Gravity and Optimal Transport

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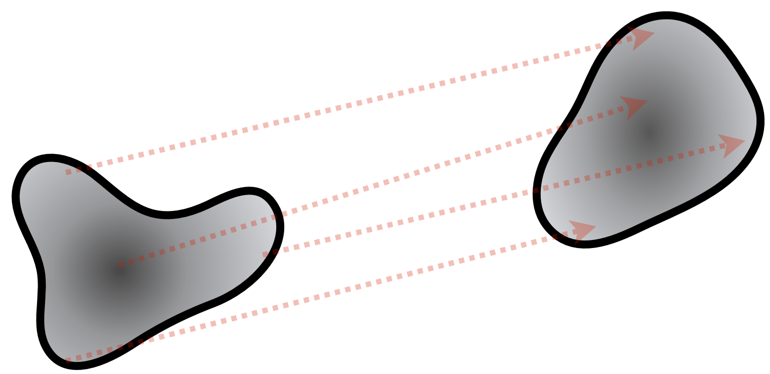
based on 2104.12773 with G.B. De Luca,
2109.11560 + 2210.xxxxx
with De Luca, N. De Ponti, A. Mondino

Madrid, 28.9.22

Introduction

Optimal transport:
best way of moving
mass distributions



[Monge 1781, Kantorovich 1940...]
review: [Villani '08]

In curved space, each bit of mass should move along geodesics

the whole motion can also be understood as a geodesic
in the space of probability distributions

Observation: the tensor $R_{mn}^{N,f} \equiv R_{mn} - \nabla_m \nabla_n f - \frac{1}{N-n} \partial_m f \partial_n f$ [Bakry, Émery '85]

appears both in optimal transport and in warped compactifications

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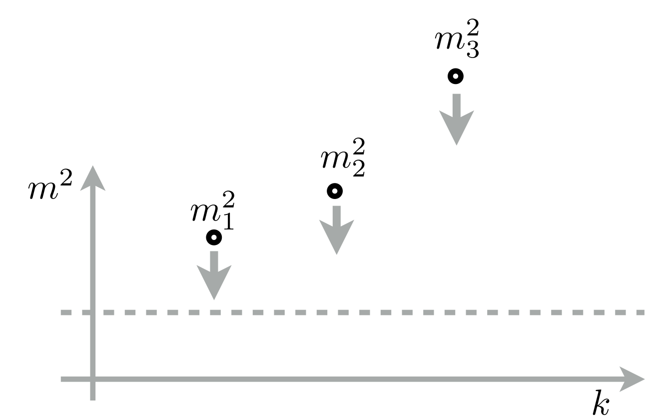
Applications in this talk:

- Gravitational equations $\Longleftrightarrow$ concavity of 'Tsallis entropy'

- Bounds on spin-2 KK masses

  ⇨ a *swampland theorem* for large c.c.

  and a challenge for small c.c.



Plan

- Entropy and transport
  - Entropy and (classical) gravity
    - Bounds on eigenvalues
      - Application: spin-2 conjectures

Entropy and transport

Entropy and transport

- Consider a distribution of particles: $\rho(x)$ such that $\int_M \sqrt{g}\rho = 1$

Entropy: $S = -\int_M \sqrt{g}\rho \log \rho$

if particles move, generically it should grow.
What about its second time derivative?

'expansion': infinitesimal volume change

$$\partial_t^2 S = -\int_M \sqrt{g}\rho \left(\nabla_U \nabla \cdot U - \tfrac{1}{2}\nabla^2 U^2\right)$$

U^μ: velocity field of geodesics

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Bochner identity/Raychaudhuri equation

$$R_{mn} \geqslant 0 \;\Rightarrow\; \partial_t^2 S \leqslant 0$$

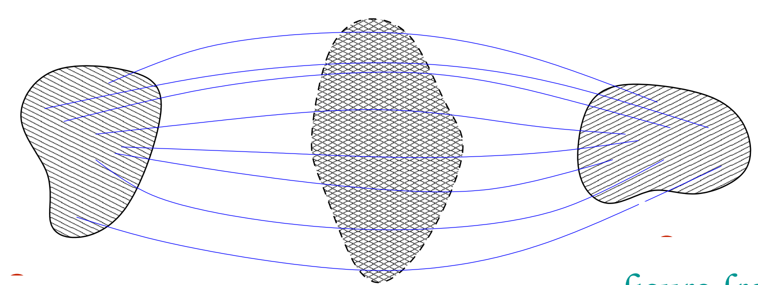

figure from [Villani '08]

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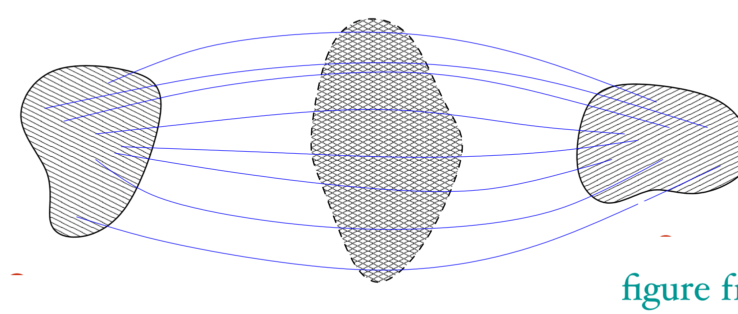

figure from [Villani '08]

one can even use this to reformulate Einstein's equations

[McCann '19; Mondino, Suhr '19]

- If measure is 'weighted': $\sqrt{g} \to \mathrm{e}^{f}\sqrt{g}$ $\qquad$ $\nabla \cdot U \to \mathrm{e}^{-f}\nabla \cdot (\mathrm{e}^{f} U)$

'weighted expansion'

$$\Rightarrow \qquad \partial_t^2 S = -\int_M \sqrt{g}\rho \left(\nabla_m U_n \nabla^m U^n + \underline{R^{\infty,f}_{mn}} U^m U^n\right)$$

$$R^{\infty,f}_{mn} = R_{mn} - \nabla_m \nabla_n f$$

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- 'Tsallis entropy': homogeneous (rather than extensive) $\qquad$ [$\sim$ log Rényi entropy]

[Havrda, Charvat '67; Patil, Taillie '82; Tsallis '88]
from axioms in [Suyari '95, Furuichi '05]

$$S_N = N\left(1 - \int_M \sqrt{g} e^f \rho^{\frac{N}{N-1}}\right)$$

$$\Rightarrow \qquad R^{\infty,f}_{mn} \to R^{N,f}_{mn}$$

[De Luca, De Ponti, Mondino, AT '22 *to appear*]

Entropy and gravity

Consider a higher-dimensional gravity $m_D^{D-2} \int \mathrm{d}^D x \sqrt{-g_D} R_D$ + matter

and a compactification $\mathrm{d}s_D^2 = \mathrm{e}^{2A}(\mathrm{d}s_d^2 + \mathrm{d}s_n^2)$

max. symmetric

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sign?

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["Reduced Energy Condition"]

- for all bulk fields in type II and $d = 11$ sugra
- for brane sources

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$R_{mn}^{N,f}$

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$f = (D-2)A$

$N = 2 - d < 0$

actually still good!

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- we can reformulate this equation in term of entropy:

Theorem:

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Tsallis entropy quantifying our ignorance of particle position in internal space

velocity field of geodesics

$$\frac{\mathrm{d}S_N}{\mathrm{d}t^2} \leqslant \int \sqrt{g}\,\mathrm{e}^f \rho^{\frac{N-1}{N}} \left(\Lambda g_{mn} + \tfrac{1}{2} m_D^{2-D} (T_{mn} - \tfrac{1}{d} g_{mn} T_{(d)}) U^m U^n\right)$$

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- purely internal equation: in terms of first derivative of Shannon entropy

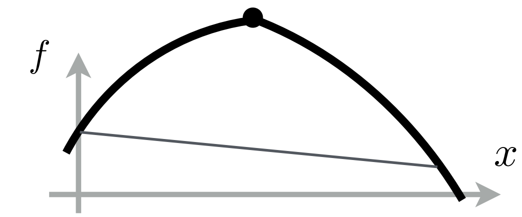
‘synthetic’ point of view: generalization to singularities

Inspiration: functions of one variable

$$f'' \leqslant 0$$

generalize to non-smooth functions:

concavity



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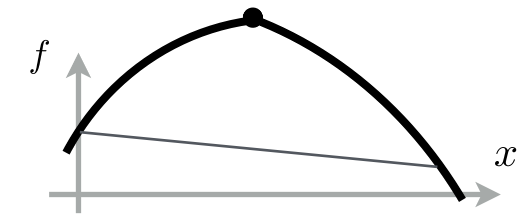

Roughly similar:

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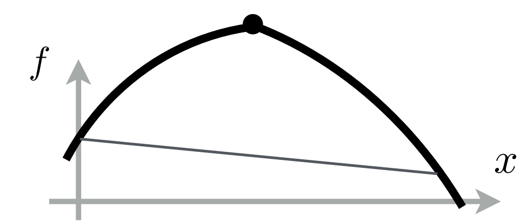

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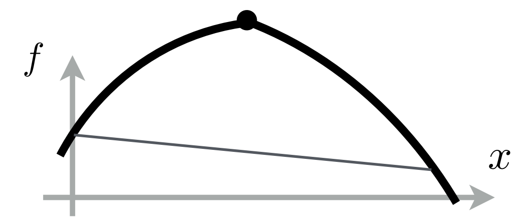

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This applies to D-branes

[De Luca, De Ponti, Mondino, AT ’21; ’22 *to appear*]

Bounds on masses

Hard to compute KK spectrum.

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A universal result:

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weighted Laplacian

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[Csaki, Erlich, Hollowood, Shirman'00; Bachas, Estes '11]

total dimension

$$f = (D-2)A$$

$$ds^2_D = e^{2A}(ds^2_d + ds^2_n)$$

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- Eigenvalue bounds can be obtained by optimal transport methods

on spaces with bounds for the $R^{N,f}_{mn}$ tensors.

Luckily, the 'REC' does imply such Ricci bounds!

$$R_{mn} + (D-2)(-\nabla_m \nabla_n A + \partial_m A \partial_n A) = \Lambda g_{mn} + \tfrac{1}{2} m_D^{2-D} \underbrace{(T_{mn} - \tfrac{1}{d} g_{mn} T_{(d)})}_{\text{non-negative}}$$

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$K \equiv |\Lambda| + \frac{\sigma^2}{D-2}$

$\sigma \geqslant (D-2)|\mathrm{d}A|$

'sup of the warping'

$$\Rightarrow \quad R^{\infty,f}_{mn} \geqslant -K g_{mn}$$

We can now prove some theorems:

empirical bound on d among SE's:
[Collins, Jafferis, Vafa, Xu, Yau '22]
and among sphere quotients:
[Gorodski, Lange, Lytchak, Mendes '19]

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[De Luca, AT '21] using [Setti '98]

for now, M_n smooth.

WIP: extension using RCD sing.
with $N < 0$ $\to$ get rid of σ

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- $m_1^2 \geqslant \frac{c(d)}{\mathrm{diam}^2}$

so small diameter does imply scale separation
for spin-2. For now, no O-planes

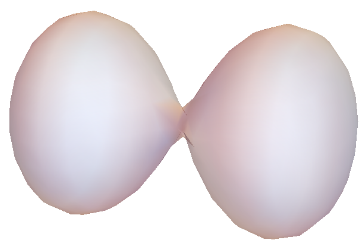
[Calderon '19;
De Luca, De Ponti,
Mondino, AT: '22 *to appear*]

- bounds in terms of **Cheeger constant** $h_1(M_n) \equiv \inf_B \frac{\int_{\partial B} \sqrt{\bar{g}_{\partial B}}\, \mathrm{e}^{(D-2)A}\, \mathrm{d}^{n-1}x}{\int_B \sqrt{\bar{g}}\, \mathrm{e}^{(D-2)A}\, \mathrm{d}^n x}$

[De Luca, De Ponti, Mondino, AT '21]

'min. of $\frac{\text{perimeter}}{\text{area}}$'

a space where h_1 is small has a small 'neck':



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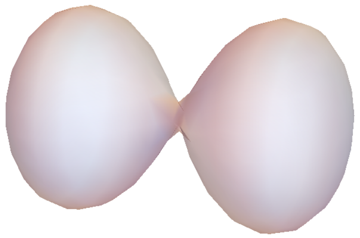

- smallest mass:

$$\frac{1}{4} h_1^2 \leqslant m_1^2 \leqslant \max\left\{ \frac{21}{10} h_1 \sqrt{K},\ \frac{22}{5} h_1^2 \right\}$$

adapting [De Ponti, Mondino '19]

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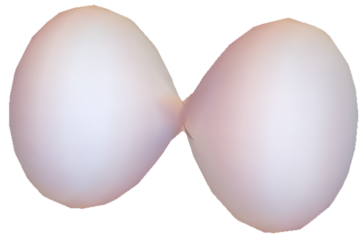

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broad class, including O-planes

[De Luca, De Ponti, Mondino, AT '22 *to appear*]

RCD(K,∞) sing.

[recall: includes D-branes]

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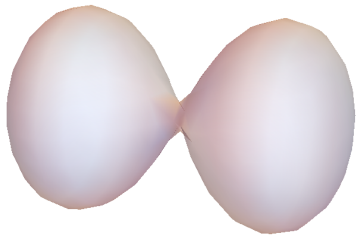
'min. of $\frac{\text{perimeter}}{\text{area}}$'

a space where h_1 is small has a small 'neck':



- smallest mass: $\frac{1}{4} h_1^2 \leqslant m_1^2 \leqslant \max\left\{ \frac{21}{10} h_1 \sqrt{K},\ \frac{22}{5} h_1^2 \right\}$

broad class, including O-planes

[De Luca, De Ponti, Mondino, AT '22 *to appear*]

RCD(K, ∞) sing.

[recall: includes D-branes]

adapting [De Ponti, Mondino '19]

$$K \equiv |\Lambda| + \frac{\sigma^2}{D-2}$$

- similar bounds on higher eigenvalues

Theorem: $m_k^2 < 600k^2 \ \mathbf{max}\left\{m_1^2, |\Lambda| + \frac{\sigma^2}{D-2}\right\}$

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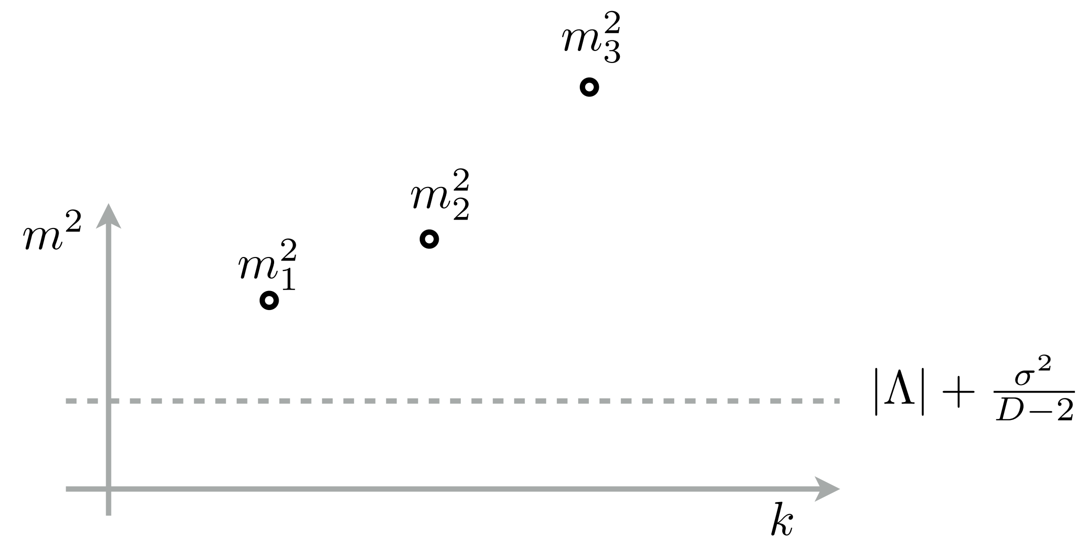

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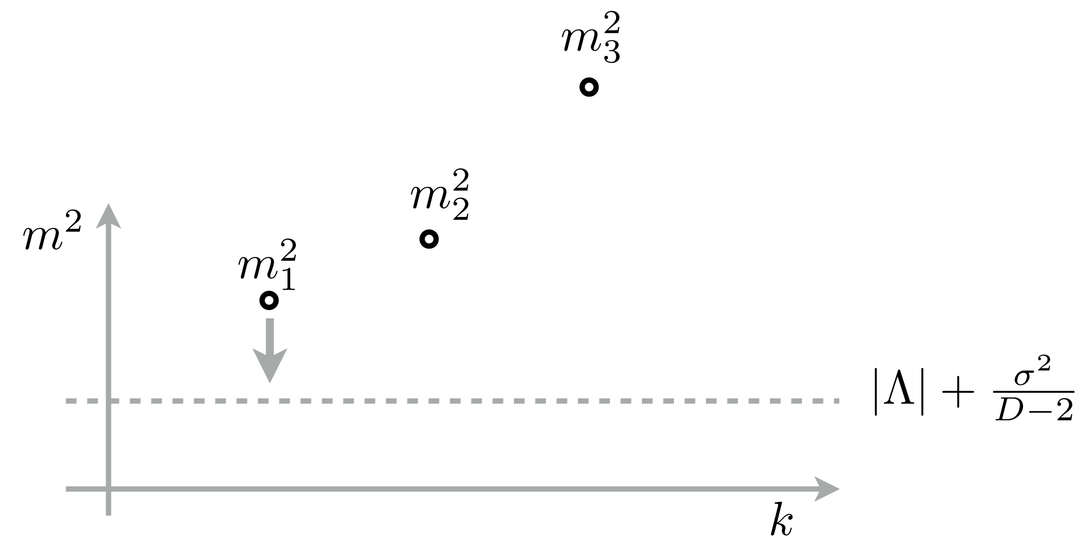

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If one lowers m_1^2,
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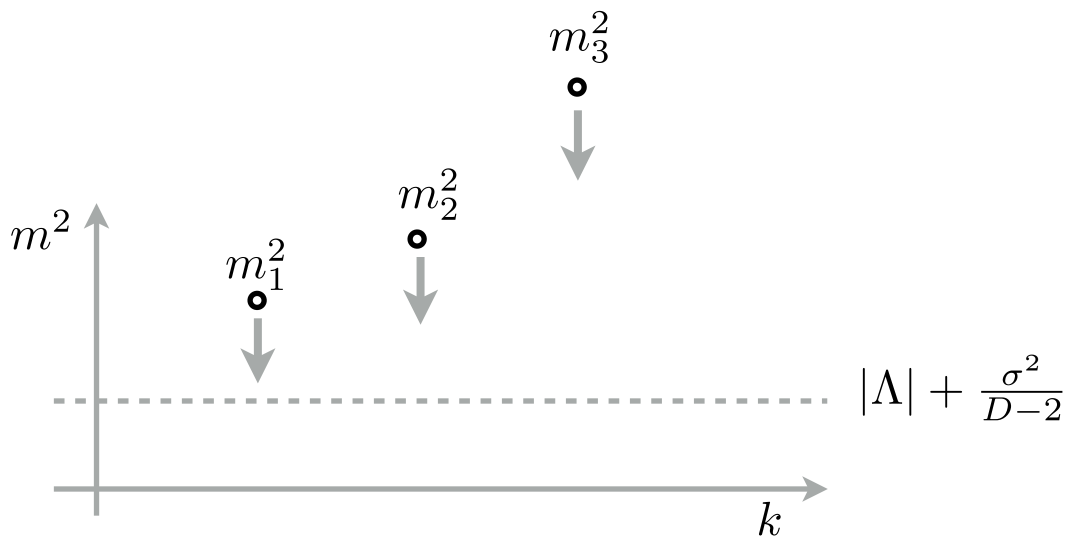

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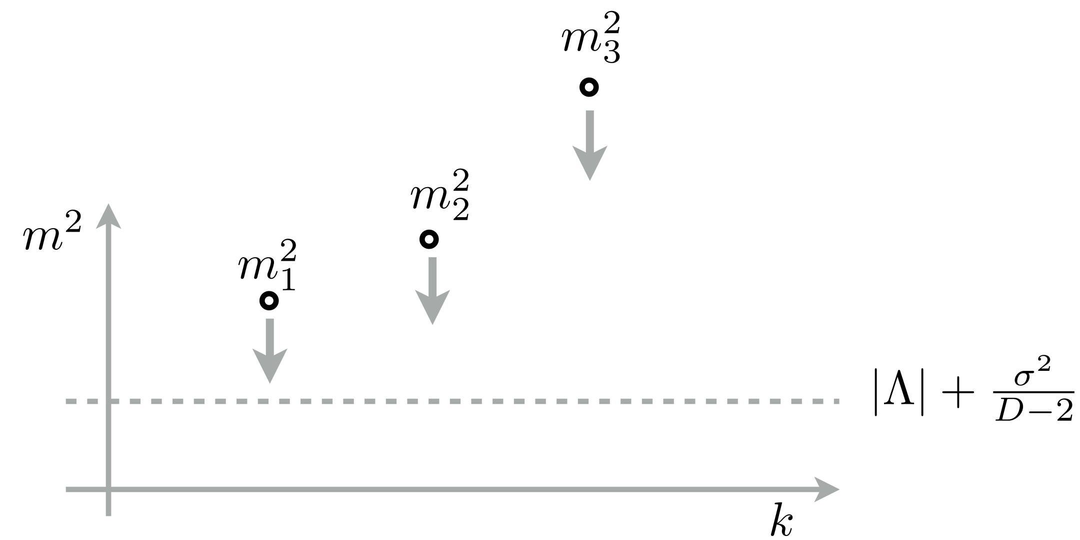

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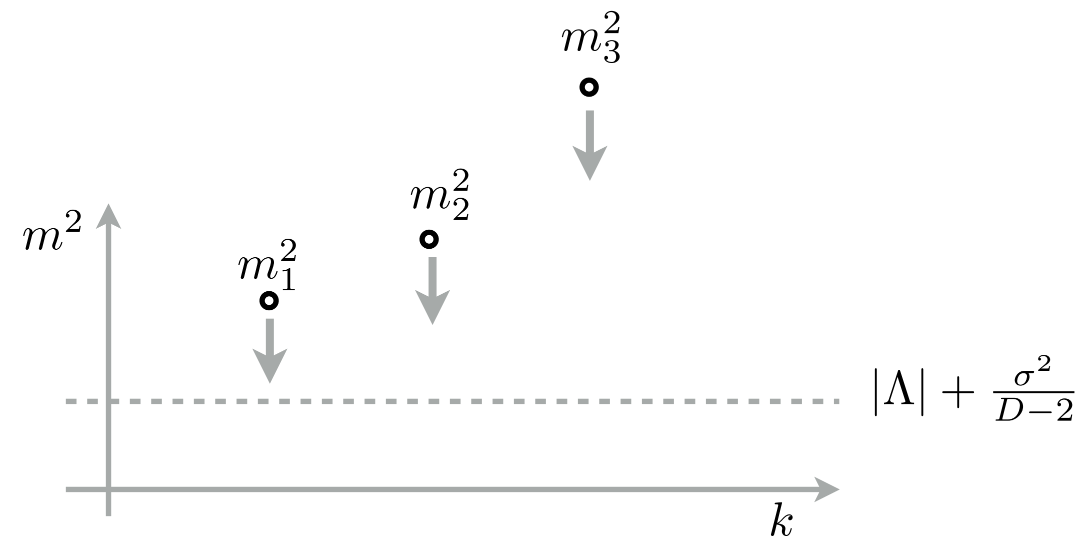

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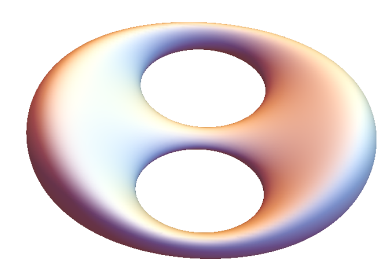
- For arbitrarily small m_1^2, counterexamples exist

Maldacena–Nuñez solutions with Σ_g with small necks

Other light spin-2 fields?

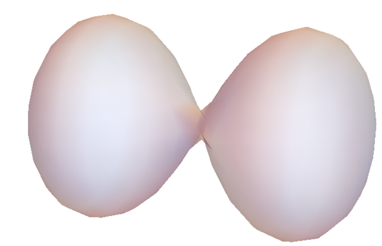


Conclusions

- Optimal transport in curved space depends on a 'weighted Ricci tensor'

- Einstein equations for compactifications equivalent to 'concavity' for Tsallis entropy

- Bounds on spin-2 KK masses in terms of diameter or Cheeger constant



⇨ proves spin-2 conjectures, in appropriate regime

Backup Slides

Entropy

$S(p_1, \ldots, p_n)$ such that:

1 is completely symmetric

2 $S(p, 1-p)$ continuous in p

3 $S(tp_1, (1-t)p_2, p_2, \ldots, p_n)$
$= S(p_1, \ldots, p_n) + p_1 S(t, 1-t)$

[implies extensivity]

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[up to constants]

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If we replace 3 with

[Suyari '95, Furuichi '05]

3′ $S(tp_1, (1-t)p_2, p_2, \ldots, p_n) =$
$S(p_1, \ldots, p_n) + p_1^{\alpha} S(t, 1-t)$

$\Rightarrow$ $S = \frac{1}{\alpha - 1}\left(1 - \sum_i p_i^{\alpha}\right)$

Tsallis entropy
[up to constants]

Are string theory singularities RCD?

[De Luca, De Ponti, Mondino, AT '21]

$$\mathrm{d}s^2 = \mathrm{e}^{2A}(\mathrm{d}s_d^2 + \underbrace{\mathrm{d}s_n^2}_{\mathrm{d}x^2_{p+1-d} + H(\mathrm{d}r^2 + r^2 \mathrm{d}s^2_{\mathbb{S}^{8-p}})})$$

[with usual caveats about supergravity singularities]

$$\sigma \geqslant (D-2)|\mathrm{d}A|$$

'sup of the warping'

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[De Luca, De Ponti, Mondino, AT '21]

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- Op-planes:

  $R^{\infty,f}_{mn} < 0$ for $p \geqslant 5$;
  $R^{2-d,f}_{mn} < 0$ for all p

  likely $\in \mathrm{RCD}(-K, 2-d)$

  [De Luca, De Ponti, Mondino, AT: *WIP*]

$$\mathrm{RCD}(K, N<0) \cup \mathrm{RCD}(K, \infty) \cup \mathrm{RCD}(K, N>0)$$