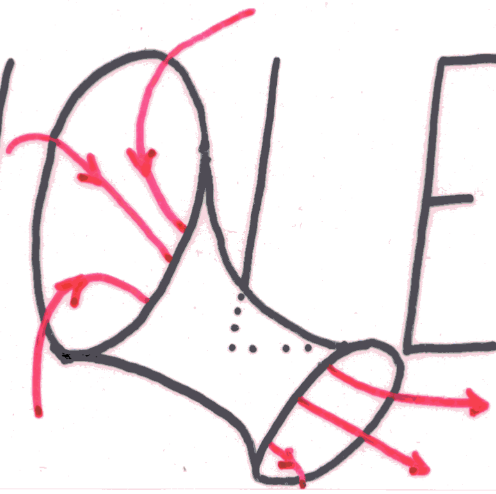
ELECTRIC-MAGNETIC DUALITY & STRINGY BLACK HOLES



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INTRODUCTION

- String Theory contains the only (known) consistent theory of Quantum Gravity.
  What is this STQG theory like?

  → Compare low energy String Theory with (General Relativity) gravity coupled to matter fields:
  - Classically:
    → - Classical solutions (singularities? black holes? cosmological solutions?)
    - Motion of test "objects"
  - Semiclassically:
    → - Hawking radiation in black holes (T(emperature), S(entropy), C(specific heat)...)
    - Gravitational collapse etc (CGHS, RST, ⋮)

E.T.C.

WE ARE GOING TO STUDY

i) Classical solutions

ii) to the low energy (super-)string effective action ($O(1)$ in α')

iii) dimensionally reduced $d=10 \rightarrow d=4$

$N=4$, $d=4$ ungauged SUGRA

iv) truncated, keeping only the bosonic sector

v) and only a few fields in it

$g_{\mu\nu} \rightarrow$

$\phi \rightarrow$ dilaton

$a \rightarrow$ axion

$A_\mu, B_\mu \rightarrow$ U(1) vector fields

vi) $g_{\mu\nu}$ will be a static black-hole-type metric

THE ACTION (I)

$$S = \int d^4x\sqrt{-g}\Big\{-R + 2(\partial\phi)^2 + \frac{1}{2}e^{4\phi}(\partial a)^2 -$$

$$- e^{-2\phi}\left[F^2 + G^2\right] + ia\left[F{}^*F + G{}^*G\right]\Big\}$$

$F = dA$; $G = dB$

${}^*F^{\mu\nu} = \frac{1}{2}(-g)^{-1/2}\,\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ $\quad$ $\mathrm{diag}\, g_{\mu\nu} = (+---)$

$\varepsilon^{0123} = i$

THE EQUATIONS OF MOTION (I)

1) $\nabla_\mu\left(e^{-2\phi}F^{\mu\nu} - ia\,{}^*F^{\mu\nu}\right) = 0$ $\quad$ (same G)

2) $\nabla_\mu {}^*F^{\mu\nu} = 0 \iff dF = 0 \Rightarrow F = dA$

3) $\nabla^2\phi - \frac{1}{2}e^{4\phi}(\partial a)^2 - \frac{1}{2}e^{-2\phi}\left[F + G\right] = 0$

4) $\nabla^2 a + 4\partial_\mu\phi\,\partial^\mu a - ie^{-4\phi}\left[F{}^*F + G{}^*G\right] = 0$

5) $R_{\mu\nu} + 2\partial_\mu\phi\,\partial_\nu\phi + \frac{1}{2}e^{4\phi}\partial_\mu a\,\partial_\nu a -$

$$-2e^{-2\phi}\left[F_{\mu\rho}F_\nu{}^\rho - \frac{1}{4}g_{\mu\nu}F^2 + (G)\right] = 0$$

EARTHQUAKE TRANSPARENCY

- There exist spherically symmetric solutions. For them SUSY plays the role of cosmic censor.
- Extreme solutions are supersymmetric.
- There can be many extreme black holes in equilibrium $\Rightarrow$ possible quantum splitting of BHs

SUSY $\rightarrow$ BOGOMOLNYI BOUND $\rightarrow$ EQ. OF FORCES

- There exists an electric-magnetic duality of the equations of motion that produces new solutions and preserves SUSY $\Rightarrow$ most general solutions with ϕ, a
- q_i; $i = 1, \ldots, 4$ SUSY CHARGES, $\Delta_i = |q_i|^2$

$$\Longrightarrow \quad S = \pi(\Delta_1 + \Delta_2)^2 \quad ; \quad T = \frac{1}{2\pi M} \frac{\Delta_1 \Delta_2}{(\Delta_1 + \Delta_2)^2}$$

BLACK-HOLE EVAPORATION $\Rightarrow$ SUSY RESTORATION

- Non-renormalisation theorems

BOSONIC CASE, $\exists\ \xi$ KILLING VECTOR $\Rightarrow \exists\ \eta \,\Big|\, \frac{\partial g_{\mu\nu}}{\partial \eta} = 0 \Rightarrow \int d^4x\, \mathcal{L} \propto \int d\eta \rightarrow \infty$

SUGRA CASE, $\exists\ \varepsilon$ KILLING SPINOR $\Rightarrow \exists\ \zeta \,\Big|\, \frac{\partial \mathcal{L}}{\partial \zeta} = 0 \Rightarrow \int d^4\theta\, d^4x\, \mathcal{L} \propto \int d\zeta = 0$

BEREZIN

THE CHARGES

INTUITIVE DEFINITIONS GOOD FOR STATIC SPHERICALLY SYMMETRIC AND ASYMPTOTICALLY FLAT SPACES:

$r \to$ radial coordinate

MASS: $g_{tt} \sim 1 - \frac{2M}{r}$

DIL. CHARGE: $\phi \sim \phi_0 + \frac{\Sigma}{r}$ $\left(e^{-2\phi} \sim e^{-2\phi_0}\left(1 - \frac{2\Sigma}{r}\right)\right)$

AX. CHARGE: $a \sim a_0 - \frac{2e^{-2\phi_0}\Delta}{r}$

EL. CHARGE: $F_{tr} \sim \frac{Q_F}{r^2}$

"MAG. CHARGE": ${}^*F_{tr} \sim i\frac{P_F}{r^2}$ $(\epsilon^{0123} = i)$

LOOKING AT THE EQUATIONS OF MOTION:

$$\Sigma \sim -(Q_F^2 + Q_G^2)\ (P_F^2 + P_G^2) \qquad (F^2 + G^2)$$

$$\Delta \sim Q_F P_F + Q_G P_G \qquad (F^*F + G^*G)$$

THEY ARE NOT INDEPENDENT QUANTITIES

SIMPLEST SOLUTIONS

— $A=B=\phi=a=0$

Schwarzschild $\quad ds^2=\left(1-\frac{2M}{r}\right)dt^2-\left(1-\frac{2M}{r}\right)^{-1}dr^2-r^2d\Omega^2$

— $B=\phi=a=0$

$\Rightarrow \quad F^2=F{}^*F=0 \Rightarrow Q_F=P_F=0$

Reissner-Nordström is not a solution.

— $\phi=a=0$

$$F^2+G^2=F{}^*F+G{}^*G=0 \Rightarrow \begin{cases} Q_F=P_G\neq 0;\ Q_G=P_F=0 \\ F\leftrightarrow G \end{cases}$$

$$\left.\begin{array}{l} F_{tr}=\frac{Q_F}{r^2} \\ {}^*G_{tr}=\frac{iP_G}{r^2} \end{array}\right\} \quad ds^2=\left[\frac{(r-r_+)(r-r_-)}{r^2}\right]dt^2-\left[\ \ \right]^{-1}dr^2-r^2d\Omega^2$$

R-N metric $\quad r_\pm=M\pm\sqrt{M^2-(Q_F^2+P_G^2)}$

— $B=a=0$

GHS:

stringy BH

G.
G.M.
G.H.S.

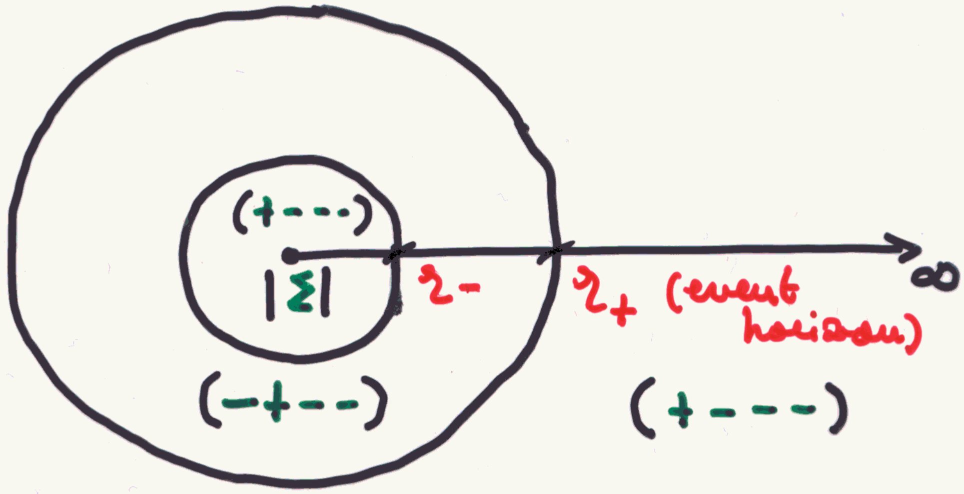
$$e^{2\phi}=e^{2\phi_0}\frac{r+\Sigma}{r-\Sigma} \qquad \Sigma=-\frac{e^{-2\phi_0}Q_F^2}{2M}$$

$$F_{tr}=Q_F\frac{e^{-2\phi}}{R^2}$$

$$ds^2=\left[\frac{(r-r_+)(r-r_-)}{R^2}\right]dt^2-\left[\ \ \right]^{-1}dr^2-R^2d\Omega^2$$

$$R^2=r^2-\Sigma^2 \qquad r_\pm=M\pm\underbrace{\sqrt{M^2+\Sigma^2-e^{-2\phi_0}Q_F^2}}_{r_0}$$

$r_0^2 > 0$

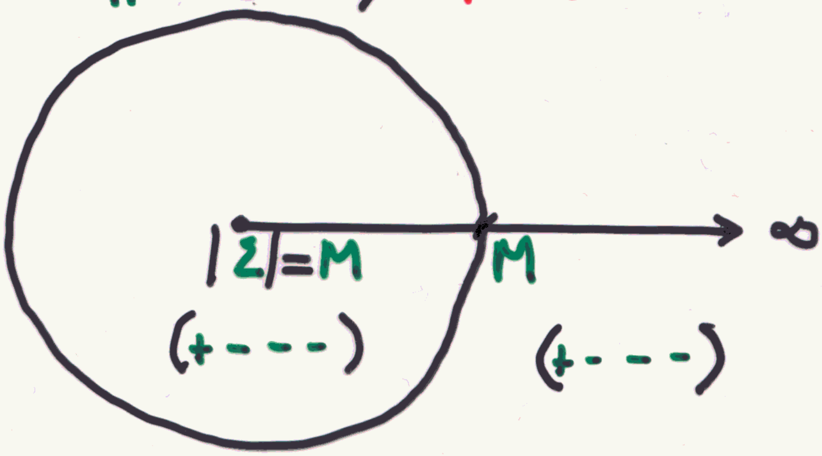


$r_0^2 = 0 \Rightarrow M^2 + \Sigma^2 - e^{-2\phi_0} Q_F^2 = 0 \Rightarrow r_+ = r_- = M$

$\Downarrow$

$M^2 = \frac{e^{-2\phi_0}}{2} Q_F^2$

$\Rightarrow M = |\Sigma|$



Point-like object

Zero area horizon

Singular horizon

HOLON

$r_0^2 < 0 \rightsquigarrow$ Naked singularity

— $a=0$ (G, G.M., KLOPP)

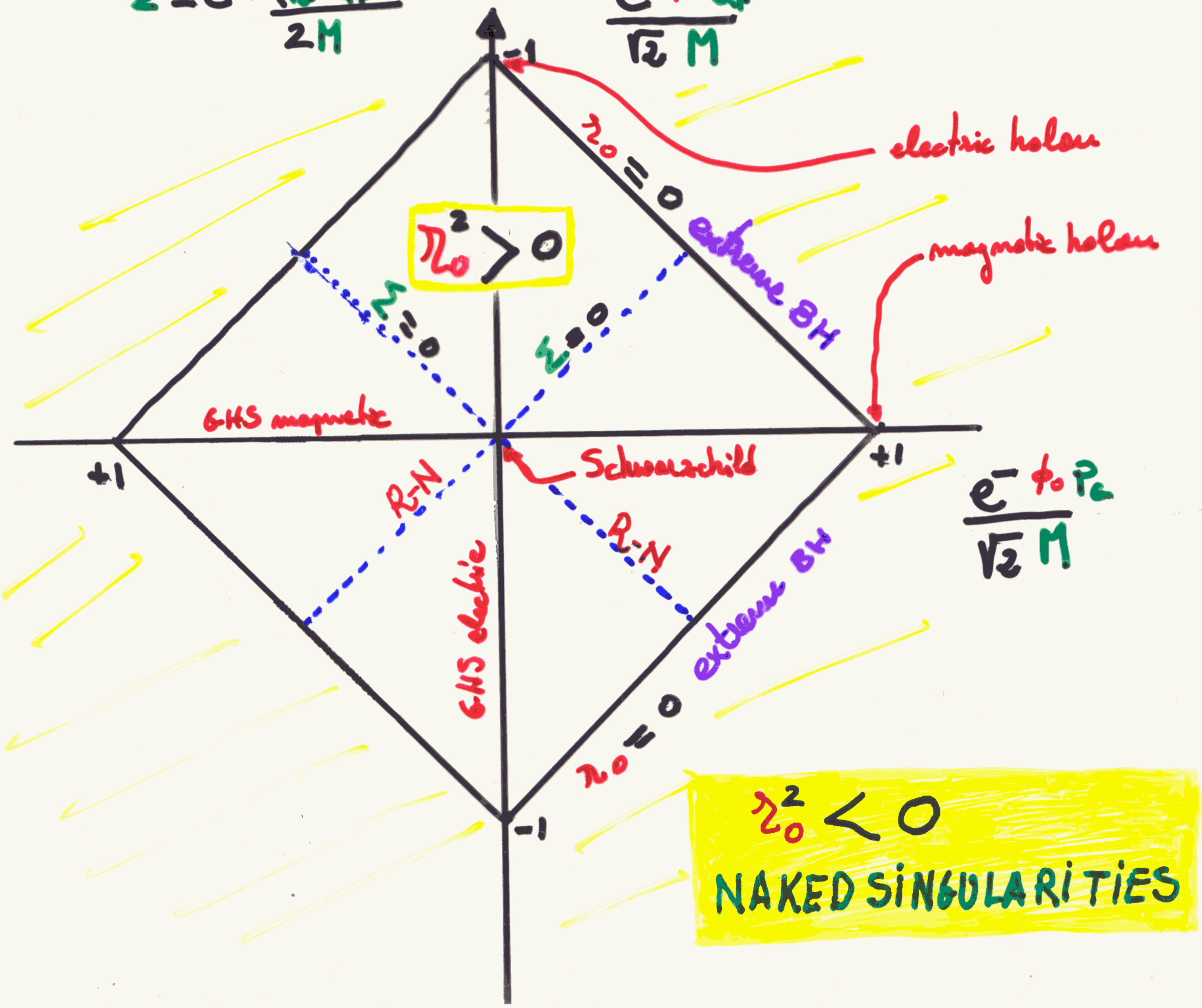
$$F_{tr} = Q_F \frac{e^{2(\phi-\phi_0)}}{R^2} \; ; \quad e^{2\phi} = e^{2\phi_0} \frac{r+\Sigma}{r-\Sigma}$$

$$^{*}G_{tr} = \frac{i P_G}{R^2} \; ; \quad ds^2 = e^{2U} dt^2 - e^{-2U} dr^2 - R^2 d\Omega^2$$

$$e^{2U} = \frac{(r-r_+)(r-r_-)}{R^2} \; ; \quad R^2 = r^2 - \Sigma^2$$

$$r_\pm = M \pm r_0 \; ; \quad r_0^2 = M^2 + \Sigma^2 - e^{-2\phi_0}(Q_F^2 + P_G^2)$$

$$\Sigma = e^{-2\phi_0} \frac{(P_G^2 - Q_F^2)}{2M}$$



THERMODINAMICS

Temperature

$$\begin{cases} T = \dfrac{\kappa}{2\pi} & \kappa \text{ surface gravity} \\ T = \beta^{-1} & \beta \text{ l. of Euclidean time} \\ T = \left(\dfrac{\partial S}{\partial M}\right)^{-1} & S \text{ entropy} \end{cases}$$

$$T = \frac{1}{2\pi} \frac{r_0}{r_+^2 - \Sigma^2} \Rightarrow \underset{r_0 \to 0}{T \to 0} \quad \text{Extreme BHs}$$

$$\lim_{\substack{r_0 \to 0 \\ r_+ \to \Sigma}} T \quad \text{does not exist}$$

(holous)

Entropy

$$\begin{cases} S = \dfrac{A}{4} & A \text{ area of the horizon} \\ \text{Euclidean path-integral in saddle point approximation} \end{cases}$$

$$S = \pi \left(r_+^2 - \Sigma^2\right)$$

EXTREME DILATON BHs HAVE ENTROPY AT $T = 0$ ⇒ DEGENERACY ?

SUPERSYMMETRY

TO WHICH EXTENT CAN WE IGNORE THE $N=4$, $d=4$ SUGRA FERMIONS IN OUR SOLUTIONS?

$$\delta_\epsilon \psi_I = \nabla_\mu \epsilon_I - \frac{i}{4} e^{2\phi}\partial_\mu a\, \epsilon_I - \frac{e^{-\phi}}{2\sqrt{2}}\sigma^{\rho\sigma}\left[F_{\rho\sigma}\alpha_{IJ} + G_{\rho\sigma}\beta_{IJ}\right]\gamma_\mu \epsilon^J$$

$$\delta_\epsilon \Lambda_I = -\gamma^\mu\left(\partial_\mu\phi + \frac{i}{2}e^{2\phi}\partial_\mu a\right)\epsilon_I + \frac{e^{-\phi}}{\sqrt{2}}\sigma^{\rho\sigma}\left[F_{\rho\sigma}\alpha_{IJ} - G_{\rho\sigma}\beta_{IJ}\right]\epsilon^J$$

If $\delta_\epsilon\psi_I = \delta_\epsilon\Lambda_I = 0$ for some spinor ϵ_I (Killing spinor), then we can ignore the corresponding fermions and have unbroken SUSY with bosons only.

IN AN APPROPRIATE BASIS

Ferrara Savoy Zumino

$$\{q_1, q_1^*\} = 2|q_1|^2 = M - |z_1| \geq 0$$
$$\{q_2, q_2^*\} = 2|q_2|^2 = M - |z_2| \geq 0$$
$$\{q_3, q_3^*\} = 2|q_3|^2 = M + |z_1| \geq 0$$
$$\{q_4, q_4^*\} = 2|q_4|^2 = M + |z_2| \geq 0$$

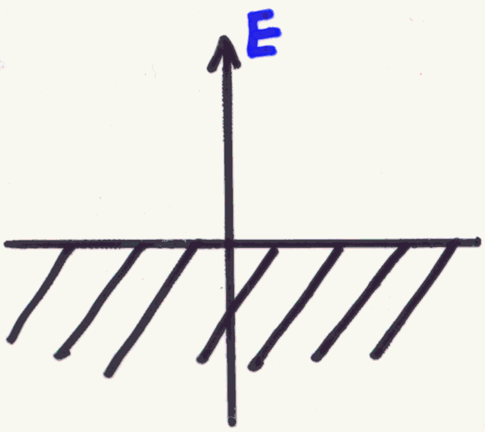
$$z_{1,2} = \frac{e^{\phi}(Q_e \mp P_e)}{\sqrt{2}}$$

Complex Central Charges

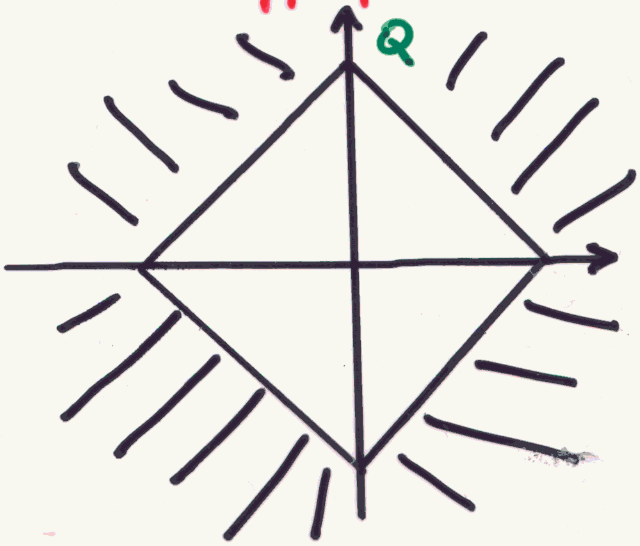
This generalizes the positivity of energy in $N=1$ SUSY

$$E = \{q, q^*\} = 2|q|^2 \geq 0$$

$N=1$



$N=4$



SUSY $\Rightarrow$ Cosmic Censorship for these solutions

$M \pm |z_i| \geqslant 0 \quad \Leftrightarrow \quad r_0^2 \geqslant 0 \Rightarrow$ NO NAKED SING.

$M = |z_i|$ for some i

$\Downarrow$

UNBROKEN SUSY

$\exists$ Killing spinor ϵ

Non-renorm. th.

$\Leftrightarrow$

$r_0^2 = 0 \Rightarrow$ Extreme BHs

$T = 0$

$M^2 + \Sigma^2 - e^{-2\phi_0}(Q_G^2 + P_F^2) = 0$

Bogomolny: Bound saturated

"Non renormal. of entropy"

IN GENERAL

Extreme BHs $\rightarrow$ 1 Unbroken SUSYs

q_1 or q_2

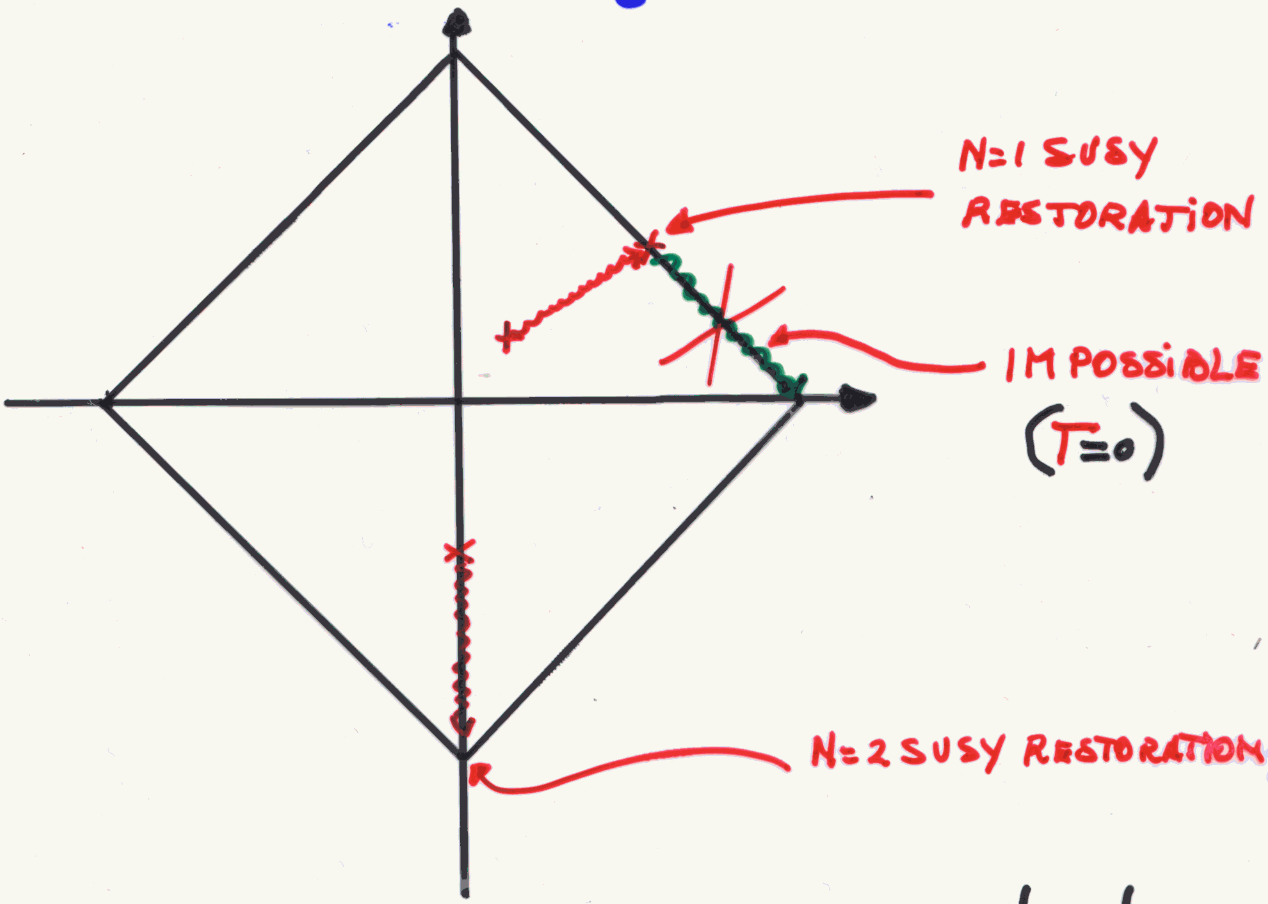
Holons $\rightarrow$ 2 Unbroken SUSYs

q_1 and q_2

$$S \sim (|q_1|^2 + |q_2|^2)^2$$

$$T \sim \frac{1}{M} \frac{|q_1 q_2|^2}{(|q_1|^2 + |q_2|^2)^2}$$

What is the picture of the evaporation of charged BHs in this setting?



There would be supersymmetric remnants storing part of the information lost in gravitational collapse.

Extreme BHs have $S \neq 0$ at $T=0$. This implies (usually) that $S \sim \ln$(degeneracy of the fundamental state).

DEGENERACY OF WHAT? HAIR?

MULTI-BH SOLUTIONS

The extremality condition $M^2 + \Sigma^2 - e^{-2\phi_0}(Q_p^2 + P_G^2) = 0$ resembles a condition of equilibrium of forces. This suggests the possibility of finding BHs in (static) equilibrium.

$r_0 = 0 \Rightarrow \rho = r - M \qquad ds^2 = e^{2U}dt^2 - e^{-2U}(d\rho^2 + \rho^2 d\Omega^2)$

Substituting this 1-BH solution into the eq. of motion we get this characterisation of the solution:

$$\left.\begin{aligned} &F = dA \quad ; \quad A_t = \frac{\sqrt{2}}{H_1} \\ &e^{-2\phi}\,{}^*G = d\tilde{A} \quad ; \quad \tilde{A}_t = \frac{\sqrt{2}}{H_2} \\ &e^{2\phi} = \frac{H_1}{H_2} \\ &e^{2U} = H_1 H_2 \end{aligned}\right\} \quad \partial_i\partial_i H_1 = \partial_i\partial_i H_2 = 0$$

$\rho = |\vec{x}|$

H_1 and H_2 are harmonic and we can choose them with any number of sources (BHs)

$$H_1 = a_1 + \sum_i \frac{b_i}{|\vec{x} - \vec{x}_i|} \quad ; \quad H_2 = a_2 + \sum_i \frac{c_i}{|\vec{x} - \vec{x}_i|}$$

After identifying the constants a_i, b_i, c_i, in terms of the charges we get

$$M_s^2 + \Sigma_s^2 - e^{-2\phi_0}\left(Q_{F,s}^2 + P_{G,s}^2\right) = 0 \quad \forall s$$

$$M = \sum_s M_s, \qquad P = \sum_s |P_s| \qquad Q = \sum_s |Q_s|$$

$$\Rightarrow M^2 + \Sigma^2 - e^{-2\phi_0}\left(Q_F^2 + P_G^2\right) = 0$$

$$F_{st} = -\frac{M_s M_t - \Sigma_s \Sigma_t + e^{-2\phi_0} Q_s Q_t + e^{-2\phi_0} P_s P_t}{|\vec{x}_s - \vec{x}_t|^2} = 0$$

- The dilaton contributes to the equilibrium of forces.
- For given total mass and charges there are ∞ configurations which are possible.
- Is there any process that changes the number of BHs?

Classically $\rightarrow$ S should increase in the process

$\Rightarrow$ It's not allowed to go from extreme BHs to holons

Q.M.ly $\rightarrow$ Why not? Huge phase space

Quantum splitting

WHEN THE BH REACHES THE EXTREME LIMIT $T=0$ AND CAN'T EVAPORATE ANY LONGER, IT CAN SPLIT

INDEFINITELY

THE END WOULD BE A CLOUD OF HOLONS

ELECTRIC-MAGNETIC DUALITY

Minkowsky vacuum Maxwell equations:

$$F = dA \Rightarrow dF = 0 \Rightarrow \nabla_\mu {}^*F^{\mu\nu} = 0 \qquad \text{Bianchi identity}$$

$$\nabla_\mu F^{\mu\nu} = 0 \Rightarrow d^*F = 0 \Rightarrow {}^*F = i\, d^*A \qquad \text{Field equation}$$

This set of equations is obviously invariant under

$$F \to -i\,{}^*F \quad \Rightarrow \quad \vec{E} \to \vec{B};\ Q \to P$$

Duality

$$\vec{B} \to -\vec{E};\ P \to -Q$$

This is what enables us to define magnetic charge because A (${}^*A$) is the dynamical field.

Duality transformations are highly non-local

$$A^\alpha \longrightarrow A'^\alpha = -i\varepsilon^{\alpha\beta\gamma\delta}\int_0^1 d\lambda\ \lambda x_\beta (\partial_\gamma A_\delta)(\lambda x) = {}^*A^\alpha$$

so we expect surprises: the action is not invariant

$$F^2 \longrightarrow -F^2$$

More general duality transformations:

$$F \longrightarrow aF - ib\,{}^*F$$

$$A \longrightarrow aA + b\,{}^*A$$

$$Q \to aQ + bP$$

Dilaton-axion system

Field equation $\nabla_\mu \underbrace{\left(e^{-2\phi} F^{\mu\nu} - i a\, {}^*F^{\mu\nu}\right)}_{{}^*\tilde{F}} = 0$

$\Rightarrow \nabla_\mu {}^*\tilde{F}^{\mu\nu} = 0 \Rightarrow d\tilde{F} = 0 \Rightarrow \boxed{\tilde{F} = i\, d\tilde{A}}$

$\tilde{F}$ defines magnetic in this system.

$\tilde{F} \sim i \dfrac{\tilde{P}_F}{r^2}$ $\qquad \tilde{P}_F = e^{-2\phi_0} P_F - i a_0 Q_F$

Duality transformation: $\quad A' = -\gamma A + \delta \tilde{A}$

$\tilde{A}' = \alpha A - \beta \tilde{A}$

This defines implicitly the following transformation law for ϕ and a:

$z \equiv e^{-2\phi} - i\, a \qquad z' = \dfrac{\alpha z - i\beta}{i\gamma z + \delta}$

The rest of the equations of motion are invariant if $\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \in SL(2, \mathbb{R})$. The action is NOT invariant.

(Excluding z in the action we have Einstein-Maxwell, $\tilde{F} = {}^*F \Rightarrow \begin{matrix} \gamma = -\beta \\ \alpha = \delta \end{matrix} \rightarrow U(1)$)

THE ACTION (II)

$$S = \int d^4x \sqrt{-g}\left\{ -R + 2\,\frac{\partial_\mu z\,\partial^\mu \bar{z}}{(z+\bar{z})^2} - F\,{}^*\tilde{F} - G\,{}^*\tilde{G} \right\}$$

$$z = e^{-2\phi} - ia$$

$$\tilde{F} = e^{-2\phi}\,{}^*F - iaF = zF^{+} - \bar{z}F^{-}$$

$${}^*F^{\mu\nu} = \frac{1}{2}(-g)^{-1/2}\,\varepsilon^{\mu\nu\alpha\rho}F_{\alpha\rho}$$

$${}^{**}F = F \;;\quad ({}^*F)^2 = -F^2$$

$$F\,{}^*\tilde{F} = {}^*F\,\tilde{F}$$

THE EQUATIONS OF MOTION (II)

i) $\nabla_\mu {}^*\tilde{F}^{\mu\nu} = 0 \Rightarrow d\tilde{F} = 0 \Rightarrow \tilde{F} = d\tilde{A}$

ii) $\nabla_\mu {}^*F^{\mu\nu} = 0 \Rightarrow dF = 0 \Rightarrow F = dA$

iii) $\nabla_\mu\left[\frac{\partial^\mu (z\bar{z})}{(z+\bar{z})^2}\right] + \frac{1}{2}\,F\,{}^*\tilde{F} = 0$

iv) $\nabla^2\left(\frac{z-\bar{z}}{z+\bar{z}}\right) - 2\left(\frac{z-\bar{z}}{z+\bar{z}}\right)R - F\tilde{F} = 0$

v) $R_{\mu\nu} + 2\,\frac{\partial_{(\mu} z\,\partial_{\nu)}\bar{z}}{(z+\bar{z})^2} - 2\left[F_{\mu\delta}\,{}^*\tilde{F}_\nu{}^\delta - \frac{1}{4}g_{\mu\nu}F\,{}^*\tilde{F}\right] = 0$

What is the effect of a duality transformation on our solutions?

1.- The charges are rotated and rescaled:

$$\begin{cases} e^{-\phi_0'}Q' = \cos\theta\, e^{-\phi_0} Q + \sin\theta\, e^{-\phi_0} P \\ e^{-\phi_0'}P' = -\sin\theta\, e^{-\phi_0} Q + \cos\theta\, e^{-\phi_0} P \end{cases}$$

$$\begin{cases} \Sigma' = \cos(-2\theta)\,\Sigma + \sin(-2\theta)\,\Delta \\ \Delta' = -\sin(-2\theta)\,\Sigma + \cos(-2\theta)\,\Delta \end{cases}$$

$\Rightarrow$ 2.- It generates non-trivial axion field (charge Δ)

3.- $\left.\begin{array}{l} e^{-2\phi_0}(Q^2+P^2) \\ \Sigma^2+\Delta^2 \end{array}\right\}$ are preserved

4.- The metric doesn't change ($\to M$ either)

5.- $M^2+\Sigma^2+\Delta^2-e^{-2\phi_0}(Q_F^2+P_F^2+Q_G^2+P_G^2)$ is invariant.

$\to$ Supersymmetric configurations are transformed into supersymmetric configurations with the same metric

$\longrightarrow$ More examples of SUSY as Cosmic Censor

In this way we have obtained the most general spherically symmetric dilaton-axion BH:

$$ds^2 = e^{2U} dt^2 - e^{-2U} dr^2 - R^2 d\Omega^2$$

$$e^{2U} = \frac{(r - r_+)(r - r_-)}{R^2} \qquad r_\pm = M \pm r_0$$

$$R^2 = r^2 - |\Upsilon|^2 \qquad r_0^2 = M^2 + |\Upsilon|^2 - e^2 \left(\sum_i Q_i^2 + \sum_i P_i^2\right)$$

$$\Upsilon = \Sigma - i\Delta$$

$$z = \frac{z_0 r - \bar{z}_0 \Upsilon}{r + \Upsilon} = e^{-2\phi} - i a$$

$$A_t = \Gamma \frac{(r + \Upsilon)}{R^2} + c.c.$$

$$i \tilde{A}_t = \Gamma \frac{(z_0 r - \bar{z}_0 \Upsilon)}{R^2} - c.c.$$

The multi-BH solutions can be found analogously.

The thermodynamics is the same,

but $\quad M\Delta \sim Q_F P_F + Q_G P_G = M + M$

Can we apply Dirac's quantization?