Cobordism, K-theory and Tadpoles I

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(RB, Cribiori, arXiv:2112.07678)

(RB, Cribiori, Kneißl, Makridou, arXiv:2205.09782 +arXiv:2208.01656)

Introduction

Introduction

Conjecture: No global symmetries in QG!

- If one seems to detect one, it actually needs to be gauged or broken
- For continuous symmetries, this means:

$$d \star F_{d-n+1} = \star J_{d-n}\,, \quad \text{or} \quad d \star J_{d-n} = I_{n+1}$$

- Non-vanishing cobordism groups $\Omega_n^{\widetilde{QG}} \neq 0$ are a source of global symmetry and thus need to be nullified eventually, i.e $\Omega_n^{QG} = 0$ (McNamara,Vafa, 1909.10355).
- Discussed many examples based on Ω_n^{Spin}, $\Omega_n^{\text{Spin}^c}$ and which defects can break the global symmetries.(see also (Dierigl,Heckmann, 2012.00013))
- Cobordism and Ricci-flow (Velázquez,De Basio,Lüst,2209.10297)

Introduction

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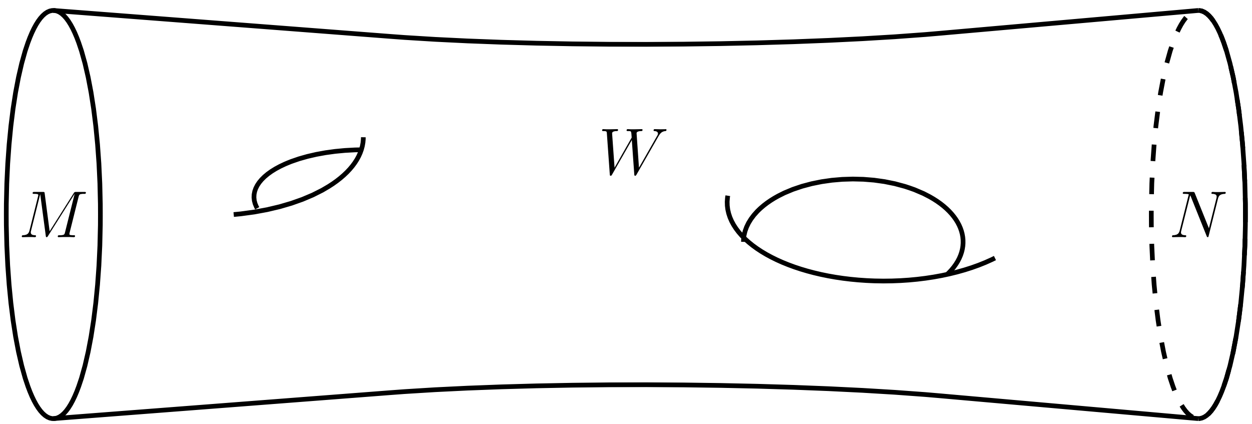
Spin-cobordism Ω_n^{Spin} are equivalence classes of n-dim. Spin-manifolds, where M and N are equivalent if

$$\partial W = M \sqcup \overline{N}\,.$$



The addition is defined via disjoint union

$$[M] + [N] = [M \sqcup N]\,.$$

Cobordism groups also appeared in Dai-Freed anomalies

(Etxebarria,Montero, 1808.00009)

Introduction

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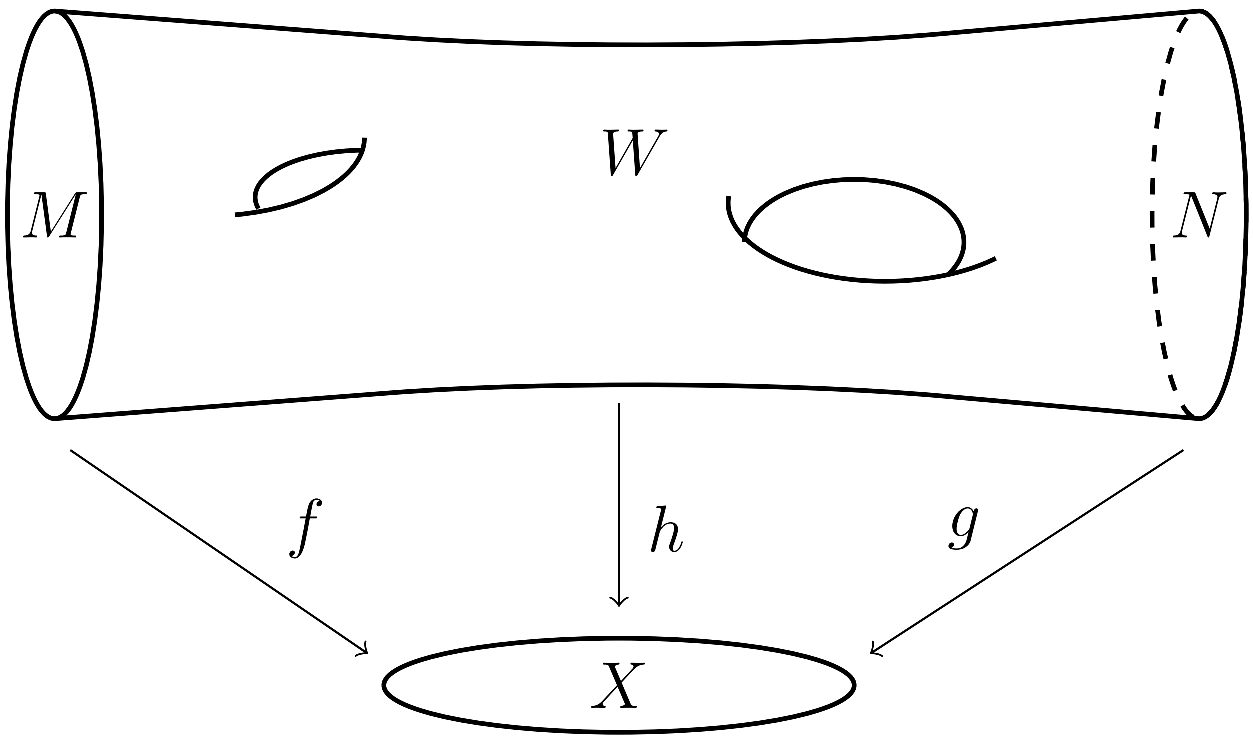
Generalization to $\Omega_n^G(X)$: cobordism groups relative to X consisting of pairs (M, f) modulo equivalence:



$X = pt$ is the former case.

Introduction

Introduction

Ω_n^{Spin} and their generators: obvious relation to $\widetilde{KO}(S^n)$

n	0	1	2	3	4	5	6	7	8
Ω_n^{Spin}	$\mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z}$	0	0	0	$\mathbb{Z}^2$
$\Sigma_{n,i}$	pt$^+$	S_p^1	$S_p^1 \times S_p^1$	0	$K3$	0	0	0	$\mathbb{B} \oplus \mathbb{HP}^2$

$\Omega_n^{\text{Spin}^c}$: Obvious relation to $\widetilde{K}(S^n)$

n	0	2	4	6
$\Omega_n^{\text{Spin}^c}$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}^2$	$\mathbb{Z}^2$
$\Sigma_{n,i}$	pt$^+$	$\mathbb{P}^1$	$\mathbb{P}^2 \oplus (\mathbb{P}^1)^2$	-
Inv.	1	c_1	td, c_1^2	td, $c_1^3/2$

K-theory and Cobordism

K-theory and Cobordism

- There exist an intricate mathematical relationship between K-theory and cobordism.
- What is the physical significance of it?

Atiyah–Bott–Shapiro (ABS): There exist ring homomorphisms

$$\alpha^c : \Omega_*^{\mathrm{Spin}^c}(pt) \to K_*(pt)\,, \qquad \alpha : \Omega_*^{\mathrm{Spin}} \to KO_*(pt)\,.$$

with $K_n(pt) = \tilde{K}(S^n)$. When restricted to a fixed grade n

$$\alpha_n^c([M]) = \mathrm{Td}(M)\,, \qquad \alpha_n^c([M]) = {}''\hat{A}(M)''.$$

The map α^c is a cobordism invariant and surjective, so that one can divide by its kernel to get an isomorphism

$$\Omega_n^{\mathrm{Spin}^c}/\ker(\alpha) \cong \widetilde{K}(S^n)$$

Gauging global symmetries

Gauging global symmetries

- Both K-theory and cobordism compute global charges
- K-theory global symmetries are all expected to be gauged (Freed, hep-th/0011220)
- For non-torsion classes ($K_n(pt) = \mathbb{Z}$) this leads to Bianchi identities of the form (for D_{9-n} branes)

$$d\tilde{F}_{n-1} = \sum_i N_i \delta^{(n)}(\Sigma_i) + \dots$$

- Proposal: the missing "geometric" piece is described by the corresponding cobordism group $\Omega_n^{\mathrm{Spin}^c}(pt)$

In string theory:

- $\Omega_n^{\mathrm{Spin}}(pt)/\Omega_n^{\mathrm{Spin}^c}(pt)$ describes the geometric contribution in type I/type IIB orientifold(F-theory) tadpole constraints

Example: Spin^c

Example: Spin^c

- All K-theory classes $K_{2n}(pt) = \mathbb{Z}$ are gauged
- The Todd classes are the natural currents

$$\star J_{d-n} = \alpha^c([M]) = \mathrm{td}_n(M),$$

- Note, there appears a proliferance of $\mathbb{Z}$ factors in higher cobordism classes $\Omega_n^{\mathrm{Spin}^c}(pt) \to$ more cobordism invariants

For $\Omega_2^{\mathrm{Spin}^c}$ one has $\alpha_2^c(M) = \mathrm{td}_2(M) = c_1(M)/2$, leading to the F-theory/type IIB orientifold relation

$$d\tilde{F}_1 = \sum_i N_i\, \delta^{(2)}(\Delta_{8,i}) - 24\, \alpha_2^c(M)\,.$$

The factor of 24 is not a priori fixed!

Example: Spin^c

Example: Spin^c

For $\Omega_6^{\mathrm{Spin}^c} = \mathbb{Z} \oplus \mathbb{Z}$, the gauging leads to a D3-brane tadpole

$$d\tilde{F}_3 = \sum_i N_i \, \delta^{(6)}(\Delta_{4,i}) + a_1^{(6)} \frac{c_2(M) c_1(M)}{24} + a_2^{(6)} \frac{c_1^3(M)}{2} \, .$$

For $a_1^{(6)} = -12$, $a_2^{(6)} = -30$ and $M = B_3$ the base of a smooth elliptically fibered CY fourfold Y, this is the $D3$ tadpole of F-theory

$$\chi(Y)/24 = \int_B \left(\frac{1}{2} c_2(B) \, c_1(B) + 15 \, c_1^3(B) \right) \, .$$

- contains precisely the two cobordism invariants td_6 and $c_1^3/2$, i.e. in particular no c_3-term.
- appearance of $c_1^3/2$ is related to the presence of $O7$-planes. Pure $O3$-planes: $M = dP_9 \times \mathbb{P}^1$

Gauging of Ω_1^{Spin}?

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Hopkins-Hovey isomorphisms: $\widetilde{KO}(S^1) \simeq \Omega_1^{\mathrm{Spin}} = \mathbb{Z}_2$

Gauging: $\mathbb{Z}_2$ valued charge neutrality condition

$$\int_M \sum_i N_i \, \delta^{(1)}(\Delta_{9,i}) = K \, \alpha_1(M) \qquad \mod 2\,.$$

What is the value of K?

- K even: r.h.s. decouples $\Rightarrow$ K-theory charge gauged + charge $\Omega_1^{\mathrm{Spin}} = \mathbb{Z}_2$ needs to be broken by 7-defects
- K odd: a single non-BPS $\widehat{D8}$-brane on the background $M = S_p^1$ would be charge neutral and allowed, unexpected $\Rightarrow$ K even

Generic situation for torsion charges?

Generalized cobordism

Generalized cobordism

The Hopkins–Hovey isomorphism (generalization of a classic theorem by Conner-Floyd)

$$\Omega_*^{\mathrm{Spin}}(X) \otimes_{\Omega_*^{\mathrm{Spin}}} KO_* \to KO_*(X),$$

$$\Omega_*^{\mathrm{Spin^c}}(X) \otimes_{\Omega_*^{\mathrm{Spin^c}}} K_* \to K_*(X)$$

are isomorphisms for any topological space X.

It involves the more refined generalized cobordism $\Omega_n^{\mathrm{Spin}^c}(X)$ and K-theory $K_n(X)$ classes.

- How to compute them?
- Physical relevance in the swampland program?

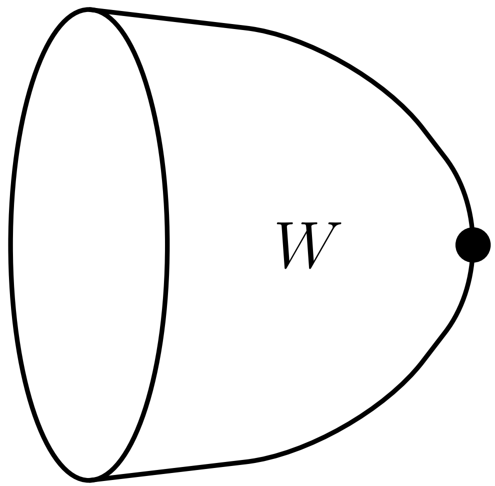
(Bhg,Cribiori, Kneißl,Makridou, 2208.01656)

(more details in talk by Niccolo Cribiori)

Dynamical Cobordism

Dynamical Cobordism

Breaking of global symmetries $\rightarrow$ existence of (new) defects



- Dynamical cobordism: (Super)gravity solutions with ETW defects (Buratti,Delgado,Uranga,2104.02091),(Buratti,Calderón-Infante,Delgado,Uranga,2107.09098)(Angius,Calderón-Infante,Delgado,Huertas,Uranga,2203.11240).
- General scaling behavior: $\Delta \sim e^{-\frac{\delta}{2}D}$, $|R| \sim e^{\delta D}$
- Time dependent backgrounds (Angius,Delgado,Uranga,2207.13108)

Dynamical Cobordism

Dynamical Cobordism

Questions: (RB, Cribiori, Kneißl, Makridou,2205.09782)

- Examples for ETW branes of higher codimension?
- (Super)gravity description of the ETW brane itself?

Backreaction of a neutral domain wall with $T > 0$

$$S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left(R - \frac{1}{2} (\partial\Phi)^2 \right) - T \int d^{10}x \sqrt{-g}\, e^{\frac{5}{4}\Phi}\, \delta(r)$$

Generalization of Dudas-Mourad (hep-th/0004165) (see also (Basile,Thomee,Raucci,2209.10553))

Examples are:

- non-BPS D8-brane of type I
- R-R tadpole free $16 \times \overline{D8} + O8^{++}$ configuration in T-dual of Sugimoto model

Dynamical Cobordism

Dynamical Cobordism

Features:

- no maximally symmetric solution
- two solutions with one non-trivial longitudinal direction, preserving 8D Poincare symmetry (Bhg,Font, hep-th/0011269)
- Solution II has topology $S^1 \times I_1$, i.e. spontaneous compactification with two end-of-the-world (ETW) walls
- Issue: extra $\log$-singularities $\rightarrow$ ETW 7-branes

Could confirm the expected relations between the geometric distance Δ, the scalar curvature R and the field space distance D:

$$\Delta \sim \mathcal{T}^{-1} \sim e^{-\frac{\delta}{2}D}\,, \qquad |R| \sim e^{\delta D}$$

for $\delta = 2\sqrt{2}$.

Dynamical Cobordism

Dynamical Cobordism

What is the nature of the ETW 7-brane?

Constraints:

- preserves 8D Poincare symmetry
- has $\log \rho$ singularities close to its core
- is non-isotropic in the two transverse directions.

Non-isotropic ansatz for metric

$$ds^2 = e^{2\hat{\mathcal{A}}(\rho,\varphi)} ds_8^2 + e^{2\hat{\mathcal{B}}(\rho,\varphi)} (d\rho^2 + \rho^2 d\varphi^2)$$

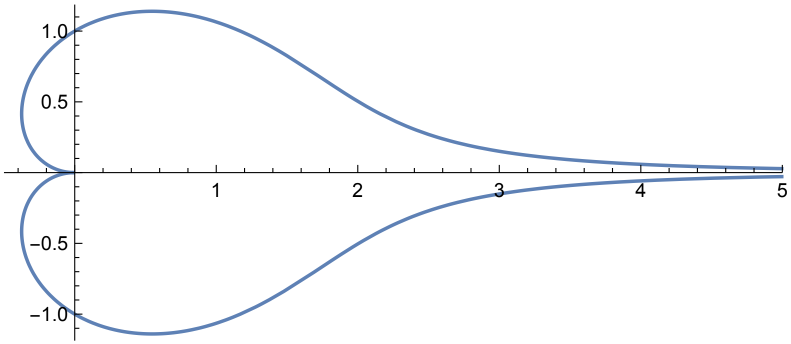
- exists solutions to the bulk eom with $\log$-singularity, closely related to domain wall
- source can be fixed as $S_7 = -T_7 \int d^{10}x \sqrt{-g}\, \frac{\delta(\rho)}{2\pi\rho}$ with $\kappa_{10}^2 T_7 = 2\pi$.

Dynamical Cobordism

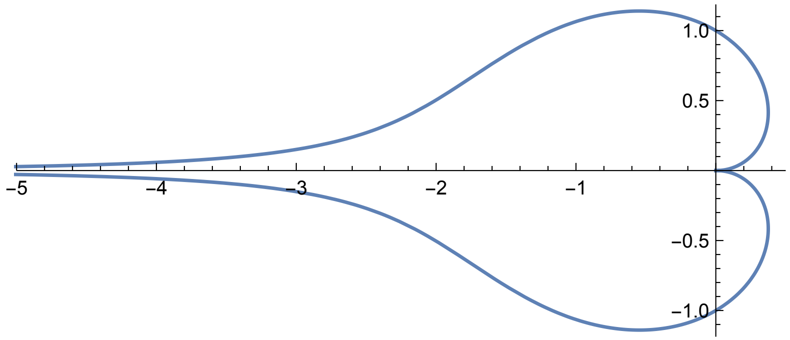
Dynamical Cobordism

A polar plot of the angle dependent warp factor:

ETW 7^- brane



ETW 7^+ brane



Open issues:

- Meaning of Solution I with topology $S^1 \times \mathbb{R}$? (no ETW)
- Unstable or related to gauging instead of breaking?
- Generalization of explicit ETW solutions?

  (Bhg,Kneißl,Wang, . . .)

Conclusions

Conclusions

- Gauging leads to tadpole cancellation conditions known from orientifolds and F-theory.
- K-theory provides the brane charges, cobordism the geometric contributions to tadpoles
- Dynamical cobordism provide a (super)gravity description of ETW branes

There are still open questions

- Generalization to type IIA?
- Explicit computation of $\Omega_n^{\mathrm{Spin+def};U(1)_p}$ classes?
- Determination of relative coefficients in tadpoles.
- Unique bottom-up result for the final $\Omega_n^{QG} = 0$?

  (Andriot,Carqueville,Cribiori, 2204.00021)