The Asymptotic Weak Gravity Conjecture for Open Strings

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based on:

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Back to the Swamp
September 27, 2022

Introduction — Weak Gravity Conjecture

The Weak Gravity Conjecture (WGC):

In a U(1) gauge theory coupled to gravity, there must exists a super-extremal state satisfying

[Arkani-Hamed, Motl, Nicolis, Vafa '06]

$$\frac{q^2 g_{YM}^2}{m^2}\Big|_{\text{state}} \geq \frac{Q^2 g_{YM}^2}{M^2}\Big|_{\text{black hole}}$$

Motivation: Extremal Black holes should be able to decay.

Alternatively: State with highest charge-to-mass ratio should be self-repulsive in order not to form bound states:

$$|F_{\text{Coulomb}}| \geq |F_{\text{grav}}|$$

$$\frac{q^2 g_{YM}^2}{m^2} \geq \frac{1}{M_P^{d-2}} \frac{d-3}{d-2}\,.$$

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$$|F_{\text{Coulomb}}| \geq |F_{\text{grav}}| \qquad + \qquad |F_{\text{Yukawa}}|$$

$$\frac{q^2 g_{YM}^2}{m^2} \geq \frac{1}{M_P^{d-2}} \frac{d-3}{d-2}\,.$$

In the presence of scalars

[Palti '17; Lee, Lerche, Weigand '18; Heidenreich, Reece, Rudelius '19]

Introduction — Weak Gravity Conjecture

Stronger Version: **Tower Weak Gravity Conjecture (tWGC)**

[Heidenreich, Reece, Rudelius '16, Montero, Shiu, Soler '16
Andriolo, Junghans, Noumi, Shiu '18]

There exists an infinite tower of states satisfying

$$\frac{q_k^2}{M_k^2} \geq \frac{1}{\Lambda_{\mathrm{WGC}}^2}$$

Magnetic weak gravity conjecture scale

$$\Lambda_{\mathrm{WGC}} = g M_{\mathrm{pl}}^{d-3}$$

Motivation: WGC should be consistent under KK reduction

[Heidenreich, Reece, Rudelius '16-'18]

WGC states can be either

- (Small) Black Holes ($M_k > M_{\mathrm{pl}}$); can be super-extremal due to α'-corrections.
- (Super-)extremal particles ($M_k < M_{\mathrm{pl}}$)
  → (t)WGC gives a constraint on the EFT spectrum.

Introduction — Asymptotic WGC

In this talk: consider asymptotic weak gravity conjecture, i.e. WGC in the limit $g_{\text{YM}} \to 0$

$\updownarrow$ Swampland Distance Conjecture

$g_{\text{YM}} \to 0$ infinite distance limit $\leftrightarrow$ tower of superextremal states (?)

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↕ Swampland Distance Conjecture

$g_{\mathrm{YM}} \to 0$ infinite distance limit ↔ tower of superextremal states (?)

Evidence for tWGC:

Closed String U(1)'s	Perturbative Heterotic Gauge Theories	Open string gauge theories
e.g. 5d N=1 -or- 4d N=2 e.g. KK-U(1)	e.g. 6d N=(1,0) -or- 4d N=1	e.g. 4d N=1 F-theory
Tower of charged BPS states (e.g. M2-brane/D2/D0-branes …)	Tower of super-extremal string excitations due to modularity	Also tower of string excitations (?)
[Grimm, Palti, Valenzuela '18, Gendler, Valenzuela '20 Bastian, Grimm, v.d. Heisteeg '20, Alim, Rudelius, Heidenreich '21	[Arkani-Hamed et al '06, Heidenreich, Reece, Rudelius '16-'18 Montero, Shiu, Soler '16, Lee, Lerche, Weigand '18-'19 Klawer, Lee, Weigand, MW '20]	[Heidenreich, Reece, Rudelius '21] + this talk! [Cota, Mininno, Weigand, MW '22]

Introduction — Asymptotic WGC

In this talk: weak coupling limits in 4d N=1 F-theory compactifications

- F-theory on elliptically fibered CY fourfolds $\pi : Y_4 \to B_3$
- Gauge theory with gauge group G realized on divisors $D \subset B_3$ with gauge coupling $g_{\rm YM}^{-2} \propto \text{vol}(D)$.
- Weak coupling limits: limits in Kähler moduli space such that $\text{vol}(D) \to \infty$.

Question: what furnishes the tower of super-extremal states?

- Candidate 1: KK tower $\to$ no charged states
- Candidate 2: excitation of D3-brane wrapped on curve C s.t. $C \,.\, D \neq 0$
  $\to$ string worldsheet theory has fermions charged under G.
  [Lawrie, Schafer-Nameki, Weigand'16]
- If perturbative string weakly-coupled: particle-like excitations are charged under G!
  cf. [Heidenreich, Reece, Rudelius '21]

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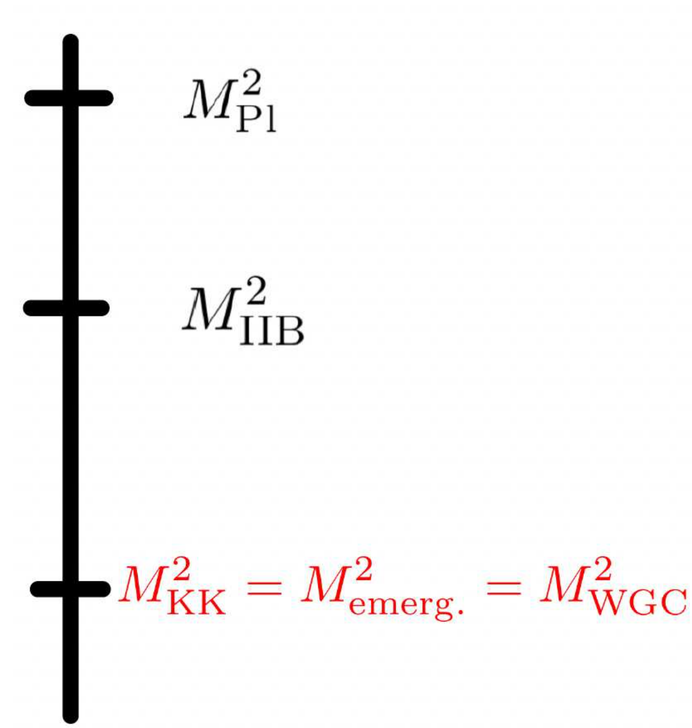
Weak Coupling Limits in F-theory

Can this work? Yes it does (at least) in emergent string limits. [Lee, Lerche, Weigand '18-'20]

- B_3 has itself rational fibration, $\rho : B_3 \to B_2$
- D3-brane on *generic* fiber string gives rise to critical string.
- Consider gauge theory on $D \sim B_2$, weak coupling limit $\hat{=}$ $\text{vol}(B_2) \to \infty$.
- Existence of tower of *marginally* super-extremal states due to modular properties of elliptic genus.
- RFC and WGC relation also work at 1-loop level.



[Kläwer, Lee, Weigand, MW '20]

What about more general settings?

Need:

1. A classification of weak coupling limits in terms of the strings.
2. Identify perturbative, weakly coupled strings that are charged under gauge group.
3. Check the WGC relation for the excitations of the string!

Main results

[Cota, Mininno, Weigand, MW '22]

1. **Provide a classification of weak coupling limits in terms of *weakly-coupled EFT* strings.**

2. **In such limits: tWGC is only satisfied by string excitations if species scale is set by tension of weakly-coupled string**

3. **tWGC is only satisfied by string excitations for emergent heterotic string limits!**

EFT String Limits

[Lanza, Marchesano, Martucci, Valenzuela '20,'21]

At **Infinite Distance Limits** in field space of 4d N=1 or 2 EFT: unbroken axionic shift symmetries

$$\text{chiral fields in 4d N=1 } T_i = s_i + i a_i : \qquad T_i \sim T_i + i c_i \qquad c_i \in \mathbb{R}$$

Shift symmetry $\longrightarrow$ can dualize axion into two-forms: $\quad a_i \leftrightarrow B_2^i$

Strings charged under 2-form

$$S = \int_{\text{string}} e_i B_2^i + \ldots$$

- Can associate 1/2-BPS cosmic string solution to string of charge (e_i)

cf. [Greene, Shapere, Vafa, Yau '90]

- Backreaction of string induces logarithmic profile for chiral fields

$$T_i(z) = T_i^0 - \frac{e_i}{2\pi} \log \frac{z}{z_0} \qquad z: \text{ 4d coordinate transverse to string}$$

- Close to string core: realize infinite distance limit $T_i \to \infty$!

- EFT string $\hat{=}$ instantons charged under shift symmetry induced by string become suppressed.

EFT String Limits — F-theory Kähler Moduli Space

[Lanza, Marchesano, Martucci, Valenzuela '20,'21]

For the F-theory 4d N=1 Kähler field space we have:

- Chiral fields: $T_i = \frac{1}{2}\int_{D_a} J \wedge J + i \int_{D_a} C_4$

J: Kähler form on B_3
D_a: Generators of $\mathrm{Eff}^1(B_3)$
C_4: Type IIB RR four-form

- Instantons: Euclidean D3-branes on effective divisors $D \in \mathrm{Eff}^1(B_3)$.
- Strings: D3-branes on curves C in movable cone $\mathrm{Mov}_1(B_3)$.

EFT string limits:

For a subset $\mathcal{I} \subset \mathrm{Eff}^1(B_3)$ of generators of effective cone:

$$\mathcal{V}_D \sim \lambda \to \infty \qquad \forall D \in \mathcal{I}$$
$$\mathcal{V}_{D'} \sim \text{finite} \qquad \forall D' \notin \mathcal{I}$$

Primitive EFT string with $\underline{e} = (0,\ldots,0,e_i,0,\ldots,0)$: $|\mathcal{I}| = 1$

Distant Axionic String Conjecture:

Primitive large volume limit for generator $D_i \in \mathrm{Eff}^1(B_3)$ $\Longleftrightarrow$ EFT string on dual generator $C^i \in \mathrm{Mov}_1(B_3)$

Classifying Weak Coupling Limits

For gauge theory on effective divisor $D \in \mathrm{Eff}^1(B_3)$:

- $D = \sum_a \kappa_a D_a$ with D_a generators of $\mathrm{Eff}^1(B_3)$ and $\kappa_a \geq 0$.
- Can take primitive EFT string limit for *one* $D_a \subset D$.
- D3-brane on dual curve $C^a \in \mathrm{Mov}_1(B_3)$ becomes weakly-coupled string.
- Can check Repulsive Force Condition for excitations of string!

Caveat:
- Primitive EFT string limit does not exist in every chamber of extended Kähler cone!
  - Reaching limit, $\mathscr{V}_{D_a} \to \infty$ $\mathscr{V}_{D'_a} \sim$ finite, might require passing through flop transition.
    $\to$ what happens at flop transition in N=1 theory?
  - EFT string spectrum differs depending on chamber of classical Kähler cone.

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Classifying EFT strings

[Cota, Mininno, Weigand, MW '22]

To avoid floppy problems: stay within fixed Kähler cone chamber!

Question: How can we classify EFT strings and their corresponding limits?

- Problem: Volume of divisors cannot be scaled independently.
- But: volume v^i of Mori cone generators $\mathscr{C}^a$ can be scaled independently!

$$J = v^i J_i \qquad v^i = \int_{\mathscr{C}^a} J$$

For (quasi-)primitive EFT string limits:

- Fix Mori cone generator $\mathscr{C}^0$.
- Consider the limit: $v^0 = \mathscr{V}_{\mathscr{C}^0} \sim \lambda \to \infty$.
- Co-scale other $v^a \sim \lambda^{-1}$ or $\sim \lambda$ such that $|\mathscr{I}|$ of homogeneously expanding divisors is minimized.
- Check that no effective divisor vanishes asymptotically due to co-scaling.

If this leads to a homogeneous limit for divisor volumes $\Rightarrow$ quasi-primitive EFT string limit!

Simple Classification

Resulting classification: [Cota, Mininno, Weigand, MW '22]

For (quasi-)primitive EFT string limit associated to Mori cone generator $\mathscr{C}^0$ dual to Kähler cone generator J_0 the associated EFT string is obtained from a D3-brane wrapped on

1. $C = \alpha J_0^2$ *if* $J_0^2 \neq 0$
2. $C = \alpha J_0 \cdot J_i$ *if* $J_0^2 = 0$ *for suitable Kähler cone generator* $J_i \neq J_0$

For curve of the form $C = D_1 \cdot D_2$ *with* $D_{1,2}$ *effective divisors define*

$$q = \Theta(D_1^2 \cdot D_2) \;+\; \Theta(D_1 \cdot D_2^2)$$

$\Longrightarrow$ ***quasi primitive EFT string limits thus come in three classes:***

$$q = 0,1,2$$

Crucial result: for primitive EFT strings the qualitative behavior in the weak coupling behavior only depends on value of q!

q - Dependence

Importance of q manifests itself in following result: [Cota, Mininno, Weigand, MW '22]

In the limit induced by a primitive EFT string, the volume of B_3 factorizes asymptotically as:

$$\mathcal{V}_{B_3}^2 \to (\text{Re}\, T_0)^{1+q} P_{2-q}(\text{Re}\, T_{i\neq 0})$$

where $\text{Re}\, T_0 \to \infty$ *and* $\text{Re}\, T_{i\neq 0}$ *finite in the asymptotic limit.*

The Kähler potential is asymptotically of the form:

$$K = -(1+q)\log(T_0 + \bar{T}_0)$$

Consider gauge theory on divisor $S = \kappa D_0 + \ldots$

In the primitive EFT string limit for $\mathscr{C}^0$ (dual to D_0), the gauge coupling behaves as

$$\frac{2\pi}{g_{\text{YM}}^2} = \kappa \text{Re}\, T_0 + \ldots \to \infty$$

Perturbative String Excitations (?)

In EFT string limit ($\text{Re}\, T_0 \to \infty$) the effective action reduces to

$$S_{4d} = \frac{M_{\text{Pl}}^2}{2} \int \left(R \star 1 - (1+q) \frac{\text{d}T_0 \wedge \star \text{d}\bar{T}_0}{(T_0 + \bar{T}_0)^2} \right) - \frac{\kappa M_{\text{Pl}}^2}{8} \int \left(\text{Re}\, T_0 \ \text{tr}|F|^2 - i \text{Im} T_0 \ \text{tr}(F \wedge F) \right) + \dots .$$

- Weak coupling limit for gauge theory.
- Weak coupling limit for EFT string on curve $C = J_0^2$ or $J_0 \cdot J_1$.
- Action very reminiscent of heterotic string at weak coupling ($q = 0$)

$T_0 \hat{=}$ heterotic axio-dilaton

Question: Can excitations of string furnish the superextremal tower?

Rough parametric dependence:

String tension

$$\frac{T_{\text{EFT}}}{M_{\text{Pl}}^2} \sim \frac{1}{\text{Re}\, T_0}$$

WGC scale

$$\frac{\Lambda_{\text{WGC}}^2}{M_{\text{Pl}}^2} \sim g_{\text{YM}}^2 \sim \frac{1}{\text{Re}\, T_0}$$

⟷ Correct behavior for expected tower of super-extremal states

cf. [Heidenreich, Reece, Rudelius '21; Kaya, Rudelius '22]

Weak Gravity Conjecture

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But what about the $\mathcal{O}(1)$ factors?

Assume EFT string has particle like excitations

'Heterotic-like' quantization: $M_k^2 = 8\pi T_{\rm EFT}(n_k - E_0)$

Modularity of elliptic genus → maximal charge at excitation level n: $q^2 \geq 4mn \qquad m = \frac{1}{2} C \cdot \mathbf{S}$

Repulsive Force Condition:

$$\frac{g_{\rm YM}^2 q_k^2}{M_k^2} \geq \frac{1}{M_{\rm Pl}^2}\left[\frac{d-3}{d-2}\bigg|_{d=4} + \frac{1}{4}\frac{M_{\rm Pl}^4}{M_k^4} g^{rs}\partial_r\left(\frac{M_k^2}{M_{\rm Pl}^2}\right)\partial_s\left(\frac{M_k^2}{M_{\rm Pl}^2}\right)\right]$$

Explicit evaluation for primitive q-EFT string: $\frac{1}{1+q} \overset{!}{\geq} \frac{1+\frac{q}{2}}{1+q}$

Satisfied only for $q=0$ EFT string ↔ *critical* heterotic string!

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Weak Gravity Conjecture vs Species Scale

How to explain this mismatch?

→ *assumption of particle-like excitations not valid for* $q \neq 0$
→ compare string tension to quantum gravity cut-off, i.e. species scale Λ_{sp}!

Species scale for different towers of light states: [Dvali '07], see also Dieter's talk!

Light string excitations: $\Lambda^2_{\text{sp,str}} \sim T_{\text{EFT}}$ KK tower for n compact dimensions: $\frac{\Lambda^2_{\text{sp,KK}}}{M^2_{\text{Pl}}} = \left(\frac{M^2_{\text{KK}}}{M^2_{\text{Pl}}}\right)^{\frac{n}{2+n}}$

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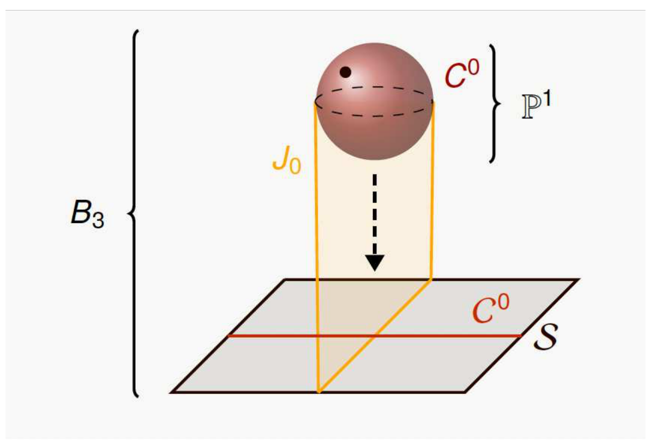
<td>
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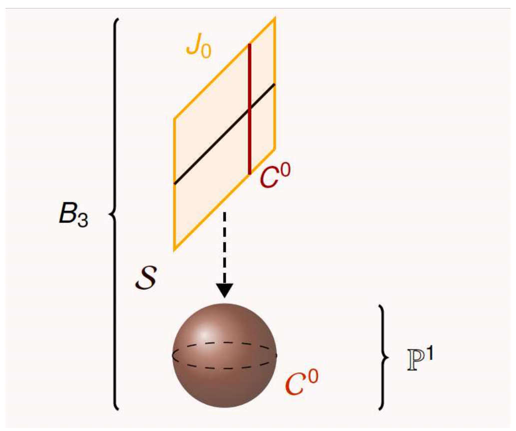
<td>
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<td>C_{EFT}
Is general curve in B_3</td>
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<tr><td>KK tower for n=4</td><td>KK tower for n=2</td><td>KK tower for n=6</td></tr>
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<td>$\frac{\Lambda^2_{\text{sp,KK}}}{M^2_{\text{Pl}}} \sim \left(\frac{\Lambda^2_{\text{sp,str}}}{M^2_{\text{Pl}}}\right)^{2/3}$</td>
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</table>

Only for $q = 0$ the species scale is set by the string!

Weak Gravity Conjecture vs Species Scale

[Cota, Mininno, Weigand, MW '22]

For $q = 0$:

- Genuine 4d weakly-coupled gauge theory: $T_{\text{EFT}} \sim \Lambda_{\text{WGC}} \ll \Lambda_{\text{sp,KK}}$
- tWGC $\hat{=}$ RFC and is satisfied by string excitations

"Emergent string limit"

For $q = 1$:

- Gauge theory effectively 6d: $T_{\text{EFT}} \sim \Lambda_{\text{WGC}} \sim \Lambda_{\text{sp,KK}}$
- 6d string and gauge theory *not* weakly-coupled, string does not yield particle-like excitations for the EFT
- Need to evaluate WGC for strongly-coupled theory
  $\rightarrow$ used form of RFC not valid

"Decompactification Limit to 6d"

For $q = 2$:

- Gauge theory becomes 8d defect theory in 10d: $T_{\text{EFT}} \sim \Lambda_{\text{WGC}} \gg \Lambda_{\text{sp,KK}}$
- No string in higher dimension (resolve the entire D3-brane)
- Gauge theory and gravity decouple entirely: WGC condition should become trivial!
- Indeed: (p,q)-string excitation super-super-extremal: $\dfrac{g_{\text{YM}}^2 q^2}{M_k^2/M_{\text{Pl}}^2} \rightarrow \infty$

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- Need to evaluate WGC for strongly-coupled theory
  $\rightarrow$ used form of RFC not valid

"Decompactification Limit to 6d"

For $q = 2$:

- Gauge theory becomes 8d defect theory in 10d: $T_{\text{EFT}} \sim \Lambda_{\text{WGC}} \gg \Lambda_{\text{sp,KK}}$
- No string in higher dimension (resolve the entire D3-brane)
- Gauge theory and gravity decouple entirely: WGC condition should become trivial!
- Indeed: (p,q)-string excitation super-super-extremal: $\dfrac{g_{\text{YM}}^2 q^2}{M_k^2/M_{\text{Pl}}^2} \rightarrow \infty$

"Decompactification Limit to 10d"

Other limits

[Cota, Mininno, Weigand, MW '22]

What about non-EFT string limits?

Gauge theory on $\mathbf{S} = \kappa D_0$ but we take a weak coupling limit that is *not* a (quasi)-primitive EFT string limit

Two possibilities:

1. In weak coupling limit gauge theory becomes defect theory in 10d or 8d.

   similar to q=2 EFT string limits.

2. In weak coupling limit gauge theory becomes strongly coupled theory in 8d.

   similar to q=1 EFT string limit

Summary: EFT string excitations only satisfy the tWGC if

1. the solitonic EFT string with $T_{\text{EFT}} = \Lambda_{\text{WGC}}$ is a heterotic string and
2. the gauge theory can be identified with a *perturbative* gauge theory in the dual heterotic string duality frame associated with the EFT string.

Conclusions

Considered weak coupling limits for open string gauge theories in F-theory compactifications to 4d N=1.

1. **Classified weak coupling limits in terms of EFT strings $\hat{=}$ D3-branes wrapped on curves $C \subset B_3$**

   → 3 types of curves depending on value of $q = 0,1,2$

2. **Three types have different behavior for species scale relative to WGC scale:**

   $q = 0: \;\; \Lambda_{\text{WGC}} \lesssim \Lambda_{\text{sp}}$ $\qquad q = 1: \;\; \Lambda_{\text{WGC}} \sim \Lambda_{\text{sp}}$ $\qquad q = 2: \;\; \Lambda_{\text{WGC}} \gg \Lambda_{\text{sp}}$

3. **tWGC satisfied by string excitations only for $q = 0$, i.e. heterotic string**

   → **all other limits are:** $i)$ non-weakly coupled higher dimensional theories
   $ii)$ higher-dimensional defect theories

Thank you!!