Sharpening the Distance Conjecture in Diverse Dimensions

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Outline

I. A Sharpened Distance Conjecture
II. Evidence for a Sharpened Distance Conjecture
    i) Dimensional reduction
    ii) Maximal supergravity
    iii) Minimal supergravity
    iv) Revaluation of naive counterexamples
III. Implications of the Conjecture
    i) Asymptotic Scalar Field Potentials
    ii) Large-field inflation

A Sharpened Distance Conjecture

The Distance Conjecture

Massless scalar fields parametrize a “moduli space” of vacua.

At large distances in moduli space, a tower of particles becomes light exponentially quickly with increasing distance:

$$m_n(\phi) \sim n e^{-\lambda \phi}$$

Ooguri, Vafa ‘06

Sharpening the Distance Conjecture

- It has long been understood that the coefficient λ appearing in the Distance Conjecture must be order-one in Planck units
- However, the precise values that are allowed has been a subject of much discussion and debate
- We propose a novel lower bound on λ for the lightest tower in a given infinite-distance limit:

$$\boxed{\lambda \geq 1/\sqrt{d-2}}$$

- More precisely, our claim is that in a given infinite distance limit, there is always at least one tower which satisfies this bound. (There may be other towers which do not)
- Our arguments also suggest an upper bound of the form $\lambda \leq \sqrt{(d-1)/(d-2)}$, though this talk will focus on the lower bound

Etheredge, Heidenreich, Kaya, Qiu, TR '22

Aside: The Emergent String Conjecture

- The Emergent String Conjecture will play an important role in the remainder of this talk
- This conjecture holds that any infinite-distance limit in moduli space is either a decompactification limit (accompanied by a tower of Kaluza-Klein modes) or an emergent string limit (accompanied by a tower of string oscillator modes)
- Neither our “sharpened" Distance Conjecture nor the Emergent String Conjecture implies the other, but together they offer a simple, coherent picture of infinite-distance limits in moduli space

Lee, Lerche, Weigand ‘19

Evidence for a Sharpened Distance Conjecture

Evidence for the Conjecture

- In this talk, I will sketch four lines of evidence in favor of this conjectured bound:
  i) Dimensional reduction
  ii) Top-down evidence from string/M-theory
  iii) Bottom-up evidence from minimal supergravity
  iv) Revaluation of naive counterexamples in 4d

Evidence for the Conjecture

- In this talk, I will sketch four lines of evidence in favor of this conjectured bound:
  i) **Dimensional reduction**
  ii) Top-down evidence from string/M-theory
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Dimensional Reduction

- Many of the most well-supported Swampland conjectures are exactly preserved under dimensional reduction (cf. Heidenreich, Reece, TR '15, TR '21)
- E.g. Weak Gravity Conjecture:

$$e_{P;D}^2 q^2 \geq \gamma_{P;D} T_P^2$$

$$\gamma_{P;D} = \gamma_{p;d} = \gamma_{P,d}\,, \quad p = P-1\,,\; d = D-1$$

Satisfied in D dimensions ⇔ satisfied in d dimensions!

- So, preservation under dimensional reduction is a useful tool for determining order-one factors in Swampland conjectures

Dim. Reduction of the Distance Conjecture

- Start with tower in D dimensions:

$$m \sim \exp(-\lambda_D \phi_D)$$

- After reduction to d=D-1 dimensions, find tower:

$$m \sim \exp\left(- \lambda_D \phi_d - \frac{1}{\sqrt{(d-1)(d-2)}} \rho \right)$$

$$\sim \exp\left(- \left(\lambda_D^2 + \frac{1}{(d-1)(d-2)} \right)^{1/2} \phi'_d \right)$$

$$\equiv \exp(-\lambda_d \phi'_d)$$

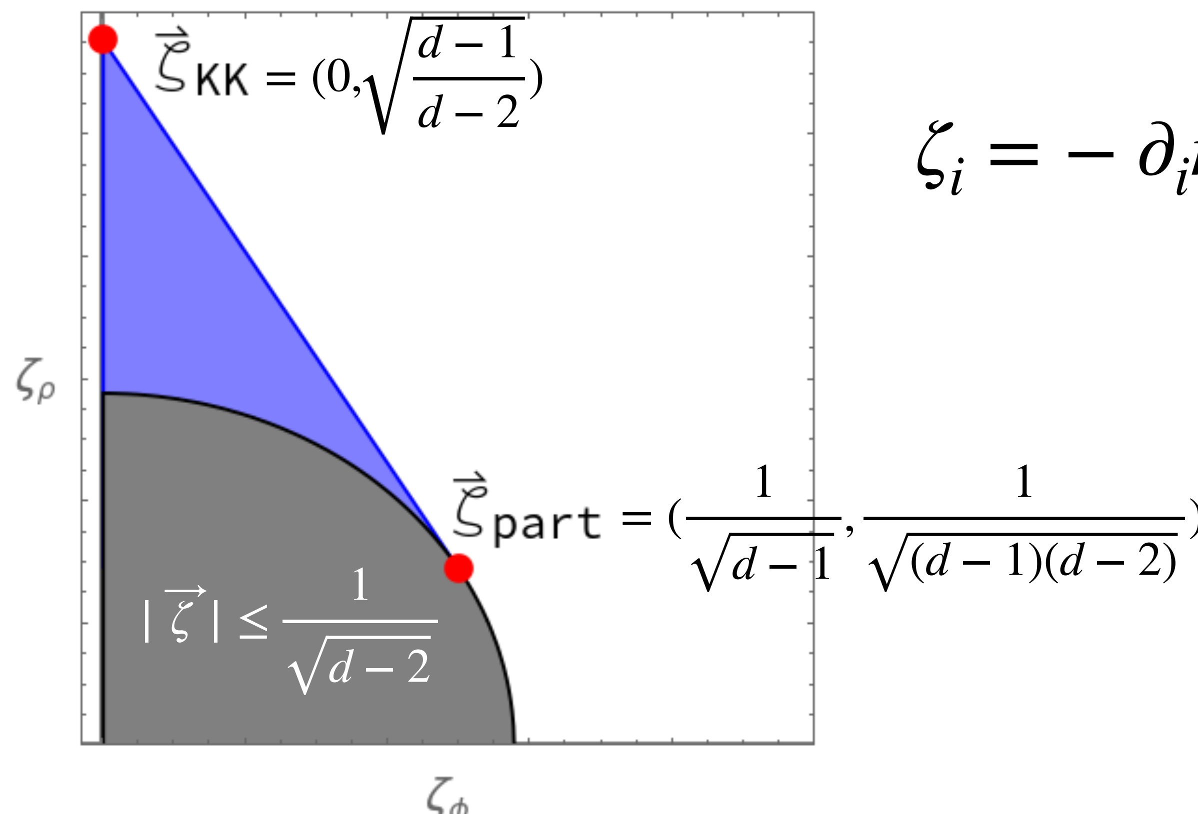
- Solved by $\lambda_D = 1/\sqrt{D-2}$, $\lambda_d = 1/\sqrt{d-2}$

$\Rightarrow \lambda_d = 1/\sqrt{d-2}$ preserved under dimensional reduction!

Dim. Reduction of the Distance Conjecture

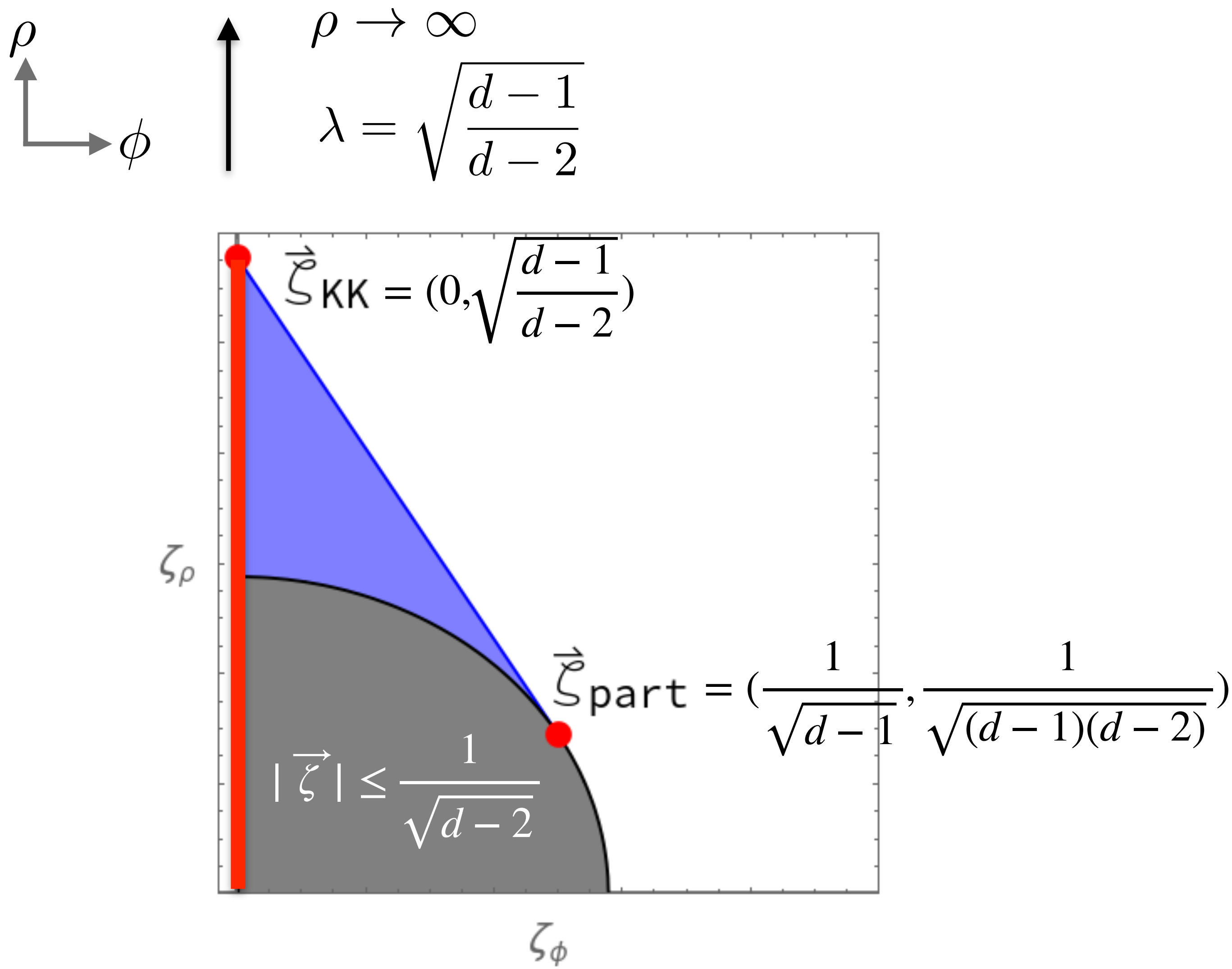
- Also have KK modes:

$$m_{\mathrm{KK}} \sim \exp(-\sqrt{(d-1)/(d-2)}\rho)$$

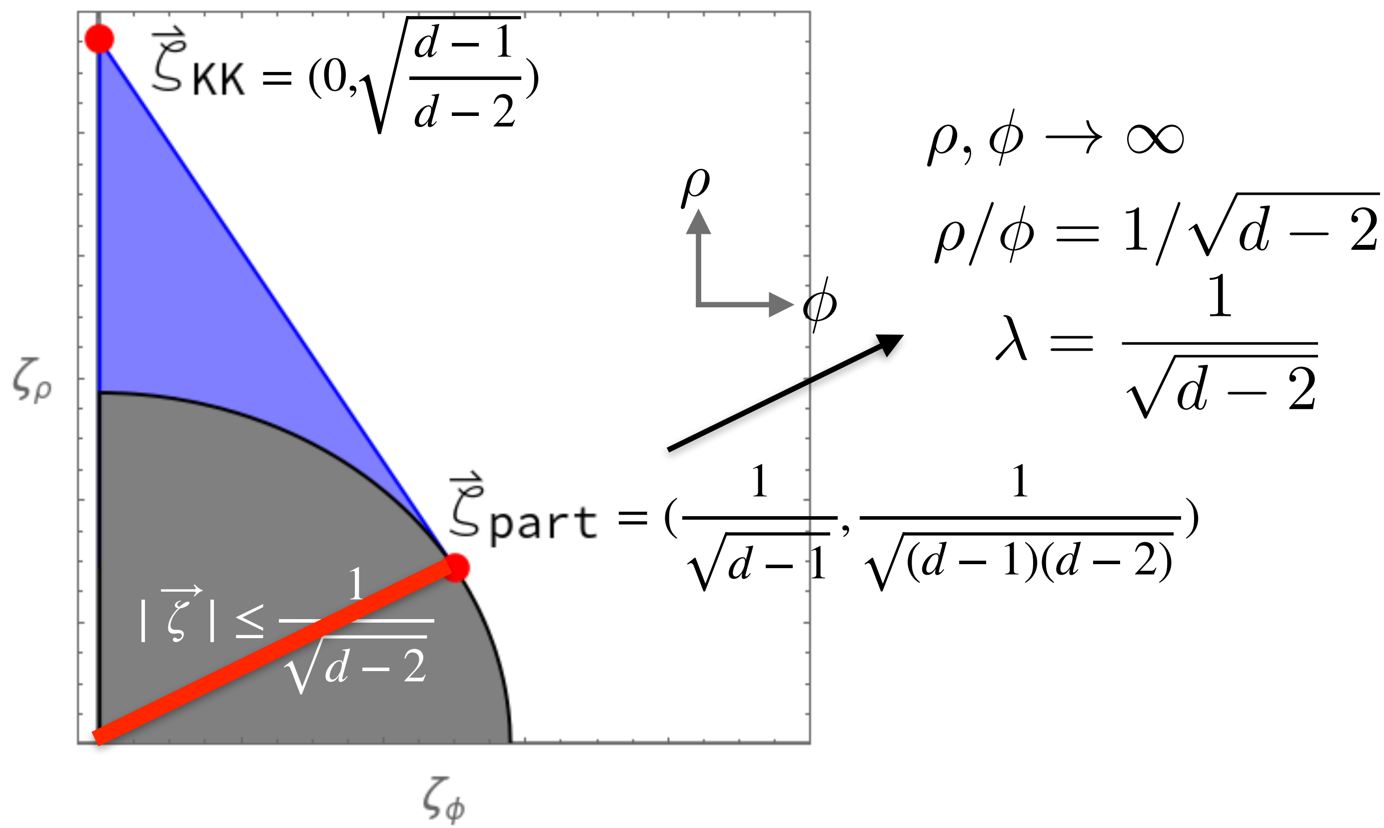


$$\zeta_i = -\partial_i m / m$$

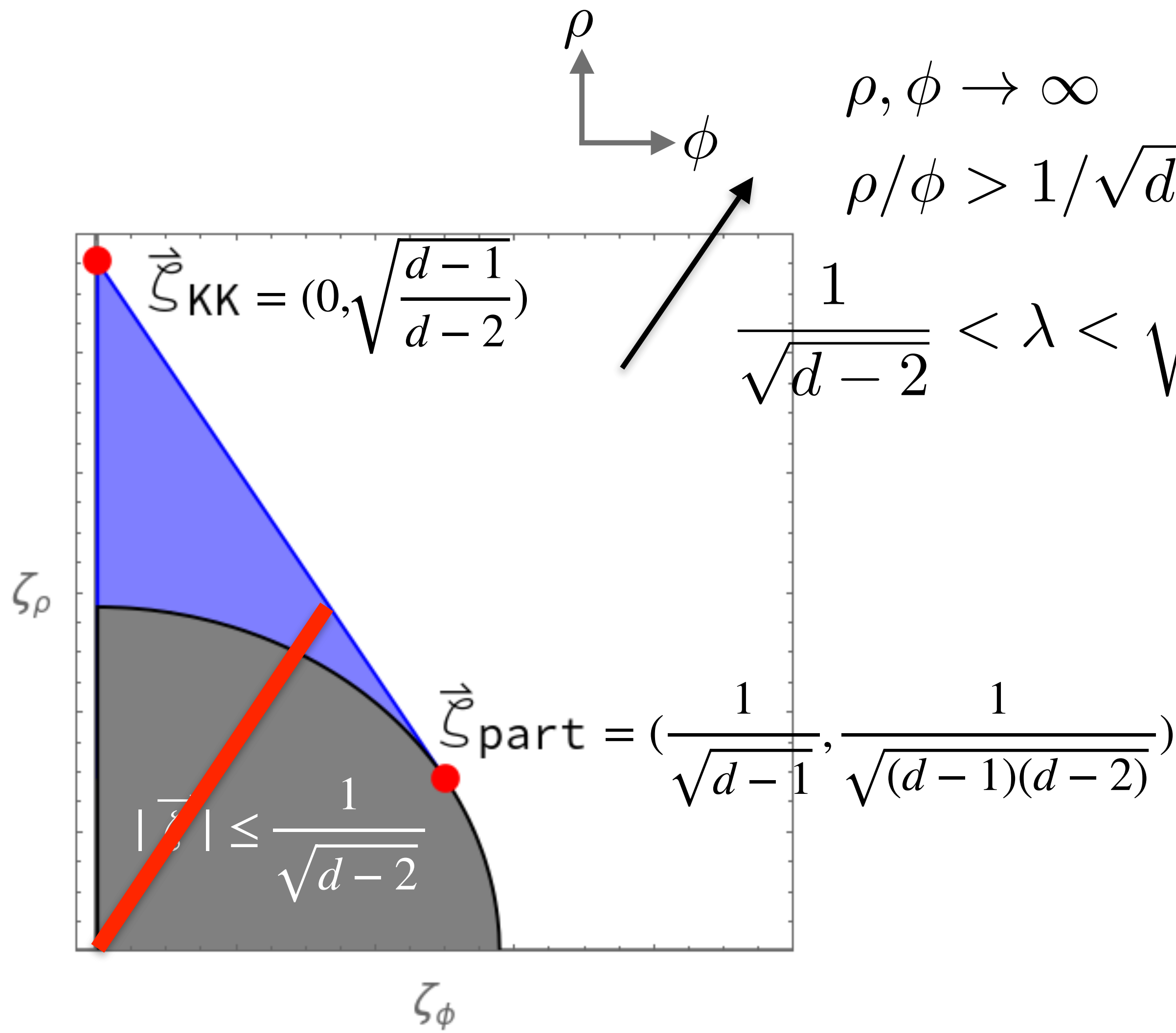
Dim. Reduction of the Distance Conjecture



Dim. Reduction of the Distance Conjecture



Dim. Reduction of the Distance Conjecture



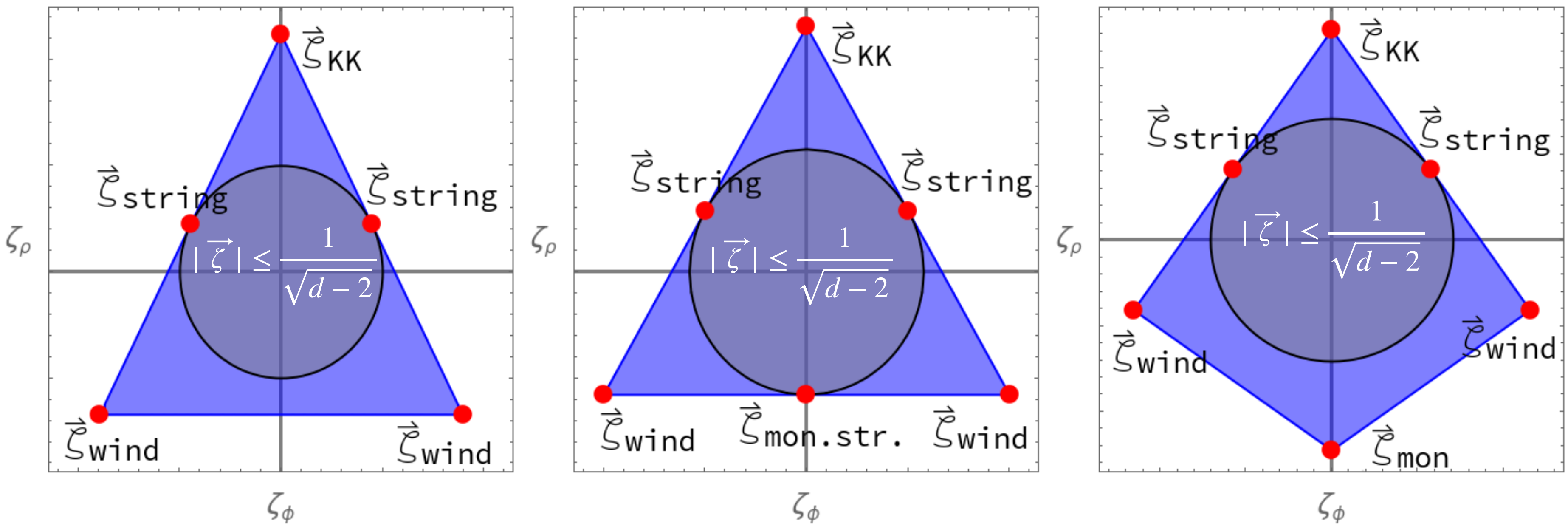
$\rho, \phi \to \infty$

$\rho/\phi > 1/\sqrt{d-2}$

$$\frac{1}{\sqrt{d-2}} < \lambda < \sqrt{\frac{d-1}{d-2}}$$

Dim. Reduction of the Distance Conjecture

- If the tower in D dimensions is a tower of string oscillator modes, then we also have winding modes (plus KK monopole strings in $d = 5$, KK monopoles in $d = 4$):



$d > 5$ $\qquad$ $d = 5$ $\qquad$ $d = 4$

Evidence for the Conjecture

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Top-Down Evidence: 10d string theory

- Weak coupling limits $\phi \to \infty$ of Type IIA, IIB, Type I, heterotic string theory:

$$T \sim \exp(-2\phi/\sqrt{8})$$

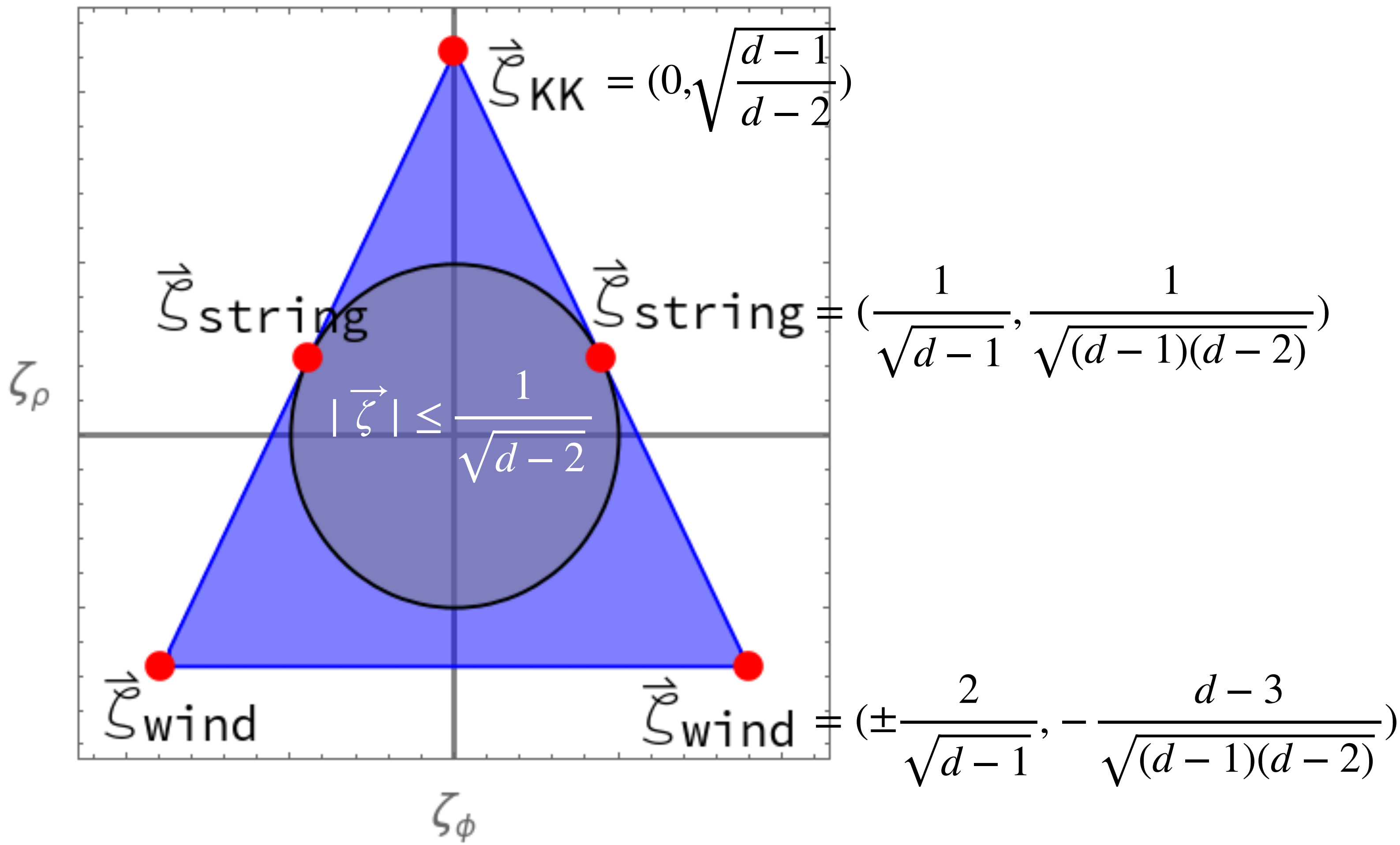
- Get string excitations:

$$m \sim M_{\mathrm{string}} \sim \sqrt{T} \sim \exp(-\phi/\sqrt{8})$$

- Saturates bound $\lambda \geq 1/\sqrt{d-2}$!
- By S-duality, find same scaling in strong coupling limits for Type IIB, Type I, SO(32) heterotic
- By duality with M-theory, strong coupling limits $\phi \to -\infty$ of Type IIA, E_8 x E_8 heterotic give KK tower:

$$m_{\mathrm{KK}} \sim \exp(-|\phi|\sqrt{9/8})$$

Top-Down Evidence: 9d string theory

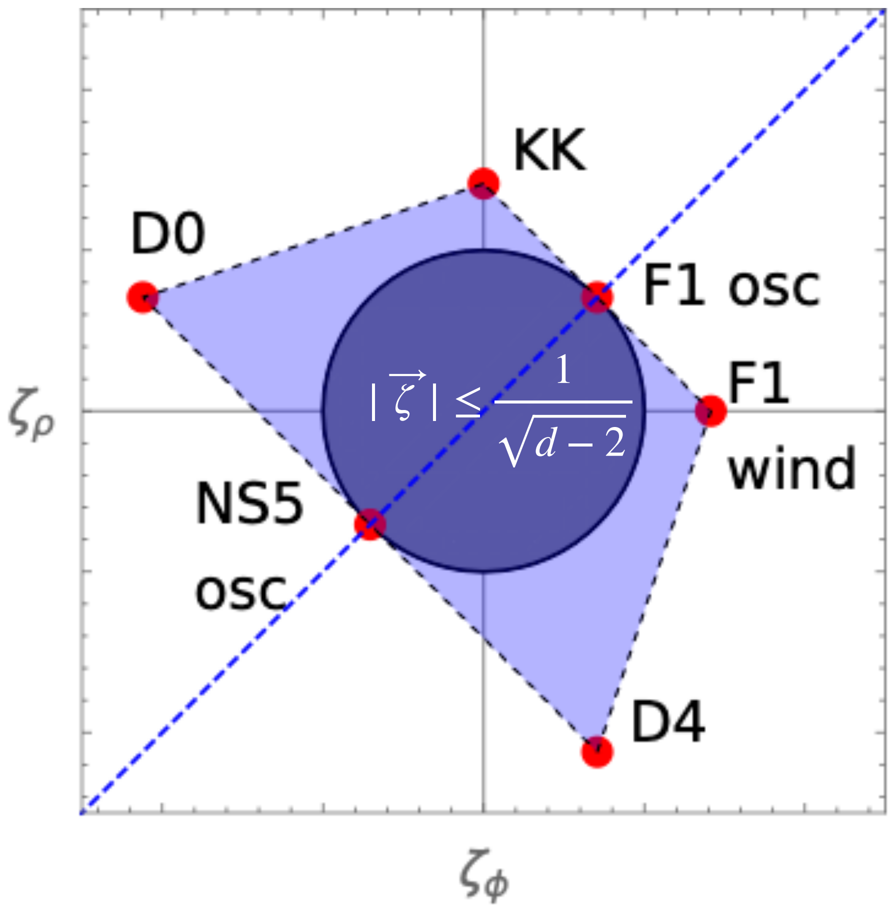


$d = 9$

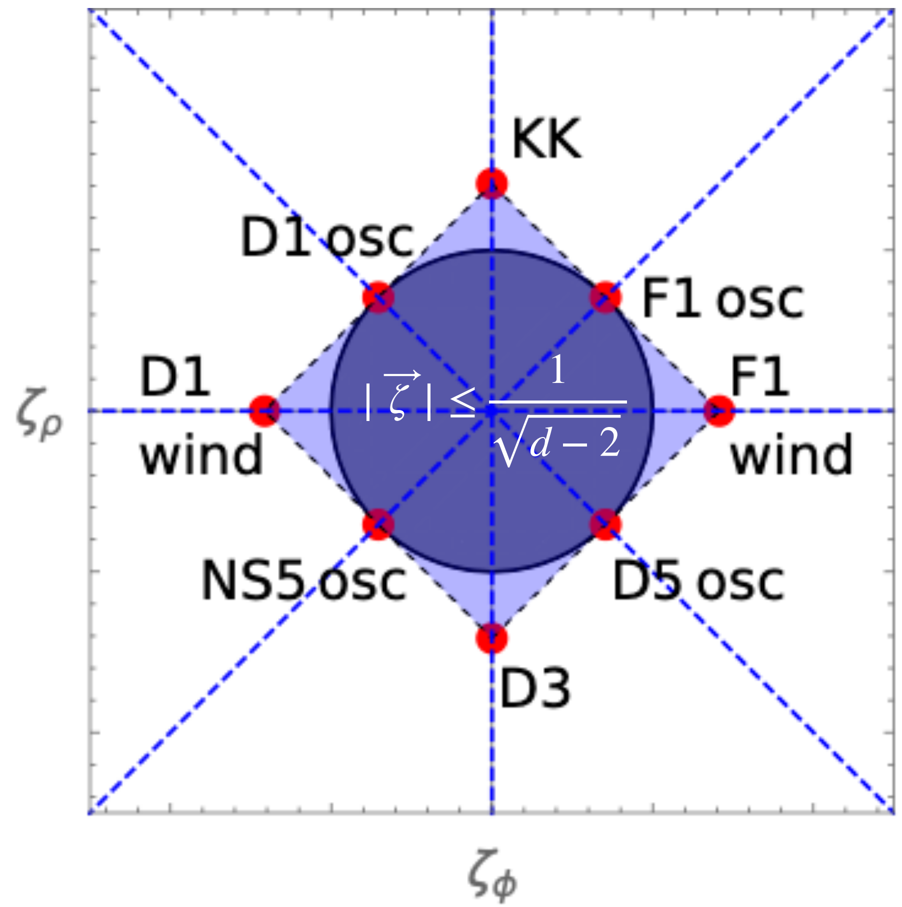
Top-Down Evidence: Maximal Supergravity in $d \geq 4$

- Through similar process, can check Distance Conjecture in all limits of moduli space for maximal supergravity in $d \geq 4$ dimensions (M-theory compactified on T^{11-d})
- Find (after significant effort!) that the bound $\lambda \geq 1/\sqrt{d-2}$ is saturated in certain limits and satisfied in all limits
- Limits that saturate the bound are always emergent string limits, which feature a tower of string oscillator modes with $\lambda = 1/\sqrt{d-2}$

Top-Down Evidence: IIA/IIB on K3



IIA



IIB

Heidenreich, TR, to appear

Evidence for the Conjecture

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Bottom-Up Evidence: Minimal Supergravity in $d = 5$

- Supergravity in 5d controlled largely by cubic prepotential:

$$\mathcal{F} = \frac{1}{6} C_{IJK} Y^I Y^J Y^K$$

- Here, Y^I are homogenous coordinates on vector multiplet moduli space, identified under simultaneous rescaling $Y^I \sim \lambda Y^I$
- Consider "straight-line" path in the space of these homogenous coordinates:

$$Y^I = Y_0^I + s Y_1^I \, , s \in [0, 1]$$

Bottom-Up Evidence: Minimal Supergravity in $d = 5$

- Assume $s = 0$ is at infinite distance $\Rightarrow$ two cases to consider:

<u>Case 1: $\mathcal{F} \sim s$</u>

$\Rightarrow$ gauge couplings scale as

$$g_{\text{min}} \sim \exp(-\frac{2}{\sqrt{3}}\rho), \quad 1/g_{\text{max}} \sim \exp(-\frac{1}{\sqrt{3}}\rho)$$

Tower WGC $\Rightarrow$

$$m_{\text{KK}} \lesssim g_{\text{min}} \sim \exp(-\frac{2}{\sqrt{3}}\rho) \sim \exp(-\sqrt{\frac{d-1}{d-2}}\rho)$$

Magnetic WGC $\Rightarrow$

$$m_{\text{string}} \sim \sqrt{T_{\text{string}}} \lesssim 1/\sqrt{g_{\text{max}}} \sim \exp(-\frac{1}{2\sqrt{3}}\rho) \sim \exp(-\frac{1}{\sqrt{(d-1)(d-2)}}\rho)$$

Expected scaling for decompactification limit!

Bottom-Up Evidence: Minimal Supergravity in $d = 5$

- Assume $s = 0$ is at infinite distance $\Rightarrow$ two cases to consider:

<u>Case 2: $\mathcal{F} \sim s^2$</u>

$\Rightarrow$ gauge couplings scale as

$$g_{\text{min}} \sim \exp(-\frac{1}{\sqrt{3}}\phi), \quad 1/g_{\text{max}} \sim \exp(-\frac{2}{\sqrt{3}}\phi)$$

Tower WGC $\Rightarrow$

$$m \lesssim g_{\text{min}} \sim \exp(-\frac{1}{\sqrt{3}}\phi) \sim \exp(-\frac{1}{\sqrt{d-2}}\phi)$$

Magnetic WGC $\Rightarrow$

$$M_{\text{string}} \sim \sqrt{T_{\text{string}}} \lesssim 1/\sqrt{g_{\text{max}}} \sim \exp(-\frac{1}{\sqrt{3}}\phi) \sim \exp(-\frac{1}{\sqrt{d-2}}\phi)$$

Expected scaling for emergent string limit!

Bottom-Up Evidence: Minimal Supergravity in $d > 5$

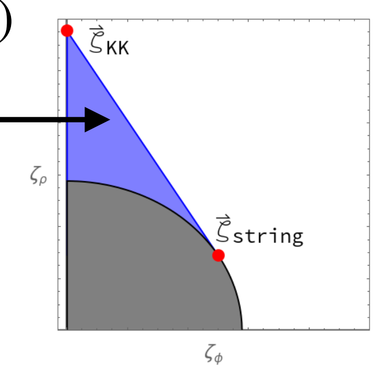
- Similar results apply to tensor multiplet moduli space in $d = 6$, moduli space in $d > 7$
- In all cases, find (assuming tower/string WGC) that infinite distance are characterized by either:
  - Charged tensionless strings with

$$M_{\text{string}} \sim \sqrt{T_{\text{string}}} \lesssim \exp\left(-\frac{1}{\sqrt{d-2}}\phi\right)$$

  - Towers of charged particles and charged strings

$$m_{\text{KK}} \lesssim \exp\left(-\sqrt{\frac{d-1}{d-2}}\rho\right), \quad M_{\text{string}} \lesssim \exp\left(-\frac{1}{\sqrt{(d-1)(d-2)}}\rho\right)$$

  - Some intermediate regime between the two



- Fits perfectly with the sharpened Distance Conjecture and the Emergent String Conjecture

Evidence for the Conjecture

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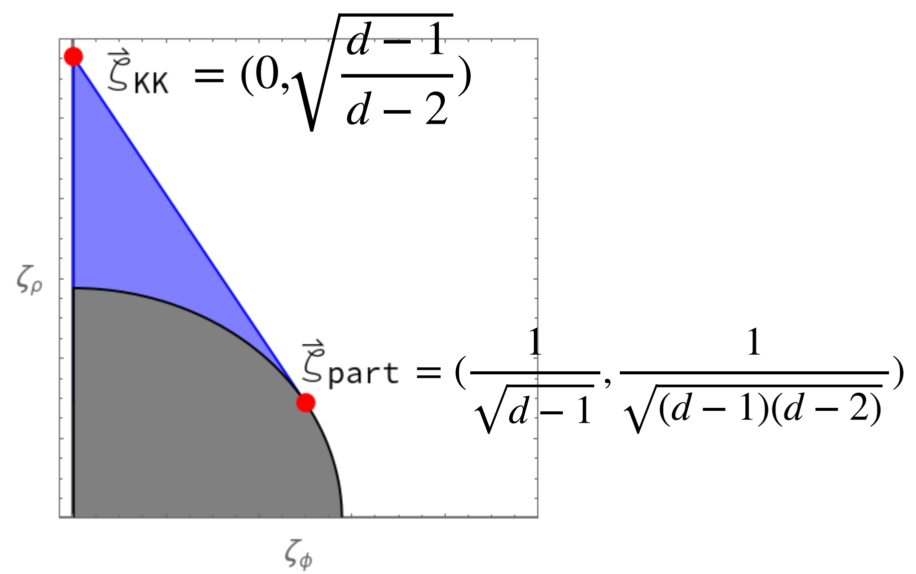
Revaluation of Naive 4d Counterexamples

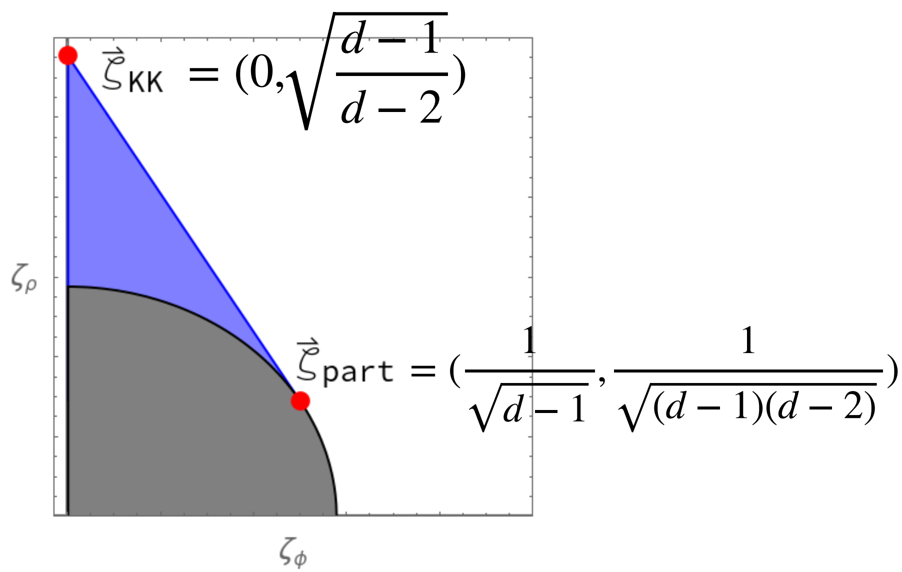
- Several previous works have found towers with $\lambda = 1/\sqrt{6}$, which naively violate our proposed bound (e.g. Grimm, Palti, Valenzuela '18, Joshi-Klemm '19, Andriot, Criori, Erkinger '20, Gendler-Valenzuela '20)
- However, the value $\lambda = 1/\sqrt{6} = 1/\sqrt{(d-1)(d-2)}$ is very familiar from our dimensional reduction analysis:

$$m \sim \exp\left(-\lambda_D \phi_d - \frac{1}{\sqrt{(d-1)(d-2)}} \rho \right)$$



Revaluation of Naive 4d Counterexamples

- In dimensional reduction, a descendent tower with $m \sim \exp(-\rho/\sqrt{(d-1)(d-2)})$ is always accompanied by a tower of Kaluza-Klein modes with $m \sim \exp(-\rho\sqrt{(d-1)/(d-2)})$
- Combined with the Emergent String Conjecture, this suggests that any tower in 4d with $\lambda = 1/\sqrt{6}$ should be accompanied by another tower with $\lambda = \sqrt{3/2}$ (satisfying our proposed bound)



A 4d Example

- Joshi, Klemm '19 studied the spectrum of particles near the "s-point" in moduli space of a Calabi-Yau threefold $X_{2,2,2,2}$
- Found mass formula in terms of integers x, y:

$$m(x,y) = m_0|y|e^{-\kappa_4\hat{\phi}/\sqrt{6}} + m_1|2iy\log(4) - x|e^{-\kappa_4\hat{\phi}\sqrt{3/2}} + \ldots$$

- $|y| = 1,2,3,\ldots \Rightarrow$ Tower of particles with $\lambda = 1/\sqrt{6}$
- $y = 0, |x| = 1,2,3\ldots \Rightarrow$ Tower of particles with $\lambda = \sqrt{3/2}$
- Exactly the scaling behavior expected for a decompactification to $D = 5$ dimensions!

Another 4d Example

- Gendler, Valenzuela ’20 considered a 4d $\mathcal{N} = 2$ theory with prepotential
$$\mathcal{F} = -\frac{(X^1)^3}{X^0}$$
- Found that this leads to gauge-kinetic matrix of the form
$$a_{IJ} = \text{diag}(e^{\sqrt{6}\kappa_4 \hat{\rho}}, 3e^{\sqrt{2/3}\kappa_4 \hat{\rho}})$$
- This implies gauge couplings which scale as
$$g_1 \sim \exp(-\kappa_4 \hat{\rho}/\sqrt{6}) \quad , \quad g_2 \sim \exp(-\kappa_4 \sqrt{3/2}\hat{\rho})$$
- Invoking the tower WGC, gives precisely the scaling behavior expected for a decompactification limit to five dimensions

Implications of the Conjecture

Asymptotic Scalar Field Potentials

- Scalar field potentials in quantum gravity seem to fall off exponentially Dine, Seiberg '85, Obied, Ooguri, Spodyneiko, Vafa '18

$$V \sim \exp(-c\phi)$$

- This suggests a bound in asymptotic regimes of scalar field space of the form

$$|\nabla V| \geq c_{\text{min}} V$$

- Like λ_{min}, the precise value of c_{min} is often debated
- We are in a position to determine this value

Asymptotic Scalar Field Potentials

- Assume that the sharpened DC applies beyond massless moduli to scalar fields with potentials Ooguri, Vafa '05, Baume, Palti '16
- Assume the bound $\lambda \geq 1/\sqrt{d-2}$ is satisfied by either KK modes or string oscillator modes (as suggested by the Emergent String Conjecture)

$$\Rightarrow M_{\mathrm{string}} \text{ or } m_{\mathrm{KK}} \lesssim \exp(-\frac{1}{\sqrt{d-2}}\phi)$$

$$V \sim \exp(-c\phi) \Rightarrow H \sim \exp(-\frac{c}{2}\phi)$$

- Thus, requiring $H \lesssim M_{\mathrm{string}}, m_{\mathrm{KK}}$ implies Hebecker, Wrase '18

$$\boxed{c \geq \frac{2}{\sqrt{d-2}}}$$

Asymptotic Scalar Field Potentials

- In asymptotic limits of scalar field space, the bound $c \geq c_{\min} = 2/\sqrt{d-2}$:
  - Is equivalent to the strong energy condition in such limits
  - Forbids accelerated expansion of the universe in such limits
  - Is exactly preserved under dimensional reduction TR '21a
  - Is satisfied in all infinite distance limits with a supersymmetric vacuum Hellerman, Kaloper, Susskind '01, TR '21b
  - Likely points to the fact that quintessence, like de Sitter space Goheer, Kleban, Susskind '03, is at best metastable

TR '21a, TR '21b, TR '22

Caveats

- Our argument for the bound $|\nabla V|/V \geq 2/\sqrt{d-2}$ deals with the *asymptotic* behavior of the potential
- The bound may be violated in principle at the expense of decompactification ($1/R = m_{\mathrm{KK}} < H$) or a stringy breakdown of EFT ($M_{\mathrm{string}} < H$) see also TR '22, Irene's talk
- If there are multiple scalar fields, the bound does not restrict the exponential falloff of the potential with respect to any given scalar field, but rather the gradient of the potential. Could have e.g. $V \sim \exp(-c_\rho \rho + c_\phi \phi)$ with $c_\rho < 2/\sqrt{d-2}$ provided $c_\rho^2 + c_\phi^2 \geq 2/(d-2)$

Large Field Inflation

- The previous argument can be specialized to four dimensions, where it may have important consequences for large-field inflation
- Assume M_{string} or $m_{\text{KK}} \lesssim \exp(-|\Delta\phi|/\sqrt{2})\, M_{\text{Pl}}$
- Plugging in $H \approx 10^{-4} < m_{\text{KK}}, M_{\text{string}}$ gives

$$|\Delta\phi| \lesssim 14 M_{\text{Pl}}$$

  - $\Rightarrow m^2\phi^2$ inflation is in the swampland
  - $\Rightarrow$ Very little room for required hierarchy of scales $H \ll m_{\text{KK}} \ll M_{\text{string}}$ in controlled model Baumann, McAllister '14
- Caveat: less clear that this argument applies to axion monodromy models Silverstein, Westphal, '08, McAllister, Silverstein, Westphal '08

Other Applications

- Proposed bound $\lambda \geq 1/\sqrt{d-2}$ has interesting consequences for
  - The Repulsive Force Conjecture Palti ’17, Heidenreich, Reece, TR ’19
  - The Scalar Weak Gravity Conjecture Palti ’17
  - Axion couplings to matter
  - The refined Distance Conjecture Baume, Palti ’16
- See Etheredge, Heidenreich, Kaya, Qiu, TR ’22, TR ’22, and our forthcoming papers for more details

Summary

Main Takeaways

- **The Distance Conjecture parameter λ of the lightest tower in an infinite-distance limit is bounded as**

$$\boxed{\lambda \geq 1/\sqrt{d-2}}$$

and saturation occurs only in emergent string limits

- Strong evidence for this bound comes from
    - Preservation under dimensional reduction
    - Examples from string theory/supergravity
    - Resolution of apparent counterexamples to the bound
    - Connection to the Emergent String Conjecture
- This bound may have important consequences for large-field inflation and scalar field potentials
    - Notably, it rules out accelerated cosmological expansion in asymptotic regions of scalar field space

Pentecostés por el Greco, Museo Nacional del Prado