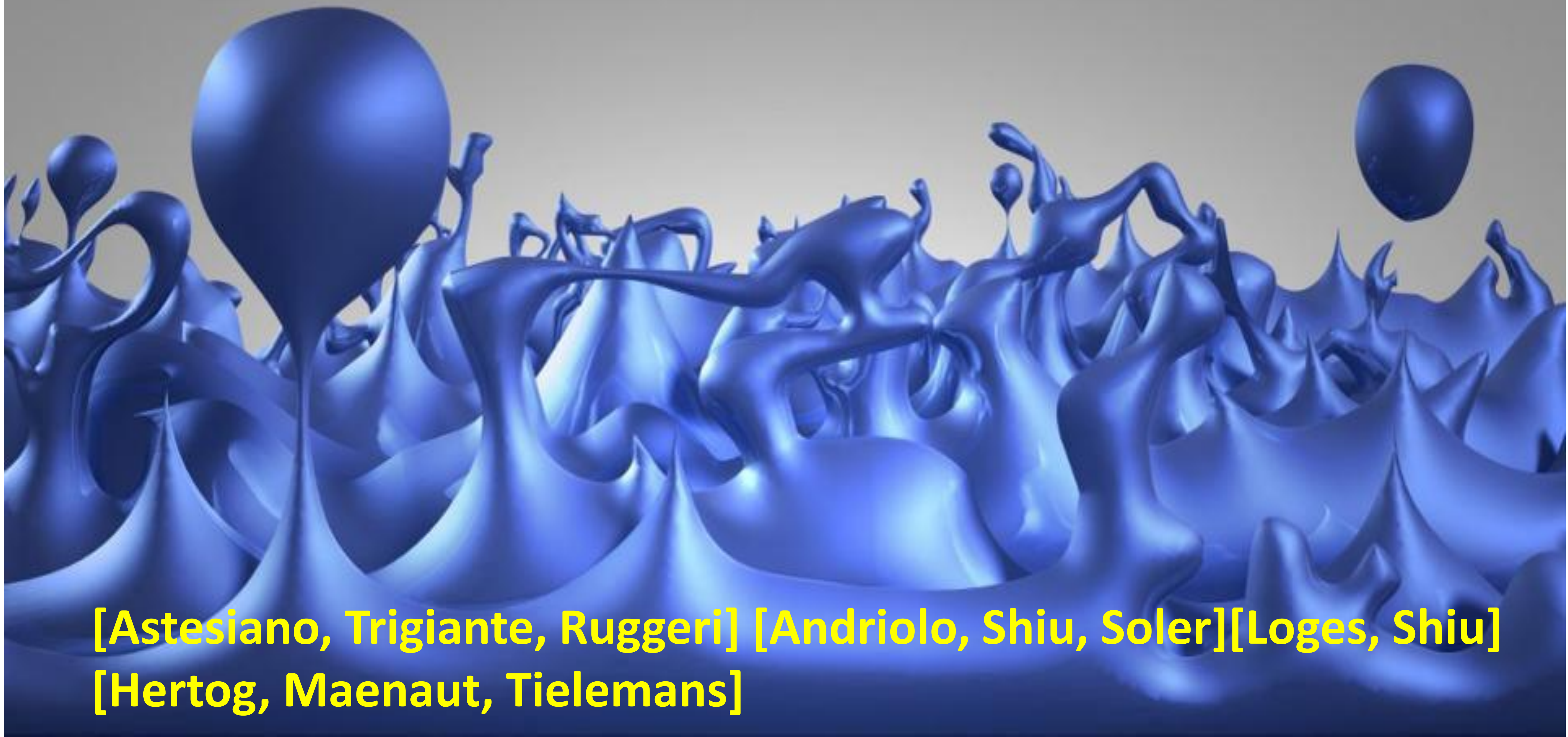
Are Euclidean wormholes in the Swampland?

Thomas Van Riet – Leuven U

Madrid 2022

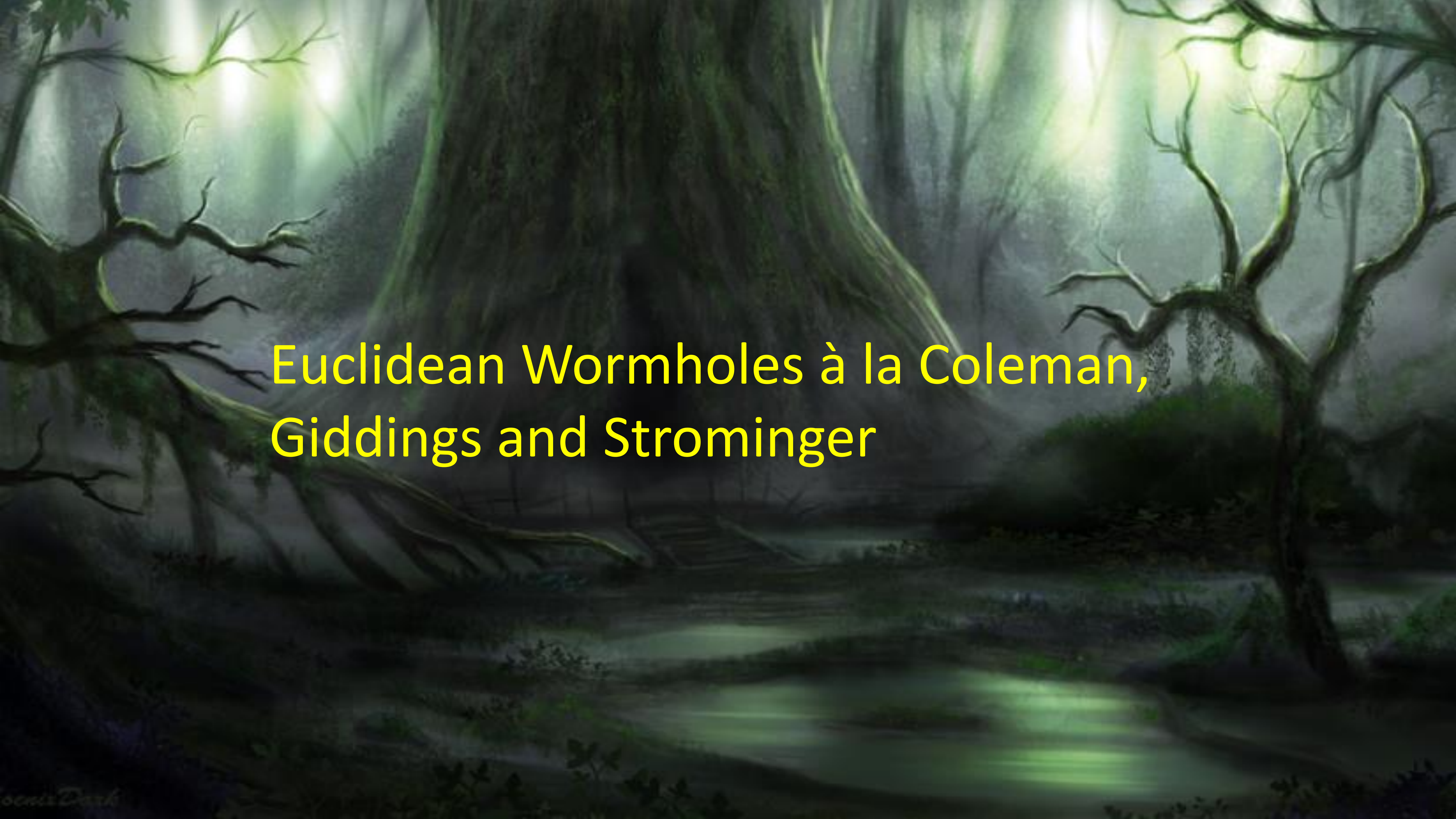
Based on collaborations with



[Astesiano, Trigiante, Ruggeri] [Andriolo, Shiu, Soler][Loges, Shiu] [Hertog, Maenaut, Tielemans]

Wormholes and the Swampland ?

- Breaking global symmetries by Planck suppressed terms (axion potential). [Kallosh, Linde, Linde, Susskind 1995]
- Wormholes & axion/instanton WGC & large field inflation [Montero-Valenzuela-Uranga 2015, Brown-Cottrell-Shiu-Soler 2015, Heidenreich-Reece-Rudelius 2015, Hebecker-Mangat-Theissen-Witkowski 2016, ….]
- (-1)-form global symmetries [McNamara-Vafa 2020]
- Derivative corrections lower wormhole actions (WGC like reasoning). [Andriolo-Huang-Noumi-Ooguri-Shiu 2020]



Euclidean Wormholes à la Coleman, Giddings and Strominger

$$\mathcal{L}_{\text{matter}} = \tfrac{1}{2}(\partial_\mu \chi)(\partial^\mu \chi) \quad \Rightarrow \quad \mathcal{L}_{\text{matter}} = \tfrac{1}{12}(F_{\mu\nu\rho}F^{\mu\nu\rho})$$

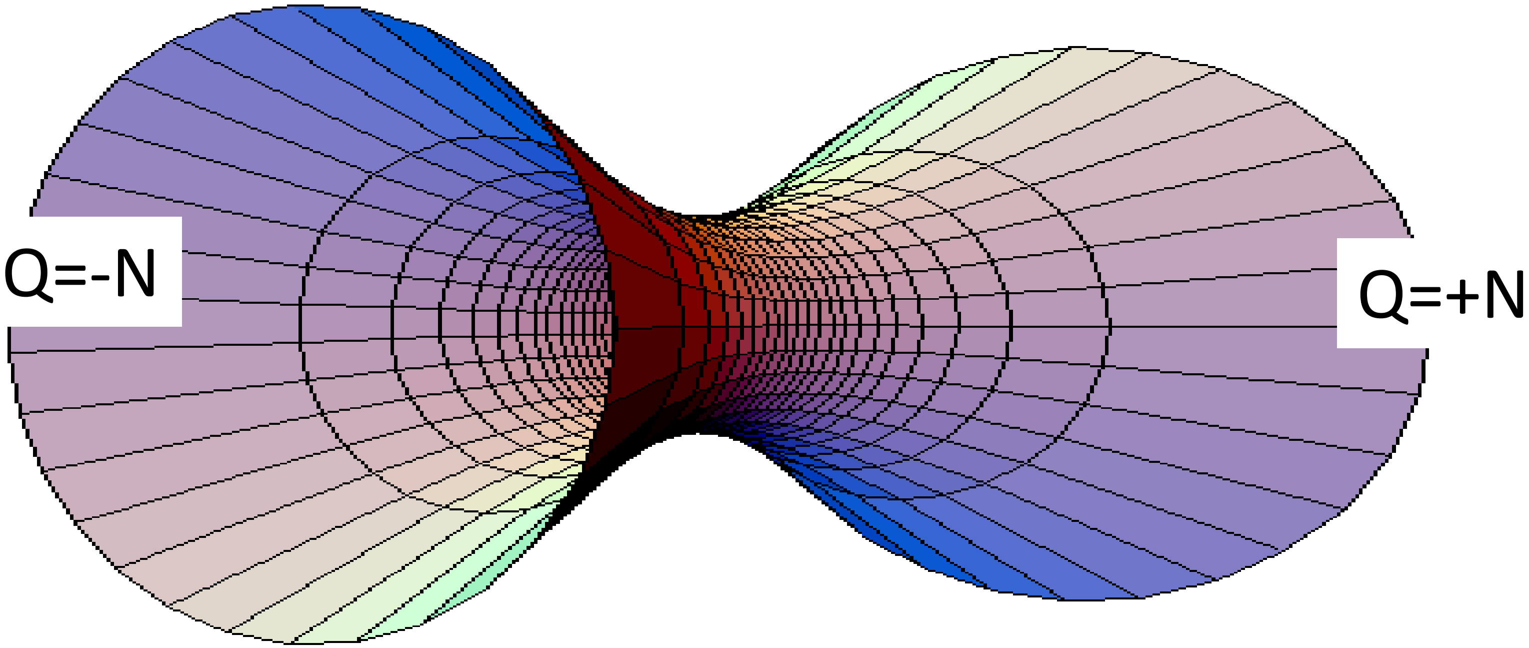
Ansatz:

$$ds^2 = f(\tau)^2 d\tau^2 + a^2(\tau)^2 d\Omega_3^2$$
$$F_3 = Q\epsilon_3$$

Wormhole? In gauge f=1, a(t) should grow, reach a minimum and then grow again.
Other gauge is easier:

$$ds^2 = \left(1 + \frac{\tau^2}{\ell^2} - \frac{Q^2}{\tau^8}\right)^{-1} d\tau^2 + \tau^2 d\Omega_3^2$$

Where I allowed AdS space asymptotics in case of negative cc $\Lambda = -\frac{1}{\ell^2}$

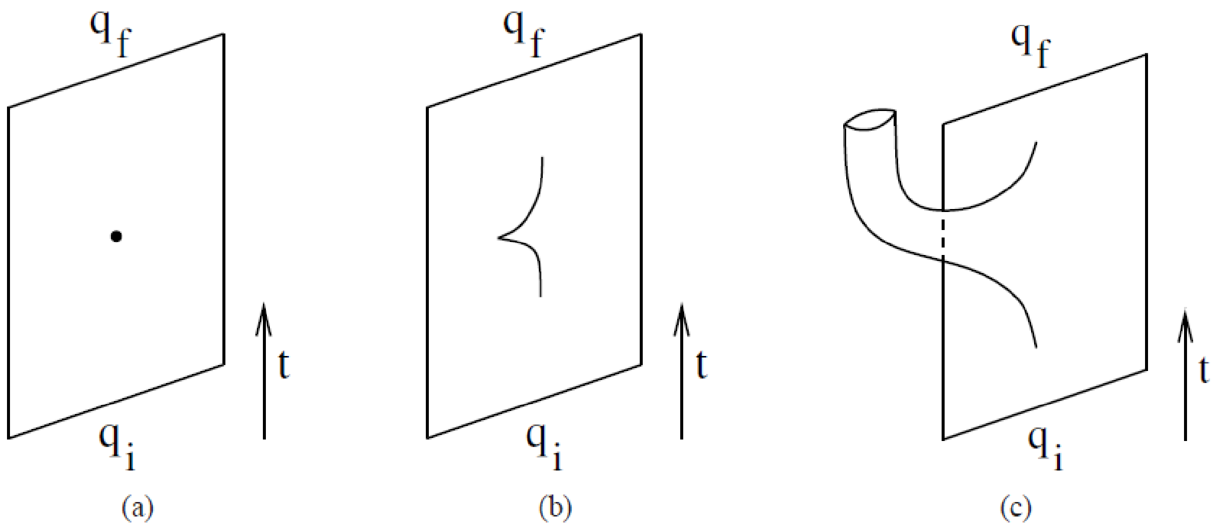


Wormhole is a dipole. There is no monopole axion charge, only locally at one side.

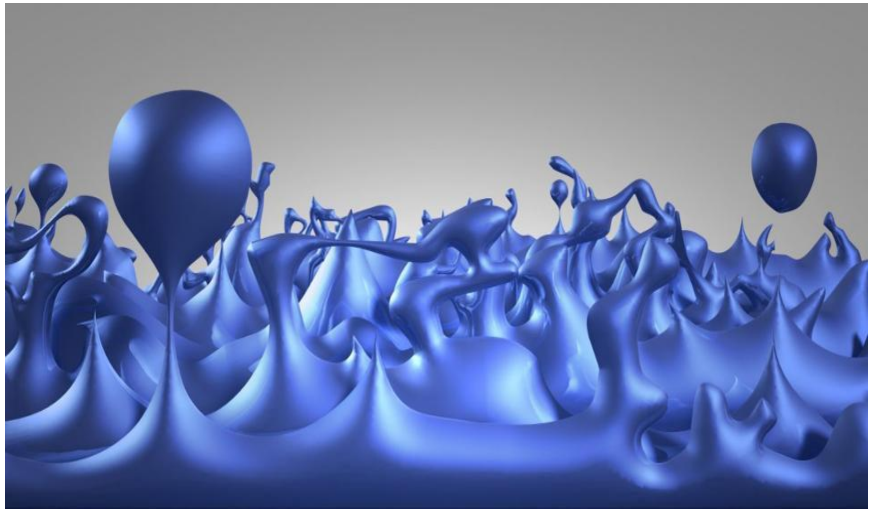
Finite action: $$S \sim |Q|$$

Very rich and long history in quantum gravity, prior to string theory. See [Hebecker, Mikhail, Soler 2018] for comprehensive review

Interpretation as instantons describing nucleation of baby universes → only if cut in half:



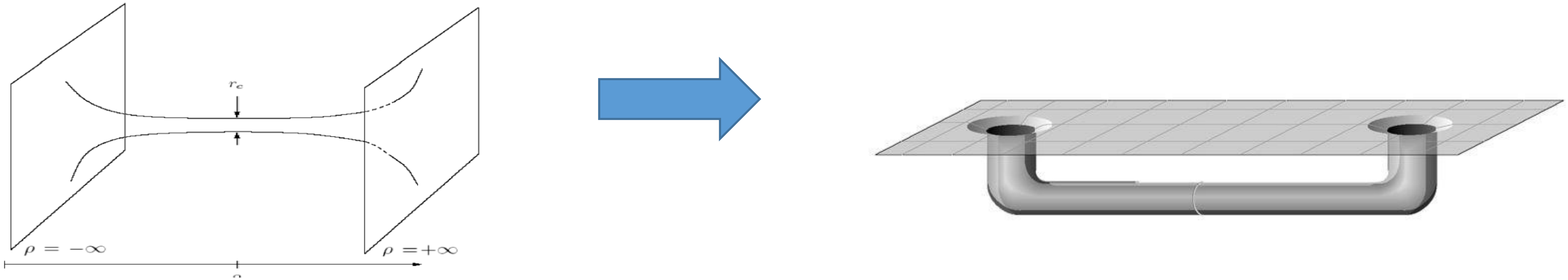
[Giddings/Strominger 1987, Lavrelashvili/Tinyakov/Rubakov 1998, Hawking 1987, …]



→Full wormhole describes emission *and* subsequent absorption of baby universe. Tunneling probability Planckian suppressed. (Planckian sized universes only)

An observer detects a violation of axion charge conservation, apparent *non-unitarity*.

If one glues the two boundaries into one space-time:



then wormholes represent a breakdown of (macroscopic) locality : the effective action would include operators of the form

$$S_{WH} = -\frac{1}{2}\sum_{IJ}\int d^Dx\, d^Dy\, \mathcal{O}_I(x) C_{IJ} \mathcal{O}_J(y)$$

[Coleman 1998]: Not really since

$$e^{-S_{WH}} = \int d\alpha_I\, e^{-\frac{1}{2}\alpha_I (C^{-1})_{IJ}\alpha_J} e^{-\int d^Dx \sum_I \alpha_I \mathcal{O}_I(x)} .$$

ENSEMBLES


String Theory embedding?

[Maldacena-Maoz 2004, Bergshoeff et al 2002-2005, Arkani-Hamed et al 2007, Hertog-Trigiante-VR 2017, Astesiano-Ruggeri-Trigiante-VR 2022, Marolf-Santos 2021,…]

Axions tend to pair up with saxions and one expects to have a multi-field system.
Two options:

1. Formal and controlled compactification with genuine moduli:

$$S = -\frac{1}{2\kappa^2}\int\sqrt{|g|}\Big(\mathcal{R} - \tfrac{1}{2}G_{ij}\partial\phi^i\partial\phi^j - \Lambda\Big)\,.$$

2. Pheno-type, less understood (?), compactification:

$$S_E = \int \mathrm{d}^4x\sqrt{g}\left[-M_P^2 R + \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{12f^2}e^{-\frac{\beta}{M_P}\phi}F^2 + V(\phi)\right].$$

[Maldacena-Maoz 2004, Bergshoeff et al 2002-2005, Arkani-Hamed et al 2007, Hertog-Trigiante-VR 2017, Astesiano-Ruggeri-Trigiante-VR 2022, Marolf-Santos 2021,…]

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Solutions?

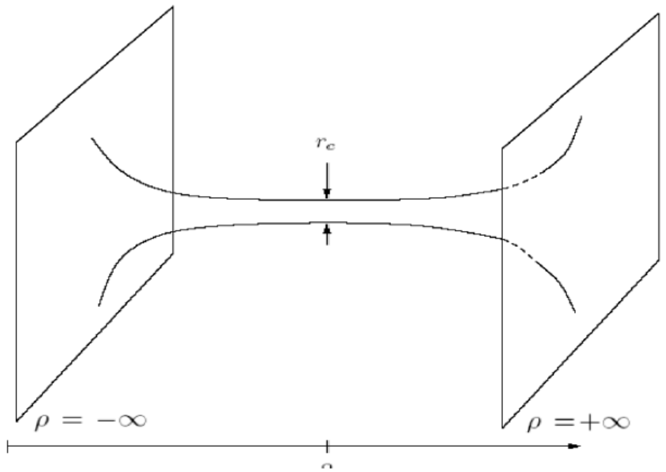
$$\frac{d^2}{dh^2}\phi^i + \Gamma^i_{jk}\frac{d}{dh}\phi^j\frac{d}{dh}\phi^k = 0 \quad \Rightarrow \quad \boxed{G_{ij}\frac{d}{dh}\phi^i\frac{d}{dh}\phi^j = c}$$

$$\boxed{ds^2 = \left(1 + \frac{\tau^2}{\ell^2} + \frac{c}{2(D-1)(D-2)}\tau^{-2(D-2)}\right)^{-1} d\tau^2 + \tau^2 d\Omega^2}$$

Works for ANY sigma model.

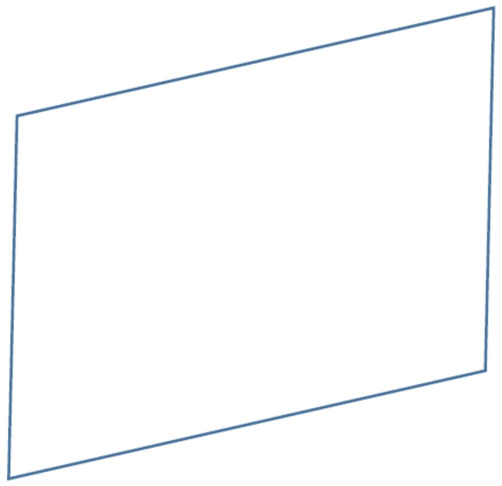
Swampland problem: how do we formulate the distance conjecture in Euclidean theories? The moduli spaces are pseudo-Riemannian.

“Over-extremal” c < 0



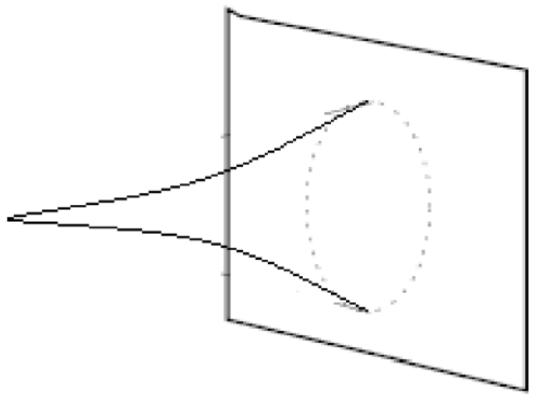
``Time-like” geodesics

“Extremal” c = 0 :



``Light-like’’ geodesics

“Under-extremal” c > 0 :



``Space-like” geodesics

→ With the extra saxion(s) we can introduce notion of extremality.

Extremality for instantons, how exactly? [VR, 2019, 2020]

Inspiration from reducing black hole in D+1 dimensions “over time” to instanton in D. The reduction of vector potential gives axion, size of extra dimension gives “saxion”:

$$G_{ij}\partial\phi^i\partial\phi^j = (\partial\phi)^2 - e^{b\phi}(\partial\chi)^2$$

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- On-shell action $$S \sim |Q| e^{-b\phi(\infty)/2}\sqrt{1+\frac{c}{Q^2}e^{b\phi(\infty)}}$$

- c=0 allows multi-center extension (no force condition). SUSY D(-1) brane is example.

- Probe extremal instantons show "repulsion" away from over-extremal instantons. Wormholes have "positive binding energy".

Over-extremal black holes are unphysical. Not over-extremal particles.

What about over-extremal instantons? There is no naked singularity to warn us.

Swampland or not?

Maybe the string theory embedding shows wormholes do not exist?

Top down constructions of

$$S = -\frac{1}{2\kappa_D^2}\int \sqrt{|g_D|}\Big(R_D - \tfrac{1}{2}G_{ij}\partial\phi^i\partial\phi^j - \Lambda\Big)$$

eg IIB on S^5 $G_{ij}\partial\phi^i\partial\phi^j = (\partial\phi)^2 + \epsilon e^{\beta\phi}(\partial\chi)^2$ With β=2 and ε=+/-

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Regular geometry but <u>singular</u> fields:

$$\chi = -\frac{2\sqrt{-c}}{\beta Q}\cot(\frac{1}{2}\sqrt{-c}\,\beta\,h) + c_0\,, \qquad e^{\frac{1}{2}\beta\phi} = \frac{Q}{\sqrt{-c}}|\sin(\frac{1}{2}\sqrt{-c}\,\beta\,h)|$$

- When β=2 , one can show that the sin function hits somewhere zero between the left and the right of the wormhole.
- If $\beta^2 < 3/2$ then regular wormholes exist! Can we get $\beta^2 < 3/2$ in string theory?

General regularity criterium? [Arkani-Hamed, Orgera, Polchinski]

$$d = \int_{r=-\infty}^{r=+\infty} \sqrt{|G_{ij}\dot{\phi}^i\dot{\phi}^j|\mathrm{d}r} = 2\int_{-\infty}^{0} |c|a^{2(1-D)}\frac{\mathrm{d}a}{\dot{a}} \approx \pi\sqrt{2\frac{D-1}{D-2}} - \mathcal{O}\left(R/l\right)$$

Smoothness requires timelike geodesics with at least the length

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→ Try Minkowski models: [Bergshoeff et al, 2004] pointed out that the universal hypermultiplet is sufficient: IIA/B on a CY gives smooth Euclidean wormholes!

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Explicit lift using a T^6 in IIA shows over-extremal version of Euclidean D2's wrapping 3 cycles in T^6 [Loges, Shiu, VR]

×	×	×	−	−	−
×	−	−	×	×	−
−	×	−	−	×	×
−	−	×	×	−	×

$$ds^2 = ab^2 \sin\left(\sqrt{3}\arctan\left(\rho^{-2}\right) + c\right)\left(1 + \frac{1}{\rho^4}\right)\left(d\rho^2 + \rho^2 d\Omega_3^2\right) + d\sum_{i=1}^{6} d\theta_i^2\,,$$

$$e^{\phi} = d^{3/2}a^{1/2}\left(\sin\left(\sqrt{3}\arctan\left(1/\rho^2\right) + c\right)\right)^{1/2}\,,$$

$$C_3 = \frac{1}{a}\cot\left(\sqrt{3}\arctan\left(\frac{1}{\rho^2}\right) + c\right)\left(d\theta^{123} + d\theta^{145} + d\theta^{256} + d\theta^{346}\right)\,.$$

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- *Despite super Planckian distances, we can arrange for everywhere weakly coupled, curved and smooth solutions*!

- Paradox is evaded: wormholes pnly there when *Wickrotation to Lorentzian D brane is prohibited*

Summary state of affairs wormhole embeddings

- In Euclidean **flat space**: generically there in universal hypermultiplet [Bergshoeff et al 2004]

  → Explicit 10d lift for T^6 [Loges, Shiu, TVR, to appear]

- In Euclidean **AdS**: for $\mathrm{AdS}_5 \times \mathrm{S}^5/\mathbb{Z}_\mathrm{k}$ [Hertog, Trigiante, VR, 2017]: but without clean 10d picture (twisted modes in 5d gauged SUGRA description)

- In Euclidean **AdS**: for AdS_5 x T^{11} [Loges, Shiu, TVR, to appear] Description in terms of 'distorted geodesic curves' since moduli space is not totally geodesic within scalar manifold. Results unclear, rather numerically heavy. At this moment: no wormholes….

Holography

Coleman wormholes have no support from AdS/CFT [Arkani-Hamed-Orgera-Polchinski 2007, Maldacena-Maoz 2004]. Dual field theory has no sign of Coleman's α parameters.

Can we say something new? *[Trigiante, Ruggeri, Katmadas, VR 2018,2020]*

Moduli space AdS are coupling constants for exactly marginal operators in the dual= *conformal manifold.* Metric Gij on moduli space corresponds to the `Zamolodchikov' metric gij defined by the two-point functions:

$$g_{ij}(\varphi) = x^{2\Delta}\langle O_i(x)O_j(0)\rangle_{S[\varphi]}$$

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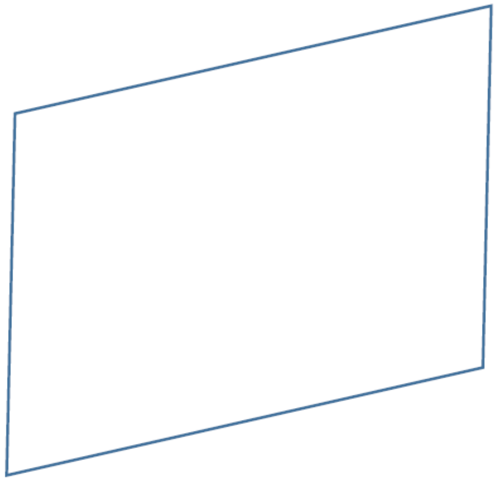
For $\mathrm{AdS}_5 \times \mathrm{S}^5/\mathbb{Z}_\mathrm{k}$ dual is N=2 necklace quiver CFT [Kachru, Silverstein '98] . Has k gauge nodes → k complex couplings

$$\mathcal{L} = \sum_{\alpha=0}^{k-1}\left(-\frac{1}{4g_\alpha^2}\mathrm{Tr}[F_\alpha^2] - i\frac{\theta_\alpha}{32\pi^2}\mathrm{Tr}[F_\alpha \wedge F_\alpha]\right)$$

moduli space $$\frac{\mathrm{SU}(1,k)}{\mathrm{S}[\mathrm{U}(1)\times \mathrm{U}(\mathrm{k})]} \Longrightarrow \frac{\mathrm{SL}(k+1,\mathbb{R})}{\mathrm{GL}(k,\mathbb{R})}.$$ 2k real scalars.

"Extremal" c = 0 :



Light-like geodesics

SUSY solutions match SUSY gauge theory instantons. (One point functions & on-shell actions)

non-SUSY solutions but extremal: Some of them can be interpreted and match so called "quasi-instantons" [Imaanpur 2008]. *These are solutions which are self-dual in each separate gauge node, but orientations differ from node to node. Very simple way of SUSY-breaking!*

$$\mathrm{Tr}[F_\alpha^2] = \mathrm{sign}(N_\alpha)\mathrm{Tr}[F_\alpha \wedge F_\alpha]$$

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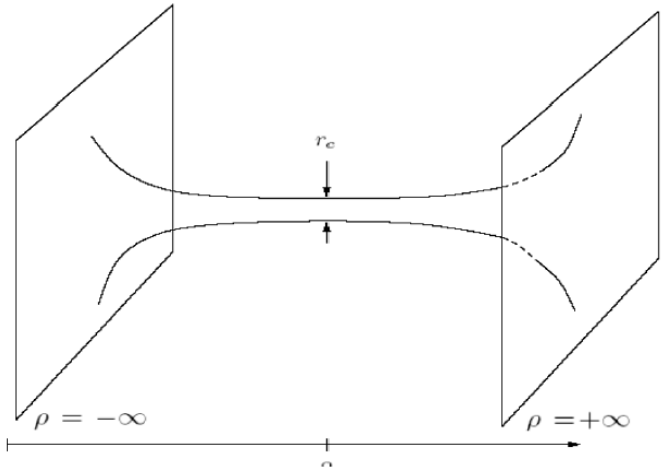

Time-like geodesics

First examples of smooth Euclidean axion wormholes in AdS!

Our explicit embedding provides another paradox: violation of positivity [Katmadas, Ruggeri, Trigiante, VR, 2018]:

$$|\mathrm{Tr}[F_\alpha^2]| < |\mathrm{Tr}[F_\alpha \wedge F_\alpha]| \, .$$

Field theories without gravity do not allow a notion of super-extremality. BPS bounds cannot be violated. It requires gravity. But AdS gravity = CFT.

→ **evidence for spurious nature of wormholes?**


Euclidean Stability

Interpretation of instantons depends on **stability**

Perform Gaussian approximation around saddle point:

$$Z = e^{-S[\Phi_0]} \int \mathrm{D}\phi \, e^{-\delta^2 S[\Phi_0, \phi] + \mathcal{O}(\phi^3)} \qquad \delta^2 S = \frac{1}{2} \int \phi \hat{\mathrm{M}} \phi$$

*Coleman: in QM & QFT we have standard instantons (all eigenvalues positive) or "bounces" with **one** negative eigenvalue. The latter describe tunneling amplitudes. **Multiple** negative eigenvalues means instanton is spurious.*

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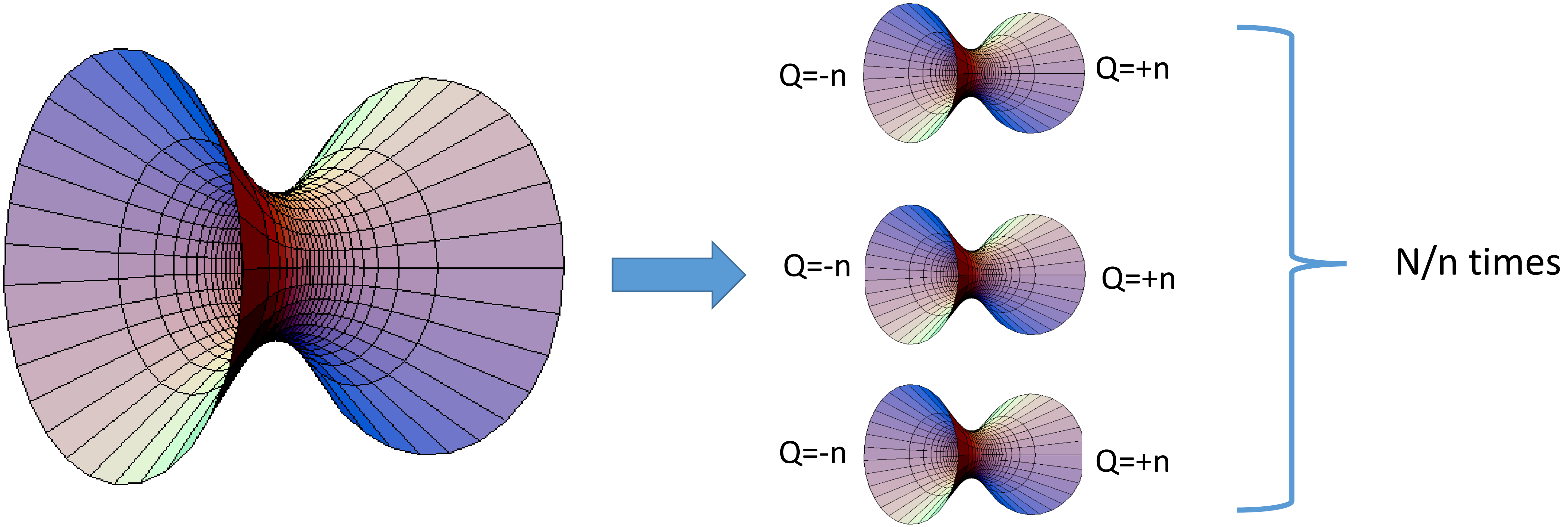
- Literature: there is possibly one negative eigenmode, which is expected from tunneling interpretation [Rubakov 1989, Kim&Lee&Myung 1997, Kim&Kim&Hetrick2003, Alonso&Urbano 2017].
- [Hertog, Truijen, VR 2018] Computations did not use the right gauge-invariant variables + Interpretation as path integral for axion-charge transitions is crucial.

Infinitely many modes lower the action. All centered close to the neck and probe the non-trivial topology. For very small wormholes those modes become sub-planckian.

→ Macroscopic wormholes do not contribute. There is a lower action saddle with same boundary conditions? Which one? → wormhole fragments into smaller wormholes.



Incorrect after all….?

Similar computation with different gauge invariant variable gives no neg modes! [Loges, Shiu, Sudhir 2022]

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How? A closer look shows that the boundary condition we wanted to impose in [Hertog, Truijen, TVR, 2018] cannot be done with the used Mukhanov variable. We now redid computation in different way and confirmed results Loges et al. [Hertog, Maenaut, Tielemans, VR, in progress]

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What now?

→ Fragmentation picture remains convincing, but arguments requires saxion. **Our guess: Instability shows in multi-field case!**

→ End-point wormhole fragmentation? → study **corrections** to small wormholes [Andriolo, Huang, Noumi, Ooguri, Shiu, 2020], [Andriolo, Shiu, Soler, VR, 2022]

These studies use models with either only axion with higher derivative corrections or massive saxion.

$$S_E = \int \mathrm{d}^4x\sqrt{g}\left[-M_P^2 R + \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{12f^2}e^{-\frac{\beta}{M_P}\phi}F^2 + V(\phi)\right].$$

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→ Indeed verified for higher derivative corrections [Andriolo, Huang, Noumi, Ooguri, Shiu, 2020],

→ And for integrating out massive saxion at 2-derivative level [Andriolo, Shiu, Soler, VR, 2022]

No clear insight about the microscopic ‘wormholes’ though ☹

Summary

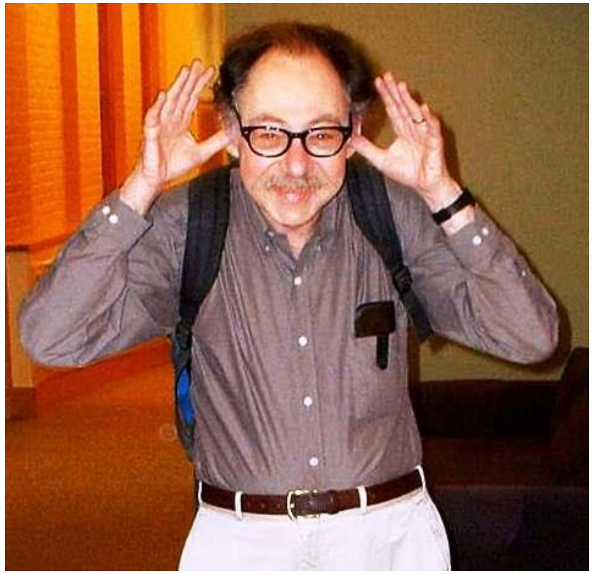
Why wormholes in the landscape

- Predicted by the axionic WGC.
- Needed to break global axion shift symmetry?
- **Clear and simple string theory embeddings in 10d.**
- **Single axion wormhole stable.**
- **Corrections to small single axion wormholes obey a WGC version.**

Why wormholes in the Swampland

- Vafa&McNamara argument
- Violate bounds in dual field theory once embedded into AdS/CFT
- No sign of Coleman α-parameters in top-down holography.
- Unstable with multiple fields?

Are Coleman's Euclidean wormholes in the Swampland?



(S. Coleman 1937-2007)

Unclear. Work in progress

Thank you!

EXTRA

$$S[A] = \int \star R - \tfrac{1}{2} \star F_p \wedge F_p$$

$$S[F, B] = \int \star R - \tfrac{1}{2} \star F_p \wedge F_p + dF_p \wedge B_{D-p-1} \qquad dB = G_{D-p}$$

With partial integration, and dropping a boundary term, we can get:

$$S[F, B] = \int \star R - \tfrac{1}{2} \star F_p \wedge F_p + (-1)^{p+1} F_p \wedge G_{D-p}$$

the EOM for F gives:

$$\star F_p = (-1)^{p(D-p)} G_{D-p}$$

$$\Rightarrow \quad S = \int \star R + \tfrac{1}{2}(-1)^t \star G_{D-p} \wedge G_{D-p}$$

$$
\begin{aligned}
Z &= \int \mathcal{D}q \; exp\left[-\int dt\left(-\frac{A}{2}\dot{q}^2 + \frac{B}{2}q^2\right)\right] \\
&= \int \mathcal{D}q\mathcal{D}p \; exp\left[-\int dt\left(\frac{A^{-1}}{2}(p - A\dot{q}^2) - \frac{A}{2}\dot{q}^2 + \frac{B}{2}q^2\right)\right] \\
&= \int \mathcal{D}q\mathcal{D}p \; exp\left[-\int dt\left(\frac{A^{-1}}{2}p^2 + \dot{p}q + \frac{B}{2}q^2\right)\right] \\
&= \int \mathcal{D}q\mathcal{D}p \; exp\left[-\int dt\left(\frac{A^{-1}}{2}p^2 + \frac{B}{2}(q + B^{-1}\dot{p})^2 - \frac{B^{-1}}{2}\dot{p}^2\right)\right] \\
&= \int \mathcal{D}p \; exp\left[-\int dt\left(-\frac{B^{-1}}{2}\dot{p}^2 + \frac{A^{-1}}{2}p^2\right)\right]
\end{aligned}
$$

Boundary

$$\mathcal{Z}_{\mathrm{QG}}(X) = \mathcal{Z}_{\mathrm{CFT}}(X).$$

$$\mathcal{H}_{\mathrm{QG}}(M_1 \sqcup M_2) = \mathcal{H}_{\mathrm{QG}}(M_1) \otimes \mathcal{H}_{\mathrm{QG}}(M_2)$$

$$\mathcal{H}_{\mathrm{BU}} = \mathcal{H}_{\mathrm{QG}}(\varnothing)$$

$$\varnothing \sqcup M = M,$$

$$\mathcal{H}_{\mathrm{QG}}(M) = \mathcal{H}_{\mathrm{BU}} \otimes \mathcal{H}_{\mathrm{QG}}(M)$$