The Strong CP Problem and the Swampland

Matt Reece
Harvard University
September 26, 2022

Strong CP Problem: Two Common Paradigms

CP-violating term: $\frac{\bar{\theta}}{8\pi^2}\int \mathrm{tr}(G \wedge G)$.

Absence of measurable neutron EDM to date: $|\bar{\theta}| \lesssim 10^{-10}$.

- CP (or other generalized parity) is a spontaneously broken symmetry of nature. Some field gets a complex VEV that leads to CP phases. Build a clever model where this gives rise to O(1) δ_{CKM} but very small $\bar{\theta}$. (*Not* easy.)

  (Original example: Nelson-Barr)

- CP is not a symmetry of nature. All CP phases are expected to be O(1). However, $\bar{\theta}$ is small because of a dynamical axion that couples to $\mathrm{tr}(G \wedge G)$ and relaxes to zero.

Strong CP vs The Swampland

The Strong CP problem is a great test case for the Swampland. Key problem in particle physics, possibly linked to the nature of dark matter.

Both approaches rely on *symmetries*, and we know symmetries become fraught in quantum gravity:

- Parity solutions: CP *itself* is a symmetry, but must be spontaneously broken. Is it gauged? What happens to CP domain walls?

- Axion solution: requires that the Chern-Simons term $\theta(x)\,\mathrm{tr}(G \wedge G)$ be *by far* the dominant source of breaking of the $\theta \mapsto \theta + c$ shift symmetry.

  "Axion quality problem"

Part 1: CP Domain Wall Stability

J. McNamara and MR, to appear soon

CP Domain Walls

If CP is a symmetry, then spontaneous CP violation leads to domain wall formation. Stable DWs are a cosmological disaster.

If CP is an *explicitly broken approximate symmetry*, the domain walls are unstable.

Often, model-building literature writes irrelevant, Planck-suppressed operators for explicit breaking "by quantum gravity."

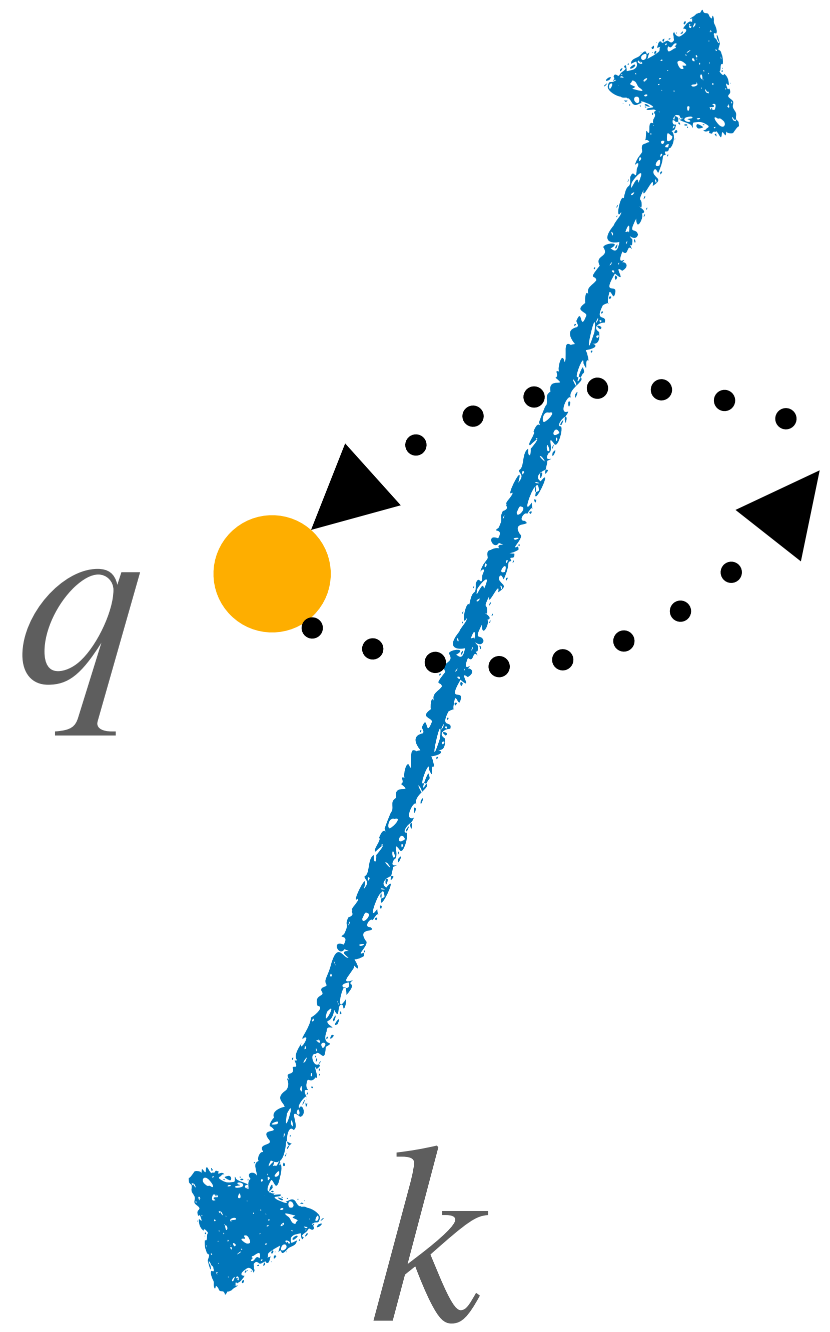
This is nonsense: if it isn't a symmetry in any sense, no reason not to write *relevant* CP-violating operators.

So consider that CP is an *exact* (gauge) symmetry of spacetime.

Internal Finite-Group Gauge Theories

Example: $\mathbb{Z}_N$ gauge theory, $\phi(x) \mapsto e^{2\pi i/N}\phi(x)$.

Lagrangian only has *N*th powers of ϕ. How to distinguish gauge from global?



Local discrete gauge symmetries come with a "***twist string***" or vortex.

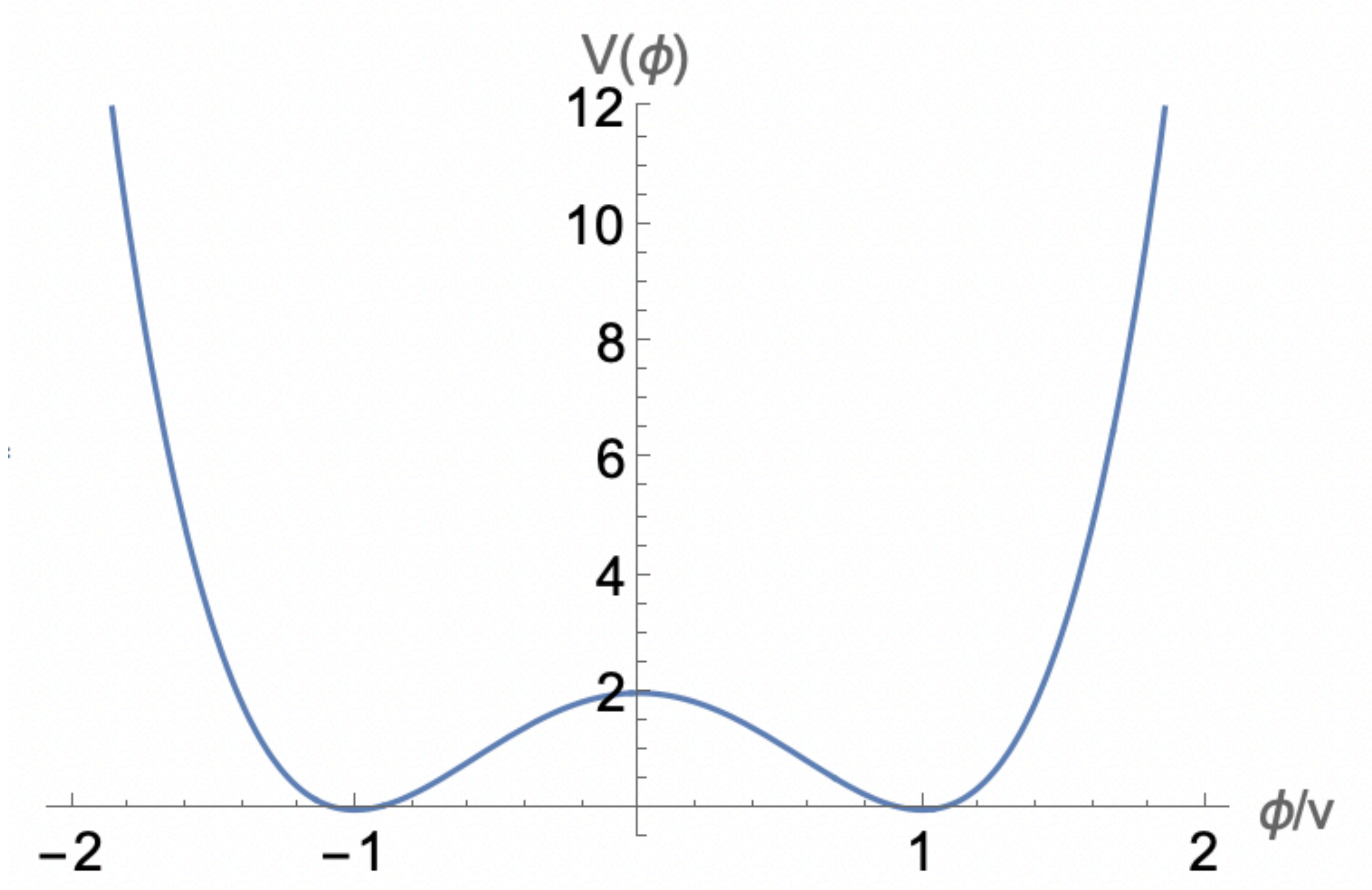
Induces an **Aharonov-Bohm phase** on charged particles that circle it.

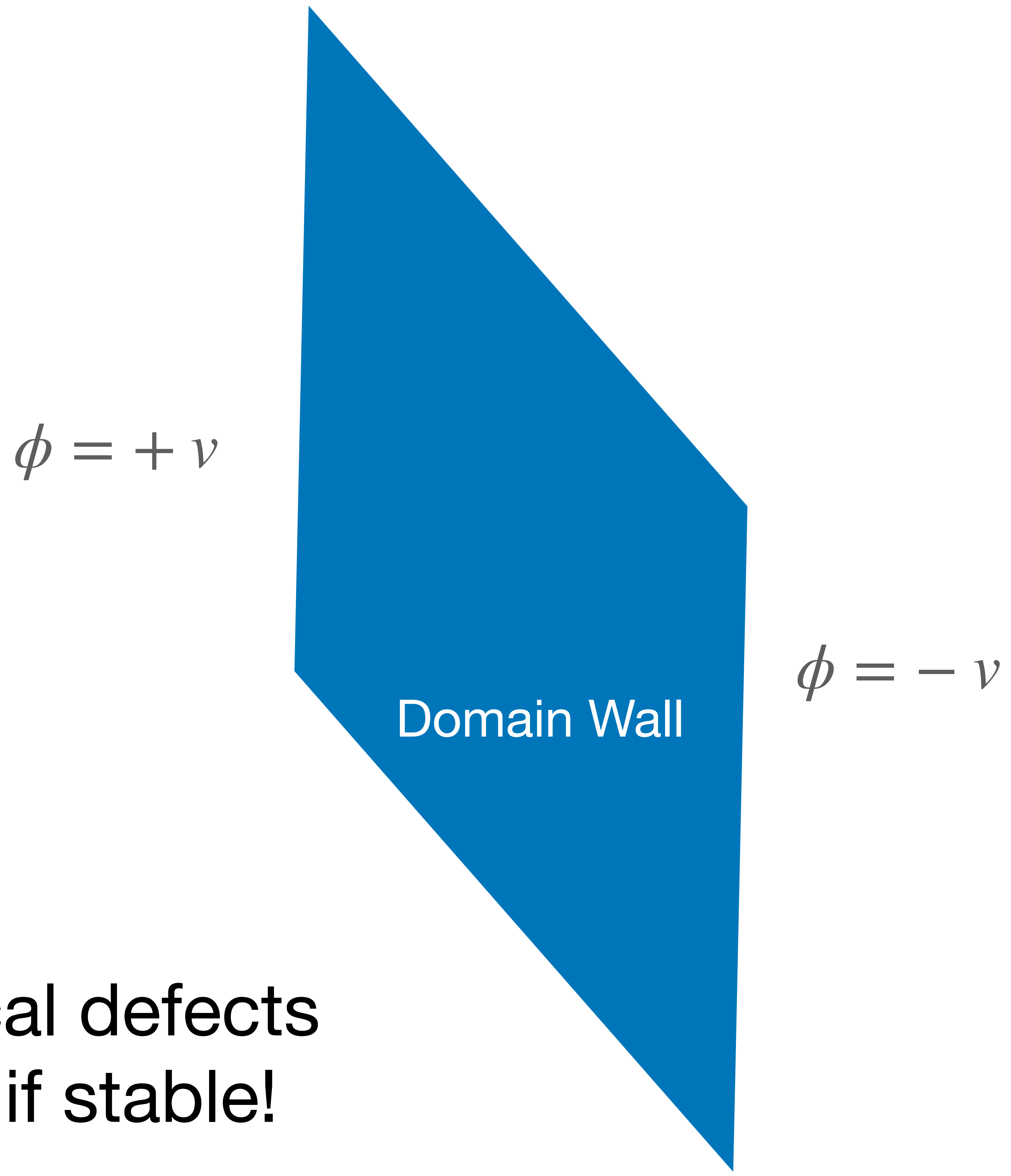
Most familiar case: Krauss-Wilczek string in $\mathbb{Z}_N$ gauge theory.

$$|\psi\rangle \to \exp(iqk/N)|\psi\rangle$$

Completeness: twist vortices must exist to eliminate global 2-form symmetry
(B. Heidenreich, J. McNamara, M. Montero, MR, T. Rudelius, I. Valenzuela 2104.07306)

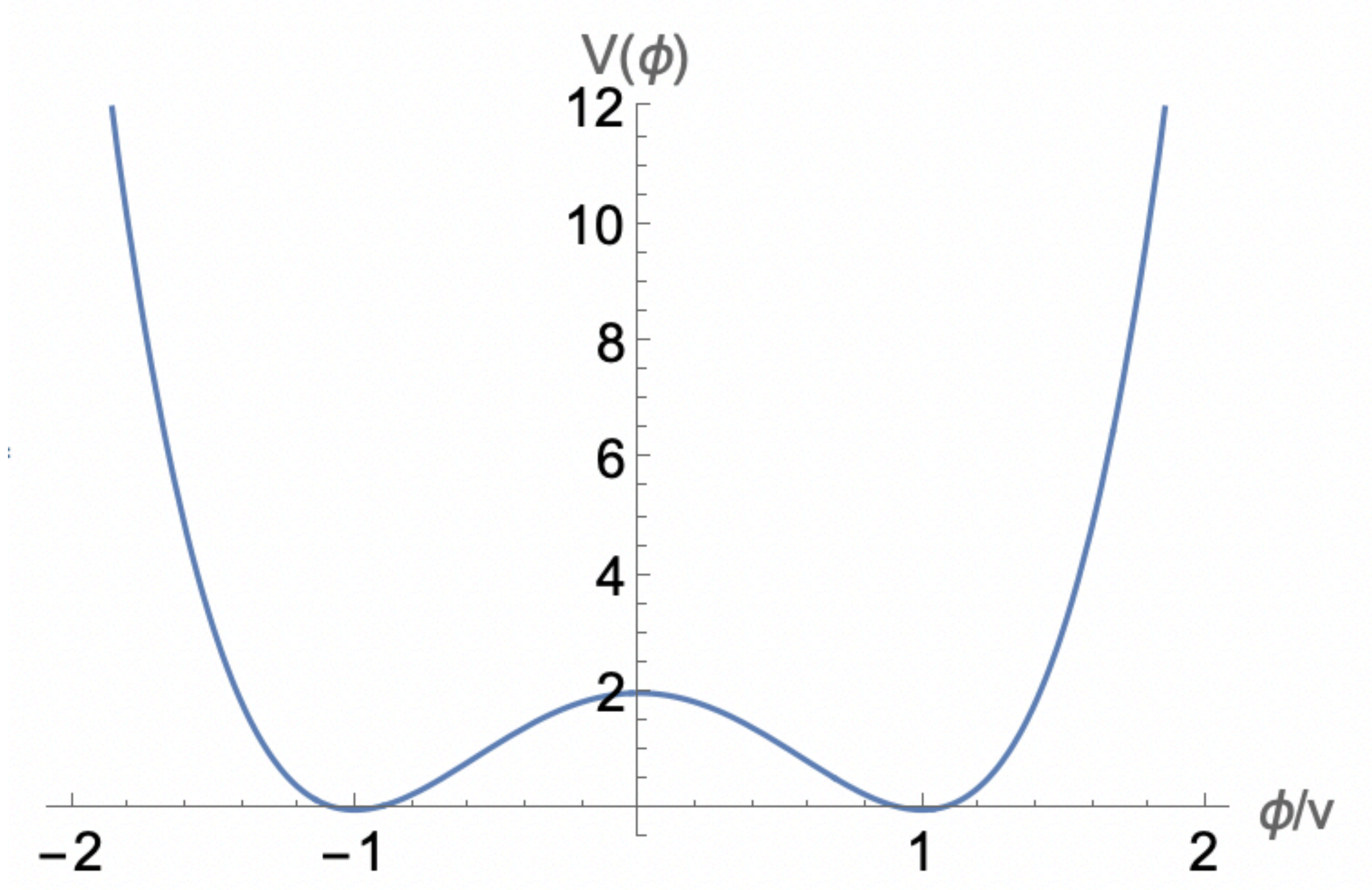
Higgsing *Internal* Finite-Group Gauge Theories

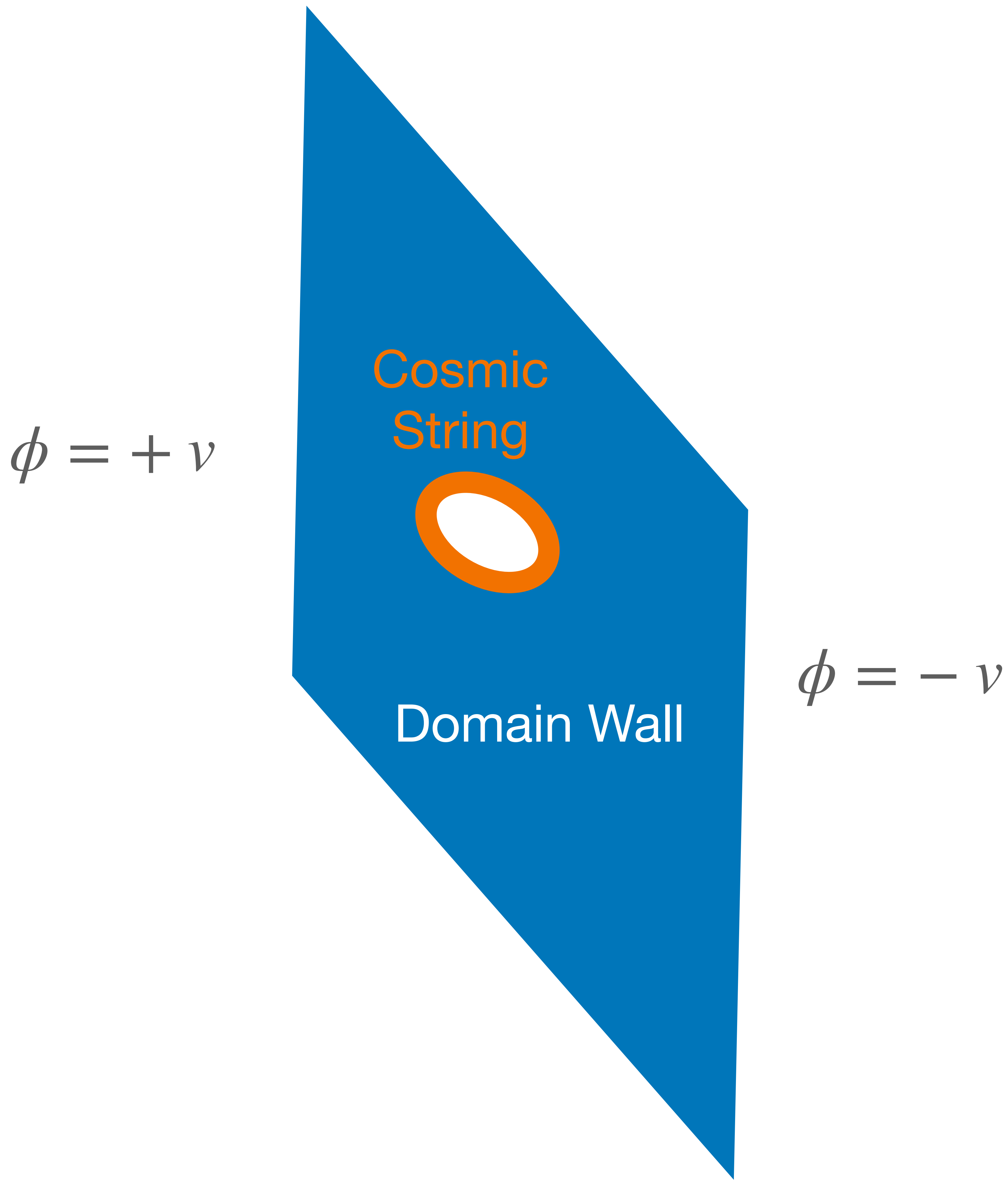




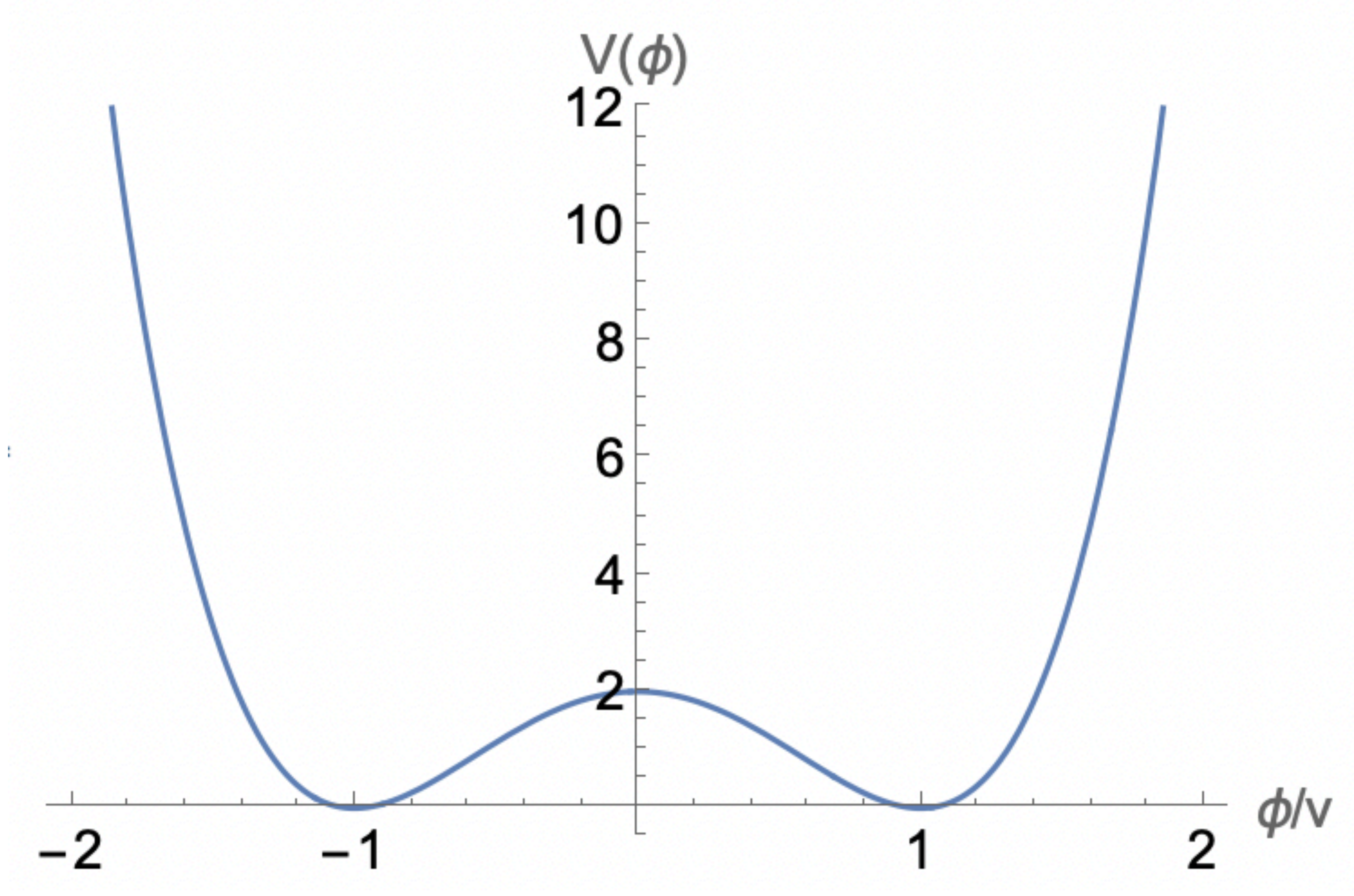
Kibble-Zurek: domain walls as topological defects in early universe. Cosmological disaster if stable!

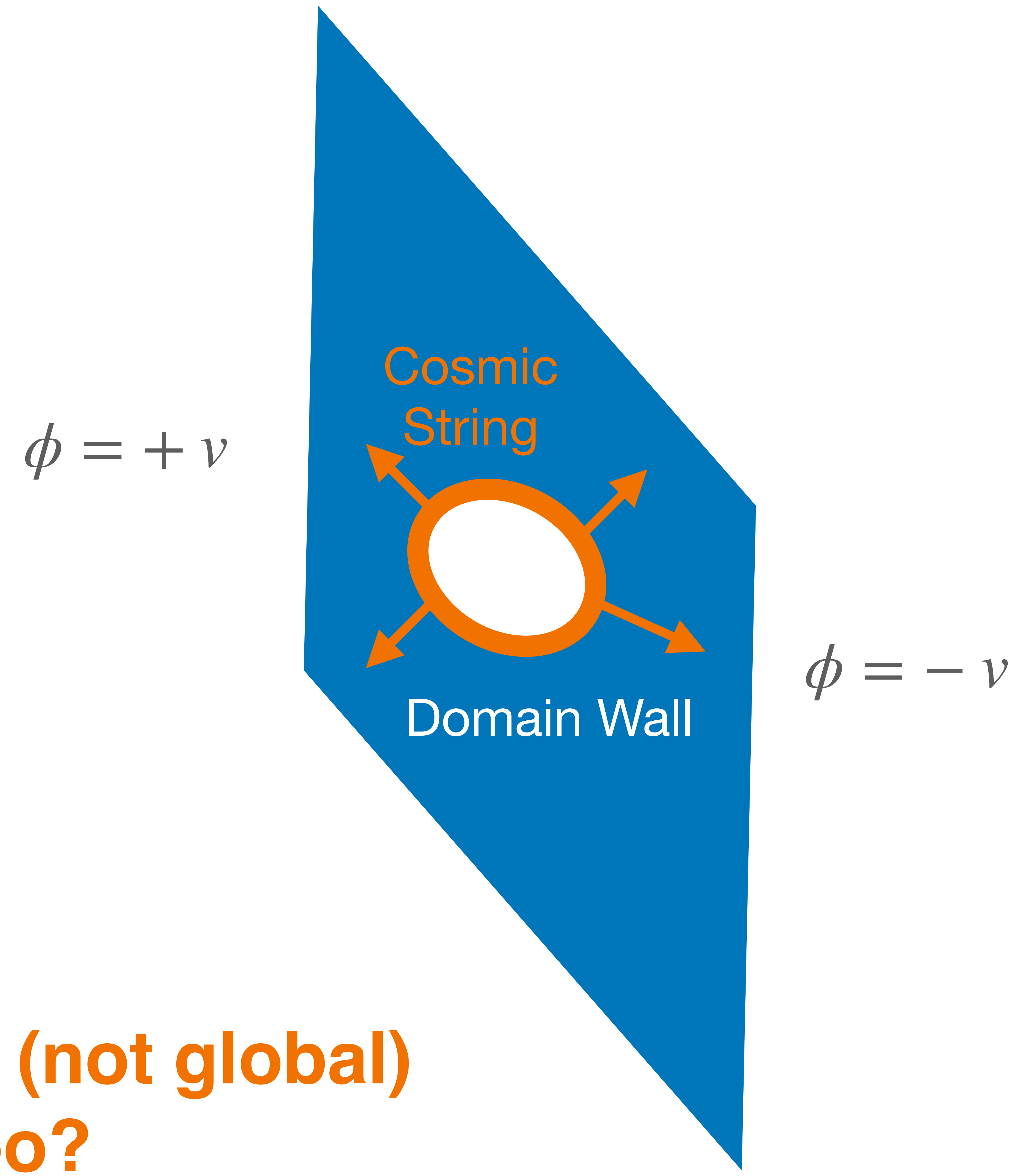
Higgsing *Internal* Finite-Group Gauge Theories





Higgsing *Internal* Finite-Group Gauge Theories





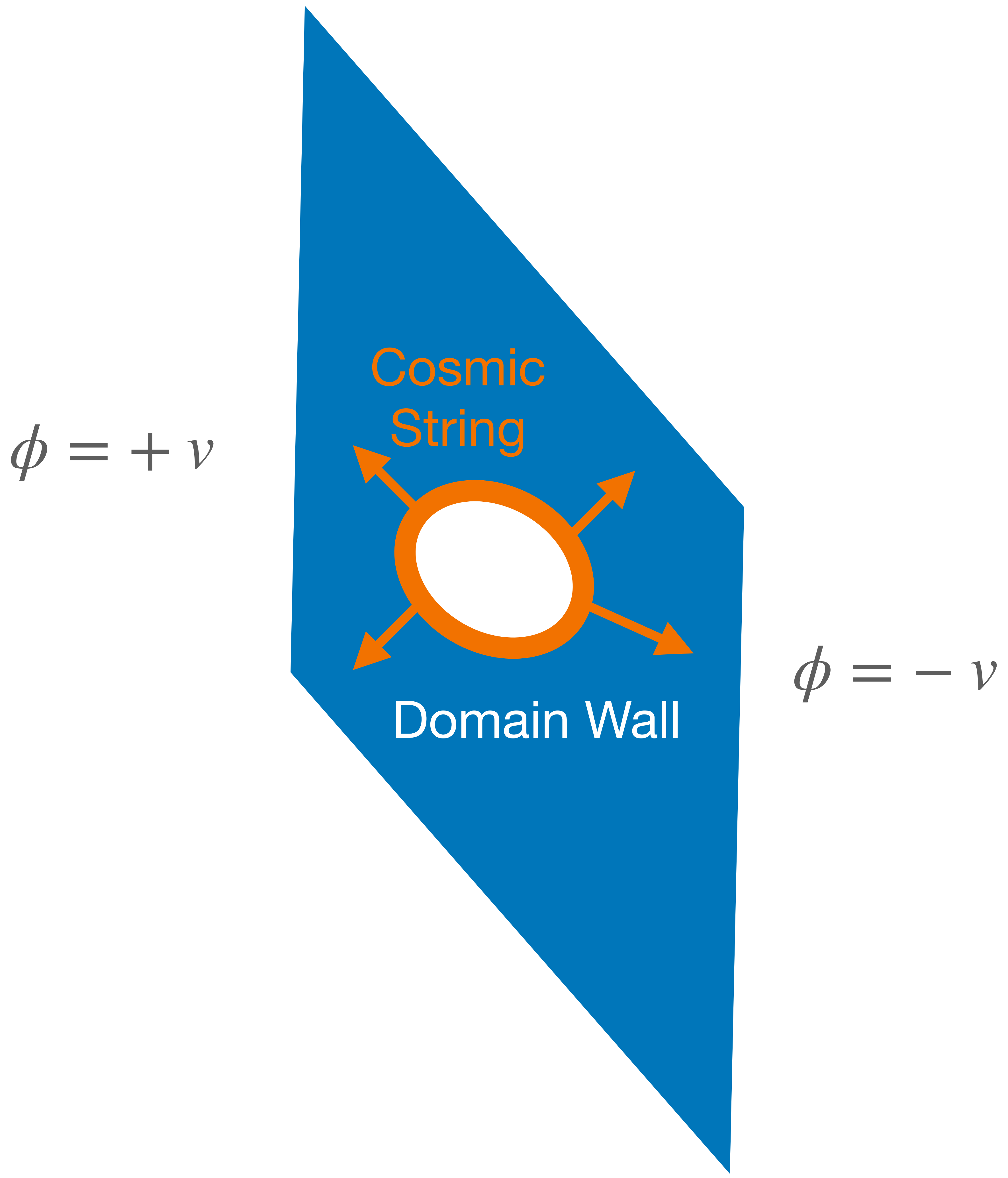
Domain walls are unstable for *gauge* (not global) symmetry! Does CP work this way too?

Higgsing *Internal* Finite-Group Gauge Theories

Our Result:

CP can *not* have such strings, so CP domain walls are exactly stable.

General mathematical fact: such strings simply do not make sense.



Parity is a Spacetime Symmetry

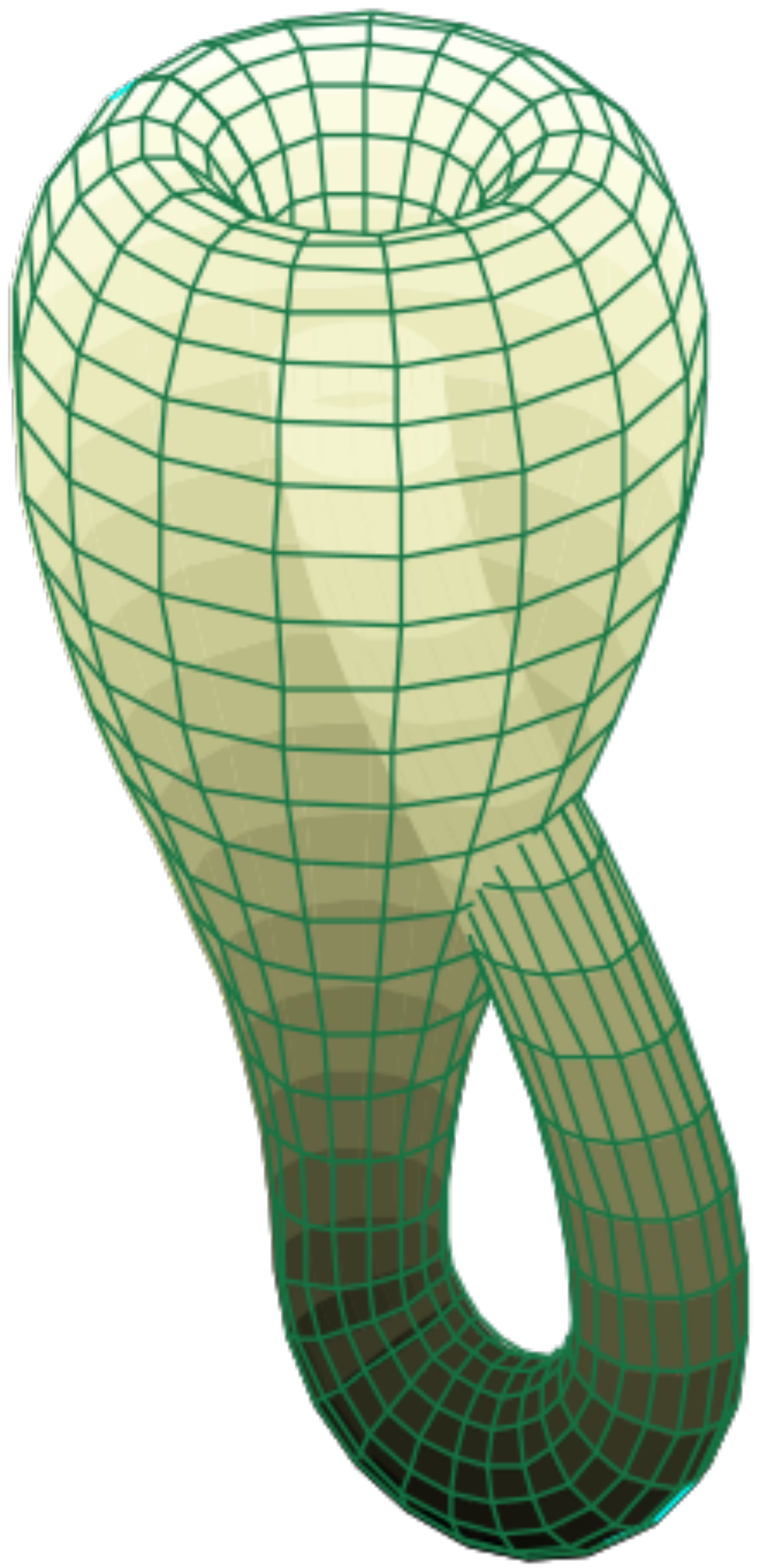
Parity/CP are orientation-reversing spacetime symmetries, $(t, \vec{x}) \mapsto (t, -\vec{x})$ in Minkowski space.

But that's nonsense in GR: what is $-\vec{x}$, anyway?

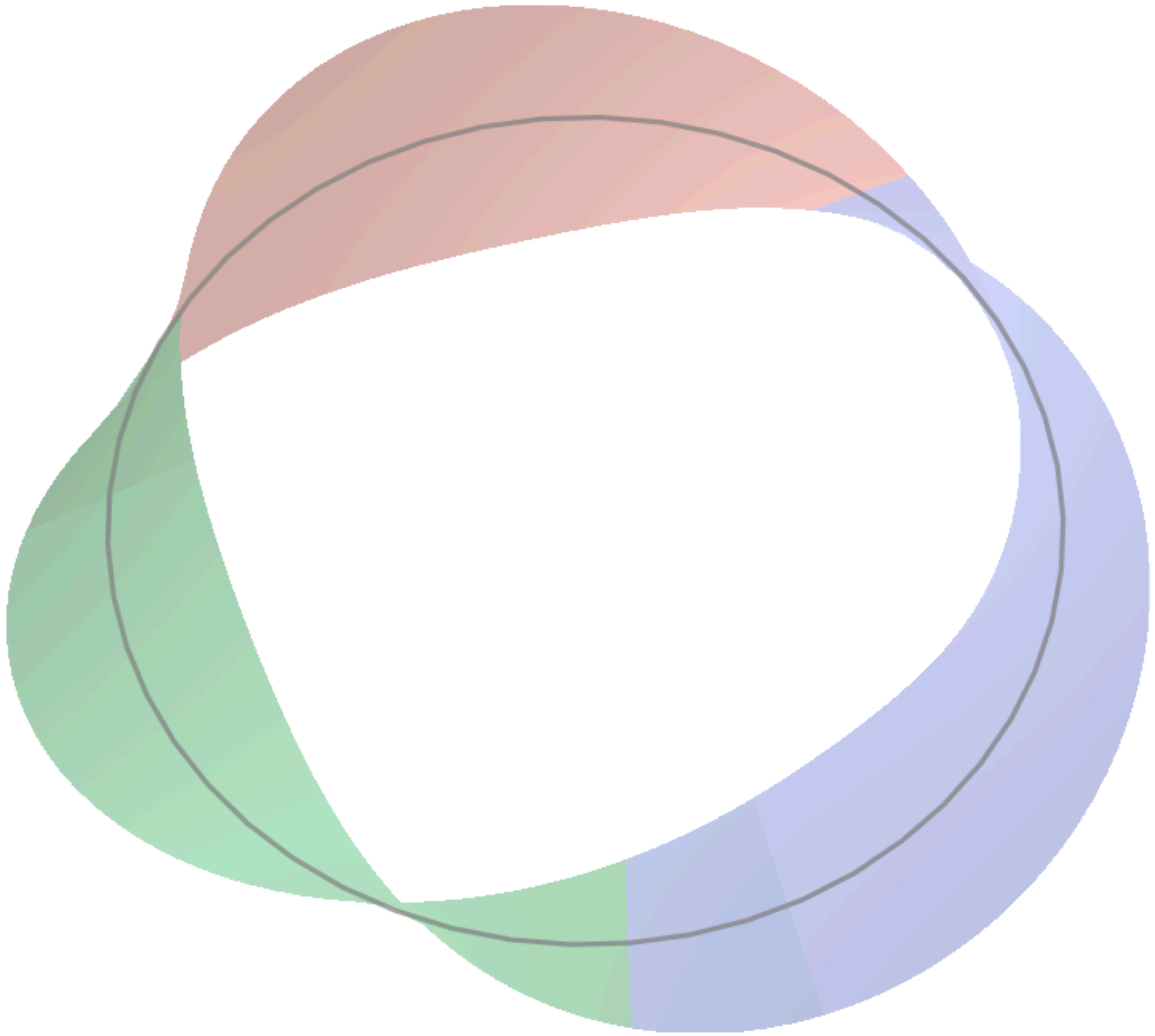
Right way to think about this (well-known):

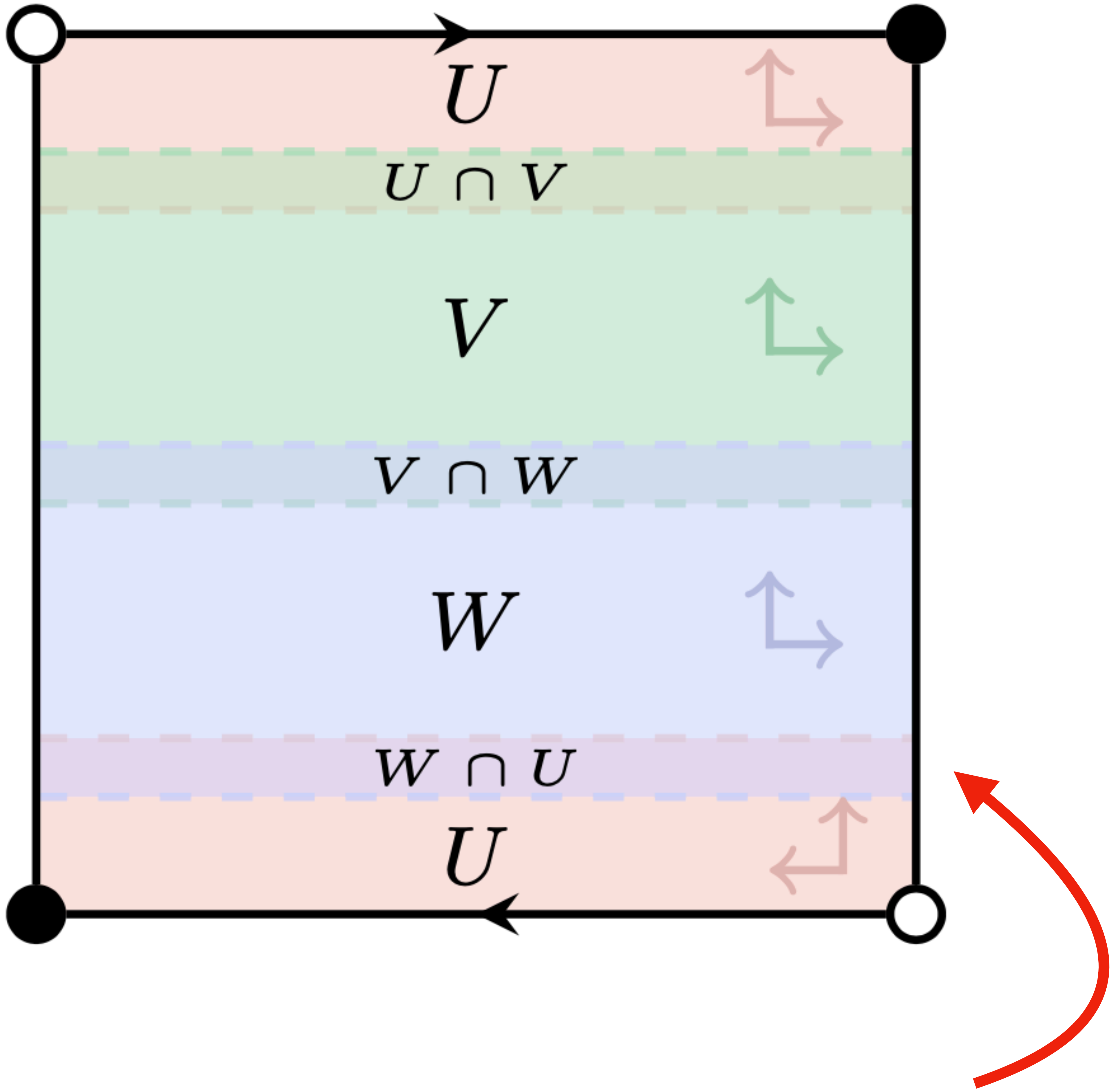
A theory with parity symmetry is a theory that makes sense when defined on non-orientable manifolds.

No one seems to have worked out what this means for phenomenology and cosmology.



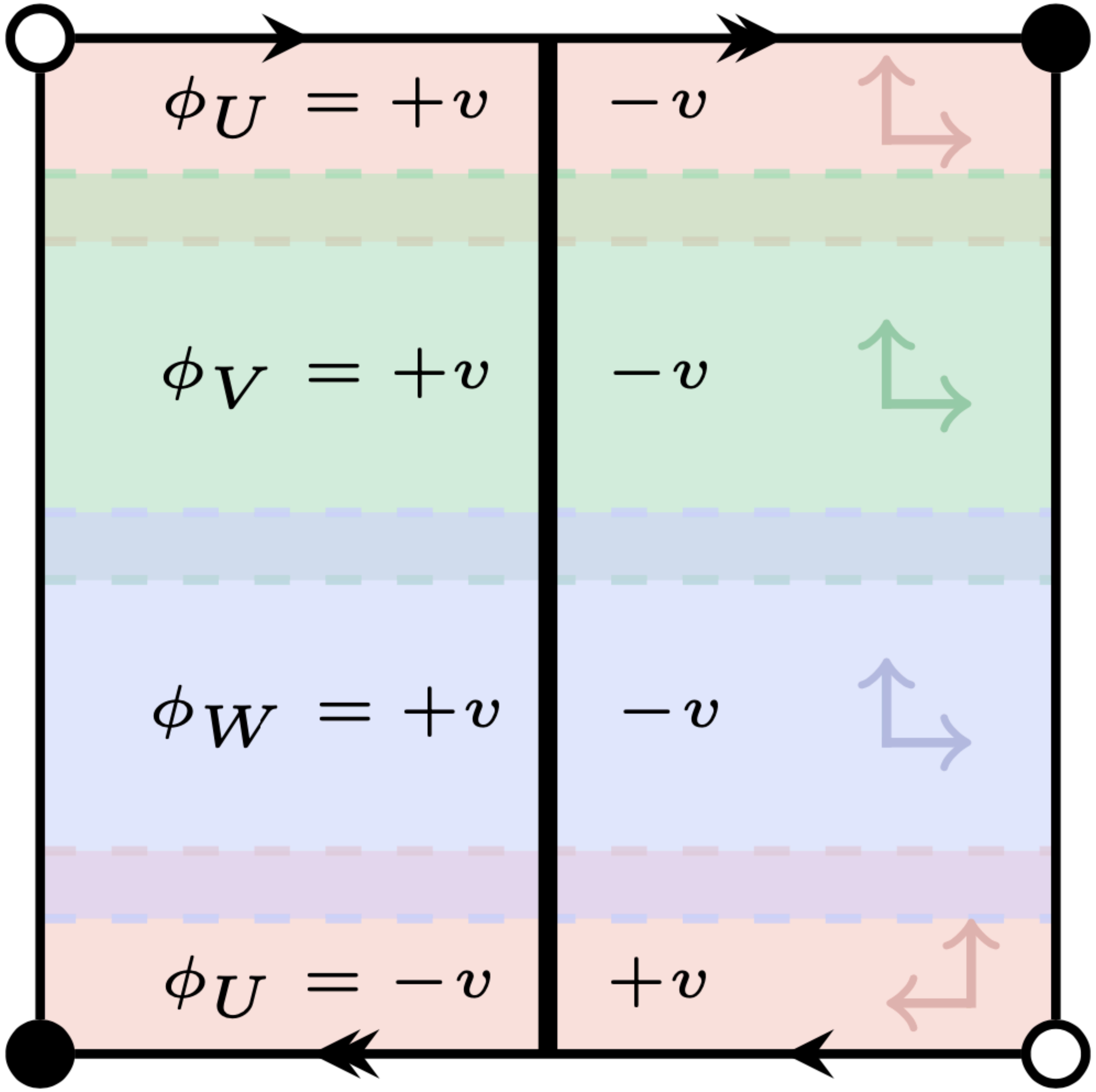
Möbius Strip: Coordinate Charts

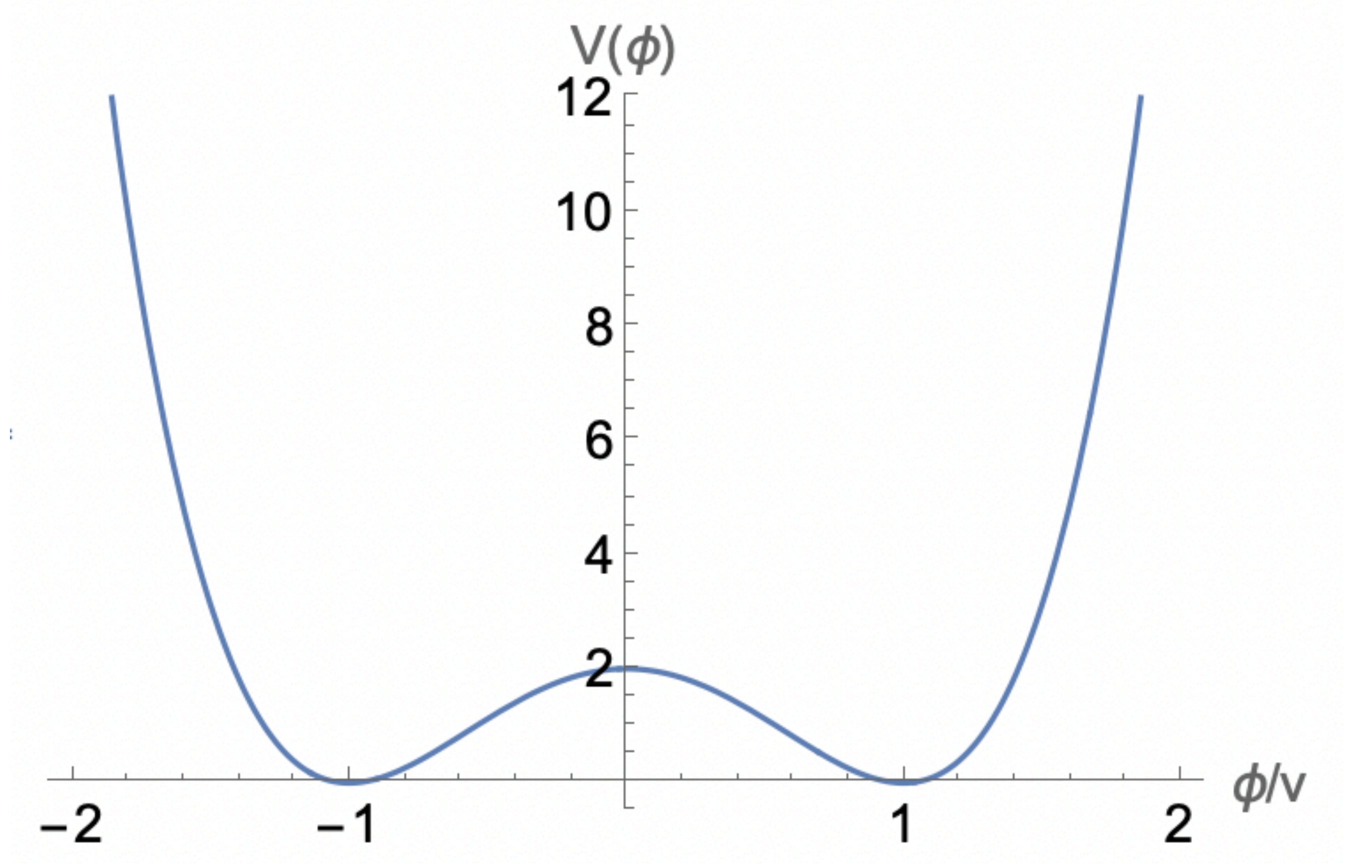




Orientation reversal!

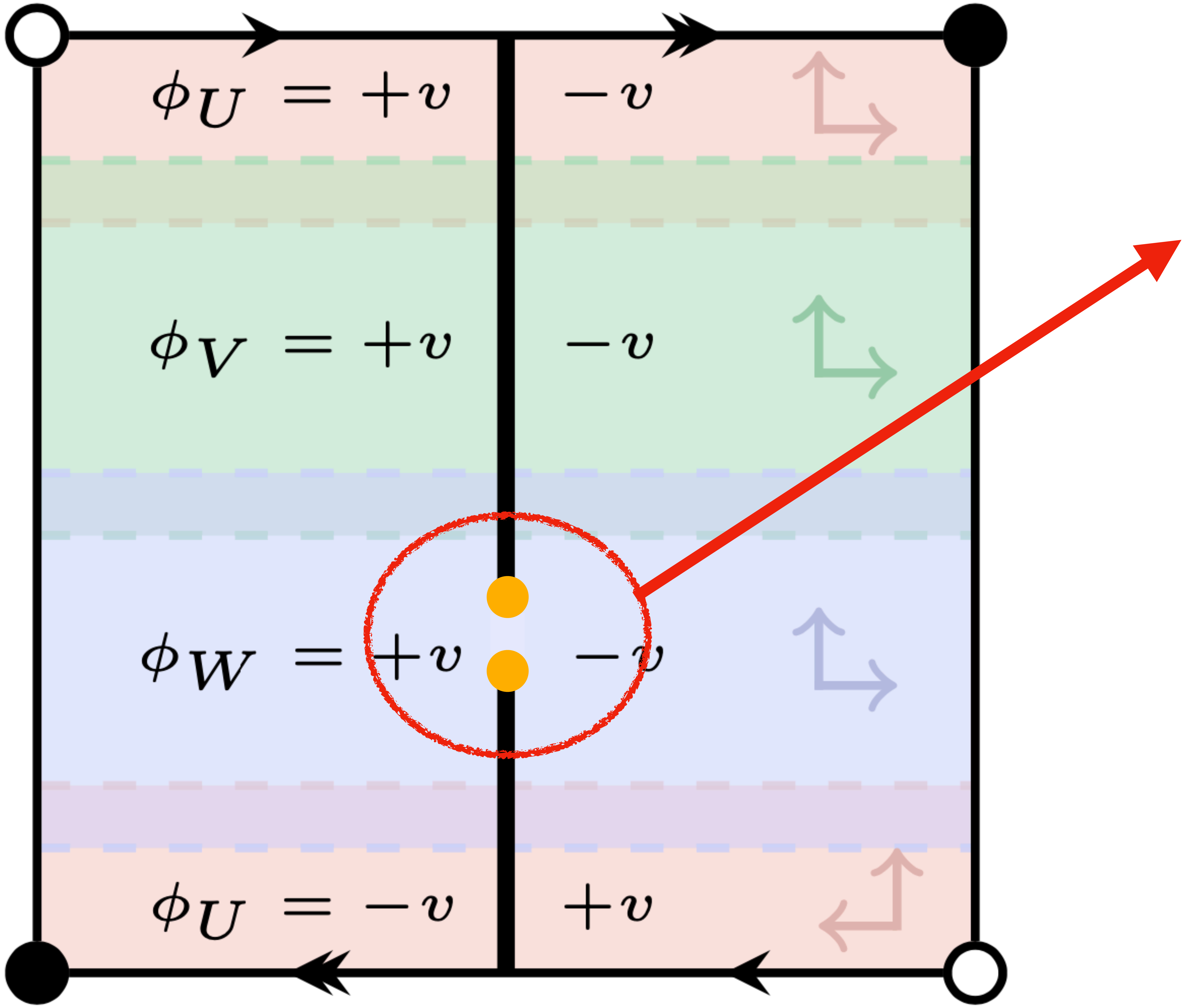
Möbius Strip: Spontaneous Parity Violation





A consistent VEV for a pseudoscalar ϕ can’t be defined everywhere — ***must*** have a parity domain wall!

Möbius Strip: Spontaneous Parity Violation



A vortex that tears a hole in the parity domain wall would lead to a well-defined orientation on the Möbius strip.

Mathematically impossible.

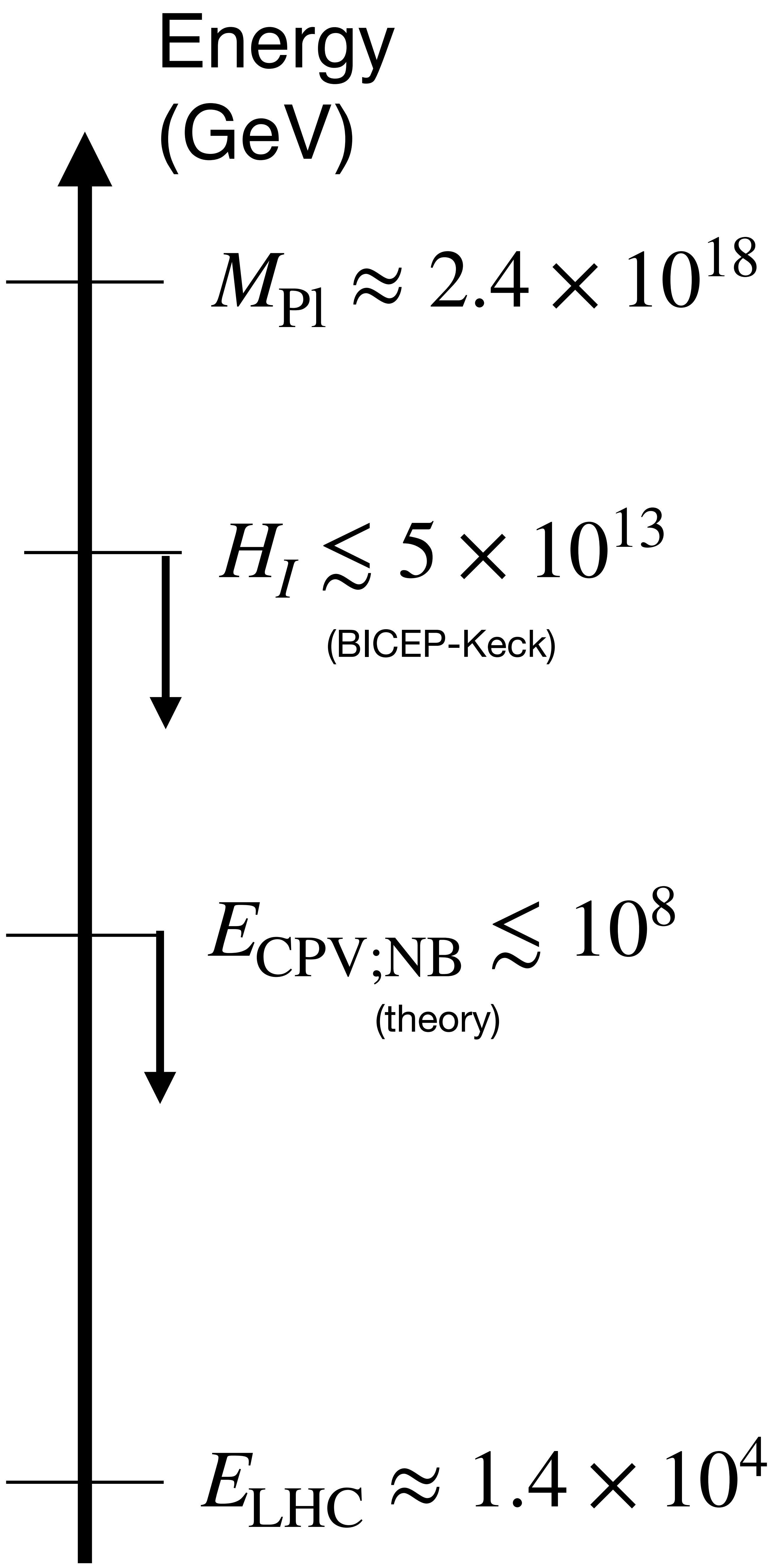
Tearing a hole in the domain wall is a *local* physical process, so conclusions we draw from the theory on non-orientable manifolds carry over to flat space.

Spontaneous CP Violation

Cosmological CP-violating phase transition produces **stable domain walls**.

These must be inflated away, so we need inflation to end **after** the phase transition: $H_I \lesssim E_{\rm CPV}$.

In many models this is an *extremely* strong constraint! Nelson-Barr wants $E_{\rm CPV} \lesssim 10^8\,{\rm GeV}$. (Dine-Draper ’15)



Sequestered CP Violation

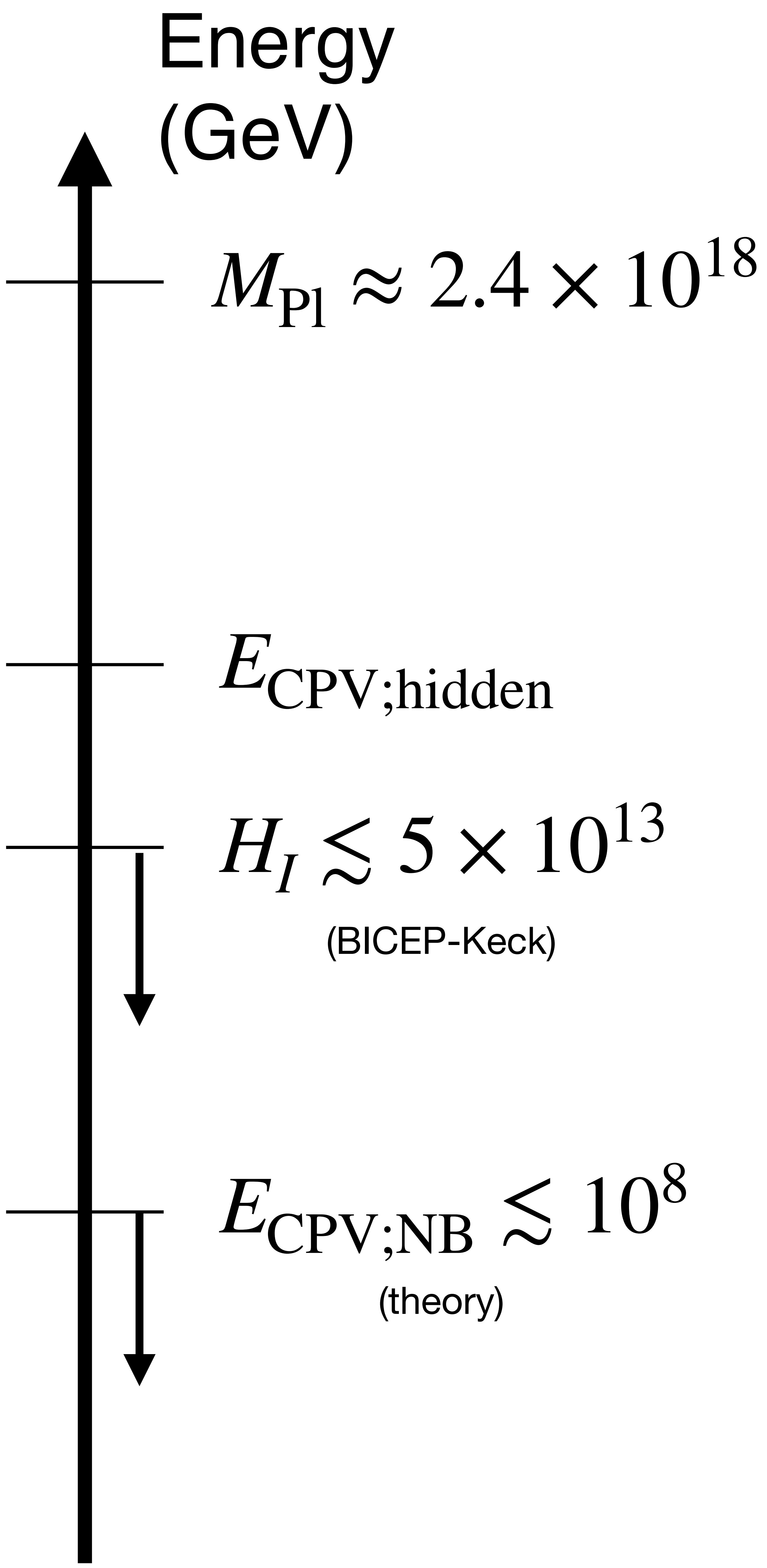
Challenge for model-building:

Break CP in a sequestered hidden sector; inflate away the stable domain walls.

Subsequently, small hidden/visible interactions make *effective* explicit CPV. (Similar to existing models, but not just “write down Planck-suppressed operators.”)

Then visible sector CPV gives *unstable* domain walls.

Solve tensions in model, predict grav. wave signals?



Part 2: Axions as Gauge Fields

B. Heidenreich, J. McNamara, M. Montero, MR, T. Rudelius, I. Valenzuela arXiv:2012.0009
+ ongoing work

Ubiquitous Axion: Lamppost or Principle?

Moduli and axions are ubiquitous in string theory compactifications. But is this an accident, or are they there for a reason?

$$\frac{1}{2}f^2(\partial\theta)^2 + \frac{\theta}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu} \quad \Rightarrow \quad \partial^\mu(f^2\partial_\mu\theta) = \frac{1}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Ubiquitous Axion: Lamppost or Principle?

Moduli and axions are ubiquitous in string theory compactifications. But is this an accident, or are they there for a reason?

instanton number density

$$\frac{1}{2} f^2 (\partial\theta)^2 + \frac{\theta}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \quad \Rightarrow \quad \partial^\mu (f^2 \partial_\mu \theta) = \frac{1}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

The axion causes a would-be “conserved quantity” (instanton number) to vanish: integral of a total derivative.

Axions as Gauge Fields

(Heidenreich, McNamara, Montero, MR, Rudelius, Valenzuela ’20)

The job of the axion in quantum gravity is to *eliminate* a Chern-Weil symmetry with current $\mathrm{tr}(F \wedge F)$ by *gauging* it.

In higher dimensions, this is a genuine *p*-form symmetry. In 4d, it is a “**(-1)-form U(1) symmetry**.” Need to understand these better!

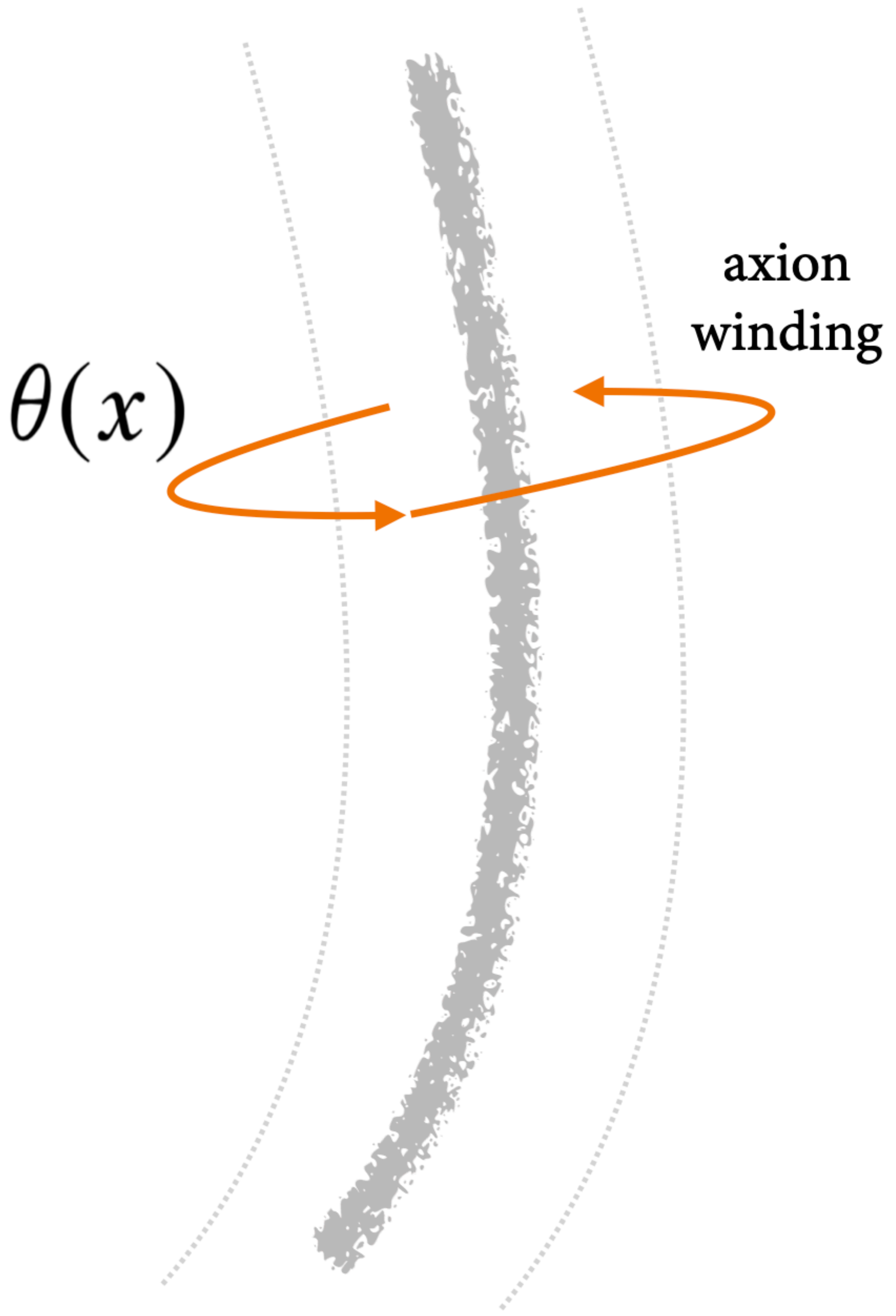
Axions in string theory often just *are* zero modes of higher dimensional gauge fields.

$$\tau(x) = \frac{1}{2\pi}\theta(x) + 4\pi \mathrm{i} S(x), \quad \theta = \int_{\Sigma_p} C_p, \quad S \sim \mathrm{Vol}(\Sigma_p)$$

Chern-Simons: $\theta F^{\mu\nu}\tilde{F}_{\mu\nu}$ from $\int C_p \wedge F \wedge F$

Interpreting the Axion Decay Constant

Standard discussion: 4d model (e.g., KSVZ) with potential $V(\phi)$ spontaneously breaking $U(1)_{PQ}$.



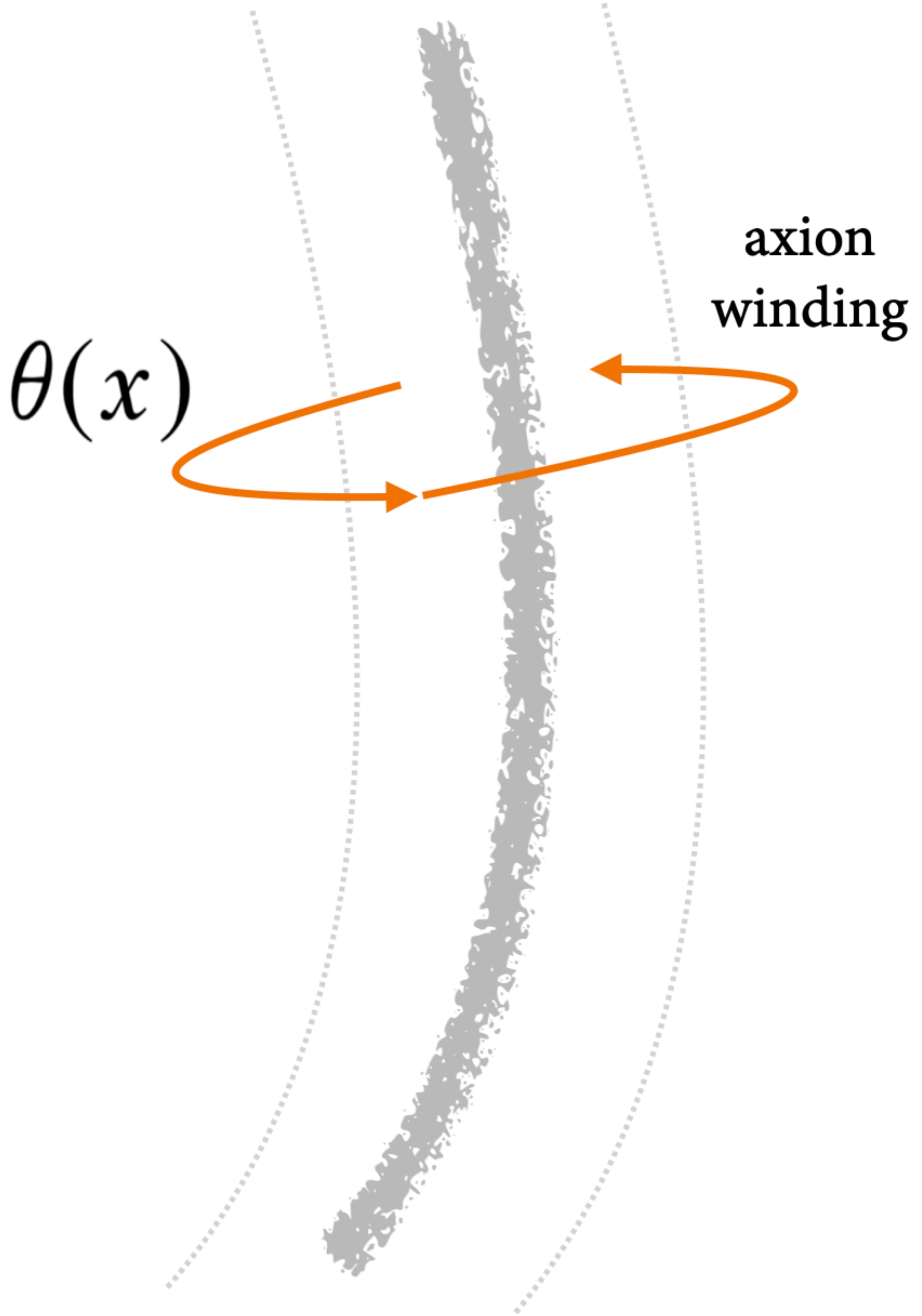
PQ breaking after inflation: **solitonic** cosmic string network produced during phase transition, affects relic abundance of axion DM.

Measuring f: learn about high-scale 4d physics, constraints on inflation (axion isocurvature), etc.

Well-developed story.

Interpreting the Axion Decay Constant

Axions from extra-dimensional gauge fields: no 4d PQ phase transition, ***fundamental*** strings (no $f \to 0$ at finite distance in field space).



Expect f to be related to fundamental scales.

Generic expectation: a discovery of an axion with, e.g., $f \sim 10^{12}\,\mathrm{GeV}$ — well below the Planck scale — indicates **large internal dimensions** and a low UV cutoff.

Interpreting the Axion Decay Constant

Various arguments (WGC, SDC / Scalar WGC, specific string constructions) suggest useful parametric relations.

e.g.: IIB, SM on D7 branes, axion from C_4, $8\pi^2/g^2$ = 4-cycle volume (τ), **axion strings are D3 branes** wrapped on intersecting 2-cycle with tension $\propto t$.
Leading order relationship:

$$M_{\mathrm{Pl}}^2 t^I = \frac{\partial^2 K}{\partial \tau_I \partial \tau_J} \tau_J \mathscr{V}$$

Interpreting the Axion Decay Constant

Various arguments (WGC, SDC / Scalar WGC, specific string constructions) suggest useful parametric relations.

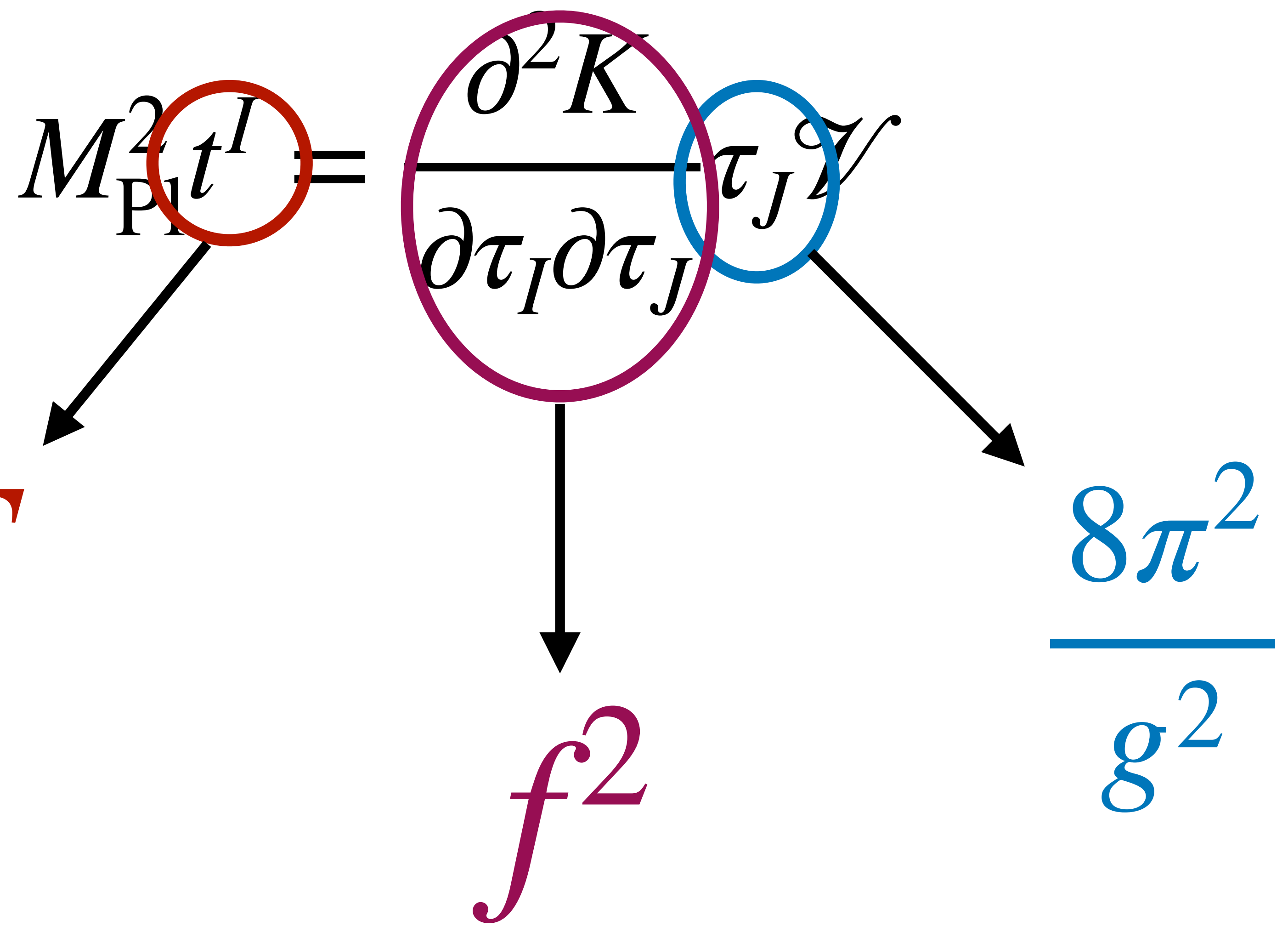
e.g.: IIB, SM on D7 branes, axion from C_4, $8\pi^2/g^2$ = 4-cycle volume (τ), **axion strings are D3 branes** wrapped on intersecting 2-cycle with tension $\propto t$.
Leading order relationship:

$$M_{\rm Pl}^2 t^I = \frac{\partial^2 K}{\partial \tau_I \partial \tau_J} \tau_J \mathcal{V}$$

T $\quad f^2$ $\quad \dfrac{8\pi^2}{g^2}$

(in string units, where the other factor $M_{\rm Pl}^2/\mathcal{V}$ is 1)

Interpreting the Axion Decay Constant



which suggests $T \sim \frac{8\pi^2}{g^2} f^2 \lesssim f M_{\mathrm{Pl}}$ (obeying naive magnetic axion WGC),

saturated only when naive electric axion WGC $f \sim M_{\mathrm{Pl}}/S_{\mathrm{inst}} = \frac{g^2}{8\pi^2} M_{\mathrm{Pl}}$ is saturated.

Low $f \Longrightarrow$ low-tension string $\Longrightarrow$ low QG cutoff.

Summary

Conventional axion theories (KSVZ, DFSZ):

f is a normal 4d scale; cosmic strings in low-energy QFT; severe axion quality problem

***Axions from higher-dimensional gauge fields (e.g., closed string axions)*:**

f close to the fundamental QG scale; cosmic strings are fundamental objects (F-strings, wrapped D-branes); mild axion quality problem

Very different paradigms, differences not usually emphasized in pheno discussions of “axiverse.”

Can we experimentally distinguish?

Conclusions

Nelson-Barr type models face a **serious problem** from the stability of CP domain walls. Needs more model-building. If tensions can be avoided, may be *predictive* of cosmological signals from a secondary phase transition.

Axions on the other hand **play a well-motivated role** as generalized gauge fields in quantum gravity. Favored models may not resemble typical 4d models of Goldstone bosons. An experimental discovery could provide hints of a low fundamental UV cutoff:

$$\Lambda_{\rm QG} \lesssim \frac{4\pi}{g} f$$

Strong CP provides an exciting arena where quantum gravity might confront data!