EFT Strings and Quantum Gravity Bounds in F-theory

- 2209.XXXX with Luca Martucci and Nicolo Risso
- Earlier works with Seung-Joo Lee and with Antonella Grassi

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Motivation

Luca's talk: General bounds on rank of gauge algebra in 4d $N=1$ supergravity theories from EFT strings

$$r(\mathbf{e}) \leq 2\langle \tilde{C}, \mathbf{e} \rangle - 2$$

in a theory with axionic couplings $S \supset -\frac{1}{96\pi} \tilde{C}_i \int_{\mathbb{R}^{1,3}} a^i \, \mathrm{tr} R \wedge R$

This talk: Concrete realisation of these bounds in F-theory

- Check of assumptions underlying EFT string spectrum in computable frameworks
- Sharpening of bound based on microscopic realisation of intrinsic interest in F-theory

Novel sharpened bound:

$$\boxed{r(\mathbf{e}) \leq \frac{5}{6} \Delta \cdot \Sigma_{\mathbf{e}} - 2}$$

for minimally $N=1$ F-theory models on smooth base spaces

EFT strings

[Lanza,Marchesano,Martucci,Valenzuela'20-21], cf talk by L. Martucci

EFT strings: strings charged under the 2-form fields,

$$S = \int_{\text{string}} e_i\, B_2^i + \ldots\,,$$

whose dual saxions become weakly coupled in the limit induced by the string backreaction:

- Backreaction of such strings induces in turn the infinite distance limit

$$T_i(z) = T_i^{(0)} - \frac{e_i}{2\pi}\log\left(\frac{z}{z_0}\right) \qquad z : \text{transverse} \subset \mathbb{R}^{1,3}$$

  $N=1$ chiral multiplets $T_i = s_i + i a_i$: $T_i \sim T_i + i\, c$

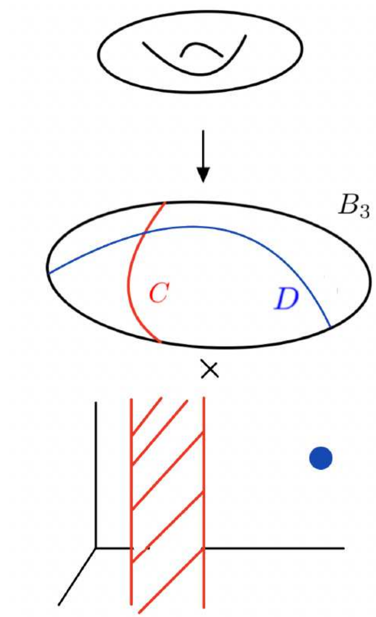
$$a_i \Longleftrightarrow B_2^i$$

- Those instantons are suppressed which are dual to the EFT string inducing the limit.

EFT strings from $\mathrm{Mov}_1(B_3)$

N=1 Kähler moduli space in F-theory: [Lanza,Marchesano,Martucci,Valenzuela'20-21]

- **Instantons:**
  Euclidean D3 on effective divisors
  $D \in \mathrm{Eff}^1(B_3)$



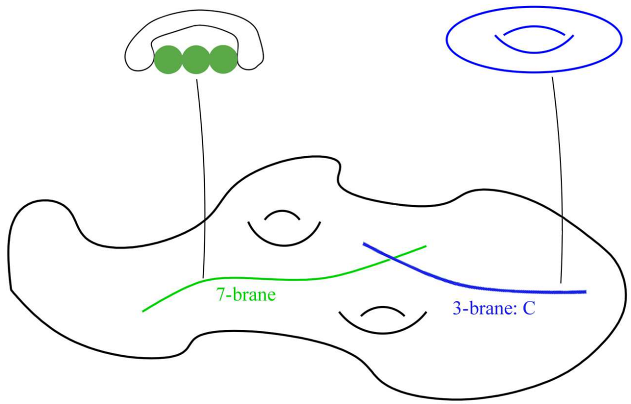
- **EFT Strings:**
  D3 on curves C in dual cone of **movable curves $\mathrm{Mov}_1(B_3)$**

- **Movable curves** can probe entire base
  (live in a family that covers dense open subset of B_3)

- EFT strings sensitive to gravity - analogous to 5d supergravity strings
  [Katz,Kim,Tarazi,Vafa,'20]

Characterisation of movable curves on B_3 and associated EFT string limits
in [Cota,Mininno,TW,Wiesner'22] see talk by M. Wiesner
see also [Alim,Heidenreich,Rudelius'22] and talk by L. McAllister

EFT strings from $\mathrm{Mov}_1(B_3)$



F-theory on elliptic CY_4 with base B_3

D3-brane on $\mathbb{R}^{1,1} \times C$

C a curve in base $C \in \mathrm{Mov}_1(B_3)$

2 important properties of movable curves C:

1. Can assume movable C is not contained in discriminant locus

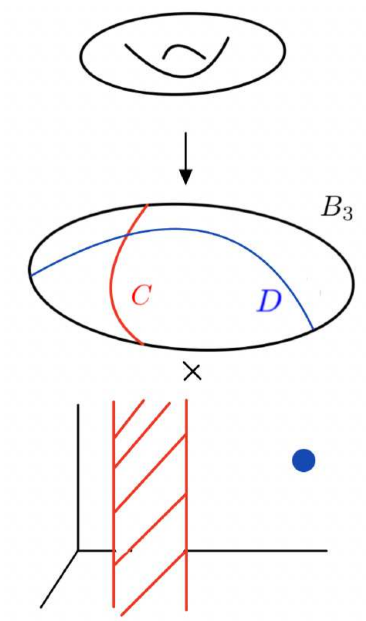
$$\Delta = 12\bar{K}_{B_3} = \text{totality of 7-branes}$$

   - C is transverse to 7-branes on B_3
   - C intersects 7-branes in isolated points on B_3
     $\Longrightarrow$ charged fermionic modes from 3-7 strings

2. Anti-canonical class $\bar{K}_{B_3} \in \mathrm{Eff}^1(B_3) \longrightarrow \bar{K}_{B_3} \cdot C \geq 0$

Worldsheet Theory

Describe EFT worldsheet theory in F-Theory [Lawrie,Schafer-Nameki,TW'16] via **topological duality twist** [Martucci'14]



Reduce $N = 4$ SYM on single D3-brane with worldvolume

$$\mathbb{R}^{1,1} \times C$$

$\Longrightarrow$ 2d N=(0,2) theory on worldsheet

$\Longrightarrow$ Massless multiplets by twisted reduction of

- gauge field A
- 6 adjoint scalars ϕ_i
- 16 fermionic partners Ψ

Massless Spectrum

Multiplets	(0,2) Type	Origin	Interpretation	Zero-mode Cohomology
U	Chiral	(ϕ_i, Ψ)	Universal	$h^0(C) = 1$
$\Phi^{(1)}$	Chiral	(ϕ_i, Ψ)	Deformations	$n_{\rm C}^{(1)} = h^0(C, N_{C/B_3})$
$\Phi^{(2)}$	Chiral	(A, Ψ)	Twisted Wilson lines	$n_{\rm C}^{(2)} = h^0(C, K_C \otimes \bar{K}_{B_3})$ $= g - 1 + \bar{K}_{B_3} \cdot C$
$\Psi^{(1)}$	Fermi	Ψ	Obstructions	$n_{\rm N}^{(1)} = h^1(C, N_{C/B_3})$ $= h^0(C, N_{C/B_3}) - \bar{K}_{B_3} \cdot C$
$\Psi^{(2)}$	Fermi	Ψ	Obstructions (?)	$n_{\rm N}^{(2)} = h^1(C) = g$
Λ	Fermi	3-7 strings	Charged	$n_{\rm F} = 8\, \bar{K}_{B_3} \cdot C$

- $n_{\rm C}^{(1)} - n_{\rm N}^{(1)} = h^0(C, N_{C/B_3}) - h^1(C, N_{C/B_3}) = \bar{K}_{B_3} \cdot C$ topological index that agrees with number of unobstructed complex geometric deformations of curve C inside B_3
- $n_{\rm C}^{(2)} - n_{\rm N}^{(2)} = \bar{K}_{B_3} \cdot C - 1$ topological index - conjectured to agree with number of unobstructed twisted Wilson line moduli

General Bound

General bound on rank of gauge group detected by EFT string of charge $\mathbf{e}$

$$r(\mathbf{e}) \leq n_{\rm F}(\mathbf{e}) + 2n_{\rm C}^{\rm eff} = 2\langle \tilde{C}(\mathbf{e}), \mathbf{e}\rangle - 2$$

- $n_{\rm F}(e)$ number of Fermi multiplets charged under 7-brane gauge group
- $n_{\rm C}^{\rm eff} = n_{\rm C} - n_{\rm N}$ number of unobstructed chiral multiplets which can experience gauged shift symmetry
- $\tilde{C}$: gravitational higher derivative coupling

Specialisation: [Martucci,Risso,TW'22]

Rank of 7-brane group detected by string from D3 brane on curve $\Sigma_{\mathbf{e}}$:

- $n_{\rm C}^{\rm eff} = (n_{\rm C}^{(1)} - n_{\rm N}^{(1)}) + (n_{\rm C}^{(2)} - n_{\rm N}^{(2)})$
  - $n_{\rm C}^{(1)} - n_{\rm N}^{(1)} = \Sigma_{\mathbf{e}} \cdot \bar{K}$
  - $n_{\rm C}^{(2)} - n_{\rm N}^{(2)} = \Sigma_{\mathbf{e}} \cdot \bar{K} - 1$
- $n_{\rm F}(e) = 8\Sigma_{\mathbf{e}} \cdot \bar{K}$

$$\boxed{r(\mathbf{e}) \leq 12\Sigma_{\mathbf{e}} \cdot \bar{K} - 2 = \Sigma_{\mathbf{e}} \cdot \Delta - 2}$$

Consistently:

$\tilde{C} = 6\bar{K}$ from effective action

[Grimm,Taylor'12]

Sharpened Bound

$$r(\mathbf{e}) \leq n_{\mathrm{F}}(\mathbf{e}) + 2n_{\mathrm{C}}^{\mathrm{eff}}$$

$$n_{\mathrm{C}}^{\mathrm{eff}} = (n_{\mathrm{C}}^{(1)} - n_{\mathrm{N}}^{(1)}) + (n_{\mathrm{C}}^{(2)} - n_{\mathrm{N}}^{(2)})$$

- $n_{\mathrm{C}}^{(1)} - n_{\mathrm{N}}^{(1)} = \Sigma_{\mathbf{e}} \cdot \bar{K}$
- $n_{\mathrm{C}}^{(2)} - n_{\mathrm{N}}^{(2)} = \Sigma_{\mathbf{e}} \cdot \bar{K} - 1$

Stronger bound for minimally SUSY F-theory over smooth base B_3

[Martucci,Risso,TW'22]

- $\Phi^{(1)}$: geometric moduli of curve $\Sigma_{\mathbf{e}}$ in B_3
  Under above assumptions, $\Phi^{(1)}$ cannot enjoy gauged shift symmetries
- $\Phi^{(2)}$: Of same origin as charged Fermis
  $\Longrightarrow$ candidates for gauged shift symmetries

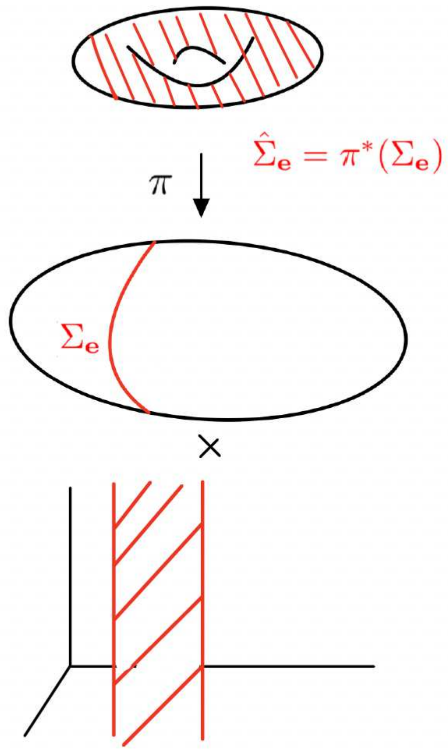
$$r(\mathbf{e}) \leq n_{\mathrm{F}}(\mathbf{e}) + 2n_{\mathrm{C}}^{\mathrm{eff},(2)} = 10\,\Sigma_{\mathbf{e}} \cdot \bar{K} - 2 = \frac{5}{6}\Sigma_{\mathbf{e}} \cdot \Delta - 2$$

Massless moduli from M-theory

EFT string from M5-brane on vertical divisor $\hat{\Sigma}_{\mathbf{e}} = \pi^*(\Sigma_{\mathbf{e}})$:

4d analogue of MSW string

[Maldacena,Strominger,Witten'96]



$\Longrightarrow$ Massless spectrum from reduction of

- self-dual 2-form B
- complex scalars Φ
- and fermionic partners

Massless moduli from M-theory

1) Reduction of chiral 2-form B

[Lawrie,Schafer-Nameki,TW'16]

- $h^{1,1}(\hat{\Sigma}_{\mathbf{e}}) - 1$ LEFT scalars
- $2h^{2,0}(\hat{\Sigma}_{\mathbf{e}}) + 1$ RIGHT scalars

$\Longrightarrow$

- $2h^{2,0}(\hat{\Sigma}_{\mathbf{e}}) + 1$ $\Phi^{(2)}$ (and U)
- $h^{1,1}(\hat{\Sigma}_{\mathbf{e}}) - 1 - (2h^{2,0}(\hat{\Sigma}_{\mathbf{e}}) + 1) = 8\bar{K} \cdot \Sigma_{\mathbf{e}}$ LEFT scalars dualised into Fermis

- LEFT scalars in $\Phi^{(2)}$ and Fermi multiplets $\Longleftrightarrow$ $H^{1,1}(\hat{\Sigma}_{\mathbf{e}})$:
  $\Longrightarrow$ charged under gauge field from C_3 $\Longleftrightarrow$ 7-brane gauge group
- RIGHT scalars in $\Phi^{(2)}$: Of different origin and hence uncharged

2) Reduction of Φ

- gives geometric moduli $\Phi^{(1)}$ $\Longrightarrow$ do not couple to C_3
- $\Phi^{(1)}$ might enjoy gauged shift symmetry, but only from metric, i.e. geometric shift symmetries

EFT vs Kodaira bounds

$$\{\Delta = 0\} = n_I \mathcal{D}^I + \mathcal{D}' \simeq 12\overline{K} \qquad \text{with} \quad n_I \equiv \text{ord}(\Delta)|_{\mathcal{D}^I}$$

Non-abelian gauge group G_I on divisor $\mathcal{D}^I$ constrained by Kodaira bound cf. [Morrison, Taylor '11]:

$$\text{rk}(G_I) < n_I \equiv \text{ord}(\Delta)|_{\mathcal{D}^I} \,.$$

	$\text{ord}_{\mathcal{D}}(f)$	$\text{ord}_{\mathcal{D}}(g)$	$\text{ord}_{\mathcal{D}}(\Delta)$	singularity
I_0	≥ 0	≥ 0	0	none
$I_n,\ n \geq 1$	0	0	n	A_{n-1}
II	1	1	≥ 2	none
III	1	≥ 2	3	A_1
IV	≥ 2	2	4	A_2
I_0^*	≥ 2	≥ 3	6	D_4
$I_n^*,\ n \geq 1$	2	3	$6+n$	D_{4+n}
IV^*	≥ 3	4	8	E_6
III^*	3	≥ 5	9	E_7
II^*	≥ 4	5	10	E_8

For every curve C in interior of movable cone $(C \cdot D_{\text{eff}} \geq 1\, \forall D_{\text{eff}})$

$$\text{rk}(G_{\text{non-ab}}) \leq \sum_I \text{rk}(G_I)(C \cdot D_I) \leq \sum_I n_I (C \cdot D_I) + C \cdot D' = C \cdot \Delta$$

Compare: For EFT curve $C = \Sigma_{\mathbf{e}}$

$$\text{rk}(\mathbf{e}) \leq \Sigma_{\mathbf{e}} \cdot \Delta - 2$$

✓ Conservative EFT bound slightly stronger than geometric upper bound

EFT vs Kodaira bounds

Geometric Kodaira bound:

$$\mathrm{rk}(G_{\mathrm{non-ab}}) = C \cdot \Delta \qquad C \text{ inside } \mathrm{Mov}_1(B_3)$$

For EFT curve $C = \Sigma_{\mathbf{e}}$:

$$\mathrm{rk}(\mathbf{e}) \leq \Sigma_{\mathbf{e}} \cdot \Delta - 2$$

What use are the EFT string bounds?

1. Kodaira bound only sensitive to non-abelian gauge algebra, but not to abelian subgroup, i.e. total rank
   By contrast, EFT string bound includes non-abelian and abelian rank
2. Proposed stronger bound

$$\boxed{\mathrm{rk}(\mathbf{e}) \leq \frac{5}{6}\Sigma_{\mathbf{e}} \cdot \Delta - 2}$$

not obvious from geometry - even for non-abelian groups

Example: $\mathbb{P}^3$

$$H^{1,1}(B_3) = \langle H \rangle \qquad \bar{K} = 4H \qquad \Delta = 12\bar{K} = 48H$$

$$\Sigma_{\rm e} = H \cdot H : \quad r_{\rm tot} \leq \begin{cases} r(\mathbf{e})_{\rm max} & = 12\,\Sigma_{\mathbf{e}} \cdot \overline{K}_X - 2 = 46\,, & n_F, n_C^{(1)}, n_C^{(2)}, \\ r(\mathbf{e})_{\rm max}^{\rm strict} & = 10\,\Sigma_{\mathbf{e}} \cdot \overline{K}_X - 2 = 38\,, & n_F, n_C^{(2)}, \\ r(\mathbf{e})_{\rm max}^{\rm F} & = 8\,\Sigma_{\mathbf{e}} \cdot \overline{K}_X \qquad = 32\,, & n_F \end{cases}$$

Maximal rank of $SU(N)$ group in Weierstrass model [Morrison, Taylor '11]

$$SU(N_{\rm max}) = SU(32) \qquad \text{geometrically}$$

Incidentally, allowed even by bound $r(\mathbf{e})_{\rm max}^{\rm F}$, but more generally, at best $r(\mathbf{e})_{\rm max}^{\rm strict}$ can be correct:

Examples:

$G = E_6 \times E_7^4 \qquad \mathrm{rank}(G) = 34$

$G = E_6^2 \times E_7^3 \qquad \mathrm{rank}(G) = 33$

Caveats:

Non-minimal fibers $\rightarrow$ blowups

Flux quantisation $\rightarrow$ non-trivial flux

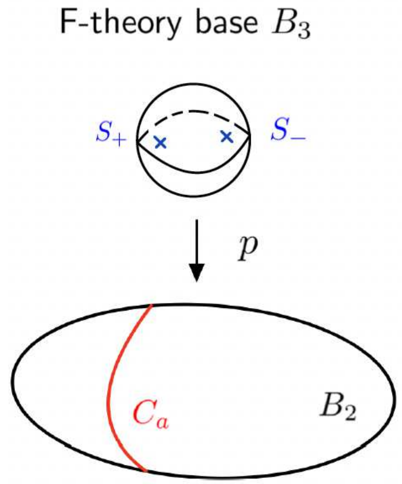
Example: Rational fibration

Rational fibration $\mathbb{P}^1 \hookrightarrow B_3 \to B_2$

- 2 sections S_-, S_+:

$$S_\pm \cdot S_\pm = \pm S_- \cdot p^* c_1(\mathcal{T})$$
$$S_- \cdot S_+ = 0$$

- $\overline{K}_{B_3} = 2S_- + p^* c_1(\mathcal{T}) + p^* c_1(B_2)$



Effective divisor cone $\mathrm{Eff}^1(B_3)$:

S_-

$p^*(D^a)$

D^a generators of

$\mathrm{Eff}^1(B_2)$

$\Longleftrightarrow$

Movable curve cone $\mathrm{Mov}_1(B_3)$:

F rational fiber

$S_+ \cdot p^*(C_a)$

C_a generators of

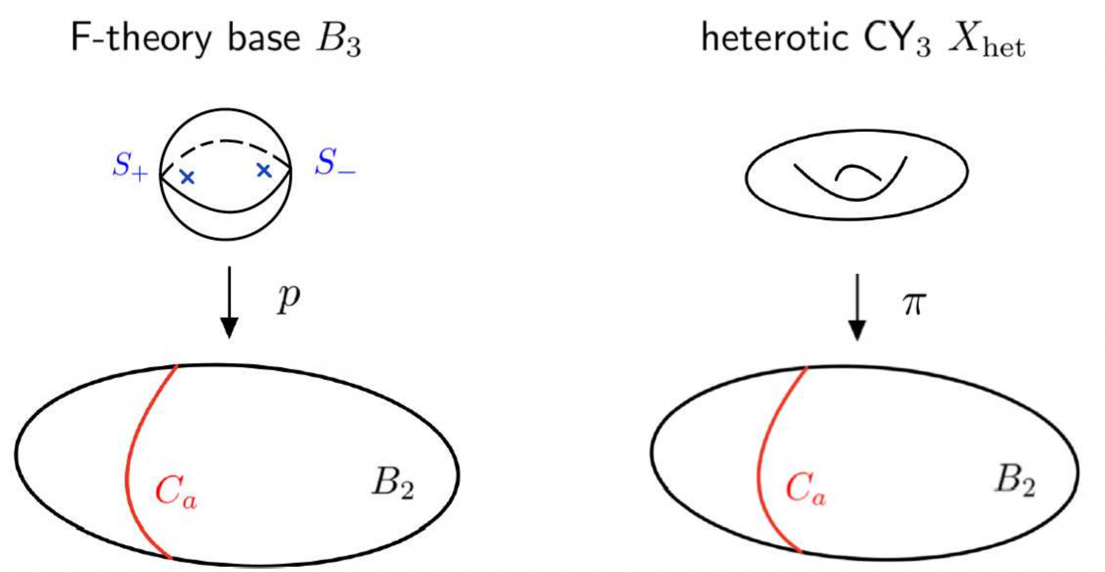
$\mathrm{Mov}_1(B_2) \equiv \mathrm{Nef}^1(B_2)$

Example: Rational fibration

Apply bound to EFT string from rational fiber F

D3-brane wrapped on rational fiber F

heterotic string of dual theory on X_{het}



Massless spectrum:

$n_{\mathrm{C}}^{(1)} = \bar{K}_{B_3} \cdot F = 2$

$n_{\mathrm{C}}^{(2)} = \bar{K}_{B_3} \cdot F - 1 = 1$

$n_{\mathrm{N}}^{(1)} = n_{\mathrm{N}}^{(2)} = 0$

4 real moduli of het. string along B_2

2 real moduli of het. string along T^2 fiber

No $U(1)_N$ charged Fermi multiplets

Example: Rational fibration

Bounds on rank of gauge group detected by EFT heterotic string:

$$r(\mathbf{e}) \leq \begin{cases} r(\mathbf{e})_{\max} & = 12\,\Sigma_{\mathbf{e}} \cdot \overline{K}_X - 2 = 22\,, \qquad n_F, n_C^{(1)}, n_C^{(2)}, \\ r(\mathbf{e})_{\max}^{\text{strict}} & = 10\,\Sigma_{\mathbf{e}} \cdot \overline{K}_X - 2 = 18\,, \qquad n_F, n_C^{(2)}, \\ r(\mathbf{e})_{\max}^{\text{F}} & = 8\,\Sigma_{\mathbf{e}} \cdot \overline{K}_X \qquad = 16\,, \qquad n_F \end{cases}$$

Claim: $r(\mathbf{e})_{\max}^{\text{strict}}$ **is indeed correct bound**

$r(\mathbf{e})_{\max}^{\text{strict}}$ can be saturated:

Example: $B_3 = \mathbb{P}^1 \times B_2$ (trivial fibration)

Various rank 18 non-abelian gauge groups possible

$$G_2 = E_6^3 : \qquad f \equiv 0\,, \qquad g = p_1^4(u,v)\, q_1^4(u,v)\, r_1^4(u,v)\, s_{6\bar{K}_{B_2}}$$

Non-minimal fibers at $s \cap s$ on B_2 avoided for $B_2 = \mathrm{dP}_9$ with $\bar{K}^2_{\mathrm{dP}_9} = 0$

Example: Rational fibration

Interpretation from dual heterotic perspective:

$$18 = 16_{\mathrm{E}_8 \times \mathrm{E}_8} + 2_{\mathrm{KK}}$$

Extra contribution from 2 KK U(1)s along 'torus fiber' of heterotic X_{het}

- Requires X_{het} to be degenerate and at (partial) orbifold point
- X_{het} is Schoen manifold and does admit orbifold degenerations [Donagi,Wendland '08]

For B_2 smooth and minimally supersymmetric, no comparable KK U(1)s from base

$\Longrightarrow$ explains stricter bound

$$r(\mathbf{e})^{\mathrm{strict}}_{\mathrm{max}} = 10\,\Sigma_{\mathbf{e}} \cdot \overline{K}_X - 2 = 18\,, \qquad n_F, n_C^{(2)}$$

for F-theory on smooth minimally SUSY setups

Compare: $B_3 = T^4 \times \mathbb{P}^1$: N=4 SUSY and $r \leq 22$ [Kim,Tarazi,Vafa'19]

Universal bounds in 6d

Bounds constrain rank of gauge algebra to which give EFT string couples

Absolute bounds on (7-brane) group require minimal $\Sigma_{\mathbf{e}}$ in interior of Mov_1:

$$\Sigma_{\mathbf{e}} \cdot D_{\mathrm{eff}} \overset{!}{\geq} 1 \quad \forall D_{\mathrm{eff}} \text{ effective}$$

Simplification for abelian (non-Cartan) U(1)s: [Lee,TW'19]
Suffices to find curve $\Sigma_{\mathbf{e}}$ such that $\Sigma_{\mathbf{e}} \cdot \bar{K}_B \geq 1$

Can be achieved for F-theory on elliptic 3-folds (6d):
Bases of elliptic 3-folds very constrained

B_2: $\mathbb{P}^2$ or (blowup of) Hirzebruch: $B_2 = \mathrm{Bl}^k(\mathbb{F}_n)$ (or Enriques)

Explicit analysis of spectrum $\Longrightarrow$ bound detected by string from curve $\Sigma_{\mathbf{e}}$:

$$r(\mathbf{e})^{\mathrm{strict}}_{\mathrm{max}} = 10\, \Sigma_{\mathbf{e}} \cdot \overline{K}_{B_2} - 2$$

Universal bounds in 6d

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For (non-Cartan) U(1) groups in 6d this gives a universal bound [Lee,TW'19]:

- $\mathbb{P}^2 : n_{U(1)} \leq 28$
- $\mathrm{Bl}^k(\mathbb{F}_n) : n_{U(1)} \leq 18$

$\Longrightarrow$ bound on rank of Mordell-Weil group of rational sections on ell. CY_3

Current Record: Schoen manifold of Namikawa type [Grassi,TW'21]

$$n_{U(1)} \leq 10$$

Generic Schoen: $n_{U(1)} = 8$ [Schoen'88]
Special Schoen: $n_{U(1)} = 9$ [Morrison,Park,Taylor'18] (12 I_2 fibers in codim-two)
Namikawa type: $n_{U(1)} = 10$ [Namikawa'02] [Grassi,TW'21]

(6 Type IV fibers in codim-two: terminal, non-$\mathbb{Q}$-factorial)

Conclusions

Applied general bottom up bounds on rank of gauge group in 4d N=1 supergravity theories to F-theory on CY_4

Novel sharpened bound: $$\boxed{r(\mathbf{e}) \leq \frac{5}{6}\Delta \cdot \Sigma_{\mathbf{e}} - 2}$$

✓ Stronger than geometric Kodaira bound

✓ Applies to abelian and non-abelian gauge group (from 7-branes)

✓ Matches expectations from dual heterotic strings, but more general

Many open questions:

- Prove assumptions on role of uncharged Fermi multiplets or at least argue that generators of movable cone do not give rise to such Fermis
- Goal: Translate this into universal bound for rank of gauge group in all 4d N=1 theories comparable to bound to bound on abelian rank in 6d
- What about matter? 6d: cf. [Tazari,Vafa'21]

Appendix: Topological Twist

Describe theory directly in language of F-Theory [Lawrie,Schafer-Nameki,TW'16] via **topological duality twist** [Martucci'14]

Theory on single D3-brane:
$\mathcal{N} = 4$ SYM

- gauge field A
- 6 adjoint scalars Φ
- 16 fermionic partners Ψ

- $G_{\text{total}} = SO(1,3)_L \times SU(4)_R \times \mathbf{U(1)_D}$
- $A_\mu \,:\, (\mathbf{2},\mathbf{2},\mathbf{1})_{*} \quad \phi_i \,:\, (\mathbf{1},\mathbf{1},\mathbf{6})_{\mathbf{0}} \quad \Psi^I_\alpha \,:\, (\mathbf{2},\mathbf{1},\mathbf{4})_{\mathbf{1}} \quad \widetilde{\Psi}_{\dot\alpha I} \,:\, (\mathbf{1},\mathbf{2},\overline{\mathbf{4}})_{-\mathbf{1}}$

$U(1)_D$: Duality symmetry incorporating $SL(2,\mathbb{Z})$ of $\mathcal{N} = 4$ SYM

- Decompose:

$$\begin{aligned} SU(4)_R &\rightarrow SO(2)_T \times SU(2)_R \times \underline{U(1)_R} \\ SO(1,3)_L &\rightarrow SO(1,1) \times \underline{U(1)_C} \end{aligned}$$

- Perform two topological twists

$$T_C^{\text{twist}} = \tfrac{1}{2}(T_C + T_R), \qquad T_D^{\text{twist}} = \tfrac{1}{2}(T_D + T_R)$$

- 2d N=(0,2) supersymmetry and massless matter transforming in various bundle cohomology groups

Appendix: Heterotic duality

F-theory:

$D_a^{\mathrm{F}} = p^*(C_a)$

$p_a := \int_{D_a^{\mathrm{F}}} c_1(\mathcal{T})$

Heterotic:

$D_a^{\mathrm{het}} = \pi^*(C_a)$

$p_a := \int_{D_a^{\mathrm{het}}} \frac{1}{2} c_2(X_{\mathrm{het}}) - \lambda(E_2)$

$\lambda(E) = -\frac{1}{16\pi^2} \mathrm{tr} F_2 \wedge F_2$

Match of EFT strings in Kähler sector $\quad$ (C_a: basis of $\mathrm{Mov}_1(B_2)$)

$\Sigma_{\mathbf{e}} = a_F F + e^a S_+ \cdot D_a^{\mathrm{F}}$

$(a_F + p_a e^a) \times (\text{fundamental})$
$+ e^a \times$ (M5 on D_a^{het})

✓ Matches direct analysis on heterotic side using curvature corrections

- Strings of charge e^a sensitive to non-pert. sectors on heterotic side
- Heterotic M5-brane on B_2 dual to 7-brane on B_3 sensitive to gauge group on D3-branes in F-theory
- Can be extended to F-theory blowups/heterotic 5-branes