Swampy Perspectives on De Sitter Quantum Gravity

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Motivation

Motivation

In recent years, the (in)stability of de Sitter space in quantum gravity has been widely debated.

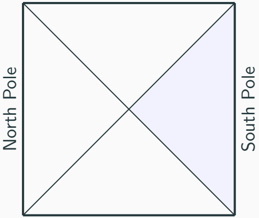
Does metastable de Sitter space belong to the landscape or swampland?
[cf. Mariana's and Severin's talk]

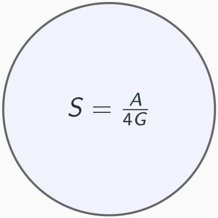
One angle: start from universal properties that are widely accepted, such as finiteness of S_{dS}.

The **goal** of this talk is to explore (some of) the constraints this poses on physics in de Sitter space.

Symmetries and Entropy

Consider the static patch of d-dimensional de Sitter space.





A static observer measure a thermal spectrum of particles with $T_{\mathrm{dS}} = \frac{1}{2\pi\ell}$ and associates an entropy to the horizon.

However, the isometries of the static patch are smaller than the whole maximally symmetric spacetime: $SO(d-1) \times R \subset SO(d,1)$.

Symmetries of De Sitter Quantum Gravity

- $\ell_p/\ell = \textit{finite} \Rightarrow$ Finite entropy leads to a discrete spectrum of energy eigenstates.
- It has been proven [Goheer, Kleban, Susskind '02] that this is inconsistent with having symmetry generators that mix different static patches.
- Thus, de Sitter quantum gravity does not have the symmetries of classical de Sitter.

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De Sitter space is in the swampland?

Depends on the time scale when these effects appear.

E.g. $t \sim \ell e^{S_{\rm dS}}$, $t \sim \ell S_{\rm dS}$ or $t \sim \ell \log S_{\rm dS}$.

JT Gravity in dS_2

Two-Dimensional Gravity Models

We will be interested in studying (gravitational) entropy in de Sitter space. Convenient to work in two dimensions.

$$I = \frac{1}{2\kappa^2}\int \mathrm{d}^2x\sqrt{-g}\Phi(R - 2/\ell^2) + (\text{matter})$$

Leads to EOM:

$$\begin{aligned} -\nabla_a\nabla_b\Phi + g_{ab}\Box\Phi + \frac{2\Phi}{\ell^2}g_{ab} - \kappa^2\langle T_{ab}\rangle = 0 \ , \\ R - 2/\ell^2 = 0 \ . \end{aligned}$$

These (backreacted) equations of motion can be solved analytically.

Dilaton Solutions

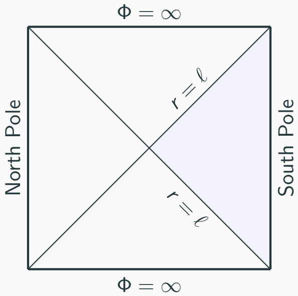
First we consider classical solutions: $T_{ab} = 0$. Different solutions exist, given by the Killing vectors manifest in different coordinate systems.

Static coordinates:

$$\mathrm{d}s^2 = -(1 - r^2/\ell^2)\mathrm{d}t^2 + (1 - r^2/\ell^2)^{-1}\mathrm{d}r^2 \ ,$$

$$\Phi = \phi_0 \frac{r}{\ell} \ .$$

Can think of $\Phi =$ *area* such that the entropy is $S_{\mathrm{dS}} = \frac{2\pi}{\kappa^2}\Phi(r = \ell)$.



Including Quantum Effects

We can also compute quantum effects by including a coupling to conformal matter. A choice of quantum state needs to be made.

Useful to consider Kruskal coordinates:

$$\mathrm{d}s^2 = -\frac{4\ell^4}{(\ell^2 - x^+x^-)^2}\mathrm{d}x^+\mathrm{d}x^- \qquad \text{Bunch-Davies:} \quad \langle T_{\pm\pm}\rangle = 0$$

$$\langle T_{+-}\rangle = \frac{c}{24\pi\ell^2}g_{+-}$$

Using coordinates natural for a static observer: $\sigma^{\pm} = \pm\ell\log(\pm x^{\pm}/\ell)$.

$$\langle : T_{\pm\pm} : \rangle = \frac{\pi c}{12\beta_{\mathrm{dS}}^2}\ , \quad \beta_{\mathrm{dS}} = 2\pi\ell$$

Static observers measure a thermal spectrum of particles.

Dilaton solution is:

$$\Phi = \frac{c\kappa^2}{24\pi} + \phi_0\frac{\ell^2 + x^+x^-}{\ell^2 - x^+x^-}\ .$$

Central Dogma of Cosmological Horizons

Thermodynamically, it is expected that the entropy of radiation grows linearly.

$$S_{\mathrm{rad}} \sim \frac{c}{\ell} t \ .$$

This can potentially lead to a violation of the "Central Dogma" of cosmological horizons:

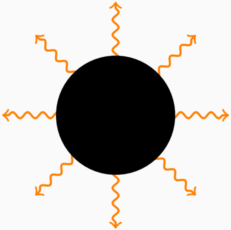
A static patch of de Sitter space can be described by a quantum system with $e^{S_{\mathrm{dS}}}$ states.

When $S_{\mathrm{rad}} > S_{\mathrm{dS}}$ this is violated. If true, we therefore expect corrections to semi-classical physics around $t \sim \ell S_{\mathrm{dS}}$.

Island Transitions

Generalized Entropy

In semi-classical gravity we are interested in generalized entropy, e.g. for a black hole.



$$S_{\mathrm{BH}} = \frac{A}{4G_N} + S_{\mathrm{matter}}$$

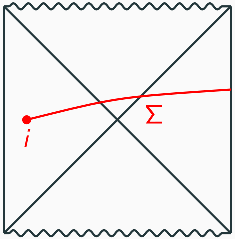
It is this quantity that obeys the usual thermodynamic laws, such as

$$\mathrm{d}S_{\mathrm{BH}} \geq 0 \ .$$

The matter entropy is the von Neumann entropy defined on a spatial slice.

Generalized Entropy

We now motivate a general expression for the entropy (see [Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini '20] for a review).



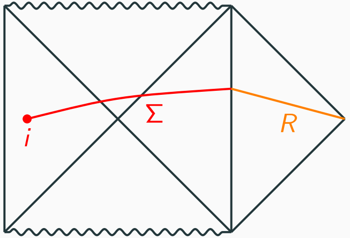
$$S_{\rm gen}(\Sigma) = \text{min,ext}_i \left[\frac{A(i)}{4G} + S_{\rm vN}(\Sigma)\right]$$

The entropy of Σ is found by extremizing this formula. Assuming a pure state ($S_{\rm gen}(\Sigma_{\rm tot})$=0), the region i shrinks to zero size.

Can there be non-trivial quantum extremal surfaces? Let us couple this system to a non-gravitational bath.

The Island Formula

If the system is in a pure state $S_{\rm gen}(\Sigma_{\rm tot}) = S_{\rm gen}(R)$. This suggests that the same formula holds for the entropy of R.



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Finally, making use of purity we can rewrite this as the island formula.

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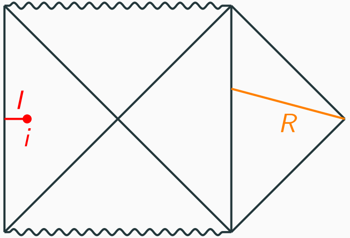

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Non-Perturbative Transition

The island formula can be derived from the Euclidean gravitational path integral. [Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini '19]

There, it arises as a new saddle point that is non-perturbatively suppressed as $\sim e^{-S}$.

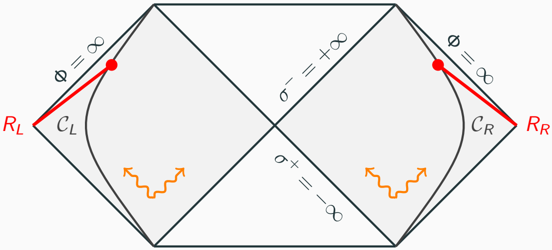
- When $S_{\mathrm{vN}}(R \cup I)$ is small, $I = \emptyset$.
- When $S_{\mathrm{vN}}(R \cup I)$ is large, a non-trivial island gives the dominant (lowest entropy) contribution.

Around a time $t \sim S/r_h$ this non-perturbative transition takes place.

Islands in de Sitter Space

Back to de Sitter. Within the static patch, there is no natural non-gravitational bath region.

Proposal: modify the geometry by glueing Rindler wedges. [LA, Aguilar-Gutierrez, Sybesma (WIP)]



We now have asymptotic non-gravitational regions, from which the cosmological horizon can be probed.

Rindler Solution and Junction Conditions

For this patched spacetime to be a solution to Einstein's equations, the Israel junction conditions need to be satisfied. [Engelhardt, Folkestad '22]

For a junction at location $x^- = 0$:

$$[\Phi]|_{x^-=0} = 0 \ ,$$
$$\kappa^2 T_{ab} l^a l^b + [l^a \nabla_a \Phi]\delta(x^-) = 0 \ .$$

The dilaton solutions are given by

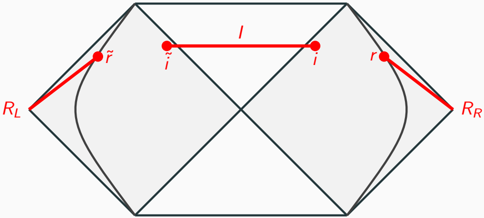
$$\text{de Sitter:} \quad \Phi = \frac{c\kappa^2}{24\pi} + \phi_0 \frac{\ell^2 + x^+ x^-}{\ell^2 - x^+ x^-} \ ,$$
$$\text{Rindler:} \quad \Phi = \frac{c\kappa^2}{24\pi} + \phi_0 - \lambda^2 x^+ x^- \ .$$

We can now compute the generalized entropy.

Searching for Islands

The generalized entropy of region R is given by

$$S_{\text{gen}}(R) = \frac{2\pi}{\kappa^2}\Phi(x_i^{\pm}) + \frac{c}{6}\log\left[\frac{x_{ir}^{+}x_{ir}^{-}}{\epsilon^2\Omega(x_i^{\pm})\Omega(x_r^{\pm})}\right] + (i \leftrightarrow \tilde{i}, r \leftrightarrow \tilde{r})$$



We now need to extremize this to find islands.

Island Results

At early times, the dominant island is the trivial one.

$$S_{\mathrm{gen}}(R) = S_{\mathrm{vN}}(R) = \frac{c}{3\ell} t \ .$$

At late times, a non-trivial island appears that saturates the growth.

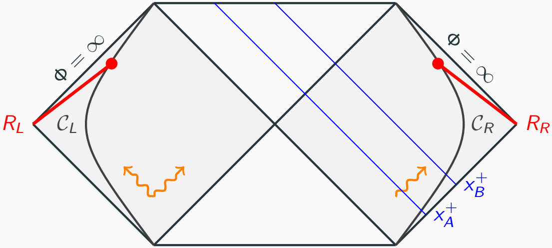
$$x_i^- = 0 \ , \qquad x_i^+ = -\frac{c\kappa^2}{12\pi\lambda^2 x_r^-} \ , \qquad \Rightarrow \qquad S_{\mathrm{gen}}(R) = \tfrac{c}{6} + \tfrac{4\pi\phi_0}{2\kappa^2} = 2S_{\mathrm{dS}} \ .$$

The island therefore shows there is a non-perturbative correction at $t \sim S_{\mathrm{dS}}/\ell$ to the entropy.

Breaking Thermal Equilibrium

We can go one step further and show that information can be recovered.

- In [LA, Sybesma '21], it was argued that a non-equilibrium state can be used that removes left-moving radiation.
- In our Rindler setup, this can be done without destroying the observer. [LA, Aguilar-Gutierrez, Sybesma (WIP)]



Information recovery in a controlled setting from the de Sitter horizon.

Universal Non-Locality of Quantum Gravity

- The main lesson is that the entanglement wedge of Hawking radiation can include regions behind the horizon.
- This seems to be true irrespective of the precise background under consideration and constitutes holography beyond AdS/CFT. [Bousso, Penington '22]
- Entropy is sensitive to a notion of non-locality present in quantum gravity.
- Are there low-energy observables that have this property? Interesting question for the swampland.

Outlook

Conclusions

- I've reviewed a simple argument that de Sitter space is in the swampland, in the sense that its isometry group cannot be realized.
- This suggests corrections to semiclassical physics in de Sitter space.
- I've argued that generalized entropy is a probe that is sensitive to these corrections in the form of islands.
- Are there other observables sensitive to this effect?
- A relatively unexplored area of the swampland.