Applications of Flops

Liam McAllister
Cornell

Back to the Swamp, IFT Madrid, September 27, 2022

Goal

Flop curves in Calabi-Yau threefolds
to explore Kähler moduli space.

Based on:

Weak Gravity and Moduli Space Reconstruction

`Naomi Gendler, Ben Heidenreich, L.M., Jakob Moritz, Tom Rudelius, 221N.NNNNN`

Superpotentials from Singular Divisors

`Naomi Gendler, Manki Kim, L.M., Jakob Moritz, Mike Stillman, 2204.06566`

building on:

Conifold Vacua with Small Flux Superpotential

`Mehmet Demirtas, Manki Kim, L.M., Jakob Moritz, 2009.03312`

Computational Mirror Symmetry

`Mehmet Demirtas, Manki Kim, L.M., Jakob Moritz, Andres Rios-Tascon, 221N.NNNNN`

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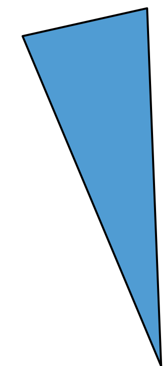

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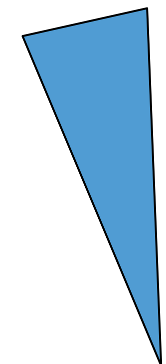

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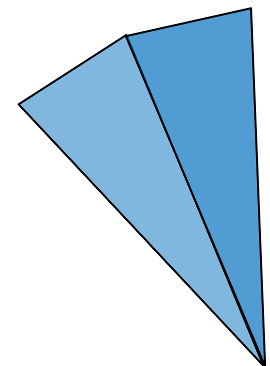

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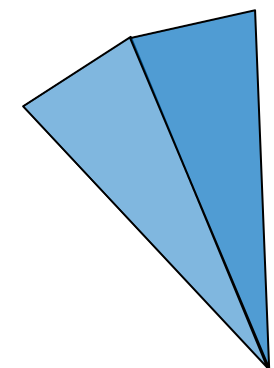

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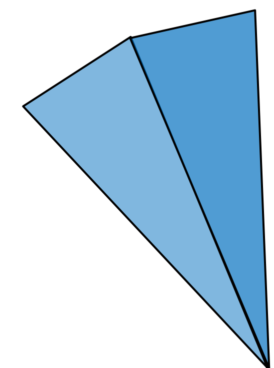

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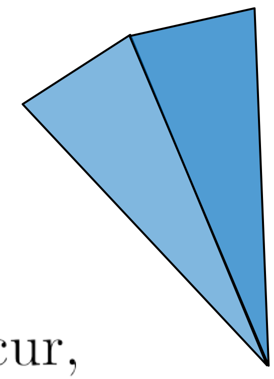

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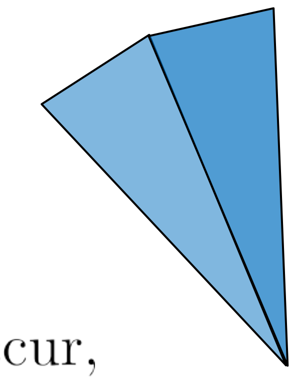

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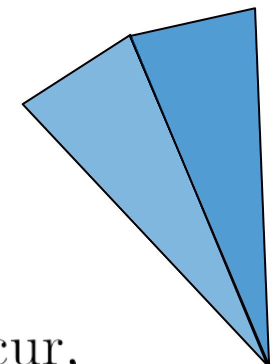

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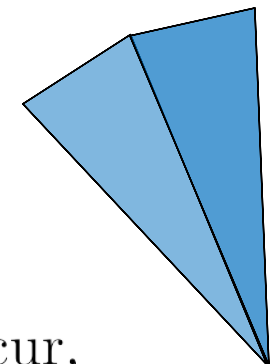

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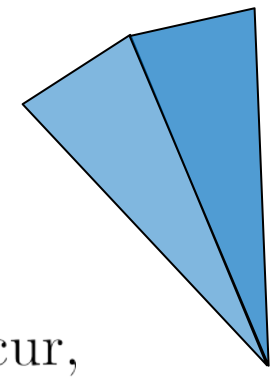

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cf. Brodie, Constantin, Lukas, Ruehle 21

Plan

1. Finding flops
2. Application: testing the WGC
3. Application: desingularizing divisors

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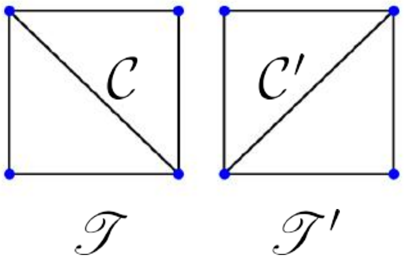

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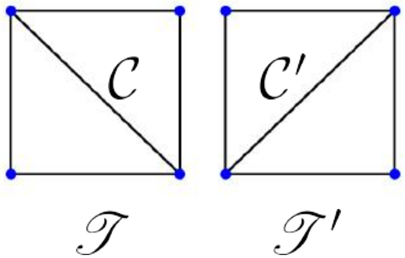

Such flops are numerous and easy to find.
But not all flops are of this sort.

Which curves can be flopped?

At a wall of $\mathcal{K}(X)$, one of the following occurs. Witten 96

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By identifying flops a different way, we find the extended Kähler cone and thus all effective divisors, cf. Lanza, Marchesano, Martucci, Valenzuela 21
Alim, Heidenreich, Rudelius 21

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But wait! What if infinitely many instantons of charges $\neq k\mathbf{q}$ must also be continued?

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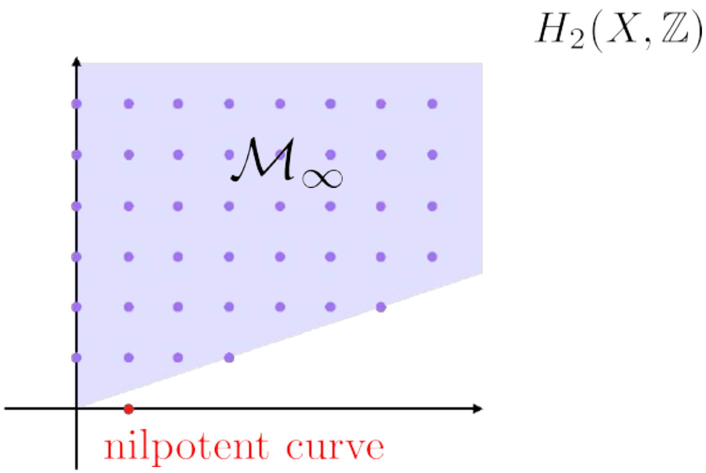

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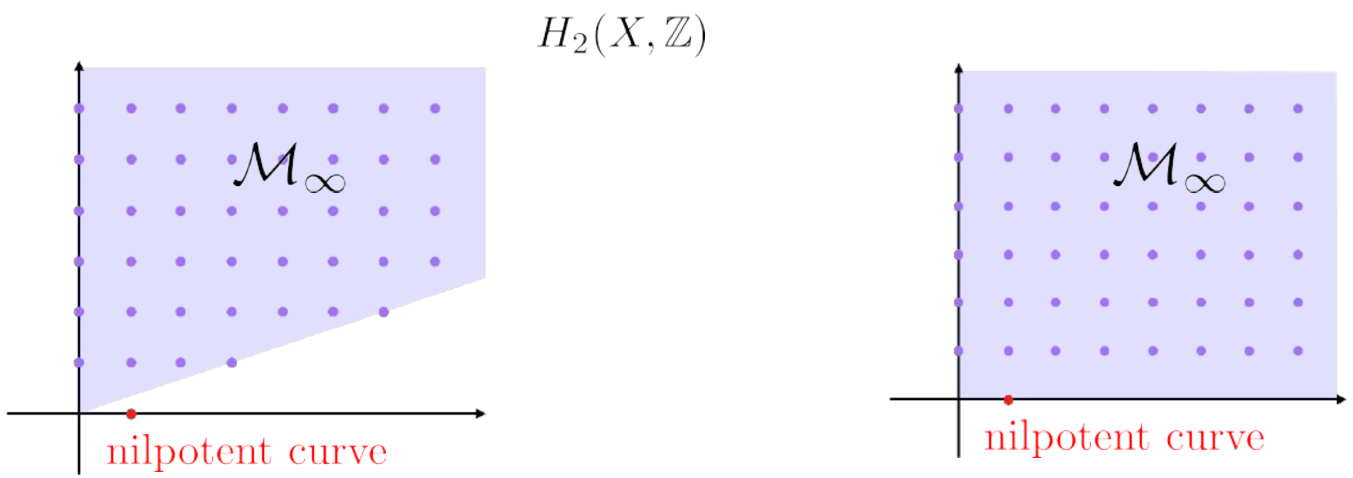

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Proof:

If $\mathcal{C}$ is *strictly outside* $\mathcal{M}_\infty$, no potent curves shrink on the wall where $\mathcal{C}$ shrinks.
$\Rightarrow$ continue $\mathcal{F}$ by continuing finitely many trilogarithms.

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Proof continued:

If $\mathcal{C}$ is *strictly inside* $\mathcal{M}_\infty$, one or more potent curves shrink on the wall where $\mathcal{C}$ shrinks.

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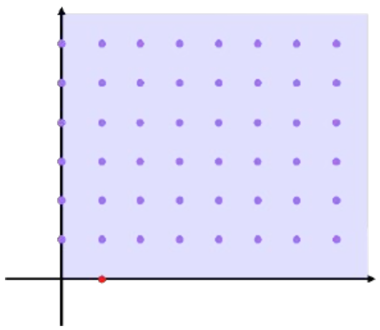
What if $\mathcal{C} \in \partial\overline{\mathcal{M}}_\infty$?

When any $\mathcal{C}$ shrinks, terms in $\mathcal{F}$ differing by $k\mathcal{C}, k \in \mathbb{Z}$ are summed.

If $\mathcal{C} \in \partial\overline{\mathcal{M}}_\infty$, these sums are infinite (infinite degeneracy).

$\Rightarrow$ cannot simply continue $\mathcal{F}$. $\square$



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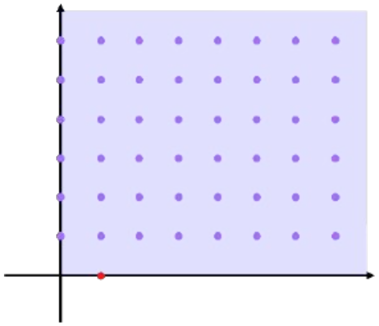

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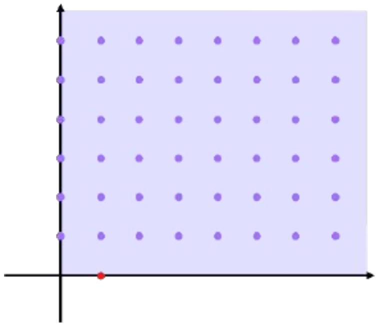
$$\mathcal{F} \sim \sum_{n,m\geq 0} \mathrm{GV}_{(n,m)}\, e^{-2\pi\,(nt_x+mt_y)}$$

absolutely convergent for large enough t_x, t_y.



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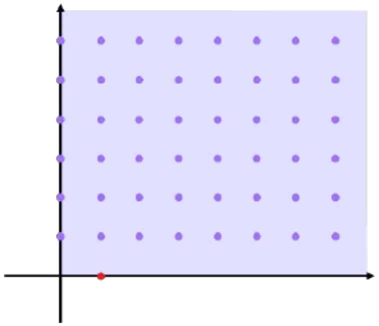

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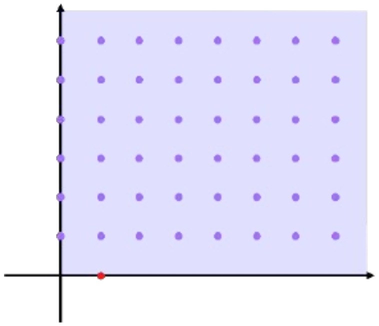

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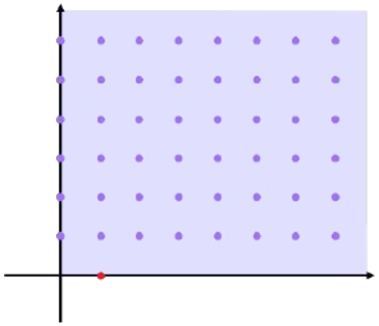

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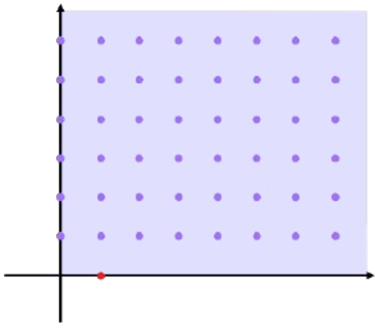
For each $m > 0$, all but finitely many $\mathrm{GV}_{(n,m)}$ are nonzero integers.

$\Rightarrow$ sum does not converge.



Implementing the flop criterion

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Demirtas, Kim, L.M., Moritz, Rios-Tascon, 221N.NNNNN

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Method: extension of approach of Hosono, Klemm, Theisen, Yau 95

> Compute fundamental period, use intersection data of mirror.
> We handle general case where $\mathcal{M}(X)$ is not simplicial.
> Implementation in `CYTools` involves significant advances:
> not computing GV of non-effective curves $\Rightarrow$ massive speedup.

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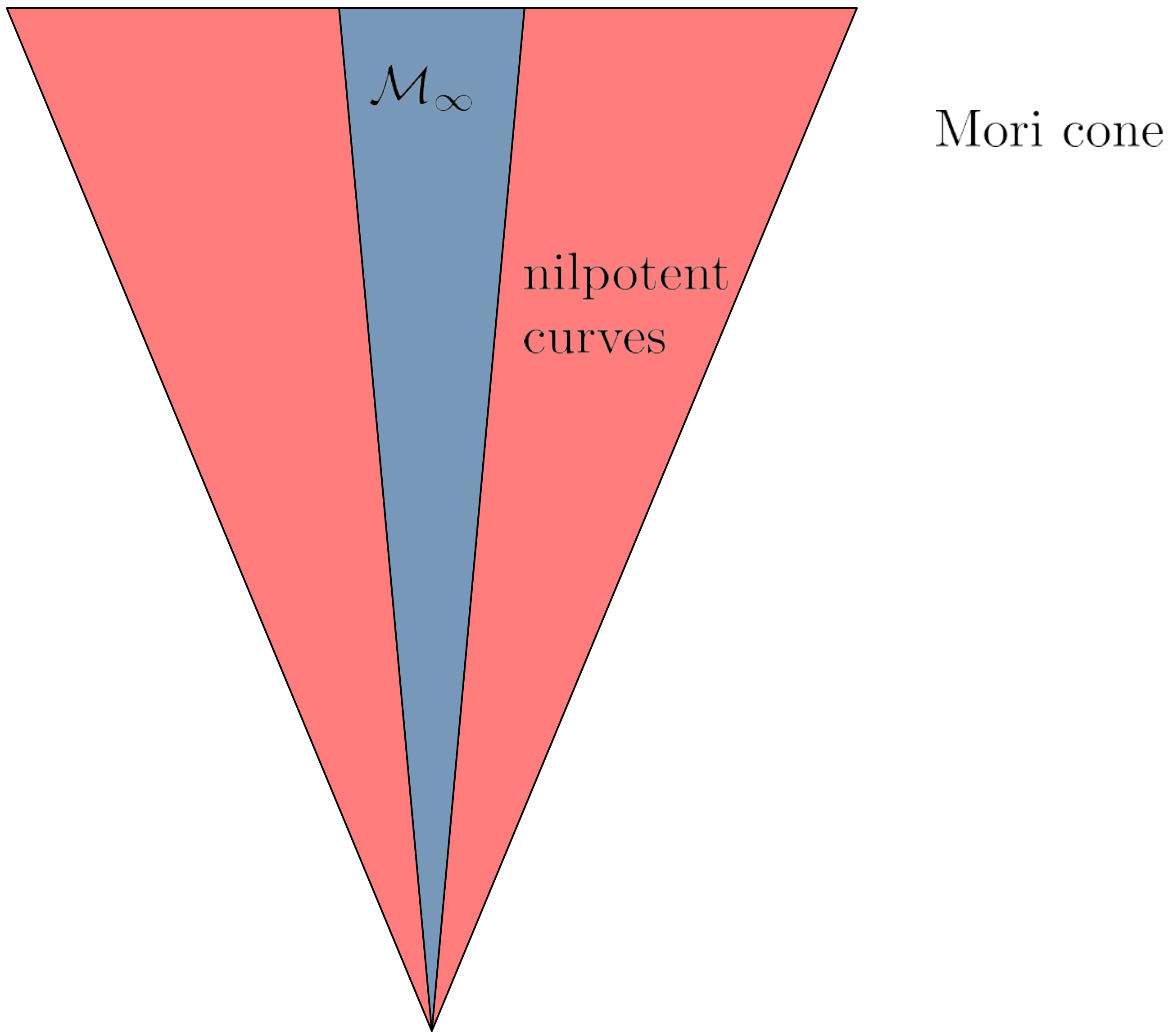
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Mori cone
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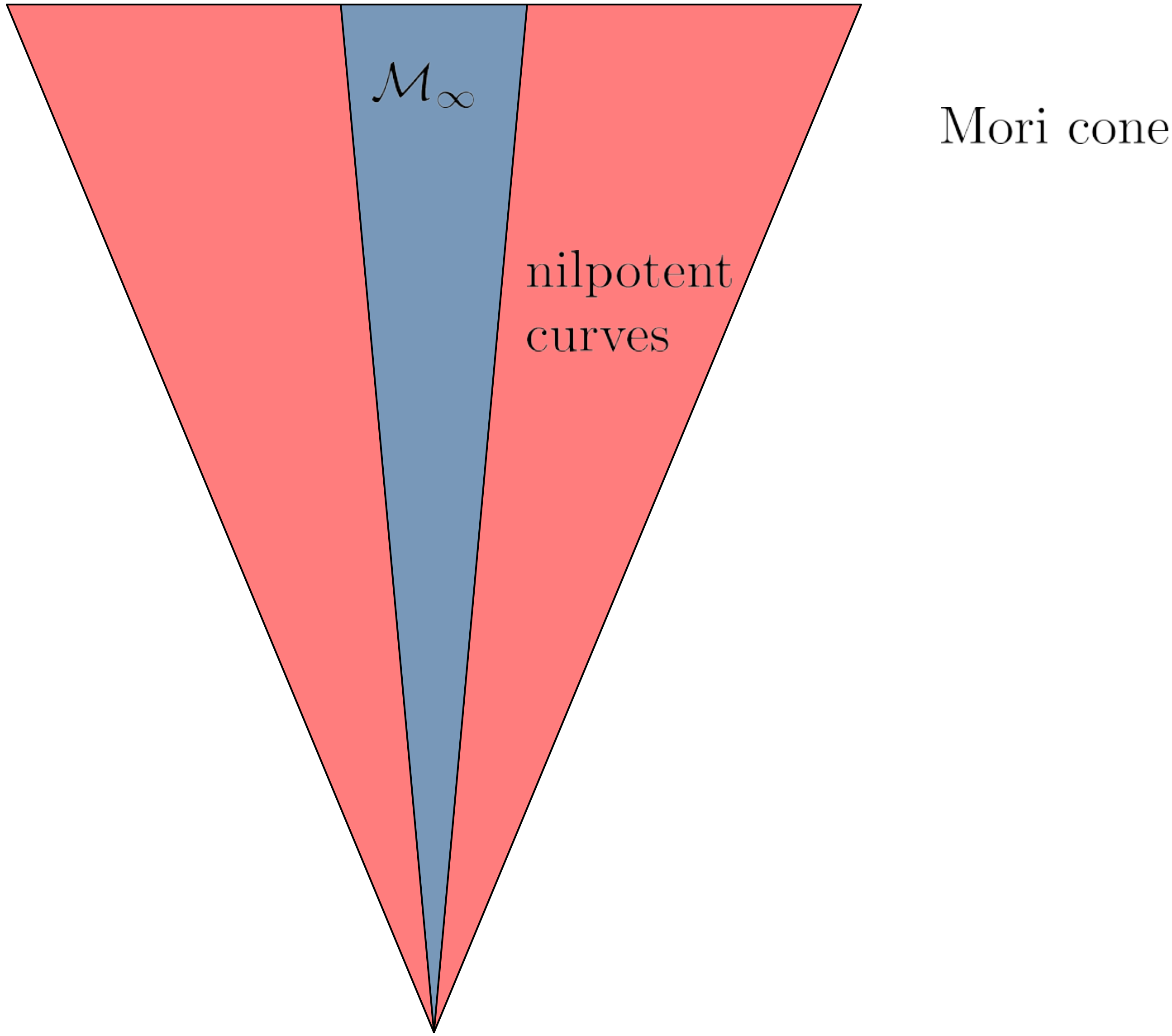
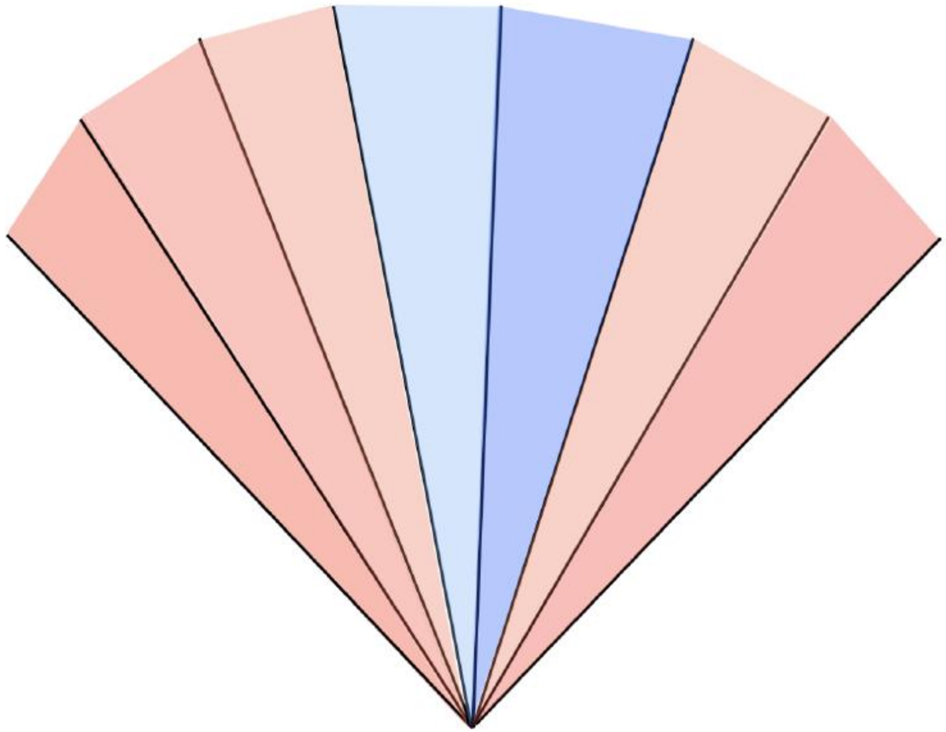


At large $h^{1,1}$, many possible flops $\Leftrightarrow$ many nilpotent curves

Flopping out the extended Kähler cone

By computing GVs and thus detecting flops,
we can map out the extended Kähler cone.



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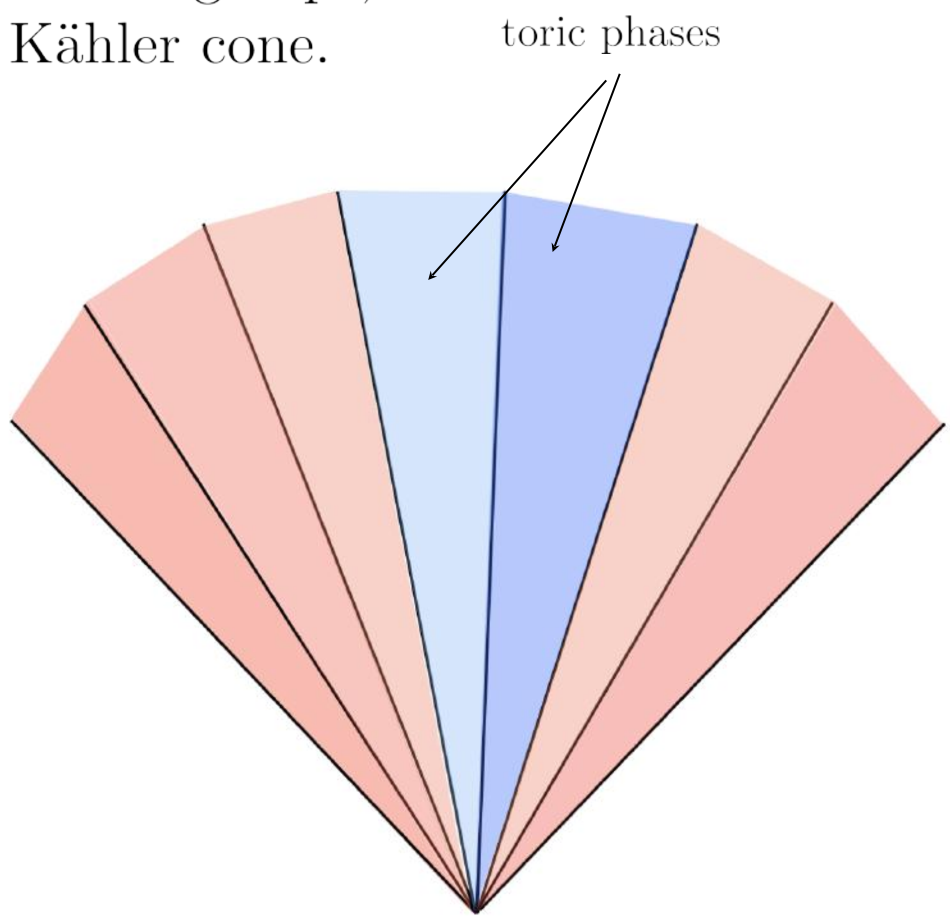

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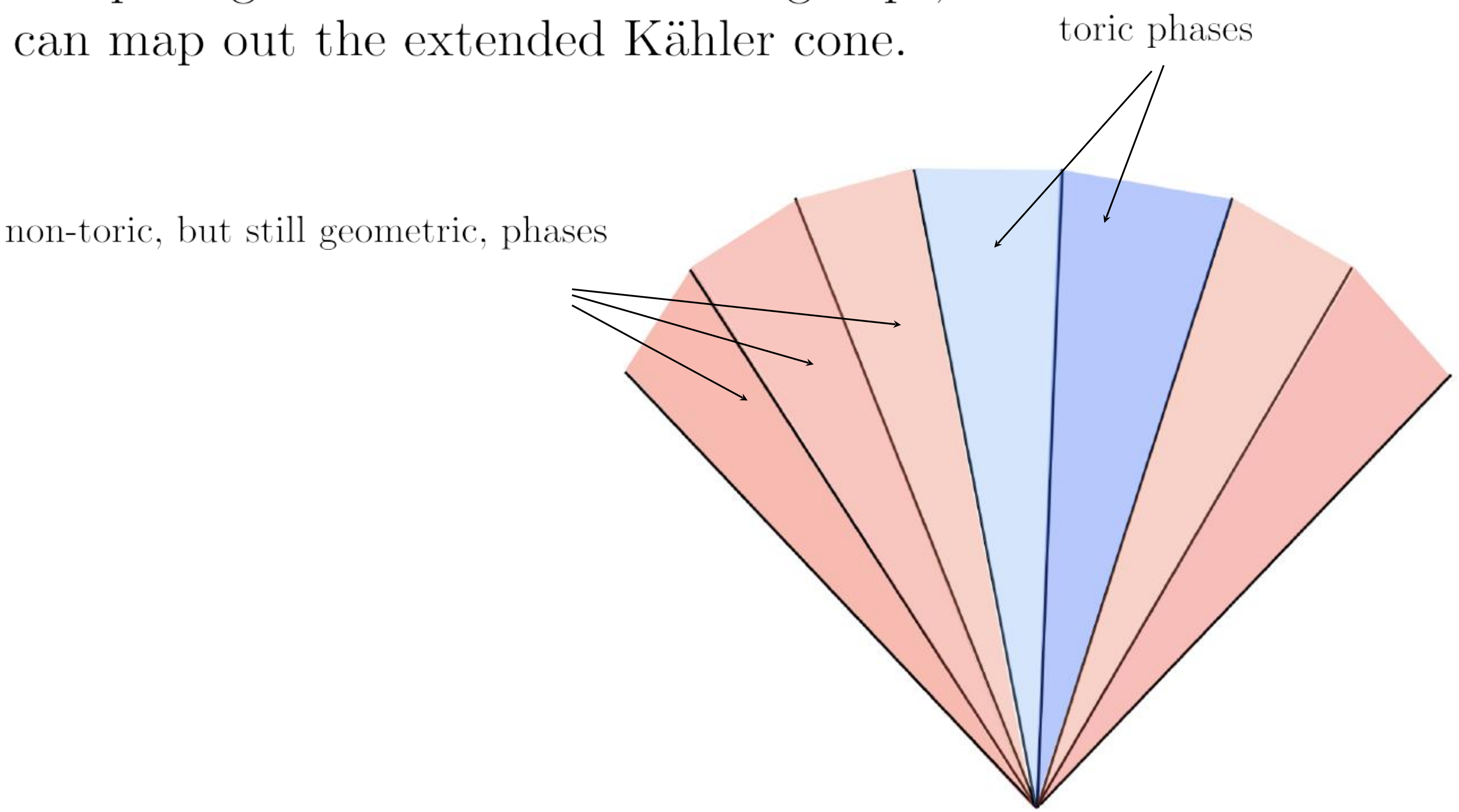

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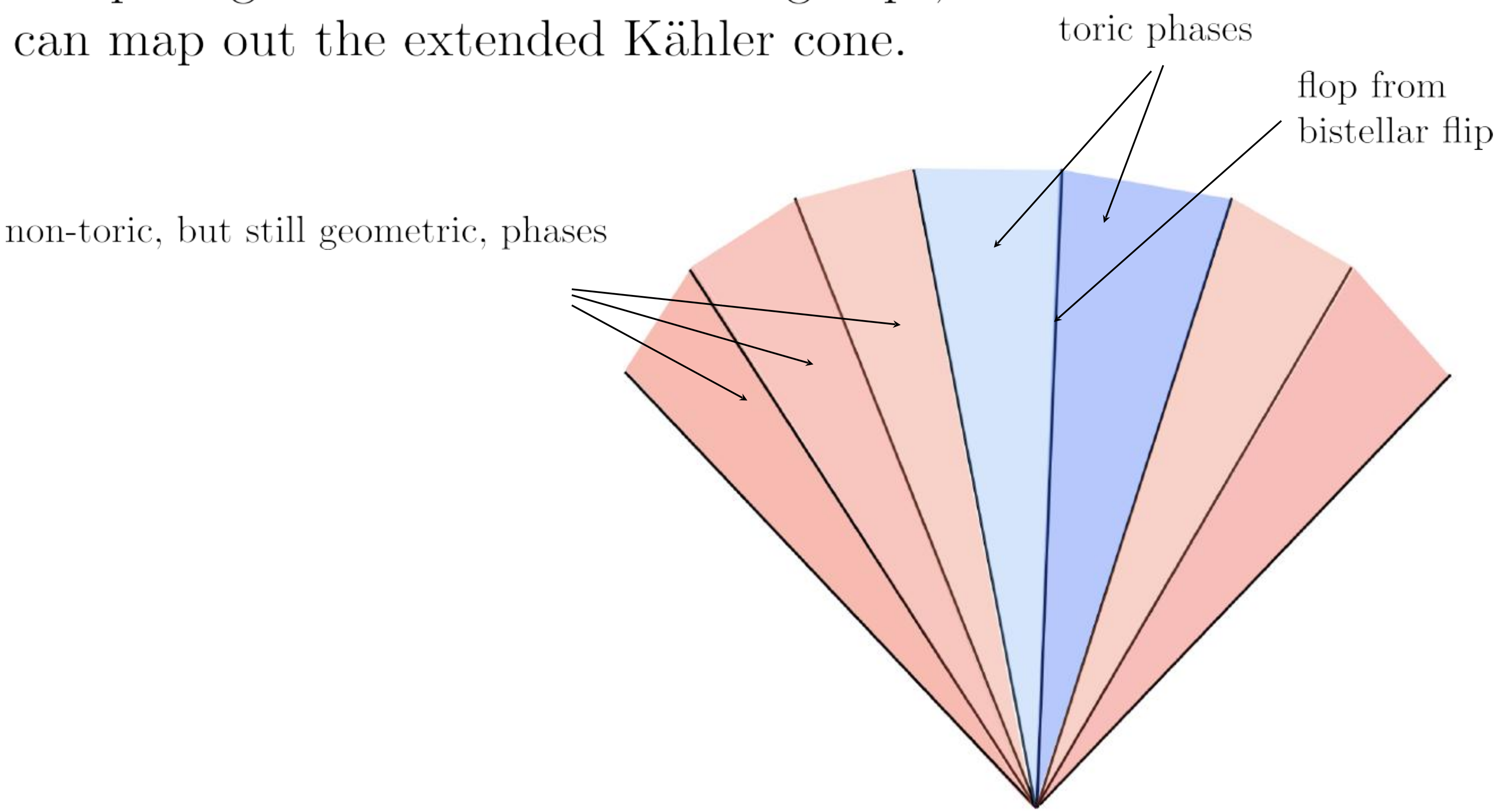

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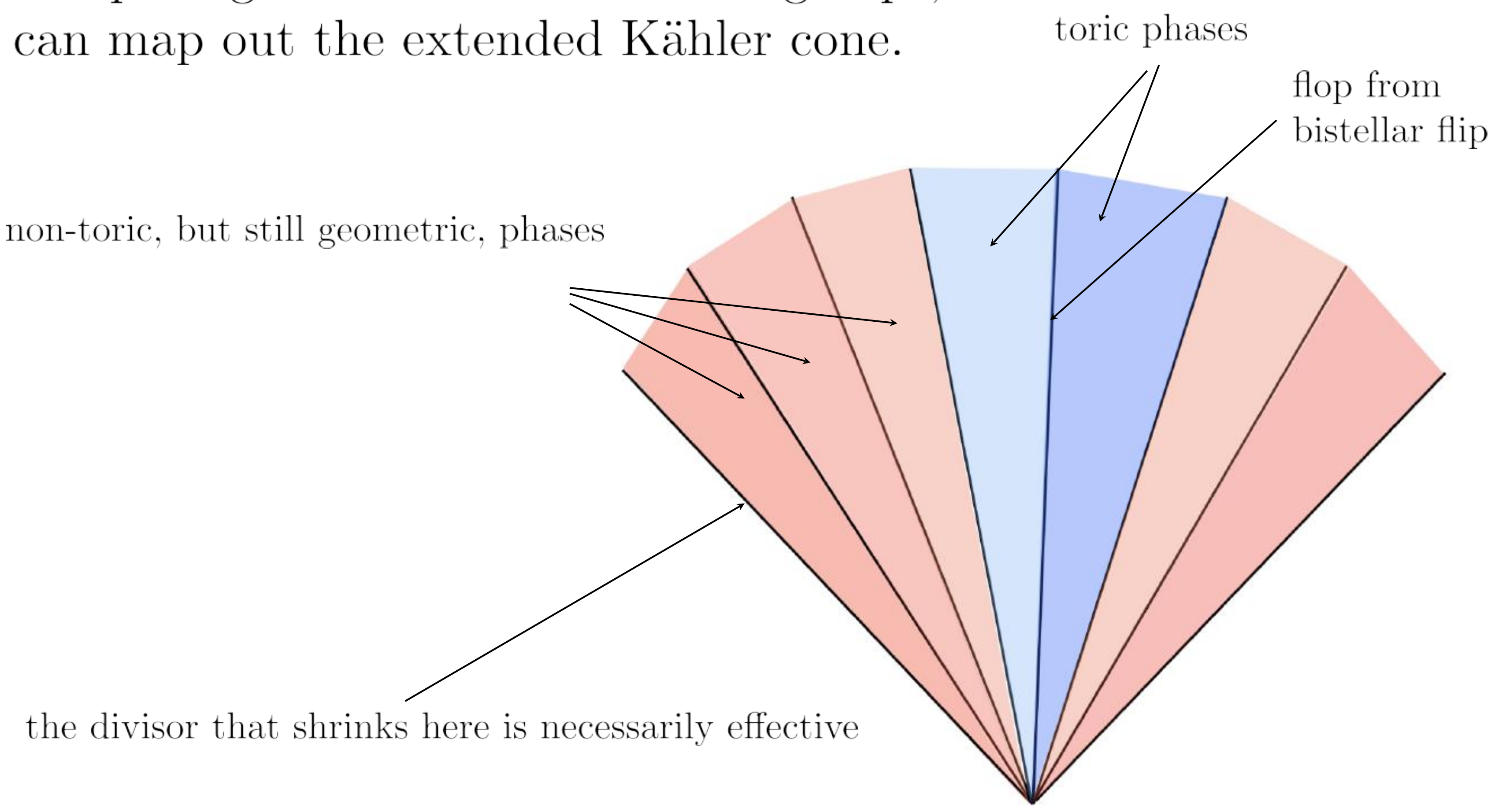

Application 1: WGC

The tower and sublattice WGC make highly nontrivial predictions for BPS states.

For M-theory or IIA on CY_3, GV invariants are BPS indices.

$$\mathrm{GV}_{\mathbf{q}} \neq 0 \Rightarrow \exists \geq 1 \text{ BPS state of electric charge } \mathbf{q}$$

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Can we use GV invariants to test strong forms of the WGC?

This approach could exclude, but cannot prove, a WGC:

- cancellations possible: $\mathrm{GV} = 0 \not\Rightarrow \nexists$ BPS state
- no information about non-BPS directions in charge space

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For each CY phase, certain curves are effective.
$\Rightarrow$ define BPS directions in charge space.
Where does WGC make predictions?

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The BPS sublattice WGC

There exists an integer $k \geq 1$ such that for any $\vec{q}$ in a direction in which the BPS and black hole extremality bounds coincide, there is a BPS particle of charge $k\vec{q}$.

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Thus, T/sLWGC predict that $\mathcal{E}^* \subseteq \mathcal{M}_\infty$.

Alim, Heidenreich, Rudelius 21

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Algorithm to test this:

1. Start with a CY_3.
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We find no violations of the lattice WGC.

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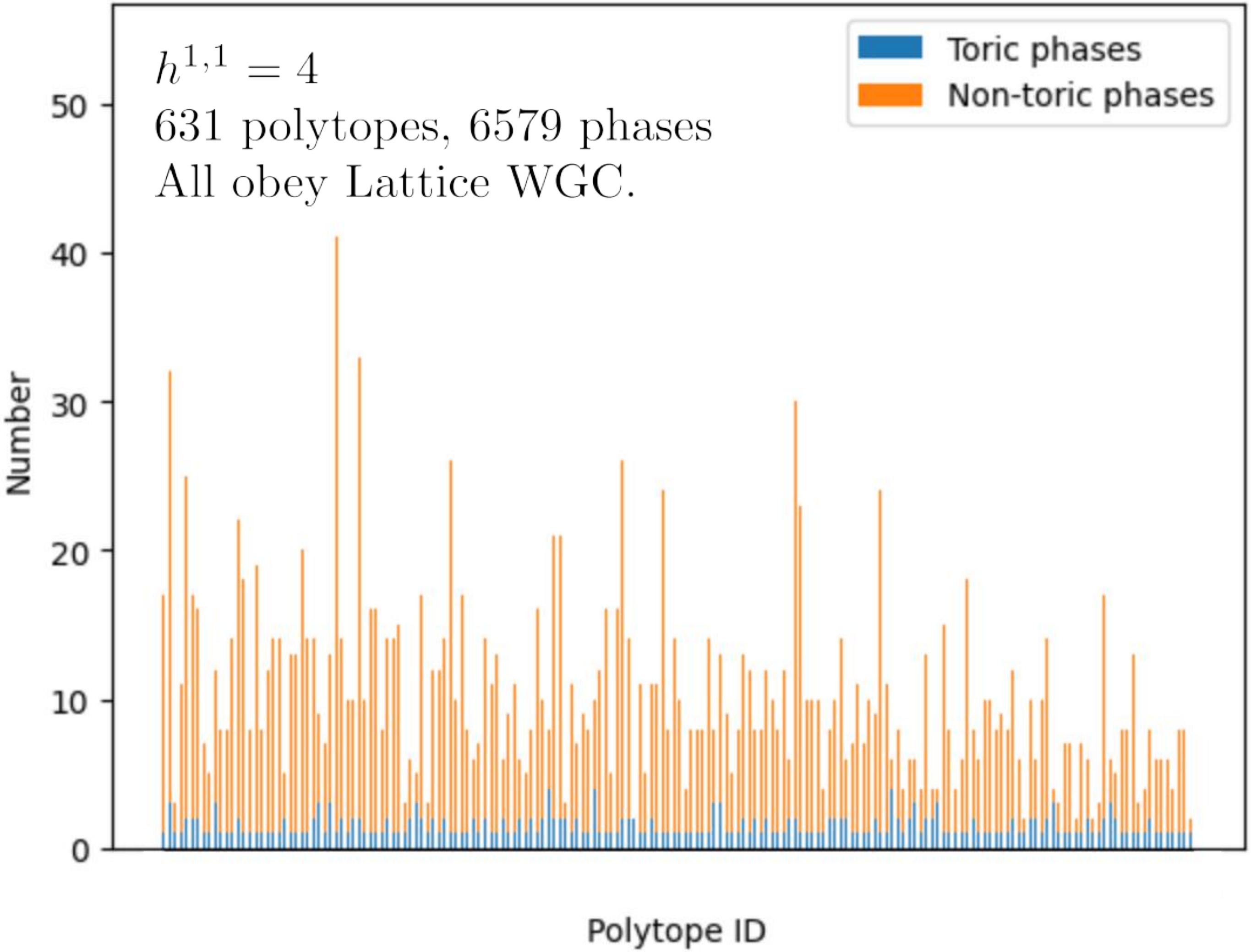
Very preliminary results



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and D is smooth,

then the number of fermion zero-modes is 2, and ED3s on D contribute

Witten 96

$$W \supset \mathcal{A}(z,\tau) e^{-2\pi \mathrm{Vol}(D) - 2\pi i \int_D C_4}$$

with $\mathcal{A}(z,\tau)$ not identically zero.

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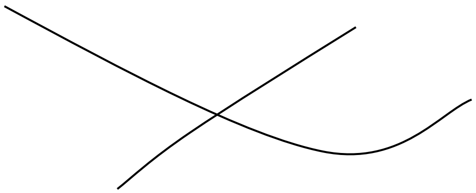

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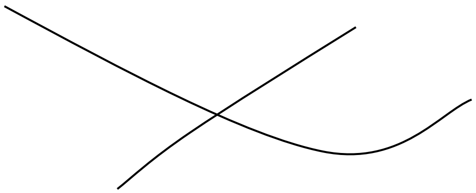

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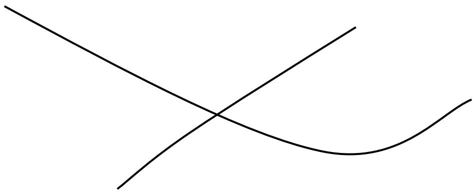

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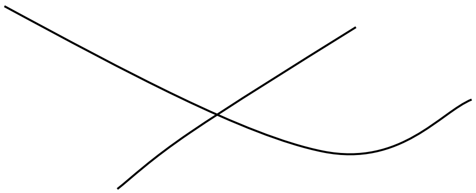

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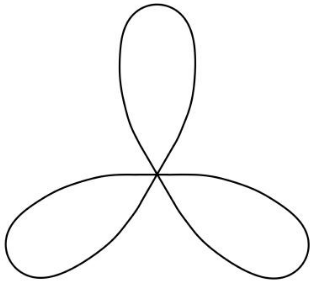

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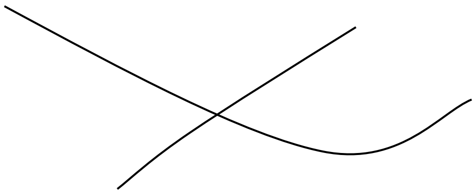

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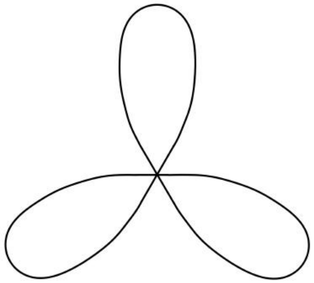

Can we count zero-modes in this case?

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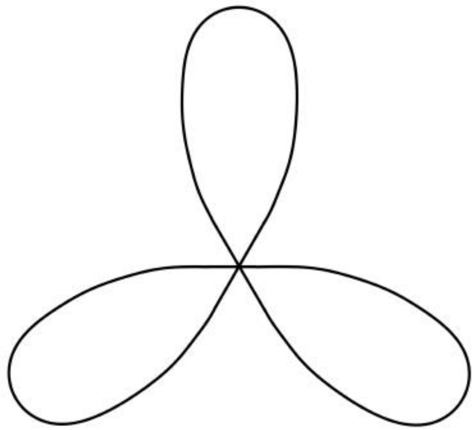

$k = 3$

Star-crossing singularities

Consider a divisor D that has singularities at codimension 1, along some locus $\mathcal{C} \subset D$.

(Codimension-2 singularities are arguably ignorable.)

If $\mathcal{C}$ is a rational curve, we call the singularity 'star-crossing'.

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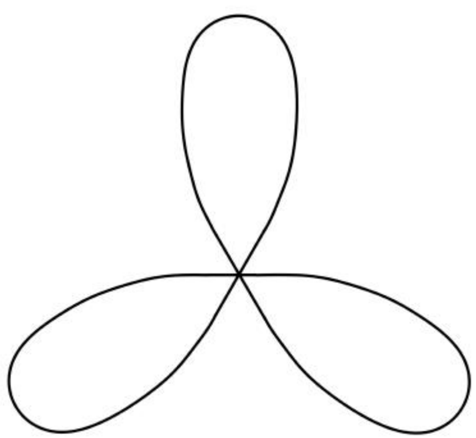

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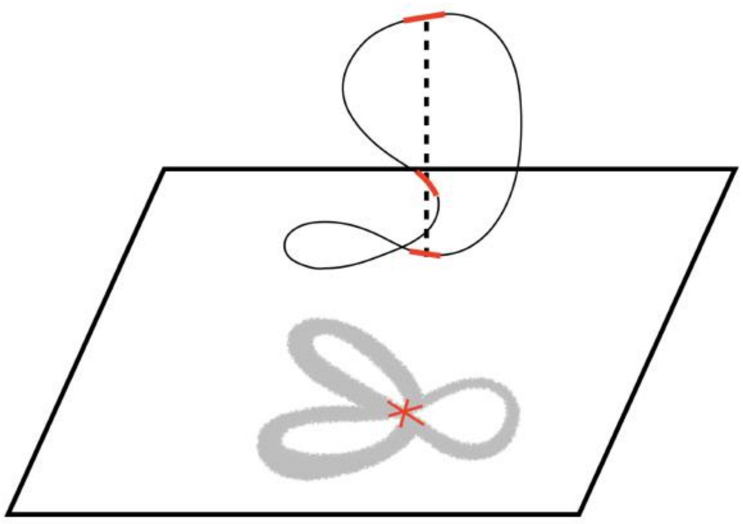

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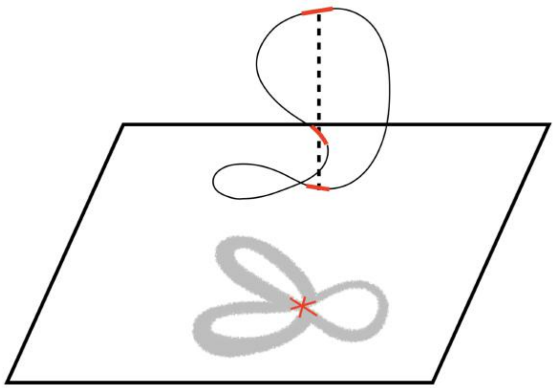

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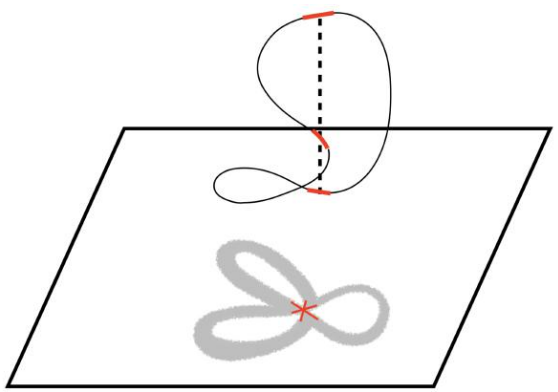

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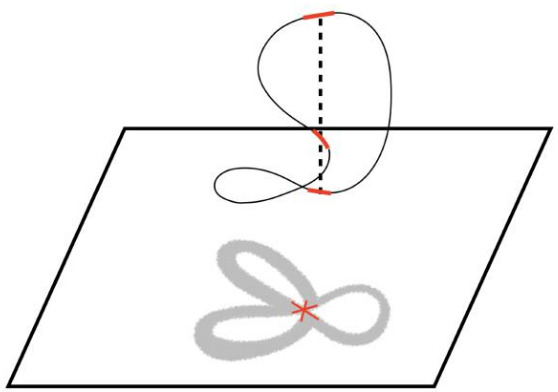
Can we learn to count fermion zero-modes in this generic case?



Unwinding star-crossing singularities

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So if $\qquad h^{\bullet}_{+}(D_{\text{smooth}}) = (1,0,0)\,, \qquad h^{\bullet}_{-}(D_{\text{smooth}}) = 0\,,$

there is a contribution in the smooth configuration, and so there must be one in the singular configuration!

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So

$$\boxed{\ast^{\bullet}_{+}(D) = (1,0,0)\,, \quad \ast^{\bullet}_{-}(D) = 0}$$

is a sufficient condition for an ED3 contribution.

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Recap:

given a D with singularities along rational curves,

if a series of flops unwinds the singularity,
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In this case our claim is proved, by continuing W
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so our formula counts the zero-modes of the various components.

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Extension of usual sufficient condition to singular divisors.


Thanks!