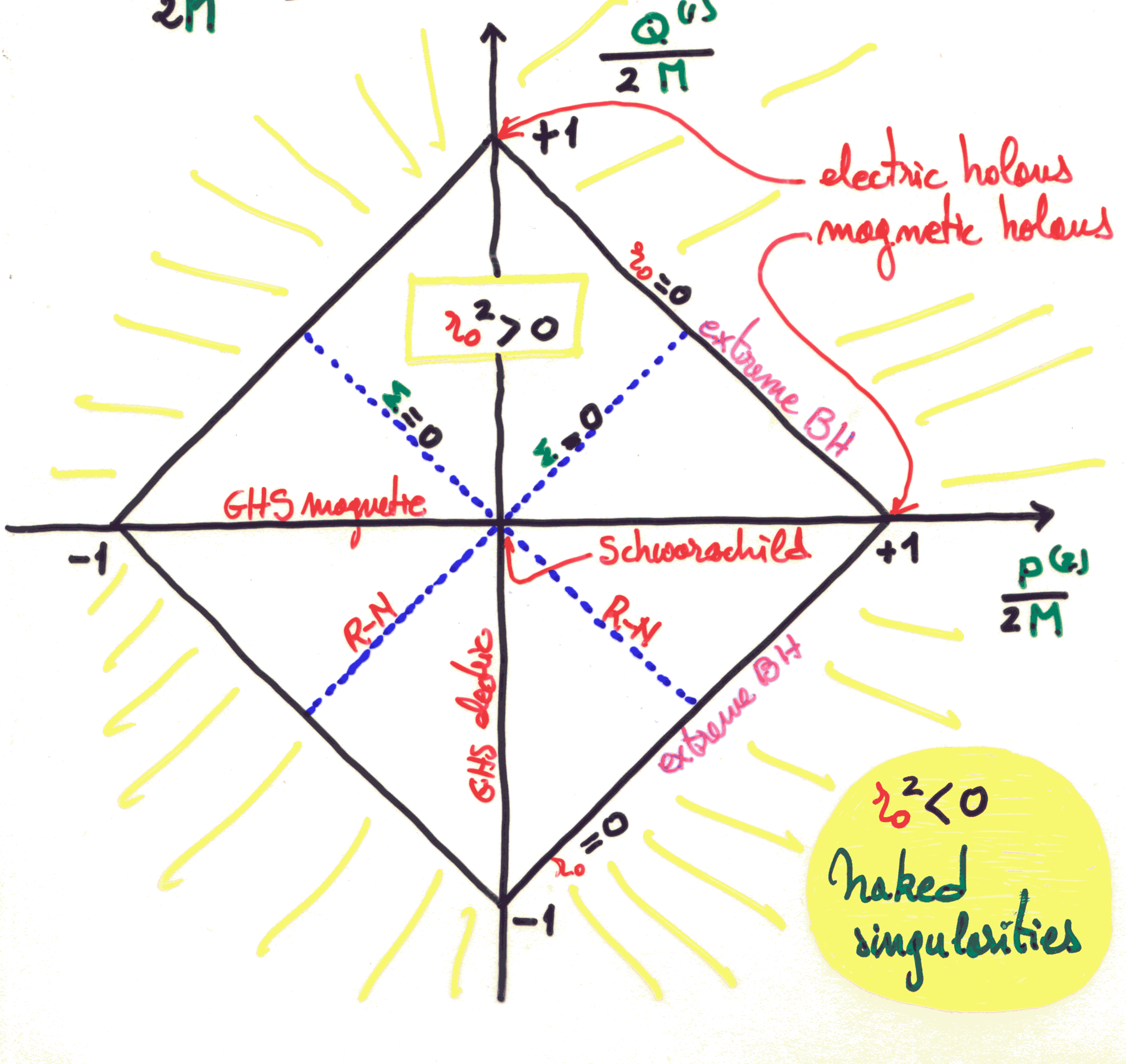
$$^{*}F_{tr}^{(2)} = \frac{i e^{\phi_0} P^{(2)}}{R^2} ;$$

$$ds^2 = e^{2U} dt^2 - e^{-2U} dr^2 - R^2 d\Omega^2$$



$$e^{2U} = \frac{(r-r_+)(r-r_-)}{R^2} ; \quad R^2 = r^2 - \Sigma^2$$

$$\Sigma = \frac{P^{(2)2} - Q^{(1)2}}{2M} ; \quad r_\pm = M \pm r_0 ; \quad r_0^2 = M^2 + \Sigma^2 - \left(Q^{(1)2} + P^{(2)2}\right)$$



ELECTRIC-MAGNETIC DUALITY

Minkowskian Maxwell equations in the vacuum:

$$F = dA \overset{\text{locally}}{\Longleftrightarrow} dF = 0 \Longleftrightarrow \nabla_\mu {}^*F^{\mu\nu} = 0 \quad \left\{ \begin{matrix}\text{Bianchi} \\ \text{identity}\end{matrix} \right.$$

$$\nabla_\mu F^{\mu\nu} = 0 \Longleftrightarrow d{}^*F = 0 \underset{\text{locally}}{\Longleftrightarrow} {}^*F = i\, d{}^*A \quad \left\{ \begin{matrix}\text{Field} \\ \text{equation}\end{matrix} \right.$$

This set of equations is obviously invariant under

$$F \rightleftarrows -i{}^*F \quad \Longrightarrow \quad \begin{matrix} \vec{E} \to \vec{B} \; ; \; Q \to P \\ \vec{B} \to -\vec{E} \; ; \; P \to -Q \end{matrix}$$

This is what enables us to define the magnetic charge because the dynamical fields are A $({}^*A)$.

Duality transformations are highly non-local

$$A^\alpha \longrightarrow A'^\alpha = -i\, \varepsilon^{\alpha\beta\gamma\delta} \int_0^1 d\lambda\; \lambda\, x_\beta (\partial_\gamma A_\delta)(\lambda x) = {}^*A^\alpha$$

so we expect surprises: the action is not invariant

$$F^2 \longrightarrow -F^2$$

More general duality transformations are

$$F \to aF - ib{}^*F$$
$$A \to aA + b{}^*A$$
$$Q \to aQ + bP \quad ; \quad a, b \in \mathbb{R}$$

Invariance of the energy-momentum tensor $\Rightarrow a^2 + b^2 = 1$

$\longrightarrow$ U(1) duality group

In our dilaton-axion system ...

Field equation $\nabla_\mu \underbrace{\left(e^{-2\phi} F^{\mu\nu} - i a \, {}^*F^{\mu\nu}\right)}_{{}^*\tilde{F}} = 0$

$$\Downarrow$$

$$\nabla_\mu {}^*\tilde{F}^{\mu\nu} = 0 \Leftrightarrow d\tilde{F} = 0 \Rightarrow \boxed{\tilde{F} = i\, d\tilde{A}}$$

$\tilde{F}$ defines the meaning of magnetic in this system

$$\tilde{F} \sim i \frac{\tilde{p}}{r^2} \; ; \quad \tilde{p} = e^{-\phi_0} P - a_0 e^{+\phi_0} Q$$

Duality transformation $\begin{cases} A' = -\gamma A + \delta \tilde{A} \\ \tilde{A}' = \alpha A - \beta \tilde{A} \end{cases}$

$$\Rightarrow \lambda' = \frac{\alpha\lambda + \beta}{\gamma\lambda + \gamma} \; ; \qquad \lambda = a - i e^{-2\phi} \; ;$$

The rest of the equations are invariant if

$$\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \in \boxed{SL(2,\mathbb{R})} = \boxed{Sp(2,\mathbb{R})}$$

The action (again) is not duality invariant.

$$S = \int d^4x \sqrt{-g} \left\{ -R + \frac{1}{2} \frac{\partial_\mu \lambda \, \partial^\mu \bar{\lambda}}{2(\operatorname{Im}\lambda)^2} - \sum_n F^{(n)} \, {}^*\tilde{F}^{(n)} \right\}$$

Schwarz & Sen → new duality-invariant actions
not manifestly Lorentz-invariant

$SL(2,\mathbb{R})$ duality

– Generated by $T: \; a \to a + b$ (Peccei-Quinn)

$S: \; \lambda \to 1/\lambda \; ; \; a = 0 \Rightarrow e^{-2\phi} \to e^{+2\phi}$

S interchanges weak and strong coupling regimes from the point of view of string theory.

– If we define the charges

$$F_{tr} \sim q/r^2 \; ; \quad {}^{*}F_{tr} \sim i\,p/r^2 \; ;$$

$$\tilde{F}_{tr} \sim i\,\tilde{p}/r^2 \; ; \quad {}^{*}\tilde{F}_{tr} \sim \tilde{q}/r^2 \; ;$$

We can build representations of $SL(2,\mathbb{R}) = Sp(2,\mathbb{R})$

$\begin{pmatrix} \tilde{q} \\ p \end{pmatrix}$ vector $\quad (q, \tilde{p})$ form

$$\begin{pmatrix} \tilde{q}' \\ p' \end{pmatrix} = R \begin{pmatrix} \tilde{q} \\ p \end{pmatrix} \; ; \quad (q', \tilde{p}') = (q, \tilde{p})\, R^{-1} \; ; \quad R = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$

and invariants:

scalar product $\quad (q_1, \tilde{p}_1) \begin{pmatrix} \tilde{q}_2 \\ p_2 \end{pmatrix} = Q_1 Q_2 + P_1 P_2 \; ;$

exterior product $\quad (q_1, \tilde{p}_1) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \tilde{q}_2 \\ p_2 \end{pmatrix} = Q_1 P_2 - Q_2 P_1 \; ;$

simplectic form

– Duality rotations generate magnetic from electric charge and axionic from dilatonic charge

$\longrightarrow$ NEW SOLUTIONS

Performing the most general duality rotation and eliminating its parameters we get the most general solution of this kind:

$$
\begin{cases}
ds^2 = e^{2U} dt^2 - e^{-2U} dr^2 - R^2 d\Omega^2 \ ; \quad (\text{same metric}) \\
\lambda(r) = \dfrac{\lambda_0 r + \bar{\lambda}_0 \Upsilon}{r + \Upsilon} \ ; \\
A_t^{(n)}(r) = e^{\phi_0} R^{-2} \left[\Gamma^{(n)} (r + \Upsilon) + c.c. \right] \ ; \\
\tilde{A}_t^{(n)}(r) = - e^{\phi_0} R^{-2} \left[\Gamma^{(n)} (\lambda_0 r + \bar{\lambda}_0 \Upsilon) + c.c. \right] \ ;
\end{cases}
$$

where

$$e^{2U} = R^{-2} (r - r_+)(r - r_-) \ ;$$

$$R^2 = r^2 - |\Upsilon|^2 \ ;$$

$$r_\pm = M \pm r_0 \ ;$$

$$r_0^2 = M^2 + |\Upsilon|^2 - 4 \sum_n |\Gamma^{(n)}|^2 \ ;$$

$$\lambda \sim \lambda_0 - i\, e^{-2\phi_0}\, 2\Upsilon / r \ ;$$

$$F^{(n)+} \sim e^{+\phi_0} \Gamma^{(n)} / r^2 \ ;$$

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Any further duality transformation will act only on the boundary conditions (the charges and λ_0) keeping the functional form of the solution invariant.

SL(2,ℝ)- invariant set of solutions

The charges of these BHs are quantum-mechanically restricted:

If we have two BHs with electric and magnetic charges (dyons) moving in each other's distant field so that the BH structure can be ignored:

Dirac Schwinger Zwanziger

$$(q_1, \tilde{p}_1)\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}\tilde{q}_2\\ p_2\end{pmatrix} = \frac{n}{2}\ ;\ n\in\mathbb{Z}$$

SL(2,ℝ) invariant

To get the spectrum of allowed states we need an elementary electric charge c

Asymptotically our Lagrangian is

$$\sim e^{-2\phi_0}F^2 + i\, a_0\, F^*F\ ;$$

$$\Rightarrow e^{-2\phi_0} = \frac{4\pi}{g^2}\ ;\quad a_0 = \frac{\theta}{2\pi}\ ;$$

$$\Rightarrow \boxed{c = \frac{g}{\sqrt{4\pi}} = e^{+\phi_0}} \Rightarrow$$ it transforms under SL(2,ℝ)

Witten effect $\Rightarrow$ $$Q = nc - \frac{\theta}{2\pi}c^2 P = e^{+\phi_0}\left(n - a_0 e^{+\phi_0} P\right)$$

$$\Leftrightarrow \boxed{\tilde{q} = n}$$

DSZ $\Rightarrow \boxed{(\tilde{q}, p) = \left(n, \frac{m}{2}\right)}$ invariant under $\Gamma_2 \subset SL(2,\mathbb{Z})$

Excluding m odd $\rightarrow \boxed{(\tilde{q}, p) = (n, r)}$

$(U(1) \subset SU(2))$

SL(2, ℤ) invariant

There is no such a symmetry in the Maxwell case

SUPERSYMMETRY

To which extent can we ignore the $N=4$, $d=4$ SUGRA fermions?

The solutions break the symmetry of the equations of motion:

$$\begin{cases} \delta_\varepsilon \psi_I = \nabla_\mu \varepsilon_I - \frac{i}{4} e^{2\phi} \partial_\mu a \, \varepsilon_I - \frac{e^{-\phi}}{2\sqrt{2}} \sigma^{\rho\sigma} \left[F_{\rho\sigma} \alpha_{IJ} + G_{\rho\sigma} \beta_{IJ} \right] \gamma_\mu \varepsilon^J \\ \delta_\varepsilon \Lambda_I = -\gamma^\mu \left(\partial_\mu \phi + \frac{i}{2} e^{2\phi} \partial_\mu a \right) \varepsilon_I + \frac{e^{-\phi}}{\sqrt{2}} \sigma^{\rho\sigma} \left[F_{\rho\sigma} \alpha_{IJ} - G_{\rho\sigma} \beta_{IJ} \right] \varepsilon^J \end{cases}$$

If $\delta_\varepsilon \psi_I = \delta_\varepsilon \Lambda_I = 0$ for some spinors ε_I (Killing spinors) then we can ignore the corresponding fermions and have unbroken SUSY with bosons only (" ").

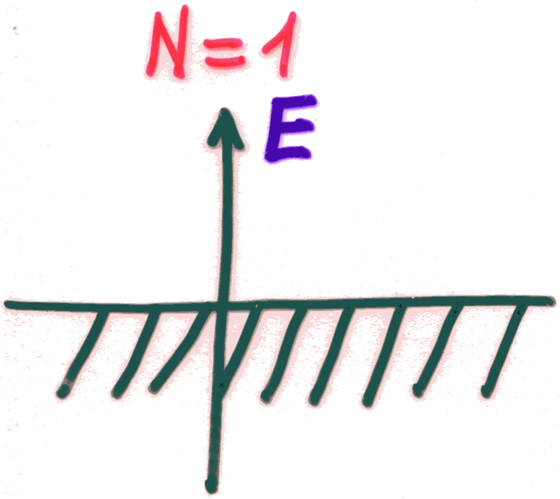
Ferrara
Savoy
Zumino

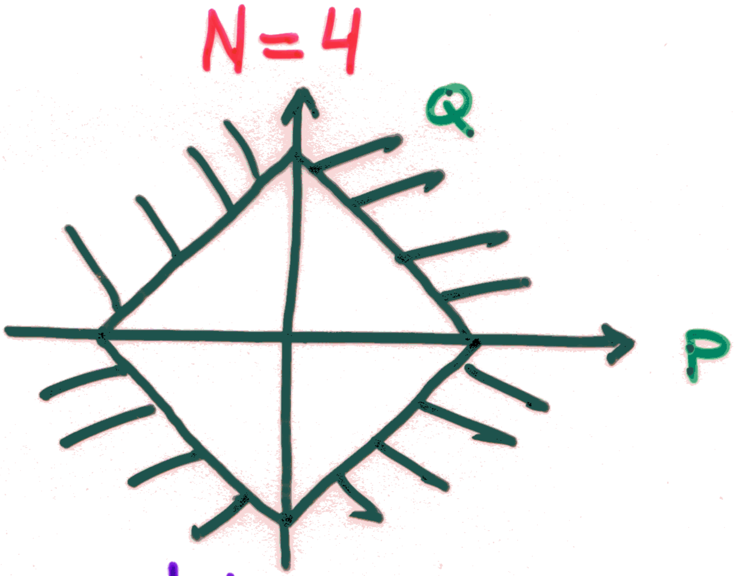
IN AN APROPRIATE BASIS

$$\left.\begin{aligned} \{q_1, q_1^*\} &= 2|q_1|^2 = M - |z_+| \geq 0 \\ \{q_2, q_2^*\} &= 2|q_2|^2 = M - |z_-| \geq 0 \\ \{q_3, q_3^*\} &= 2|q_3|^2 = M + |z_+| \geq 0 \\ \{q_4, q_4^*\} &= 2|q_4|^2 = M + |z_-| \geq 0 \end{aligned}\right\}$$

$z_\pm = \Gamma^{(1)} \pm i \Gamma^{(2)}$ for two $U(1)$ fields are the complex central charges

N=1 SUSY $\qquad E = \{q, q^*\} = 2|q|^2 \geq 0$ →





SUSY $\Rightarrow$ Cosmic censorship FOR THESE SOLUTIONS

$$M \pm |Z_\pm| \geq 0 \iff r_0^2 \geq 0 \Rightarrow \text{no naked sing.}$$

$$\left\{ \begin{array}{c} M = |Z_i| \text{ for some } i \\ \Downarrow \\ \text{UNBROKEN SUSY} \\ \exists \text{ Killing spinors } \varepsilon_I \\ \text{non-renormaliz. th} \end{array} \right\} \iff \left\{ \begin{array}{c} r_0^2 = 0 \Rightarrow \text{Extreme BHs} \\ T = 0 \\ M^2 + \Sigma^2 - (Q^2 + P^2) = 0 \\ \text{(Bogomolnyi bound saturated)} \\ \text{"non-ren. of the entropy"} \end{array} \right\}$$

SUSY classification of stringy black holes

$$S = \pi \left(|q_1|^2 + |q_2|^2 \right)^2 ;$$

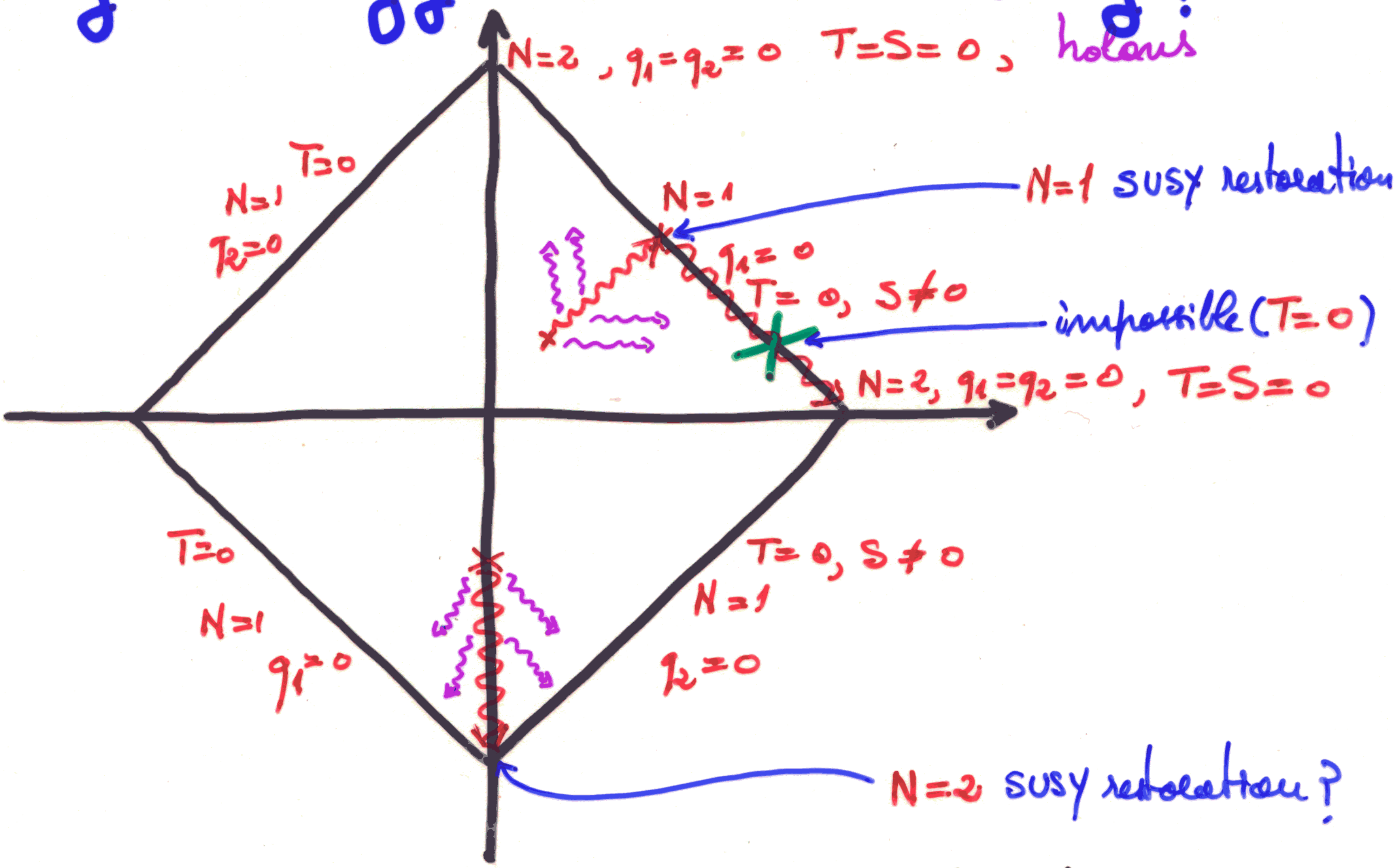
$$E = |q_1|^2 + |q_2|^2 = M ;$$

$$T = \frac{1}{2\pi E} \frac{|q_1 q_2|^2}{\left(|q_1|^2 + |q_2|^2 \right)^2} ;$$

$N=1$ unbroken $\Rightarrow T = 0 \not\Rightarrow S = 0$

$N=2$ unbroken $\Rightarrow S = 0$, holons

What is the picture of the evaporation of charged stringy BHs in this setting?



Black hole evaporation ~ SUSY restoration

Supersymmetric remnants storing information

$S\neq 0,\ T=0$

(If $S = \ln$(degeneracy of the ground state)

$\Rightarrow$ degeneracy of which degrees of freedom?)

multi-BH solutions (extreme BHs in equilibrium)

$\Rightarrow$ N=1 $\xrightarrow[\text{Splitting}]{}$ N=2 holons ?

Huge phase space, Q-M process

SUPERSYMMETRY & DUALITY

The condition $r_0^2 \geq 0 \rightarrow M^2 + |\Upsilon|^2 - \sum_m \left(Q^{(m)2} + P^{(m)2}\right) \geq 0$ is related to SUSY.
It is invariant under duality rotations $(g_{\mu\nu})$.
Will SUSY be also "invariant" under $SL(2,\mathbb{R})$?

$\longrightarrow$ A less strong statement

$$\boxed{\begin{array}{l}\text{If } \exists\, \epsilon_I \;/\; \delta_\epsilon \psi_I = \delta_\epsilon \lambda_I = 0 \\ \text{then } \exp[i \operatorname{Arg}(\gamma\lambda + \delta)]\, \epsilon_I \text{ are Killing spinors} \\ \text{for the rotated solutions.}\end{array}}$$

The number of trivial and non-trivial ϵ_I's will be the same and they will be equally constrained. Then:

The number of unbroken SUSYs is duality invariant.

This is consistent with the invariance of this diagram.

Thermodynamics $\leftrightarrow$ Geometry

SUSY

$\left(\xi^\mu = \epsilon^I \gamma^\mu \epsilon_I \quad \text{Killing vector} \right)$

The end