Asymptotic Puzzles and Accelerated Expansion

Irene Valenzuela

CERN

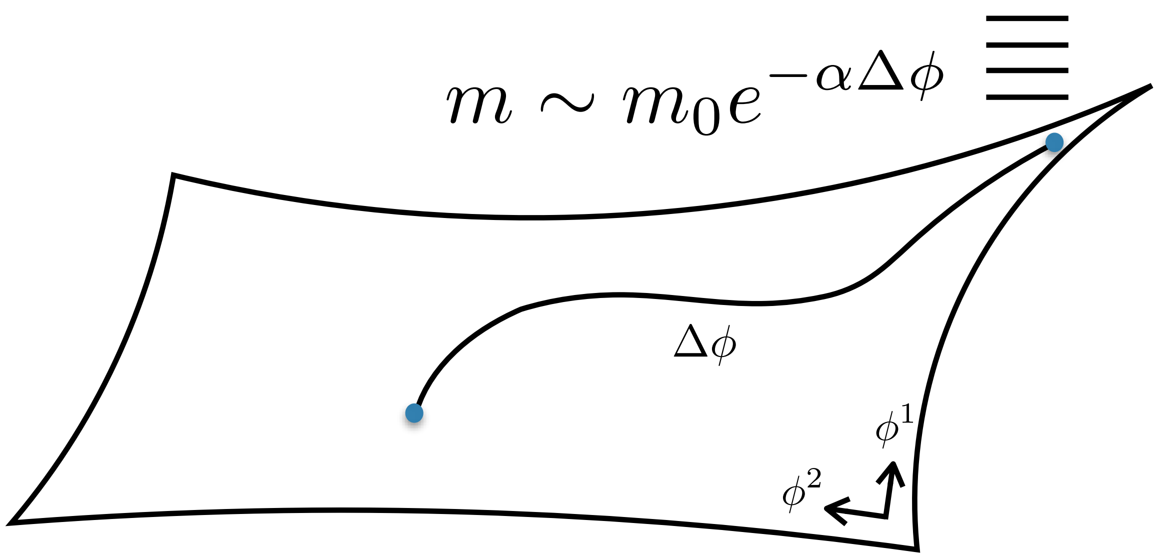
IFT UAM-CSIC

Based on 2209.xxxxx with Jose Calderon-Infante and Ignacio Ruiz
(to appear tomorrow!)

Back to the Swamp, IFT, Sept 2022

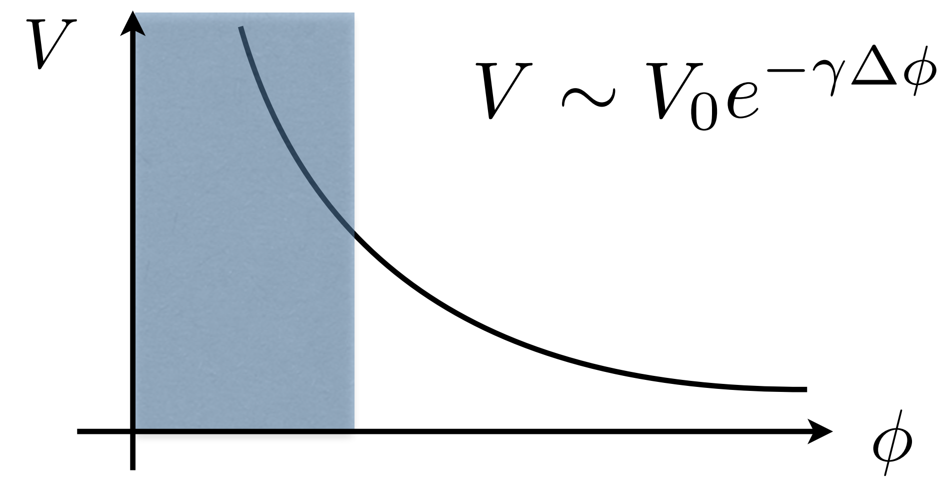
Swampland Program

Universal quantum gravity properties emerge at infinite field distance:



Distance conjecture

[Ooguri, Vafa'05]



Asymptotic dS conjecture

[Obied et al'18][Ooguri et al'18]

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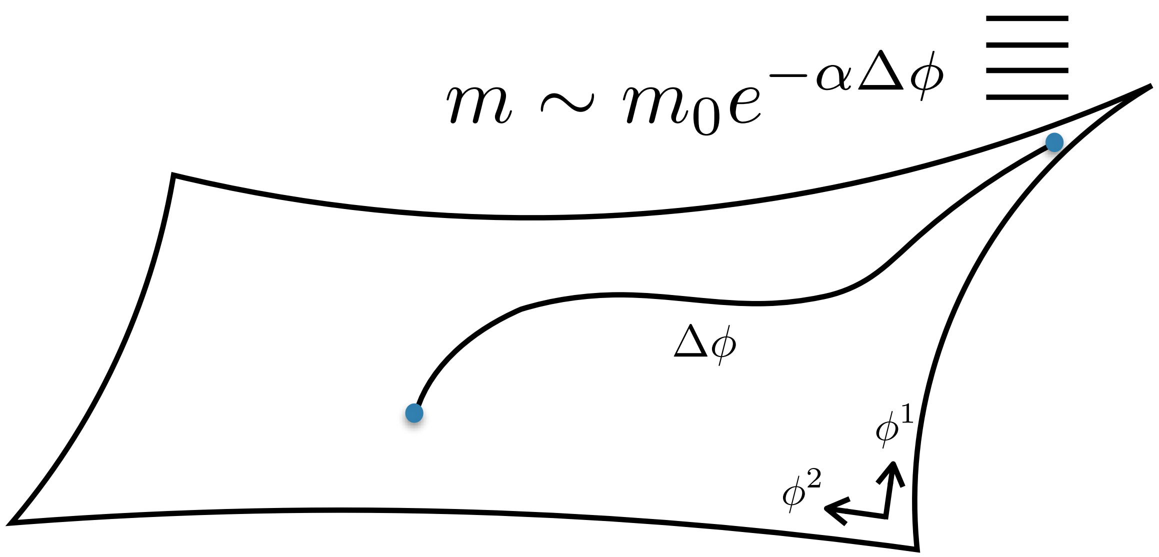

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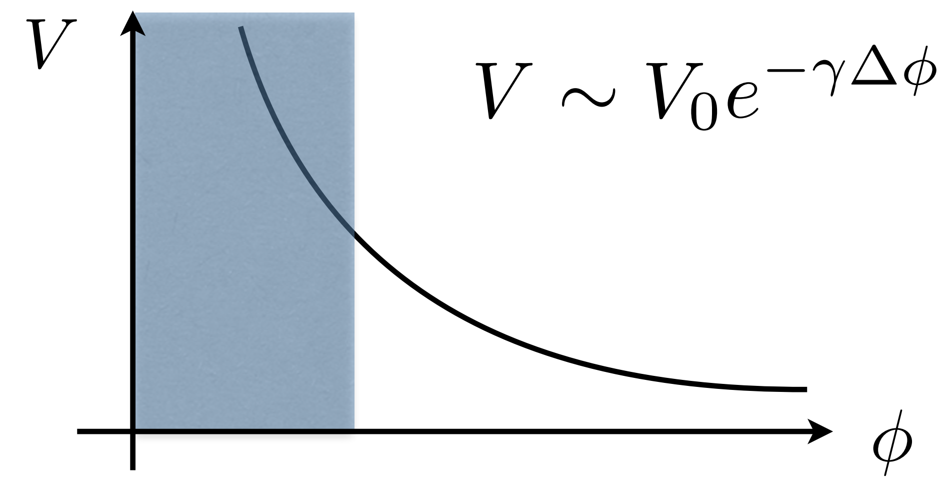

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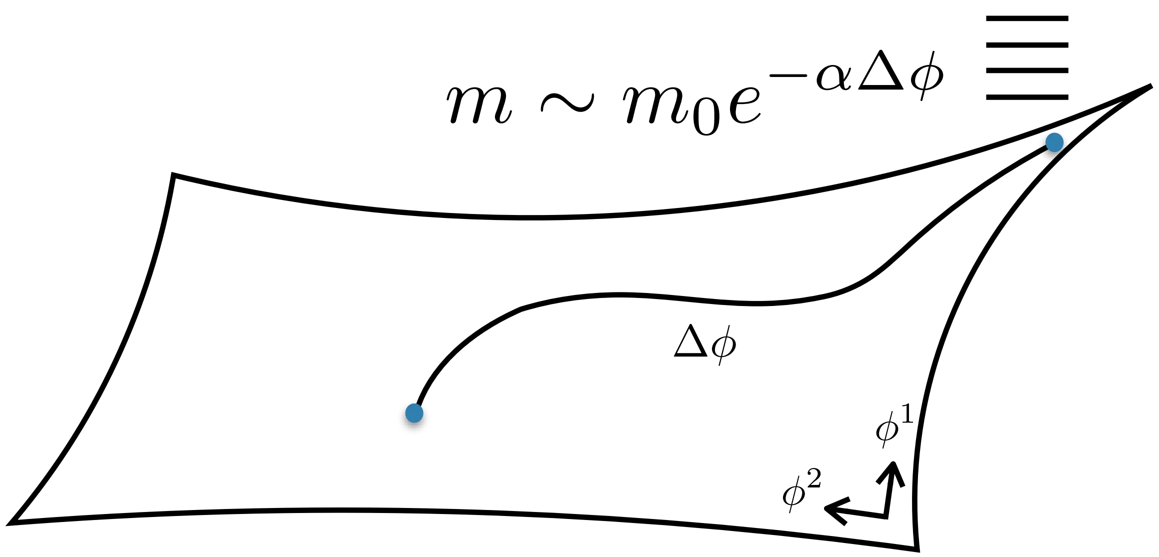

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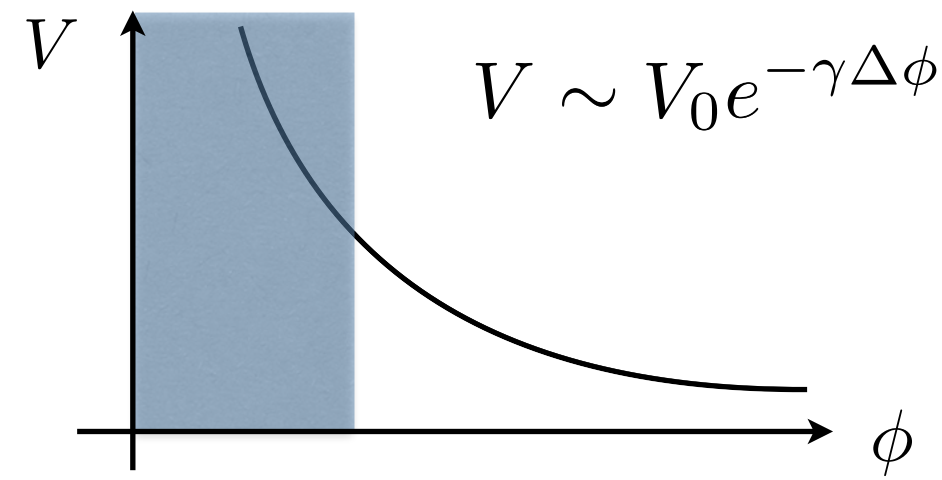

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Good for phenomenological implications and to classify QG resolutions (inflation, Dark Dimension…)

[Montero,Vafa,IV'22]

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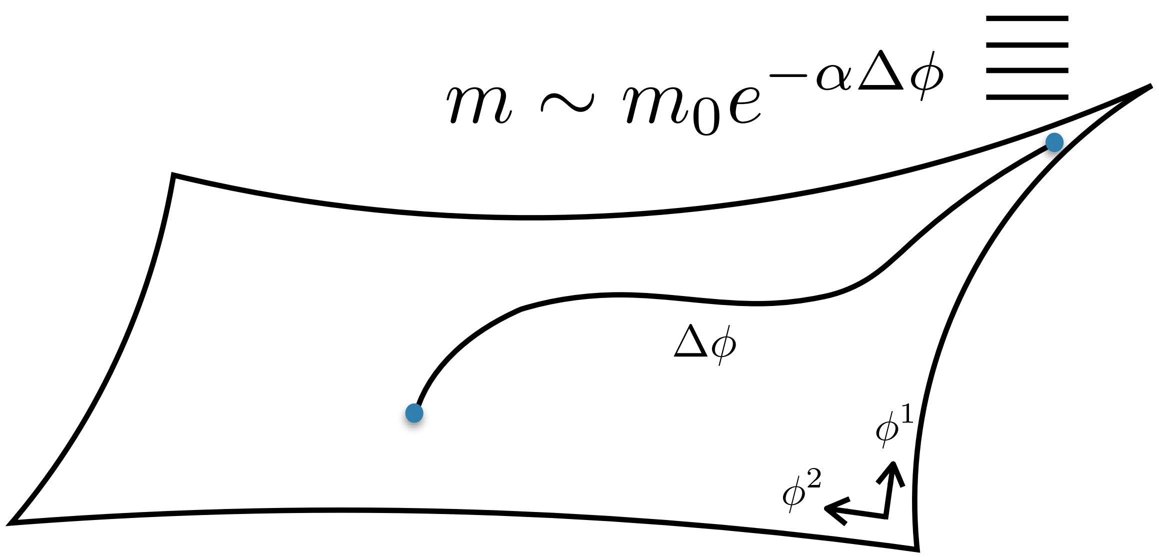

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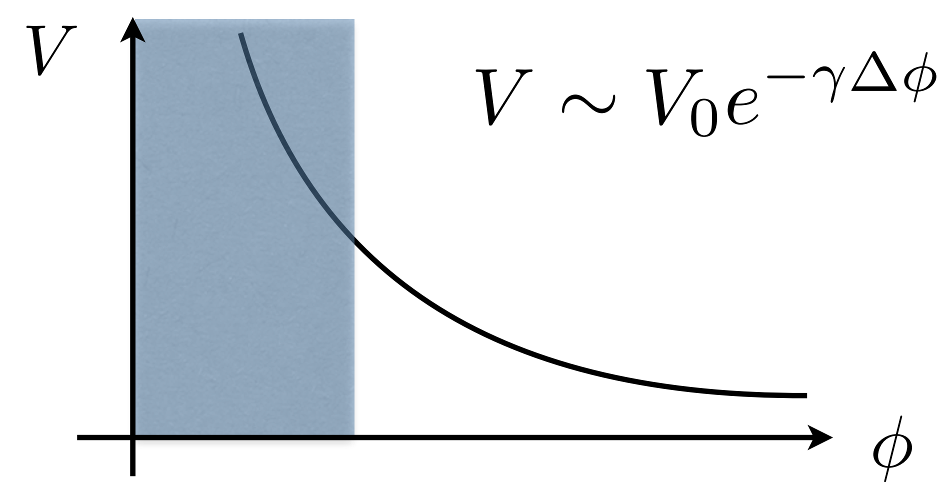

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In this talk: Can we get asymptotic accelerated expansion?

Asymptotic Quintessence

Take asymptotic runaway behaviour of potential:

$$\frac{\|\nabla V(\varphi)\|}{V(\varphi)} \geq c_d \qquad \textsf{as} \qquad D(\varphi_0, \varphi(t)) = \int_{t_0}^{t} \sqrt{G_{ab}\varphi^{a\prime}\varphi^{b\prime}}dt \to \infty$$

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To get accelerated expansion (for gradient flows):

$$\gamma = \frac{\|\nabla V(\varphi)\|}{V(\varphi)} < \frac{2}{\sqrt{d-2}} \quad = \sqrt{2} \quad \text{in four dimensions}$$

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Proposed Swampland bounds:

[Rudelius'21]

$$c_d^{\mathrm{TCC}} = \frac{2}{\sqrt{(d-1)(d-2)}} \qquad\qquad c_d^{\mathrm{strong}} = \frac{2}{\sqrt{d-2}}$$

[Bedroya, Vafa'19]

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No accelerated expansion!

Asymptotic Trajectories

Given $V = \sum_{l \in \mathcal{E}} V_l$ with $V_l = A_l \prod_{i=1}^{n} (s^i)^{l_i}$

$K = -\log(P(s^i) + \dots)$

How to compute gamma?

- **Gradient flow trajectory:** $s^i(\lambda) = \alpha^i \lambda^{\beta^i}$, $\alpha^i, \beta^i > 0$

  as $\lambda \to \infty$

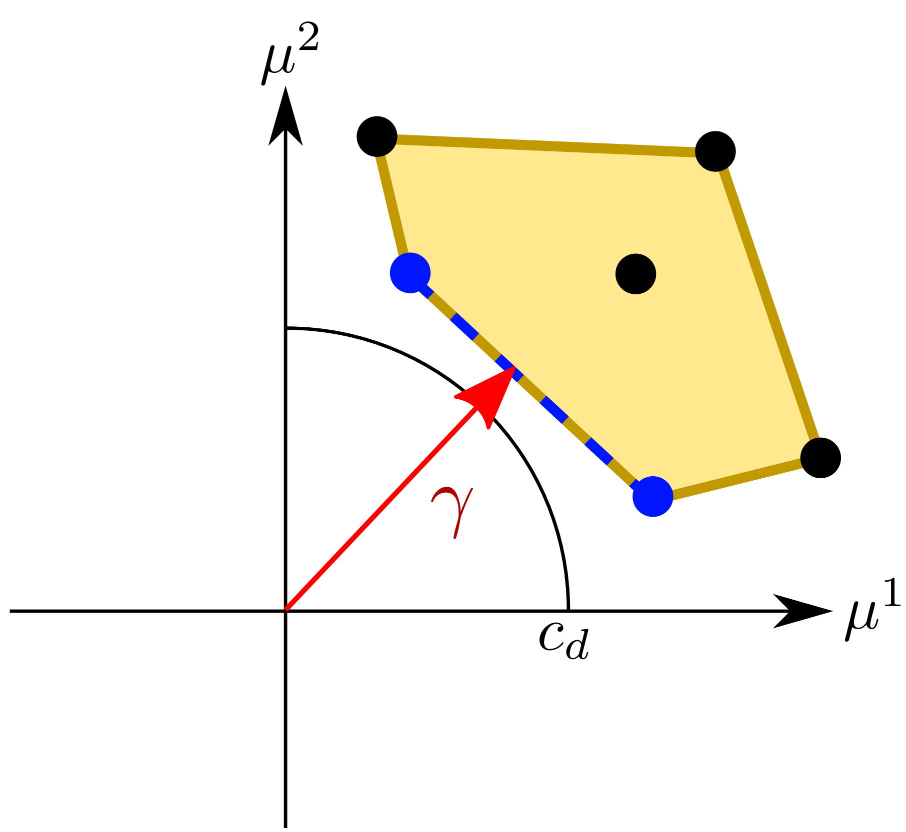
- **Optimization problem:**

$$\min_{\hat{\beta} \in \mathbb{S}^n} \left\{ \max_{l \in \mathcal{E}} \left\{ \hat{\beta}^i l_i \right\} \right\} \quad \text{with} \quad \mathbb{S}^n = \left\{ \hat{\beta} \in \mathbb{R}^n : |\hat{\beta}|^2 = \frac{1}{2} = 1 \right\}$$

Convex Hull dS conjecture

Given $V = \sum_l V_l$, the asymptotic dS conjecture with $\frac{\|\nabla V\|}{V} \geq c_d$ will be satisfied if the convex hull of all the dS ratios $\vec{\mu}_l$

lie outside the ball of radius c_d



deSitter ratios: $\mu_l^a = -\delta^{ab} e_b^i \frac{\partial_i V_l}{V_l}$

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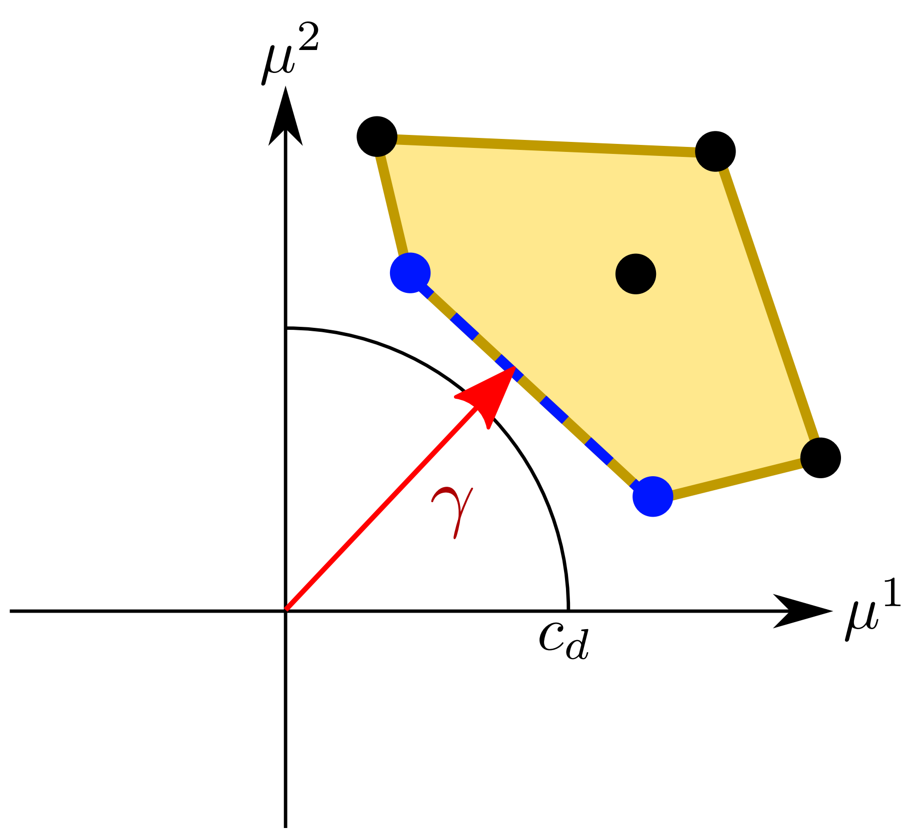

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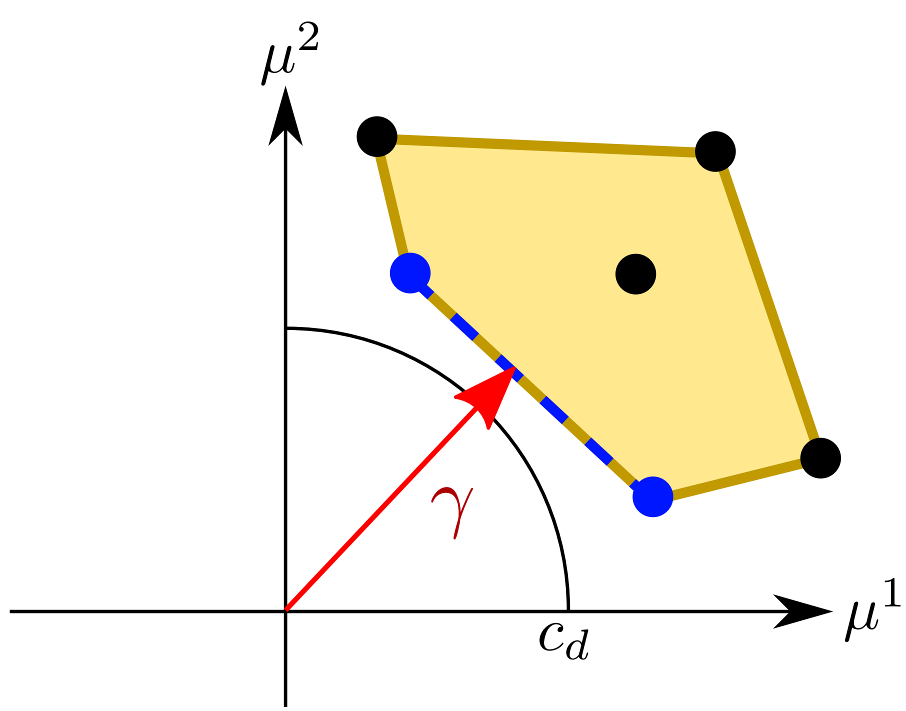

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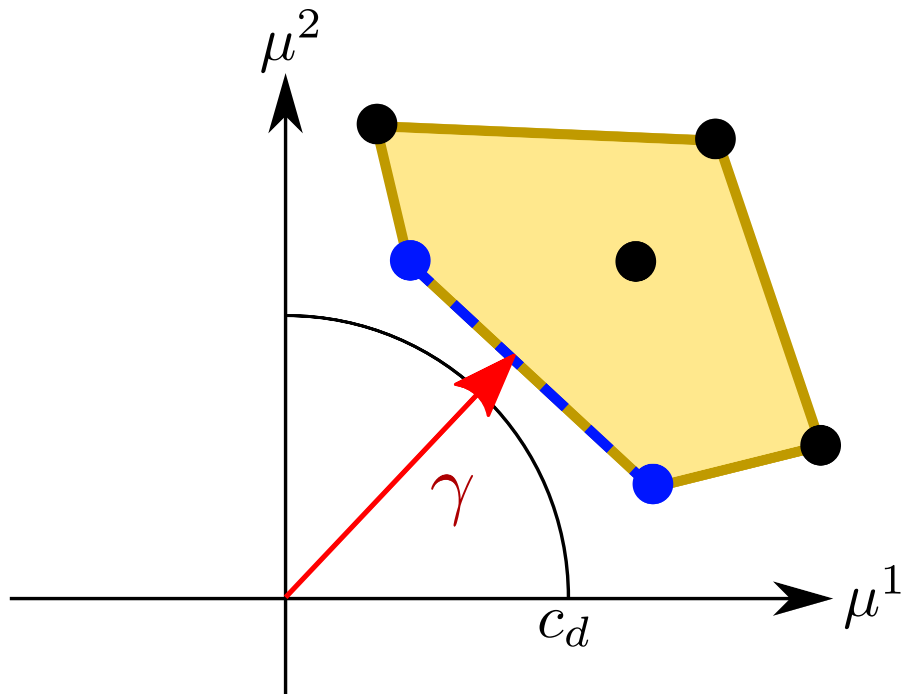

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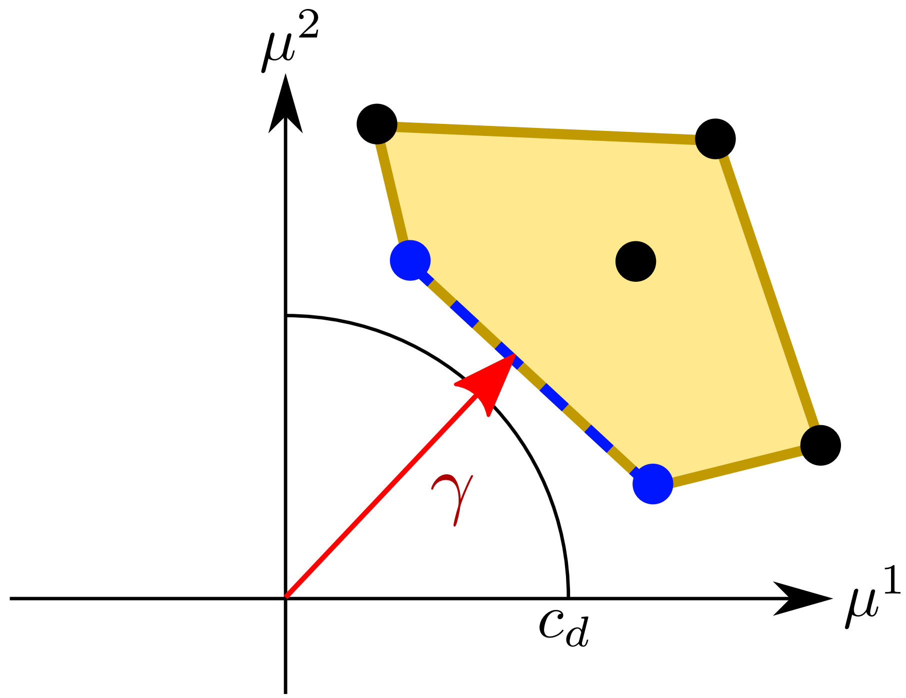

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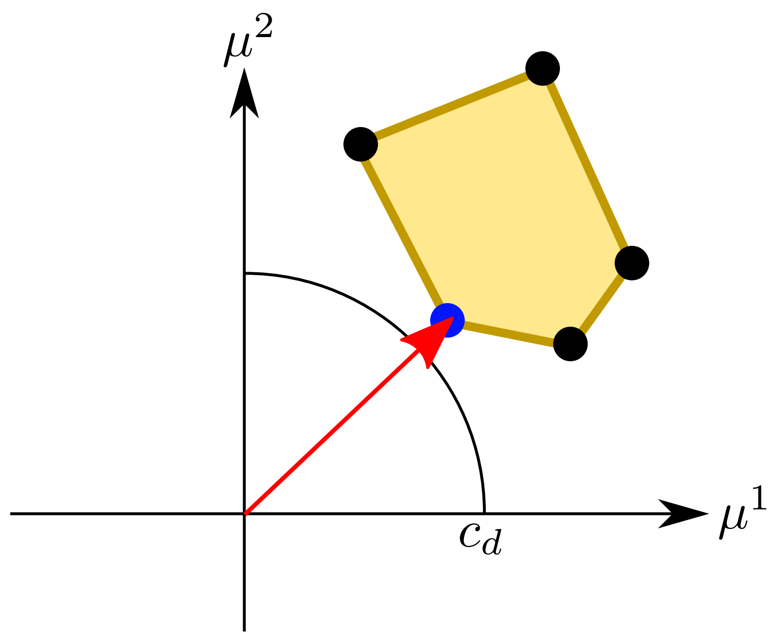
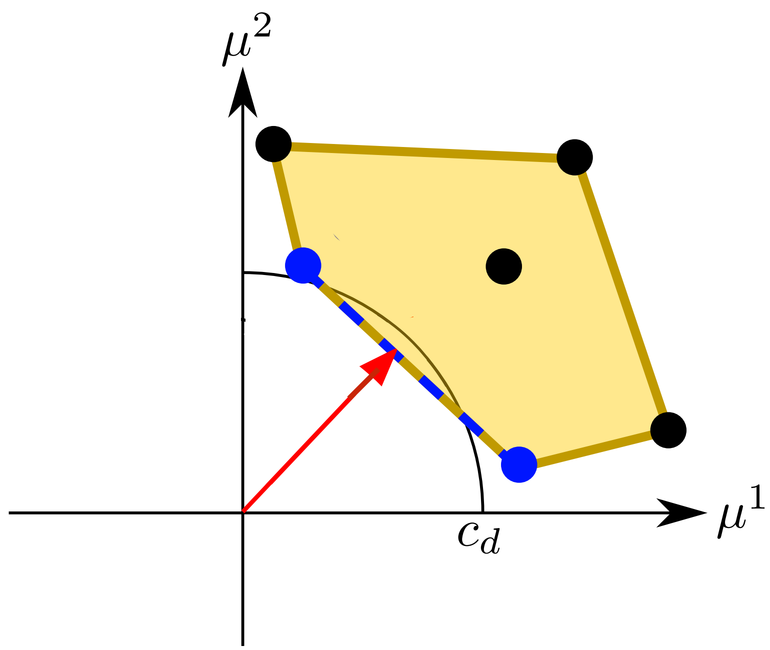
"Scalar WGC for membranes in which **all membranes** must satisfy the bound"

Scenario (I)



Scenario (II)



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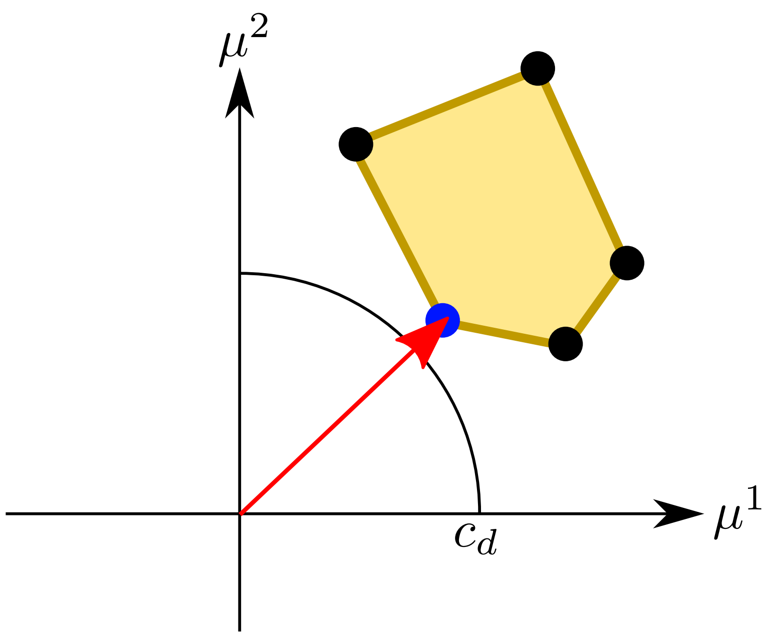
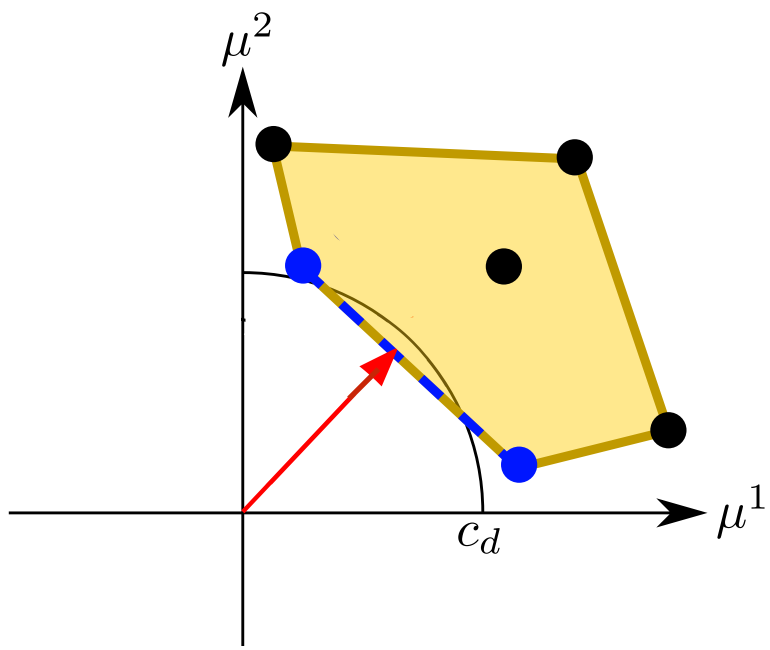


A single term dominates

Scenario (II)



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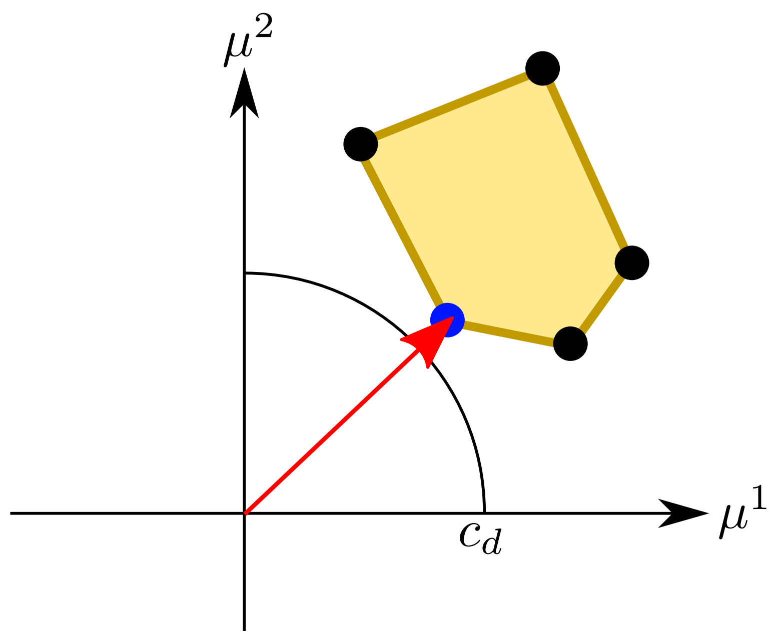

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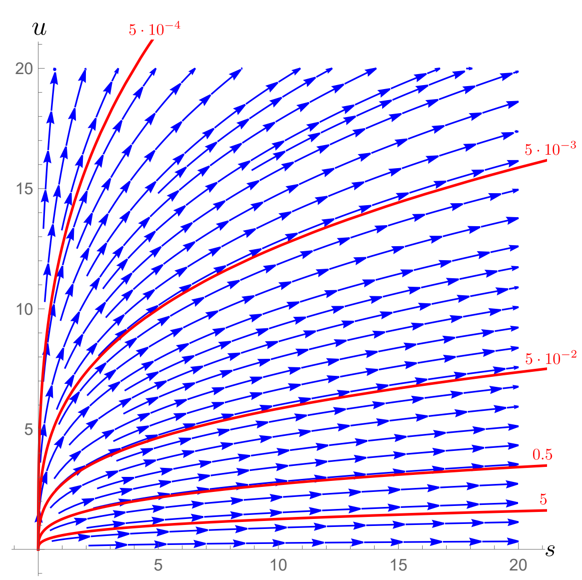
e.g. $V = \dfrac{1}{su} + \dots \ \sim \dfrac{1}{\lambda^4}$



family of gradient flow solns

$$(s, u) = (\alpha\lambda^3, \lambda)$$

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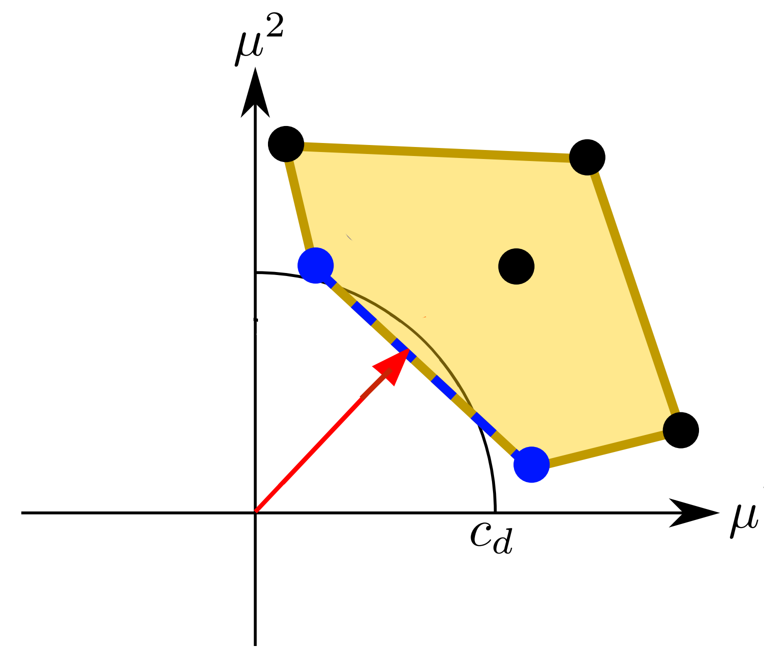

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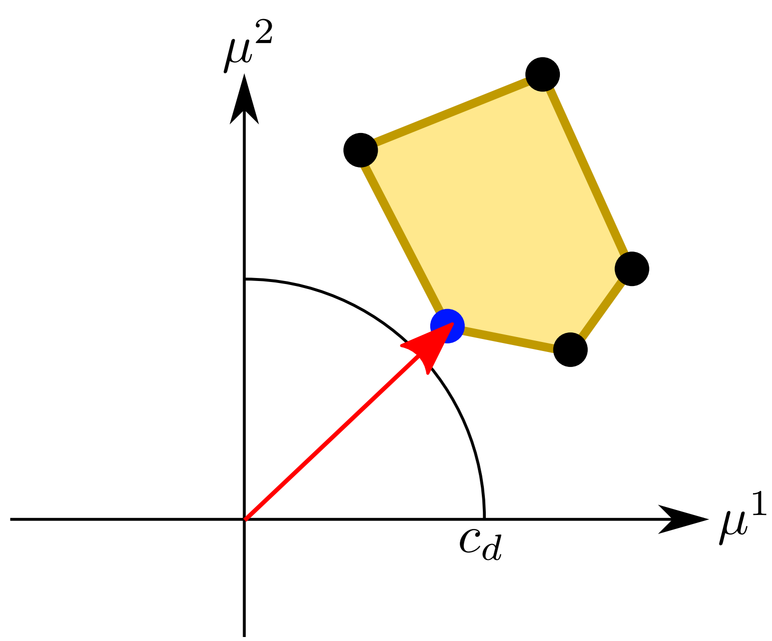

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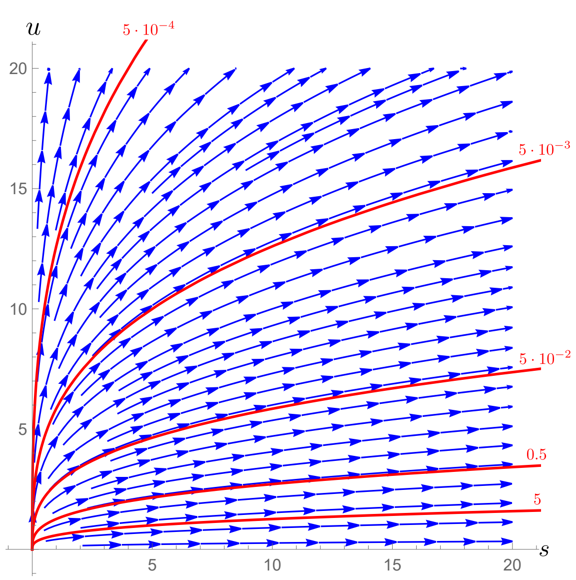

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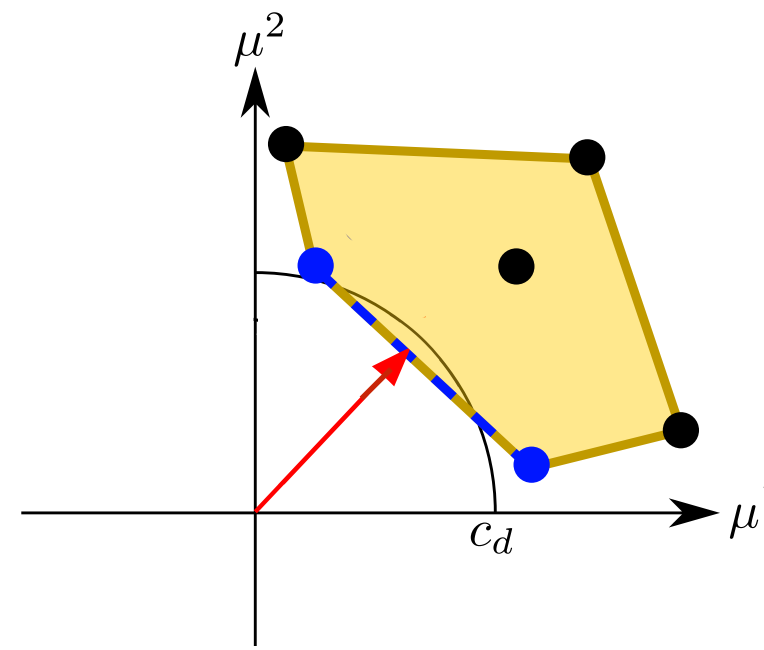

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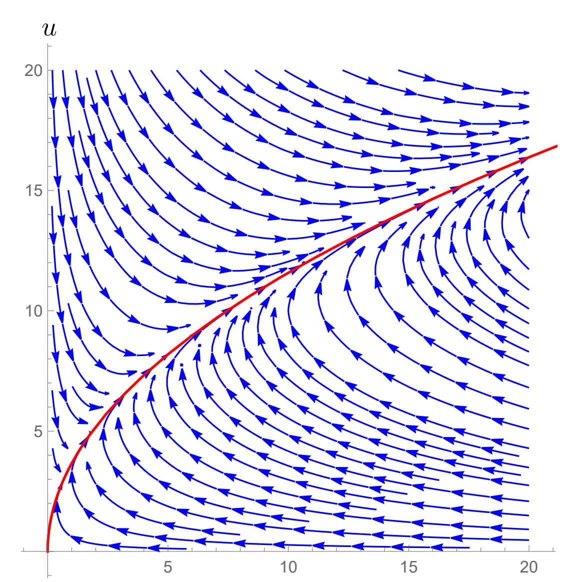

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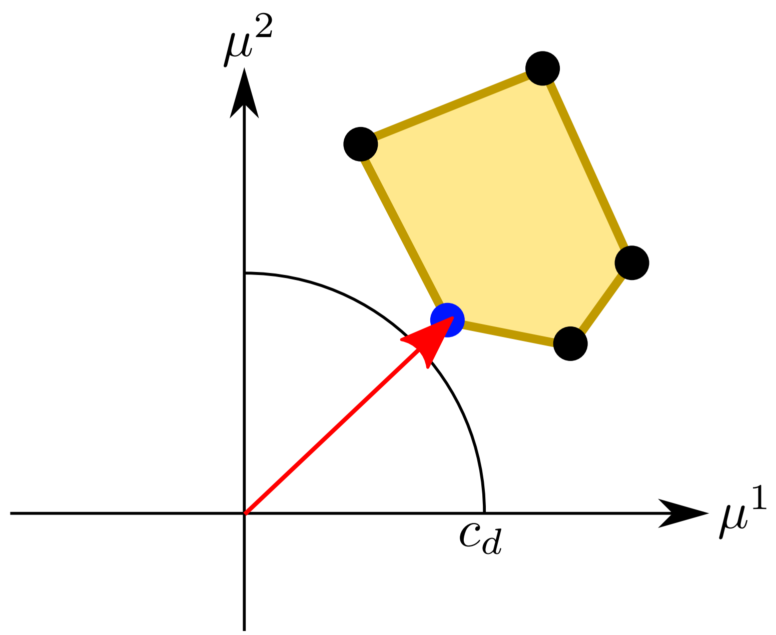

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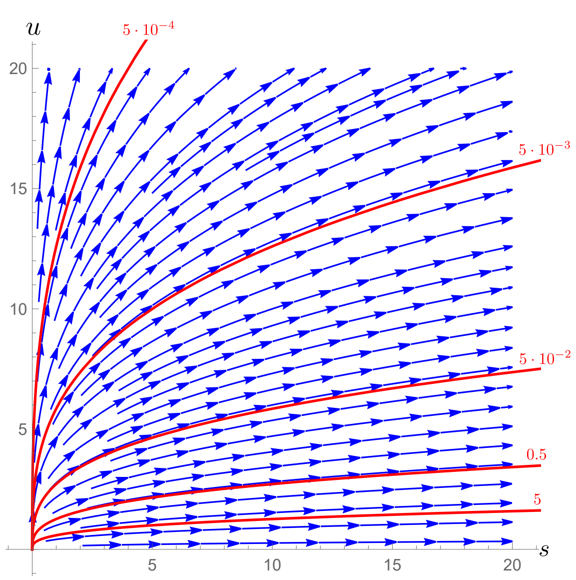

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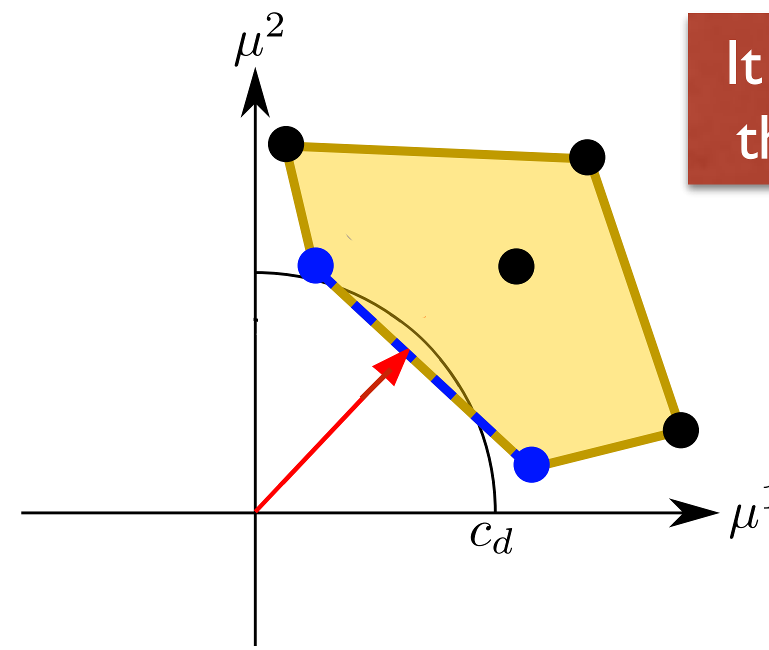
Scenario (II)

It can violate the bounds!



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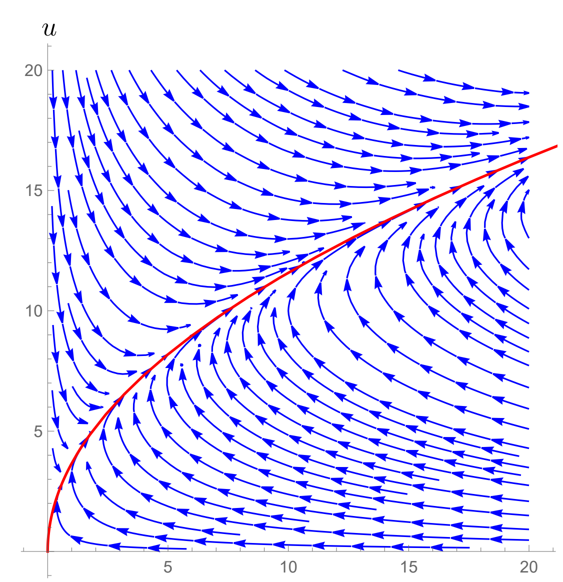

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Let us consider 4d N=1 EFTs arising from string theory

Is there in restriction / lower bound on the value

of the dS ratios $\vec{\mu}_l$?

4d N=1 supergravities

Proposed no-go: [Rudelius'21] (based on [Hellerman et al'01])

A scalar field rolling down a potential that asymptotes to a zero-energy SUSY minimum cannot yield an accelerating cosmology

Define $\mathcal{T} = e^{K/2}|W|$

$$V = \frac{1}{2}\left(\|\partial\mathcal{T}\|^2 - \frac{3}{2}\mathcal{T}^2\right) = \frac{1}{4}\left(\frac{\gamma^2}{2} - 3\right)\mathcal{T}^2$$

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Not true in Scenario (II), when several terms dominate asymptotically

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Not, gradient flows of W (or T) and V are different in Scenario (II)

4d N=1 supergravities

Scenario (II) presents a loophole to the sugra no-go for accelerated expansion.

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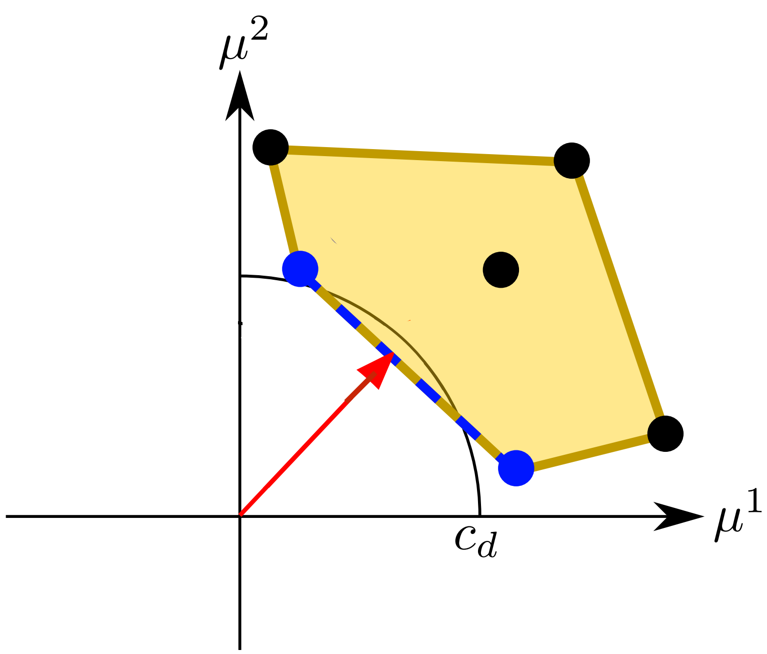
Interpretation from Convex Hull perspective:



Even if each individual term is bounded, the CH can still cut the ball

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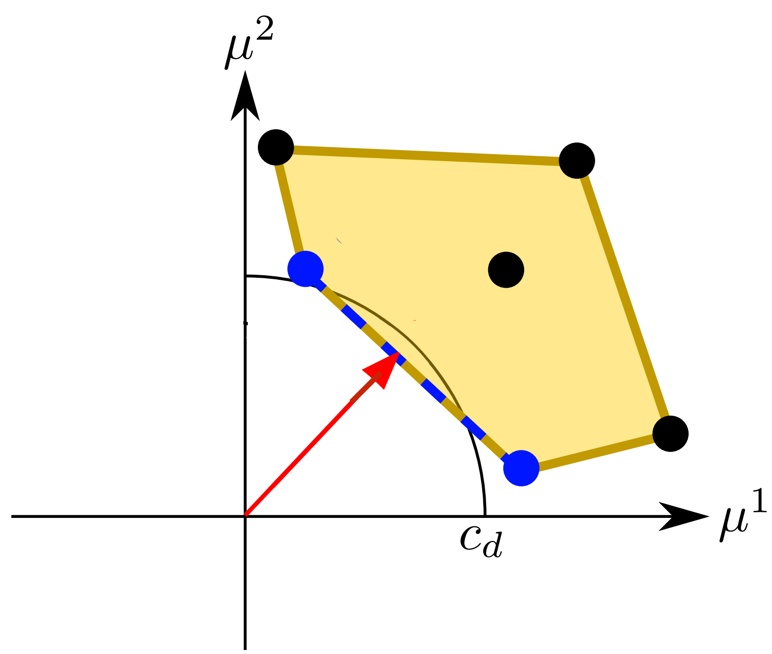

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Concrete example in IIA:

$$V_M \sim \frac{1}{s^3}\left(A_{34}\frac{u}{s} + A_{52}\frac{s}{u^3}\right) \quad \Rightarrow \quad \gamma_{\vec{f}} = 7\sqrt{\frac{2}{19}} \approx 2.2711 < \sqrt{6}$$

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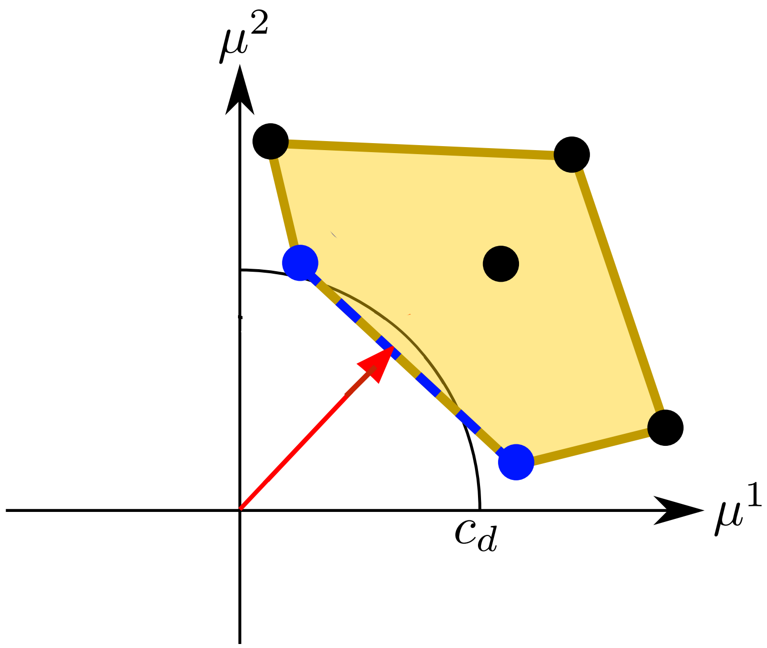

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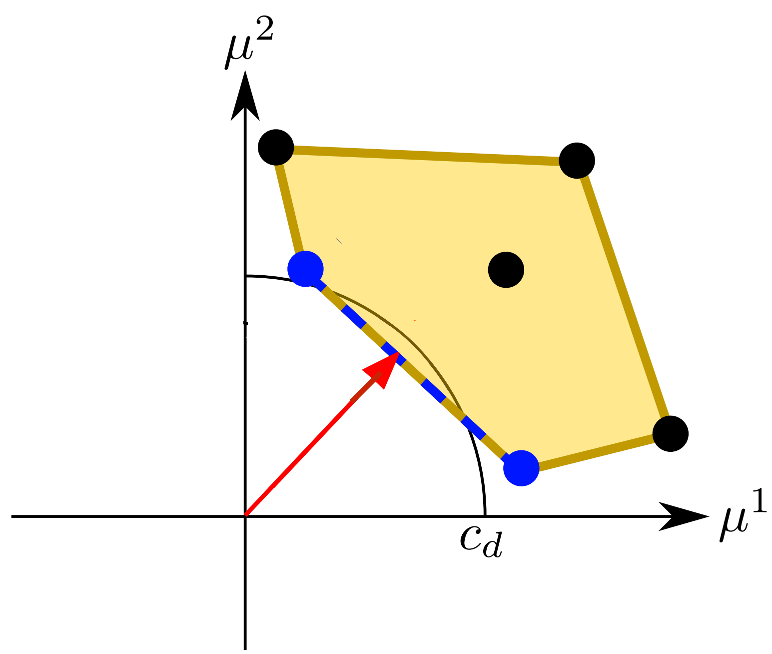

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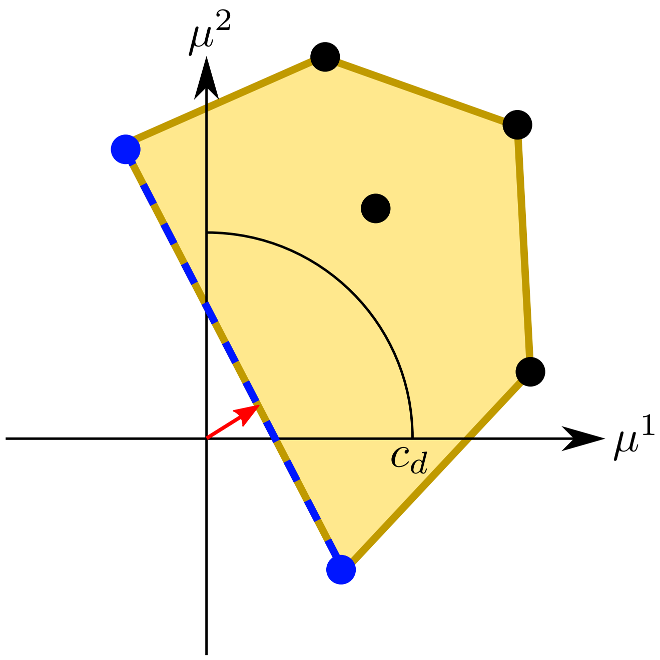
There is no lower bound for γ coming from V>0



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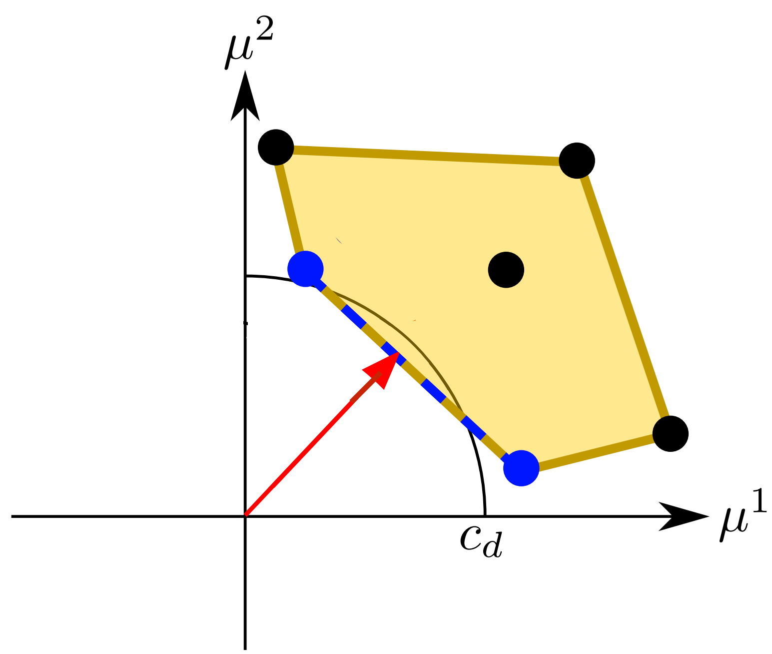

Even if each individual term is bounded, the CH can still cut the ball

No obstruction to get asymptotic accelerated expansion in SUGRA

… maybe in Quantum Gravity?

String Theory Asymptotic Limits

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Weak string coupling and large volume/large complex structure

[Valeixo et al'20] [Andriot et al'20-22] [Cicoli et al'21-22] …

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Let us study different limits in the complex structure moduli space of F-theory on CY_4 [Grimm et al'19]

$$K_{\rm cs} = -\sum_{j=1}^{n} \Delta d_j \log s^j + \dots ,$$

$$V_M = \frac{1}{\mathcal{V}_0^3}\left(\sum_{l\in\mathcal{E}} A_l \prod_{j=1}^{n} (s^j)^{\Delta l_j}\right) \qquad A_l = \|\rho_l(G_4,\phi_0)\|_\infty^2$$

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$\vec{\mu}_l$ fixed in terms of $(\Delta l_i, \Delta d_i)$ that characterizes the asymptotic limit

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Potential counterexamples

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Both realize Scenario (II). Smaller γ than previous CY bounds

[Bastian,Grimm,van de Heisteeg'20]

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Caveat: One has to stabilize the Kahler moduli, otherwise they also contribute to γ

Relation to Distance Conjecture

How does the potential compare with the mass scale of the tower of states becoming light?

At the very least, we will have a tower coming from BPS string:

$$T(\lambda) \simeq T(0)\exp(-\alpha_s D) \quad \text{with } \alpha_s = \frac{\beta_{\max}}{|\vec{\beta}|}$$

(exponential rate fixed by geometry!)

We get $\boxed{V \sim m^\chi}$ with $\chi = 1$ for example with $\gamma = \sqrt{\frac{2}{7}} < c_4^{TCC}$

$\chi = 2$ for example with $\gamma = \frac{2}{\sqrt{5}} < c_4^{\text{strong}}$

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This can be problematic, example cannot be trusted?

...but it is perturbative Type IIB with f2 and h0 flux...

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If one assumes the SDC lower bound $\alpha \geq \frac{1}{\sqrt{d-2}}$ in [Etheredge et al '22]

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However, accelerated expansion is still consistent with above SDC bound if we only require:

$$V \leq \Lambda_{\text{species}}^2$$

Is this enough?

Summary (before puzzle)

❖ We have studied whether string theory (asymptotic) runaway potentials allow for accelerating cosmologies

❖ This can be reformulated as a convex hull condition that must lie outside the ball

❖ The Strong dS bound can be violated at the level of the flux potential (in CY compactifications)

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Asymptotic Puzzle

Everything is starting to come into place in **flat space compactifications**

(even the numerical factors!)

Very useful organizing principle coming from:

Emergent String Conjecture *(all limits are either decompactifications or string perturbative limits)*

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But what about in AdS?

CFT Distance Conjecture

AdS_{d+1}/CFT_d with $d > 2$ [Perlmutter,Rastelli,Vafa,IV'20] (see also [Baume,Calderon-Infante'20])

Bulk moduli space $\longleftrightarrow$ Conformal manifold (space of exactly marginal couplings)

field metric $\longleftrightarrow$ Zamolodchikov metric $|x-y|^{2d}\langle O_i(x)O_j(y)\rangle = g_{ij}(t^i)$

tower of light states $\longleftrightarrow$ tower of operators saturating unitarity bound

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Our proposal:

$$\boxed{\exists \text{ tower of HS with } \gamma_J \sim e^{-\alpha d(\tau,\tau')} \text{ as } d(\tau,\tau') \to \infty \text{ in the conformal manifold}}$$

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By perturbation theory: $\mathcal{O}_\tau = \mathrm{Tr}(F^2+\dots)$

$$ds^2 = (24\,\mathrm{dim}G)\frac{d\tau d\bar\tau}{(\mathrm{Im}\tau)^2} \quad \text{as} \quad \mathrm{Im}\tau\to\infty$$

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$\geq \frac{1}{\sqrt{3}}$ for 4d N=2

$\geq \frac{1}{2}$ for 4d N=1

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gauge group getting free

We compute the exponential rate for all possible 4d SCFTs (N=1, N=2 and N=4) with simple gauge groups:

N=2

G	Hypermultiplets	c	α
$SU(N)$	$2N$ fund	$\frac{1}{6}(2N^2-1)$	$\sqrt{\frac{2}{3}}$
$SU(N)$	1 asym, $N+2$ fund	$\frac{1}{24}(7N^2+3N-4)$	$\sqrt{\frac{7}{12}}$
$SU(N)$	2 asym, 4 fund	$\frac{1}{12}(3N^2+3N-2)$	$\frac{1}{\sqrt{2}}$
$SU(N)$	1 asym, $N-2$ fund	$\frac{1}{24}(7N^2-3N-4)$	$\sqrt{\frac{7}{12}}$
$SU(N)$	1 sym, 1 asym	$\frac{1}{12}(3N^2-2)$	$\frac{1}{\sqrt{2}}$
$USp(2N)$	$4N+4$ $\frac{1}{2}$ fund	$\frac{1}{2}N(1+2N)$	1
$USp(2N)$	1 asym, 4 fund	$\frac{1}{12}(6N^2+9N-2)$	$\frac{1}{\sqrt{2}}$
$SO(N)$	$N-2$ vect	$\frac{1}{12}N(2N-3)$	$\sqrt{\frac{2}{3}}$

N=1

G	Theory	c	α
$SU(N)$	Table 2, #1	$\frac{1}{24}(7N^2-5)$	$\sqrt{\frac{7}{12}}$
$SU(N)$	Table 2, #5	$\frac{1}{24}(6N^2+3N-5)$	$\frac{1}{\sqrt{2}}$
$SU(N)$	Table 3, #4	$\frac{1}{24}(7N^2-4)$	$\sqrt{\frac{7}{12}}$
$SU(N)$	Table 5, #4	$\frac{1}{24}(8N^2-3)$	$\sqrt{\frac{2}{3}}$
$USp(2N)$	Table 12, #1	$\frac{1}{24}(14N^2+15N-1)$	$\sqrt{\frac{7}{12}}$
$USp(2N)$	Table 13, #9	$\frac{1}{8}(4N^2+8N-1)$	$\frac{1}{\sqrt{2}}$
$USp(2N)$	Table 13, #10	$\frac{1}{24}(14N^2+21N-2)$	$\sqrt{\frac{7}{12}}$
$SO(N)$	Table 18, #1	$\frac{1}{48}(7N^2-21N-4)$	$\sqrt{\frac{7}{12}}$
$SO(N)$	Table 18, #2	$\frac{1}{48}(7N^2-15N-2)$	$\sqrt{\frac{7}{12}}$
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Are we somehow decompactifying in a dual description??

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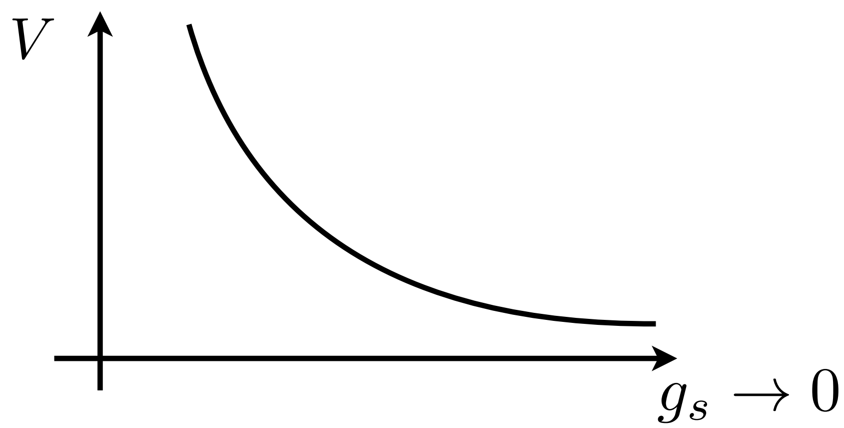
back-up slides

Non-SUSY example

SO(16)xSO(16) non-SUSY (tachyon-free) heterotic string theory:



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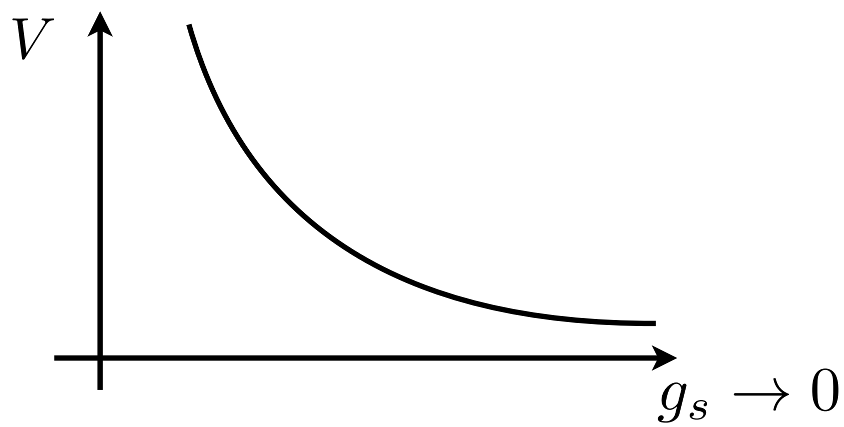

Positive runaway on the dilaton

Tower of string modes becoming light in the weak coupling limit

$$V_{\text{1-loop}} \sim -\sum_i (-1)^{F_i} \int_{\Lambda_{UV}^{-2}}^{\infty} \frac{ds}{s^6} \, \exp\left(-\frac{m_i^2 s}{2}\right)$$

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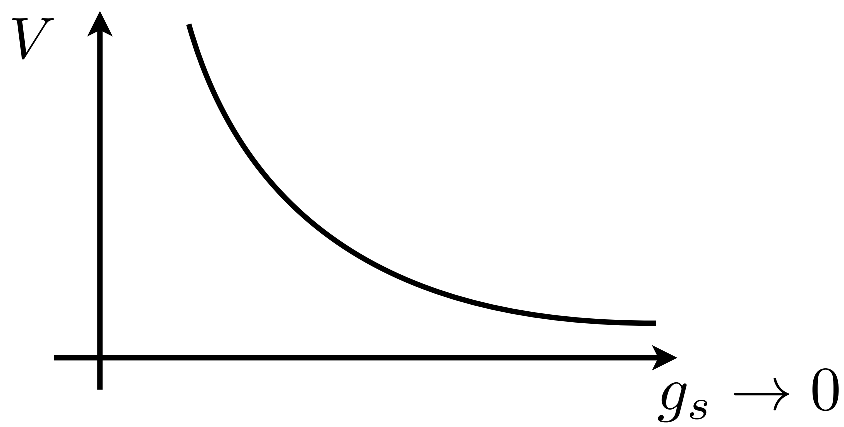

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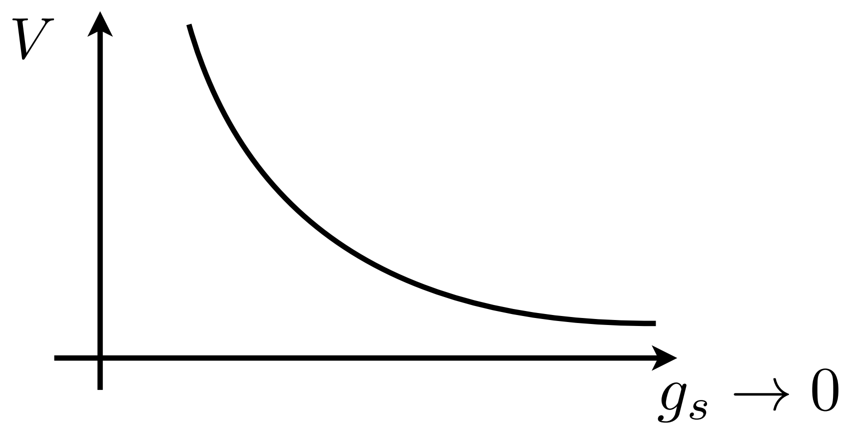

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Contribution of massive string excitations
is cut-off at Ms due to modular invariance

Relation between V and m

$V \sim m_{\rm tower}^{\alpha}$ **with** $2 \leq \alpha \leq d$

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Consistent with Light Fermion conjecture: [Gonzalo,Ibañez,IV'21]

If $V \geq 0$ there is a surplus of light fermions with $m \lesssim V^{1/d}$

(to avoid inconsistency of Casimir vacua with AdS swampland conjectures upon compactification of the theory)

Status report of SDC

Asymptotically Minkowski compactifications:			Exponential behaviour of the tower mass	Tower populated by infinitely many states	Classification of limits
More than 8 supercharges: coset spaces					
8 supercharges	4d N=2 (Type II on CY3)	Vector multiplets			
		Hypermultiplets			
	5d/6d N=1 (M/F-theory on CY3)	Vector/tensor multiplets			
		Hypermultiplets			
4 supercharges: 4d N=1					
No supersymmetry					
Asymptotically AdS compactifications:					
Weak coupling points in d>3					
Other points					

Two moduli limits

Enhancements	Potential V_M
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
$\mathrm{I}_{0,sh-2} \Longrightarrow \mathrm{I}_{0,sh-4}$ $\mathrm{I}_{0,sh-2} \Longrightarrow \mathrm{II}_{0,sh-2}$ $\mathrm{II}_{0,sh} \Longrightarrow \mathrm{II}_{0,sh-2}$ $\mathrm{I}_{0,sh-2}, \mathrm{II}_{0,sh} \Longrightarrow \mathrm{II}_{0,sh-2}$	$\frac{c_1}{\tau} + \frac{c_2}{\rho} + c_3\rho + c_4\tau - c_0$
$\mathrm{I}_{0,sh-2} \Longrightarrow \mathrm{I}_{1,sh}$ $\mathrm{I}_{0,sh-4} \Longrightarrow \mathrm{I}_{2,sh}$ $\mathrm{II}_{0,sh-2} \Longrightarrow \mathrm{III}_{0,sh-2}$	$\frac{c_1}{\rho\tau} + \frac{c_2\rho}{\tau} + \frac{c_3\tau}{\rho} + c_4\rho\tau - c_0$
$\mathrm{I}_{1,sh} \Longrightarrow \mathrm{I}_{2,sh}$ $\mathrm{I}_{1,sh} \Longrightarrow \mathrm{III}_{1,sh-2}$ $\mathrm{III}_{0,sh-2} \Longrightarrow \mathrm{III}_{1,sh-2}$ $\mathrm{I}_{1,sh}, \mathrm{III}_{0,sh-2} \Longrightarrow \mathrm{III}_{1,sh-2}$	$\frac{c_1}{\tau^2} + \frac{c_2}{\rho^2} + c_3\rho^2 + c_4\tau^2 - c_0$
$\mathrm{III}_{1,sh-2} \Longrightarrow \mathrm{V}_{2,sh}$	$\frac{c_1}{\rho^2\tau^2} + \frac{c_2}{\tau^2} + \frac{c_3}{\rho^2} + \frac{c_4\rho^2}{\tau^2} + \frac{c_5\tau^2}{\rho^2} + c_6\rho^2 + c_7\tau^2 + c_8\rho^2\tau^2 - c_0$

Enhancements	Potential V_M
$\mathrm{II}_{0,sh} \longrightarrow \mathrm{V}_{1,sh}$	$\frac{c_1}{\rho^3\tau} + \frac{c_2\rho^3}{\tau} + \frac{c_3\tau}{\rho^3} + c_4\rho^3\tau - c_0$
$\mathrm{II}_{0,sh} \longrightarrow \mathrm{V}_{1,sh-2}$	$\frac{c_1}{\rho^3\tau} + \frac{c_2}{\rho} + \frac{c_3\rho^3}{\tau} + \frac{c_4\tau}{\rho^3} + c_5\rho + c_6\rho^3\tau - c_0$
$\mathrm{II}_{0,sh} \longrightarrow \mathrm{V}_{2,sh}$	$\frac{c_1}{\rho^3\tau} + \frac{c_2}{\rho^2} + \frac{c_3\rho^3}{\tau} + \frac{c_4\tau}{\rho^3} + c_5\rho^2 + c_6\rho^3\tau - c_0$
$\mathrm{II}_{1,sh} \longrightarrow \mathrm{V}_{2,sh}$	$\frac{c_1}{\tau^2} + \frac{c_2}{\rho^3\tau} + \frac{c_3\rho^3}{\tau} + \frac{c_4\tau}{\rho^3} + c_5\rho^3\tau + c_6\tau^2 - c_0$
$\mathrm{III}_{0,sh-2} \longrightarrow \mathrm{V}_{1,sh}$	$\frac{c_1}{\rho^2\tau^2} + \frac{c_2}{\rho^2} + \frac{c_3\rho^2}{\tau^2} + \frac{c_4\tau^2}{\rho^2} + c_5\rho^2 + c_6\rho^2\tau^2 - c_0$
$\mathrm{III}_{0,sh-2} \longrightarrow \mathrm{V}_{1,sh-2}$	$\frac{c_1}{\rho^2\tau^2} + \frac{c_2}{\rho^2} + \frac{c_3}{\rho} + \frac{c_4\rho^2}{\tau^2} + \frac{c_5\tau^2}{\rho^2} + c_6\rho + c_7\rho^2 + c_8\rho^2\tau^2 - c_0$
$\mathrm{III}_{0,sh-2} \longrightarrow \mathrm{V}_{2,sh}$	$\frac{c_1}{\rho^2\tau^2} + \frac{c_2}{\rho^2} + \frac{c_3\rho^2}{\tau^2} + \frac{c_4\tau^2}{\rho^2} + c_5\rho^2 + c_6\rho^2\tau^2 - c_0$
$\mathrm{III}_{0,sh-4} \longrightarrow \mathrm{V}_{1,sh-2}$	$\frac{c_1}{\rho^2\tau^2} + \frac{c_2}{\tau} + \frac{c_3}{\rho^2} + \frac{c_4\rho^2}{\tau^2} + \frac{c_5\tau^2}{\rho^2} + c_6\rho^2 + c_7\tau + c_8\rho^2\tau^2 - c_0$
$\mathrm{III}_{0,sh-4} \longrightarrow \mathrm{V}_{2,sh}$	$\frac{c_1}{\rho^2\tau^2} + \frac{c_2}{\rho\tau} + \frac{c_3}{\rho^2} + \frac{c_4\rho^2}{\tau^2} + \frac{c_5\rho}{\tau} + \frac{c_6\tau}{\rho} + \frac{c_7\tau^2}{\rho^2} + c_8\rho^2 + c_9\rho\tau + c_{10}\rho^2\tau^2 - c_0$

We compute leading behaviour of the flux induced scalar potential for the 46 possible asymptotic limits

$$\tau, \rho \to \infty$$

Two moduli limits

Enhancements	Potential V_M
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
$\mathrm{I}_{0,\hat{m}-2} \Longrightarrow \mathrm{I}_{0,\hat{m}-4}$; $\mathrm{I}_{0,\hat{m}-2} \Longrightarrow \mathrm{II}_{0,\hat{m}-2}$; $\mathrm{II}_{0,\hat{m}} \Longrightarrow \mathrm{II}_{0,\hat{m}-2}$; $\mathrm{I}_{0,\hat{m}-2}, \mathrm{II}_{0,\hat{m}} \Longrightarrow \mathrm{II}_{0,\hat{m}-2}$	$\frac{c_1}{\tau} + \frac{c_2}{\rho} + c_3\rho + c_4\tau - c_0$
$\mathrm{I}_{0,\hat{m}-2} \Longrightarrow \mathrm{I}_{1,\hat{m}}$; $\mathrm{I}_{0,\hat{m}-4} \Longrightarrow \mathrm{I}_{2,\hat{m}}$; $\mathrm{II}_{0,\hat{m}-2} \Longrightarrow \mathrm{III}_{0,\hat{m}-2}$	$\frac{c_1}{\rho\tau} + \frac{c_2\rho}{\tau} + \frac{c_3\tau}{\rho} + c_4\rho\tau - c_0$
$\mathrm{I}_{1,\hat{m}} \Longrightarrow \mathrm{I}_{2,\hat{m}}$; $\mathrm{I}_{1,\hat{m}} \Longrightarrow \mathrm{III}_{1,\hat{m}-2}$; $\mathrm{III}_{0,\hat{m}-2} \Longrightarrow \mathrm{III}_{1,\hat{m}-2}$; $\mathrm{I}_{1,\hat{m}}, \mathrm{III}_{0,\hat{m}-2} \Longrightarrow \mathrm{III}_{1,\hat{m}-2}$	$\frac{c_1}{\tau^3} + \frac{c_2}{\rho^3} + c_3\rho^2 + c_4\tau^2 - c_0$
$\mathrm{III}_{1,\hat{m}-2} \Longrightarrow \mathrm{V}_{2,\hat{m}}$	$\frac{c_1}{\rho^3\tau^3} + \frac{c_2}{\tau^3} + \frac{c_3}{\rho^3} + \frac{c_4\rho^2}{\tau^3} + \frac{c_5\tau^2}{\rho^3} + c_6\rho^2 + c_7\tau^2 + c_8\rho^2\tau^2 - c_0$

Enhancements	Potential V_M
$\mathrm{II}_{0,\hat{m}} \longrightarrow \mathrm{V}_{1,\hat{m}}$	$\frac{c_1}{\rho^3\tau} + \frac{c_2\rho^3}{\tau} + \frac{c_3\tau}{\rho^3} + c_4\rho^3\tau - c_0$
$\mathrm{II}_{0,\hat{m}} \longrightarrow \mathrm{V}_{1,\hat{m}-2}$	$\frac{c_1}{\rho^3\tau} + \frac{c_2}{\rho} + \frac{c_3\rho^3}{\tau} + \frac{c_4\tau}{\rho^3} + c_5\rho + c_6\rho^3\tau - c_0$
$\mathrm{II}_{0,\hat{m}} \longrightarrow \mathrm{V}_{2,\hat{m}}$	$\frac{c_1}{\rho^3\tau} + \frac{c_2}{\rho^3} + \frac{c_3\rho^3}{\tau} + \frac{c_4\tau}{\rho^3} + c_5\rho^2 + c_6\rho^3\tau - c_0$
$\mathrm{II}_{1,\hat{m}} \longrightarrow \mathrm{V}_{2,\hat{m}}$	$\frac{c_1}{\tau^3} + \frac{c_2}{\rho^3\tau} + \frac{c_3\rho^3}{\tau} + \frac{c_4\tau}{\rho^3} + c_5\rho^3\tau + c_6\tau^2 - c_0$
$\mathrm{III}_{0,\hat{m}-2} \longrightarrow \mathrm{V}_{1,\hat{m}}$	$\frac{c_1}{\rho^3\tau^3} + \frac{c_2}{\rho^3} + \frac{c_3\rho^2}{\tau^3} + \frac{c_4\tau^2}{\rho^3} + c_5\rho^2 + c_6\rho^2\tau^2 - c_0$
$\mathrm{III}_{0,\hat{m}-2} \longrightarrow \mathrm{V}_{1,\hat{m}-2}$	$\frac{c_1}{\rho^3\tau^3} + \frac{c_2}{\rho^3} + \frac{c_3}{\rho} + \frac{c_4\rho^2}{\tau^3} + \frac{c_5\tau^2}{\rho^3} + c_6\rho + c_7\rho^2 + c_8\rho^2\tau^2 - c_0$
$\mathrm{III}_{0,\hat{m}-2} \longrightarrow \mathrm{V}_{2,\hat{m}}$	$\frac{c_1}{\rho^3\tau^3} + \frac{c_2}{\rho^3} + \frac{c_3\rho^2}{\tau^3} + \frac{c_4\tau^2}{\rho^3} + c_5\rho^2 + c_6\rho^2\tau^2 - c_0$
$\mathrm{III}_{0,\hat{m}-4} \longrightarrow \mathrm{V}_{1,\hat{m}-2}$	$\frac{c_1}{\rho^3\tau^3} + \frac{c_2}{\tau} + \frac{c_3}{\rho^3} + \frac{c_4\rho^2}{\tau^3} + \frac{c_5\tau^2}{\rho^3} + c_6\rho^2 + c_7\tau + c_8\rho^2\tau^2 - c_0$
$\mathrm{III}_{0,\hat{m}-4} \longrightarrow \mathrm{V}_{2,\hat{m}}$	$\frac{c_1}{\rho^3\tau^3} + \frac{c_2}{\rho\tau} + \frac{c_3}{\rho^3} + \frac{c_4\rho^2}{\tau^3} + \frac{c_5\rho}{\tau} + \frac{c_6\tau}{\rho} + \frac{c_7\tau^2}{\rho^3} + c_8\rho^2 + c_9\rho\tau + c_{10}\rho^2\tau^2 - c_0$

We compute leading behaviour of the flux induced scalar potential for the 46 possible asymptotic limits

$$\tau, \rho \to \infty$$

weak coupling + large volume limit in IIA

$$V_{\mathrm{M}} \sim \frac{1}{\mathcal{V}_4^3}\left(\sum_{p=0,2,4,6} \frac{A_{f_p}}{\rho^{p-3}\tau} + \sum_{q=0,1,2,3} \frac{A_{h_q}\tau}{\rho^{3-2q}} - A_{\mathrm{loc}}\right)$$

What is the value of the exponential rate?

❖ AdS_{d+1}/CFT_d with $d > 2$ [Perlmutter,Rastelli,Vafa,IV'20]

$$\alpha = \sqrt{\frac{2c}{\mathrm{dim}G}} \begin{array}{ll} \geq \frac{1}{\sqrt{3}} & \text{for 4d N=2} \\ \geq \frac{1}{2} & \text{for 4d N=1} \end{array}$$

[Grimm, Palti, IV'18] [Gendler,IV'20]

❖ Lower bound for BPS states in CY compactifications: $\alpha \geq \frac{1}{\sqrt{2n}}$ for CY_n

$K = -n \log \phi + \ldots$

❖ 4D N=2 theories: $\alpha^2 \geq \left.\frac{Q^2_{\mathrm{ext}}}{T^2_{\mathrm{ext}}}\right|_{\mathrm{BPS\,particles}} - \frac{1}{2}$

❖ 4D N=1 theories: $\alpha \geq \frac{1}{2} \left.\frac{Q_{\mathrm{ext}}}{T_{\mathrm{ext}}}\right|_{\mathrm{BPS\,string}}$

→ bounded by scalar contribution to WGC/extremality bound!

[Lee,Lerche,Weigand'19] [Gendler,IV'20]
[Bastian, Grimm, Van de Heisteeg'20]

❖ TCC $\alpha \geq \frac{1}{\sqrt{(d-2)(d-3)}}$ [Bedroya,Vafa'19] [Andriot et al'20]