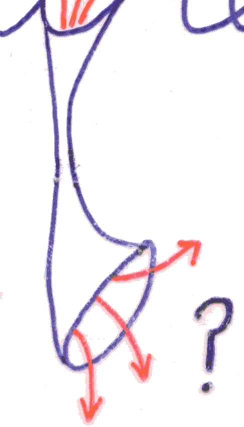
Building new supersymmetric stringy black holes



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INTRODUCTION

– What are stringy black holes?

i) Solutions to dimensionally reduced low energy effective string theory action. (N=4, d=4 SUGRA)

ii) We keep only some bosonic fields:

$g_{\mu\nu} \to$ metric

$\phi \to$ dilaton

$a \to$ axion (pseudoscalar)

$A_\mu \; B_\mu \ldots \to U(1)$ gauge fields

iii) $g_{\mu\nu}$ is a black hole metric

– Why?

i) They may provide information about non-perturbative string theory and

ii) the Q.G. ~~theory present~~ in S.T.

THE ACTION (I)

$$I = \int d^4x \sqrt{-g}\Big\{-R + 2(\partial\phi)^2 + \frac{1}{2}e^{4\phi}(\partial a)^2 -$$

$$- e^{-2\phi}\left[F^2 + G^2\right] + ia\left[F{}^*F + G{}^*G\right]\Big\}$$

$F = dA$; $G = dB$

${}^*F^{\mu\nu} = \frac{1}{2}(-g)^{-1/2}\,\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$

$\text{diag}\, g_{\mu\nu} = (+\,-\,-\,-)$

$\varepsilon^{0123} = i$

EQUATIONS OF MOTION

1) $\nabla_\mu\left(e^{-2\phi}F^{\mu\nu} - ia\,{}^*F^{\mu\nu}\right) = 0$ (resp. G)

2) $\nabla_\mu {}^*F^{\mu\nu} = 0 \quad \Leftrightarrow \quad dF = 0 \quad \Rightarrow \quad F = dA$

3) $\nabla^2\phi - \frac{1}{2}e^{4\phi}(\partial a)^2 - \frac{1}{2}e^{-2\phi}\left[F^2 + G^2\right] = 0$

4) $\nabla^2 a + 4\partial_\mu\phi\,\partial^\mu a - ie^{-4\phi}\left[F{}^*F + G{}^*G\right] = 0$

5) $R_{\mu\nu} + 2\partial_\mu\phi\,\partial_\nu\phi + \frac{1}{2}e^{4\phi}\partial_\mu a\,\partial_\nu a -$

$$-2e^{-2\phi}\left[F_{\mu\rho}F_\nu{}^\rho - \frac{1}{4}g_{\mu\nu}F^2 + (G)\right] = 0$$

WHAT CAN WE EXPECT *a priori*?

THE CHARGES

INTUITIVE DEFINITIONS GOOD FOR STATIC SPHERICALLY SYMMETRIC AND ASYMPTOTICALLY FLAT SPACES:

$r \to$ radial coordinate

MASS: $g_{tt} \sim 1 - \frac{2M}{r}$

DIL. CHARGE: $\phi \sim \phi_0 + \frac{\Sigma}{r}$ $\quad \left(e^{-2\phi} \sim e^{-2\phi_0}\left(1-\frac{2\Sigma}{r}\right)\right)$

AX. CHARGE: $a \sim a_0 - \frac{2e^{-2\phi_0}\Delta}{r}$

EL. CHARGE: $F_{tr} \sim \frac{Q_F}{r}$

MAG. CHARGE: $^*F_{tr} \sim i\,\frac{P_F}{r}$ $\quad (\varepsilon^{0123} = i)$

LOOKING AT THE EQUATIONS OF MOTION:

$$\Sigma \sim -(Q_F^2 + Q_G^2) + (P_F^2 + P_G^2) \qquad (F^2 + G^2)$$

$$\Delta \sim Q_F P_F + Q_G P_G \qquad (F^*F + G^*G)$$

THEY ARE NOT INDEPENDENT QUANTITIES

$\to$ (NOHAIR THEOREMS?)

("Toy model")

EINSTEIN-MAXWELL + SCALAR

$$I = \int d^4x \sqrt{g}\left\{-R + 2(\partial\phi)^2 + F^2\right\}$$

EQUATIONS OF MOTION

1) $\nabla_\mu F^{\mu\nu} = 0 \quad \Leftrightarrow \quad d{}^*F = 0 \quad \Rightarrow \quad {}^*F = d{}^*A$

2) $\nabla_\mu {}^*F^{\mu\nu} = 0 \quad \Leftrightarrow \quad dF = 0 \quad \Rightarrow \quad F = dA$

3) $R_{\mu\nu} + 2\partial_\mu\phi\,\partial_\nu\phi - 2\left[F_{\mu\rho}F_\nu{}^\rho - \frac{1}{4}g_{\mu\nu}F^2\right] = 0$

WHAT DO WE KNOW IN GENERAL ?

i) FOUR PARAMETERS: M, Q, P, J. (*)

ii) THE CHARGE OF ϕ IS ZERO

iii) ONLY ONE FAMILY OF SPHERICALLY SYMMETRIC SOLUTIONS ($\phi = 0$)
REISSNER-NORDSTRÖM BLACK HOLES

iv) THE EQUATIONS ARE SYMMETRIC UNDER
$F \to F' = i\,{}^*F \quad (\Rightarrow Q \leftrightarrows P)$ Bianchi ⇄ Gauss
BUT THE ACTION IS NOT $\left(({}^*F)^2 = F^2\right)$

v) THERE IS MORE SYMMETRY ⟶ ▷

(*) NO-HAIR THEOREMS

UNDER

$$F \to F' = \cos\alpha\, F + i \sin\alpha\, {}^*F$$

$$\Rightarrow \begin{pmatrix} Q \to Q' = \cos\alpha\, Q + \sin\alpha\, P \\ P \to P' = -\sin\alpha\, Q + \cos\alpha\, P \end{pmatrix}$$

- the equations of motion are invariant (Gauss $\rightleftarrows$ Bianchi) $\left(F_{\mu\rho} {}^*F_\nu{}^\rho - \frac{1}{4} g_{\mu\nu} F {}^*F = 0 \right)$
- in the action appear terms

$$\sim \int d^4x \sqrt{-g}\; F\,{}^*F$$

which are total derivatives ($\Rightarrow$ symmetry of the equations of motion).

THE EFFECT OF THIS $U(1)$ GROUP OF DUALITY ROTATIONS IS TO SUBSTITUTE

$$\begin{Bmatrix} Q = Q(Q', P') \\ P = P(Q', P') \end{Bmatrix}$$

IN THE METRIC, WHICH IS INVARIANT ($\Leftrightarrow$ SQARE κ, A, $\int[K] \ldots$)

WHAT ABOUT THE $\underbrace{\text{ACTION}}_{\text{EUCLIDEAN}}$???

THE SOLUTIONS

- Gibbons, Maeda,
- Garfinkle, Horowitz, Strominger
- Kallosh, Linde, Peet, Van Proeyen, Ortín

$$ds^2 = e^{2U} dt^2 - e^{-2U} dr^2 - R^2 d\Omega^2$$

$$e^{2\phi} = e^{2\phi_0} \frac{(r+\Sigma)}{(r-\Sigma)}$$

$$a = a_0$$

$$F = Q_F \frac{e^{2(\phi-\phi_0)}}{R^2} dt \wedge dr - P_F \sin\theta \, d\theta \wedge d\varphi \quad (\text{resp } G)$$

$$e^{2U} = \frac{(r-r_+)(r-r_-)}{R^2} \Rightarrow$$ Event horizon at $r = r_+$; Cauchy horizon at $r = r_- \leq r_+$

$$R^2 = r^2 - \Sigma^2 \Rightarrow$$ Area of event horizon $4\pi(r_+^2 - \Sigma^2)$

$$r_\pm = M \pm r_0 \Rightarrow$$ $r_0 = 0 \Rightarrow$ only one horizon

EXTREME BLACK HOLES

$$r_0^2 = M^2 + \Sigma^2 - e^{-2\phi_0}\left(Q_F^2 + P_F^2 + Q_G^2 + P_G^2\right)$$

$$\Sigma = e^{-2\phi_0} \frac{(P_F^2 + P_G^2) - (Q_F^2 + Q_G^2)}{2M} = \Sigma_F + \Sigma_G$$

UNDER THE CONDITION

$$\Delta = -e^{-2\phi_0} \frac{(Q_F P_F + Q_G P_G)}{M} = \Delta_F + \Delta_G = 0$$

$$\Delta_F = -\Delta_G$$

(consistent with $a = a_0$)

PROPERTIES OF OUR SOLUTIONS

1) $r_0^2 = M^2+\Sigma^2 - e^{-2\phi_0}\left(Q_F^2+Q_G^2+P_F^2+P_G^2\right) \geq 0$

LOOKS LIKE A B. B.

INCLUDING Δ $(=0)$

$$M^2+\Sigma^2+\Delta^2 - e^{-2\phi_0}\left(Q_F^2+Q_G^2+P_F^2+P_G^2\right) \geq 0$$

THE B.B. OF R-N BHs $Q=P$ IS IN IT

2) THE B.B. CORRESPONDS TO THE C.C.C. of [illegible]

AND SATURATION IMPLIES EXTREMALITY

3) IT CAN BE PROVED THAT THE EXTREME ONES ARE SUPERSYMMETRIC

$$\epsilon_I = e^{\frac{1}{2}U}\,\epsilon_{I0}$$

ϵ_{I0}: constant spinor

$$N=1\begin{cases}\epsilon_1 = -\dfrac{(Q_F - iQ_G)}{\sqrt{Q_F^2\ Q_G^2}}\,\gamma^0\epsilon^2\\ \text{sign}\,Q_F = \text{sign}\,P_G\end{cases}$$

$$N=1\begin{cases}\epsilon_3 = -\dfrac{(Q_F + iQ_G)}{\sqrt{Q_F^2 + Q_G^2}}\,\gamma^0\epsilon^4\\ \text{sign}\,Q_F = -\,\text{sign}\,P_G\end{cases}$$

$$N=2\begin{cases}P_F = P_G = 0 & \text{PURELY ELECTRIC}\\ Q_F = Q_G = 0 & \text{PURELY MAGNETIC}\end{cases}$$

ELECTRIC-MAGNETIC DUALITY

IS THERE A DUALITY SYMMETRY IN OUR CASE?

$F \to i^*F$ does not work

PRINCIPLE: INTERCHANGE GAUSS BIANCHI

$$\Rightarrow \boxed{\begin{array}{l} F \to e^{-2\phi}\,{}^*F - i a F = \tilde{F} \\ \tilde{F} \to F \end{array}} \to \boxed{z \to 1/z}$$

THE EQUATIONS OF MOTION CAN BE WRITTEN IN SELF-DUAL FORM (III) BUT, AGAIN THE ACTION IS NOT INVARIANT.

THE SYMMETRY CAN BE EXTENDED TO $SL(2,\mathbb{R})$

$$z \to \frac{\alpha z - i\beta}{i\gamma z + \delta} \qquad \alpha\delta - \beta\gamma = +1$$

$$\begin{cases} \tilde{F} \to \alpha \tilde{F} - i\beta F \\ F \to i\gamma \tilde{F} + \delta F \end{cases}$$

OR $F^+ \to (i\gamma z + \delta) F^+$ (SEE II)

THE ACTION (II)

$$I = \int d^4x \sqrt{-g} \left\{ -R + \frac{\partial_\mu z \partial^\mu \bar{z}}{(z+\bar{z})^2} - \left[z\left[(F^+)^2 + (G^+)^2\right] + c.c. \right] \right\}$$

$$z = e^{-2\phi} - i a$$

$$F^\pm = \frac{1}{2}(F \pm {}^*F) \; ; \quad \overline{F^+} = F^-$$

EQUATIONS OF MOTION

1) $\nabla_\mu (z F^{+\mu\nu} + c.c.) = 0 \qquad$ (resp. G)

2) $\nabla_\mu (F^{+\mu\nu} - c.c.) = 0$

3) $\nabla^2 z - 2\frac{(\partial z)^2}{(z+\bar{z})} + (z+\bar{z})^2\left[(F^-)^2 + (G^-)^2\right] = 0$

4) $R_{\mu\nu} + \frac{\partial_{(\mu} z \, \partial_{\nu)} \bar{z}}{(z+\bar{z})^2} - (z+\bar{z})\left[F_{\mu\rho} F_\nu{}^\rho - \frac{1}{4} g_{\mu\nu} F^2 + (G)\right]$
$= 0$

THE ACTION (III)

$$I = \int d^4x \sqrt{-g} \left\{ -R + \frac{\partial_\mu z \partial^\mu \bar{z}}{(z+\bar{z})^2} - F^*\tilde{F} - G^*\tilde{G} \right\}$$
$$(-\tilde{F}^*F - \tilde{G}^*G)$$

$\tilde{F} = e^{-2\phi} {}^*F - i a F$; $\tilde{\tilde{F}} \neq F$

EQUATIONS OF MOTION

1) $\nabla_\mu {}^*\tilde{F}^{\mu\nu} = 0 \quad \Leftrightarrow \quad d\tilde{F} = 0 \quad \Rightarrow \quad \tilde{F} = d\tilde{A}$

2) $\nabla_\mu {}^*F^{\mu\nu} = 0 \quad \Leftrightarrow \quad dF = 0 \quad \Rightarrow \quad F = dA$

3) $\nabla_\mu \left[\frac{\partial^\mu (z\bar{z})}{(z+\bar{z})} \right] + \frac{1}{2} F^*\tilde{F} + \frac{1}{2} G^*\tilde{G} = 0$

4) $\nabla^2 \left(\frac{z-\bar{z}}{z+\bar{z}} \right) - 2 \left(\frac{z-\bar{z}}{z+\bar{z}} \right) R - F\tilde{F} = 0$

5 $R_{\mu\nu} + \frac{\partial_{(\mu} z \, \partial_{\nu)} \bar{z}}{(z+\bar{z})^2} - 2 \left(\frac{z+\bar{z}}{z-\bar{z}} \right) \left[F_{\mu\rho} \tilde{F}_\nu{}^\rho - \frac{1}{4} g_{\mu\nu} F\tilde{F} \right] = 0$

We use the identity $\boxed{F_{\mu\rho} {}^*F_\nu{}^\rho - \frac{1}{4} g_{\mu\nu} F^*F = 0}$ that holds in $d=4$.

THE ACTION OF $SL(2,\mathbb{R})$ ON THE CHARGES

$$\begin{cases} Q' = \sqrt{A}\left(\cos\alpha\, Q + \sin\alpha\, P\right) \\ P' = \sqrt{A}\left(-\sin\alpha\, Q + \cos\alpha\, P\right) \end{cases}$$

$$\begin{cases} e^{-2\phi_0'} = \dfrac{e^{-2\phi_0}}{A} \quad ; \quad A = \gamma^2(e^{-4\phi_0} + a_0^2) + 2\gamma\delta\, a_0 + \delta \\ a_0' = B \quad ; \quad B = \dfrac{\gamma\alpha(e^{-4\phi_0} + a_0^2) + (\gamma\delta + \gamma\beta)\, a_0 + \beta\delta}{A} \end{cases}$$

$$\cos\alpha = (\gamma a_0 + \delta)/\sqrt{A}$$

$$\sin\alpha = -\gamma e^{-2\phi_0}/\sqrt{A}$$

$$\Rightarrow \begin{cases} e^{-\phi_0'} Q' = \cos\alpha\,(e^{-\phi_0} Q) + \sin\alpha\,(e^{-\phi_0} P) \\ e^{-\phi_0'} P' = -\sin\alpha\,(e^{-\phi_0} Q) + \cos\alpha\,(e^{-\phi_0} P) \end{cases}$$

$$\begin{cases} \Sigma' = \cos(-2\alpha)\,\Sigma + \sin(-2\alpha)\,\Delta \\ \Delta' = -\sin(-2\alpha)\,\Sigma + \cos(-2\alpha)\,\Delta \end{cases}$$

$$e^{-2\phi_0'}(Q'^2 + P'^2) = e^{-2\phi_0}(Q^2 + P^2)$$

$$\Sigma'^2 + \Delta'^2 = \Sigma^2 + \Delta^2$$

$$\boxed{M^2 + \Sigma^2 + \Delta^2 - e^{-2\phi_0}(Q_F^2 + Q_G^2 + P_F^2 + P_G^2) = 0}$$

IS INVARIANT $\Rightarrow$ SUSY?

SUPERSYMMETRY

- OUR THEORY CAN BE EMBEDDED IN $N=4$, $d=4$ SUGRA (SU(4) VERSION) (UNGAUGED)
- OUR SOLUTIONS CAN BE CONSIDERED AS SOLUTIONS OF IT WITH IDENTICALLY ZERO FERMIONIC FIELDS $\Lambda_I = \psi_{I,\mu} = 0$

Q: "WILL THEY REMAIN INVARIANT UNDER SOME LOCAL SUSY TRANSFORMATIONS ϵ_I ?"

OR "ARE THERE MAJORANA SPINORS ϵ_I SUCH THAT SOME OF THESE EQUATIONS ARE SATISFIED?"

$$\left\{\begin{array}{l} \frac{1}{2}\delta_\epsilon \psi_{\mu I} = \nabla_\mu \epsilon_I - \frac{i}{4} e^{2\phi} \epsilon_I \partial_\mu a - \frac{1}{8}\sigma^{\rho\sigma} T^{(+)}_{\rho\sigma,IJ} \gamma_\mu \epsilon^J = 0 \\ \frac{1}{2}\delta_\epsilon \Lambda_I = -\gamma^\mu (\partial_\mu \phi + \frac{i}{2} e^{2\phi} \partial_\mu a)\epsilon_I + \frac{1}{4}\sigma^{\rho\sigma} T^{(-)}_{\rho\sigma,IJ} \epsilon^J = 0 \end{array}\right. \quad (*)$$

ϵ_I KILLING SPINORS, $\Rightarrow$ $\bar{\epsilon}\gamma^\mu \epsilon = \xi^\mu$ KILLING VECTOR (ϵ DIRAC)

IF SOLUTIONS EXIST WITH THE ASYMPTOTIC BEHAVIOR

$$\epsilon \sim \epsilon_0 + O(r^{-1})$$

ϵ_0 : constant spinor

THEN ... $\longrightarrow$

(*) $T_{\rho\sigma,IJ} = 2\sqrt{2}\, e^{-\phi}[F_{\rho\sigma}\alpha_{IJ} + i\, G_{\rho\sigma}\beta_{IJ}]$

1) OUR BOSONIC BACKGROUND IS INVARIANT UNDER SOME SUSY TRANSFORMATIONS
2) CONSERVED SUPERSYMMETRIC CHARGES CAN BE DEFINED (IN +/- THE SAME WAY ONE USES KILLING VECTORS TO DEFINE CONSERVED QUANTITIES IN G.R.) (DON'T ASK FOR DETAILS, PLEASE!)
3) IN GENERAL, A SUB-SUPERALGEBRA CAN BE BUILT SUCH THAT THE STATE CORRESPONDING TO THE BACKGROUND BELONGS TO ONE OF ITS REPRESENTATIONS (SUPERMULTIPLET)
4) "BOGOMOLNYI BOUNDS" CAN BE DERIVED WITH THE SUB-SUPERALGEBRA
5) AND ARE SATURATED BY THE SUPERSYMMETRIC BACKGROUND
6) IN MANY CASES OF BLACK HOLE BACKGROUNDS THE BOGOMOLNYI BOUND COINCIDES WITH COSMIC CENSORSHIP CONDITIONS

$\Rightarrow$ SATURATION OF B.B = EXTREME B.H.

we will see examples

INVARIANCE OF N=2 SUSY UNDER U(1) DUALITY

ASSUME

$$\delta_\varepsilon \psi_{\mu I} = \nabla_\mu \varepsilon_I - \frac{1}{2\sqrt{2}} \sigma^{\rho\sigma} F^{(-)}_{\rho\sigma} \varepsilon_{IJ} \gamma_\mu \varepsilon^J = 0$$

ONLY THE SELFDUAL PART OF F ENTERS

$$F \to \cos\alpha \, F + i \sin\alpha \, {}^*F \quad \Rightarrow \quad F^+ \to e^{i\alpha} F^+$$

THIS FACTOR CANBE ABSORBED IN THE SPINORS

$$\begin{cases} \varepsilon'_I = e^{+i\alpha/2} \, \varepsilon_I \\ \varepsilon'^I = e^{-i\alpha/2} \, \varepsilon^I \end{cases}$$

CHIRAL CONJUGATES

PERFORMING DUAL ROTATIONS OF KNOWN SOLUTIONS WHICH ARE SUPERSYMMETRIC (EXTREME REISSNER-NORDSTRÖM BHs) WE GET SOLUTIONS WITH THE SAME METRIC (→ AGAIN EXTREME) WHICH ARE ALSO SUPER-SYMMETRIC, WITH DIFFERENT CHARGES.

B.B. $\boxed{M^2 - Q^2 - P^2 \geq 0 \text{ is INVARIANT}}$

INVARIANCE OF N=4 SUSY UNDER SL(2,$\mathbb{R}$) DUALITY

IF THERE ARE SOME ε_I SUCH THAT

$$\delta_\varepsilon \psi_{\mu I} = 0$$
$$\delta_\varepsilon \Lambda_I = 0$$

FOR THE BACKGROUND g, z, F, G, THEN

$$\varepsilon_I' = \left(\frac{i\gamma z + \delta}{|i\gamma z + \delta|}\right)^{1/2} \varepsilon_I$$

MAKES THE EQUATIONS HOLD FOR THE SL(2,$\mathbb{R}$) ROTATED BACKGROUND

$$\begin{cases} g \\ z' = \dfrac{\alpha z - i\beta}{i\gamma z + \delta} \\ F'^{+} = (i\gamma z + \delta) F^{+} \\ G'^{+} = (i\gamma z + \delta) G^{+} \end{cases}$$

THIS EXPLAINS THE INVARIANCE OF THE B.B.

CONCLUSIONS

- We have showed a method for generating new solutions with new charges.
- It preserves supersymmetry providing more examples of the relation

$$\text{SUSY} \approx \text{C.C.C.}$$

since the metric is invariant.

SOME QUESTIONS

- Is it a symmetry of the full S.T.?
- Since the metric is invariant, the temperature T and entropy S $\left(\frac{\kappa}{2\pi}, \frac{A}{4}\right)$ of the transformed black holes is the same.

  However we have seen that the action I is not invariant in general (nor is the Euclidean I_E)

  Will the relation

$$I_E = S = \frac{A}{4}$$

  still hold for the transformed solutions?
- Are there other B.B.-SUSY "duality" transformations acting on M?