03

Dark Matter cosmological production

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2022

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Freeze-out and WIMPs

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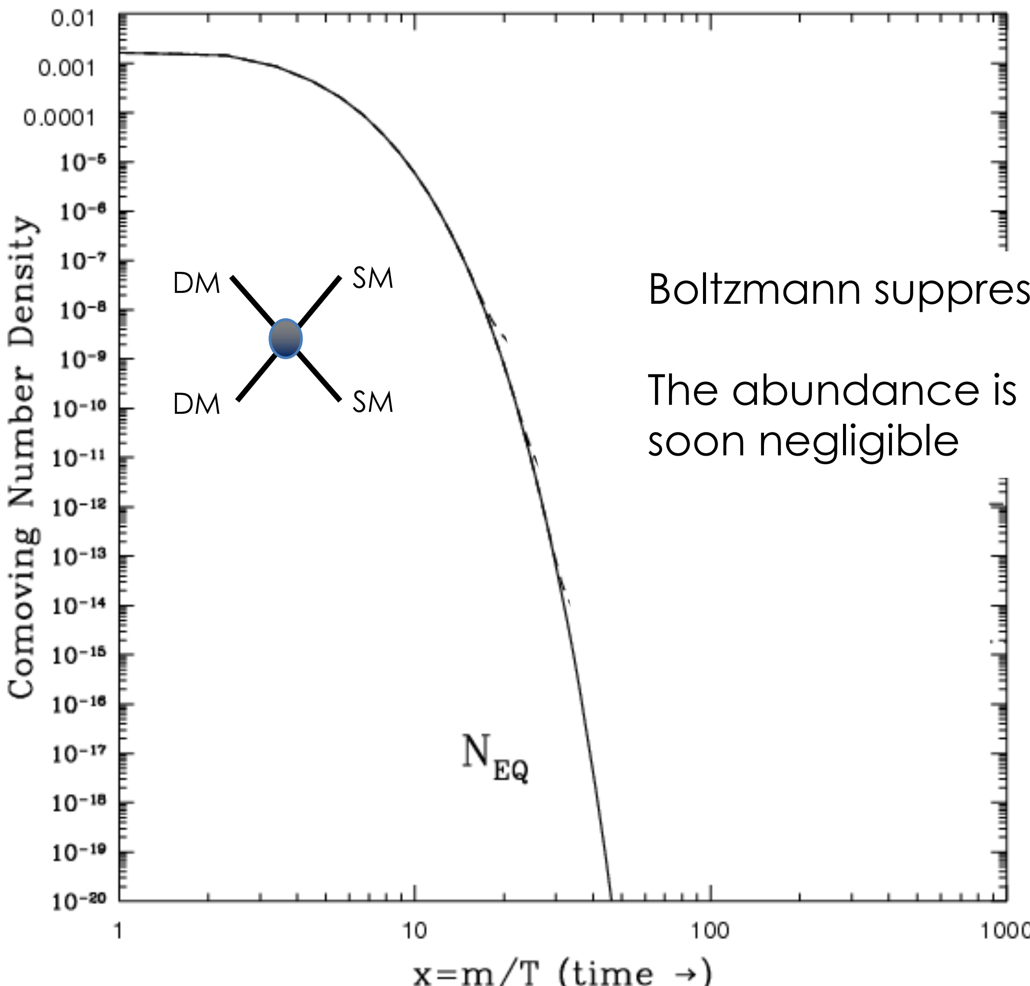
Yield at **equilibrium** for massive particles:

$$Y_{eq} = \frac{45}{2\pi^4}\left(\frac{\pi}{8}\right)^{1/2}\frac{g_{eff}}{g_{*s}}\left(\frac{m}{T}\right)^{3/2} e^{-m/T}$$



Boltzmann suppression at late times

The abundance is rapidly decreasing and is soon negligible

EXAMPLE 1.1

It is easy to estimate the value of the Yield that we need in order to reproduce the correct DM relic abundance, $\Omega h^2 \approx 0.1$, since

$$\Omega h^2 = \frac{\rho_\chi}{\rho_c} h^2 = \frac{m_\chi n_\chi h^2}{\rho_c} = \frac{m_\chi Y_0 s_0 h^2}{\rho_c} , \tag{1.9}$$

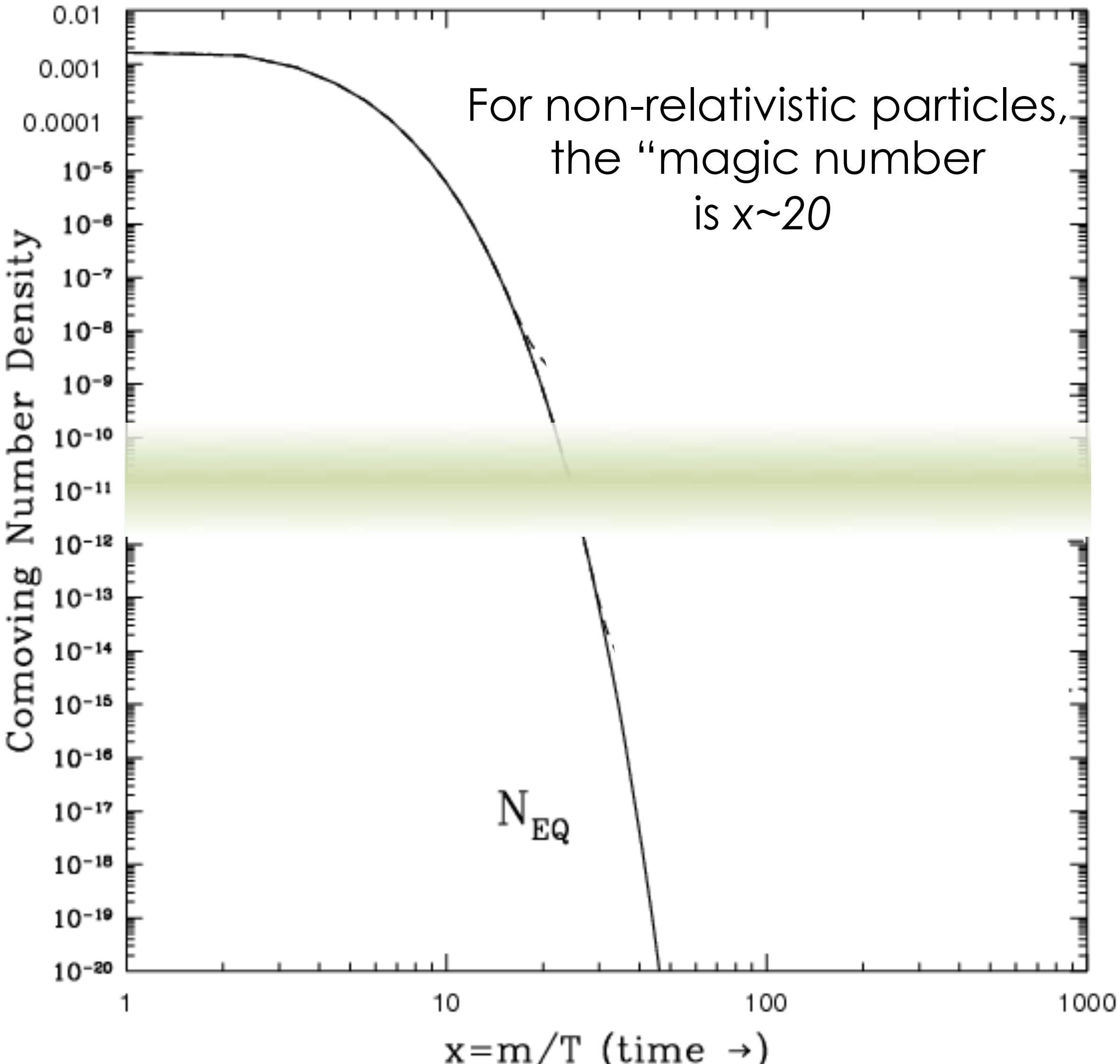
where Y_0 corresponds to the DM Yield today and s_0 is today's entropy density. We can assume that the Yield did not change since DM freeze-out and therefore

$$\Omega h^2 = \frac{m_\chi Y_f s_0 h^2}{\rho_c} . \tag{1.10}$$

Using the measured value $s_0 = 2970 \text{ cm}^{-3}$ and the value of the critical density $\rho_c = 1.054 \times 10^{-5} h^2 \text{ GeV cm}^{-3}$, as well as Planck's result on the DM relic abundance we arrive at

$$Y_f \approx 3.55 \times 10^{-10} \left(\frac{1 \text{ GeV}}{m_\chi} \right) . \tag{1.11}$$

$$Y_{eq} = \frac{45}{2\pi^4}\left(\frac{\pi}{8}\right)^{1/2}\frac{g_{eff}}{g_{*s}}\left(\frac{m}{T}\right)^{3/2} e^{-m/T}$$



For non-relativistic particles, the "magic number is x~*20*

For DM masses in the range 1 GeV – 1 TeV

The time evolution of the phase space distribution function is dictated by Liouville's operator (which ensures conservation of density in the phase space) and the Collisional operator, which encodes number changing processes

$$\hat{L}[f] = C[f]$$

The Liouville operator can be written in a covariant way

$$\hat{L} = \frac{d}{d\tau} = p^\mu \frac{\partial}{\partial x^\mu} - \Gamma^\mu_{\sigma\rho} p^\sigma p^\rho \frac{\partial}{\partial p^\mu}$$

Where the affine connection is related to derivatives of the metric as follows

$$\Gamma^\mu_{\nu\lambda} = \frac{1}{2} g^{\mu\sigma} (g_{\sigma\nu,\lambda} + g_{\sigma\lambda,\nu} - g_{\nu\lambda,\sigma})$$

Notice that this terms incorporates gravity and the actual geometry of space-time.

If we apply this to the FRW metric, which only depends on *t* and *E*

$$f(x^\mu, p^\mu) = f(t, E)$$

We find that Liouville operator can be greatly simplified

Exercise 1

$$\begin{aligned}\hat{L} &= E\frac{\partial}{\partial t} - \Gamma^0_{\sigma\rho}p^\sigma p^\rho \frac{\partial}{\partial E} \\ &= E\frac{\partial}{\partial t} - H|\mathbf{p}|^2\frac{\partial}{\partial E}\end{aligned}$$

Ultimately, we are interested in the time evolution of the number density

$$n = \frac{g}{2\pi^3}\int f(\mathbf{p})d^3p$$

Thus, we integrate Liouville's operator in the momentum space

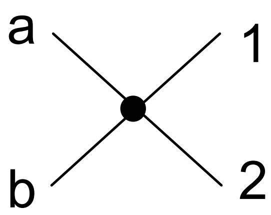
$$\frac{g}{2\pi^3}\int \hat{L}[f]\, d^3\mathbf{p} = \frac{g}{2\pi^3}\int C[f]\, d^3\mathbf{p}$$

Exercise 2

Prove the following relation

$$\frac{g}{(2\pi)^3}\int \frac{d^3\vec{p}}{E}\left[E\frac{\partial f}{\partial t} - H|\vec{p}|^2\frac{\partial f}{\partial E}\right] = \frac{dn}{dt} + 3Hn$$

Where we have divided by E for convenience



a,b=WIMP
1,2=SM (light) particles

$$d\Pi_i = \frac{g_i}{2\Pi^3}\frac{d^3\mathbf{p_i}}{2E_i}$$

No CP violation in DM sector

$$|\mathcal{M}_{12\to AB}|^2 = |\mathcal{M}_{AB\to 12}|^2$$

Energy Conservation

$$f_A f_B = f_A^{eq} f_B^{eq} = e^{-\frac{E_A+E_B}{T}} = e^{-\frac{E_1+E_2}{T}} = f_1^{eq} f_2^{eq}$$

$$\frac{g}{2\pi^3}\int \frac{C[f]}{E}\, d^3\mathbf{p} \quad = \quad - \int d\Pi_A d\Pi_B d\Pi_1 d\Pi_2 (2\pi)^4 \delta\left(p_A + p_B - p_1 - p_2\right) \left[|\mathcal{M}_{12\to AB}|^2 f_1 f_2 - |\mathcal{M}_{AB\to 12}|^2 f_A f_B\right]$$

$$= \quad -\langle\sigma v\rangle \left(n^2 - n_{eq}^2\right)$$

We have defined the thermally averaged annihilation cross section

$$\langle\sigma v\rangle \equiv \frac{1}{n_{eq}^2}\int d\Pi_A d\Pi_B d\Pi_1 d\Pi_2 (2\pi)^4 \delta\left(p_A + p_B - p_1 - p_2\right) |\mathcal{M}|^2 f_1^{eq} f_2^{eq}$$

The thermally averaged annihilation cross section can be expressed as

$$\langle \sigma v_{\mathrm{Møl}} \rangle (T) = \frac{1}{8 m_\chi^4 T K_2^2(m_\chi/T)} \int_{4m_\chi^2}^{\infty} ds \, \sigma(s)(s - 4m_\chi^2)\sqrt{s} K_1 \left(\frac{\sqrt{s}}{T} \right)$$

It is customary to Taylor-expand this expression for small T/m

$$\langle \sigma v \rangle_{ij} \approx a_{ij} + b_{ij} x.$$

$$a_{ij} = \frac{1}{m_\chi^2} \left(\frac{N_c}{32\pi} \beta(s, m_i, m_j) \frac{1}{2} \int_{-1}^{1} d \cos\theta_{CM} |\mathcal{M}_{\chi\chi \to ij}|^2 \right)_{s=4m_\chi^2}$$

$$\beta(s, m_i, m_j) = \left(1 - \frac{(m_i + m_j)^2}{s} \right)^{1/2} \left(1 - \frac{(m_i - m_j)^2}{s} \right)^{1/2}$$

Non-relativistic species

$$\frac{dn}{dt} + 3Hn - \langle\sigma v\rangle \left(n^2 - n_{eq}^2\right)$$

- $$\frac{dY}{dt} = \frac{d}{dt}\left(\frac{n}{s}\right) = \frac{d}{dt}\left(\frac{a^3 n}{a^3 s}\right) = \frac{1}{a^3 s}\left(3a^2 \dot{a} n + a^3 \frac{dn}{dt}\right) = \frac{1}{s}\left(3Hn + \frac{dn}{dt}\right)$$

- $$x = \frac{m}{T}$$

$$\frac{d}{dt}(a^3 s) = 0 \;\rightarrow\; \frac{d}{dt}(aT) = 0 \;\rightarrow\; \frac{d}{dt}\left(\frac{a}{x}\right) = 0 \quad \longrightarrow \quad \frac{dx}{dt} = Hx$$

$$\frac{dY}{dt} = \frac{dY}{dx}\frac{dx}{dt} = \frac{dY}{dx}Hx$$

$$\frac{dY}{dx} \;=\; -\frac{\lambda\langle\sigma v\rangle}{x^2}\left(Y^2 - Y_{eq}^2\right) \qquad \lambda \;\equiv\; \frac{2\pi^2}{45}\frac{M_P\, g_{*s}}{1.66\, g_*^{1/2}}\, m$$

Exercise 3

$$\frac{dY}{dx} = -\frac{\lambda\langle\sigma v\rangle}{x^2}\left(Y^2 - Y_{eq}^2\right)$$

$$\lambda \equiv \frac{2\pi^2}{45}\frac{M_P\, g_{*s}}{1.66\, g_*^{1/2}}\, m$$

$$\Delta_Y \equiv Y - Y_{eq}$$

↓

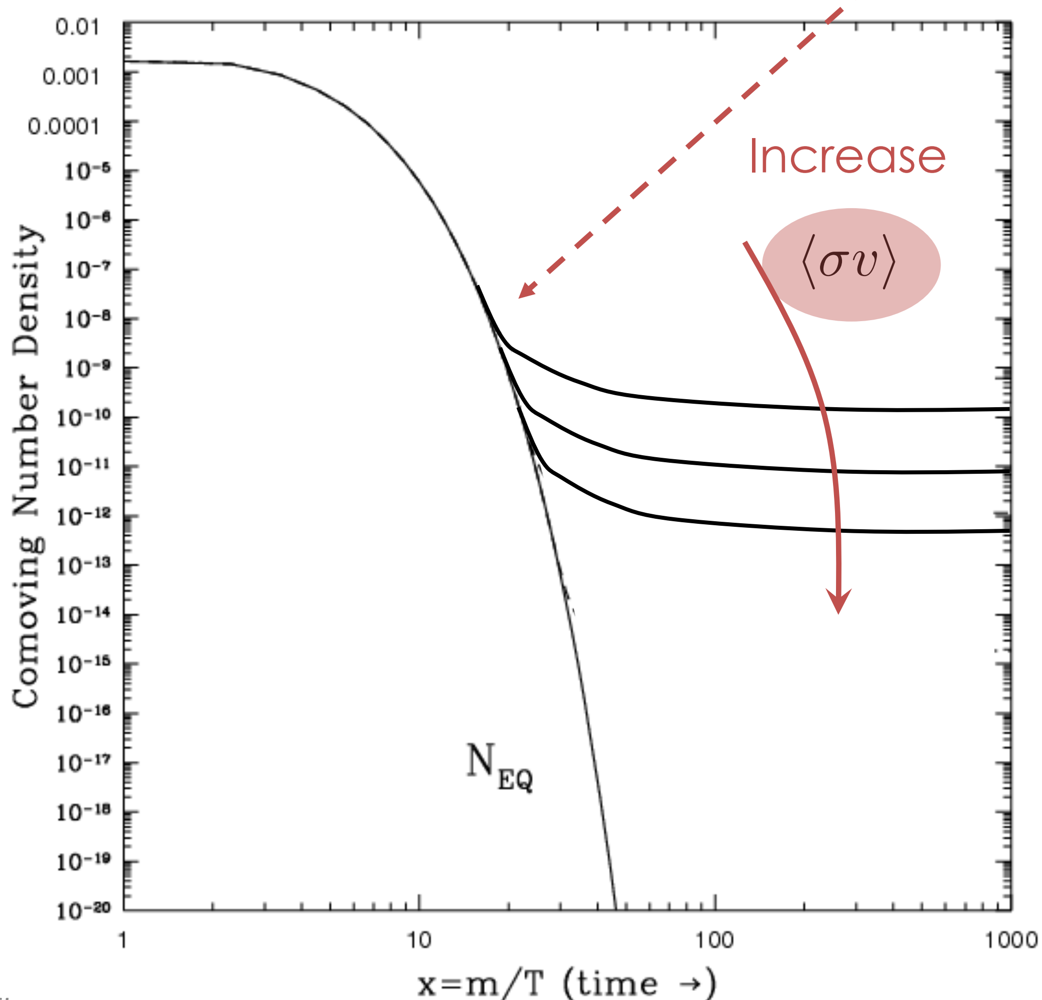
$$\Delta_Y = -\frac{\frac{dY_{eq}}{dx}}{Y_{eq}}\frac{x^2}{2\lambda\langle\sigma v\rangle}, \qquad 1 < x \ll x_f$$

$$\Delta_{Y_\infty} = Y_\infty = \frac{x_f}{\lambda\left(a + \frac{b}{3\,x_f}\right)}, \qquad x \gg x_f$$

This leads to :

$$\Omega h^2 = \frac{m_\chi Y_\infty s_0 h^2}{\rho_c} \approx \frac{10^{-10}\ \mathrm{GeV}^{-2}}{\left(a + \frac{b}{60}\right)} \approx \frac{10^{-27}\ \mathrm{cm}^3\ \mathrm{s}^{-1}}{\left(a + \frac{b}{60}\right)}$$

DM particles fall out of equilibrium at some point



The relic density reads

$$\Omega h^2 \approx \frac{3 \times 10^{-27}\,\mathrm{cm}^3\,s^{-1}}{\langle \sigma v \rangle}$$

It is related to their interaction scale!

Typical DM-SM coupling

g~ 0.01

ELECTROWEAK scale

02

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Special Cases

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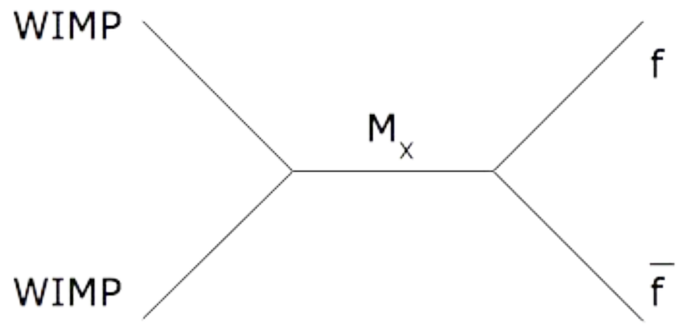
- Very different scales conjure up to lead to the electroweak scale

A typical electroweak scale cross section for a non-relativistic particle

$$\sigma v \approx \alpha^2 \frac{m^2}{M_W} = G_F^2 m^2$$

$$G_F \approx 10^{-5}\,\mathrm{GeV}^{-2}$$



Notice that this implies $$\Omega h^2 \sim \frac{1}{\langle \sigma_A v \rangle} \sim \frac{1}{m^2}$$ (non-relativistic particle)

Imposing $$\Omega \leq 1 \quad \rightarrow \quad m \leq 340\,\mathrm{TeV}$$ (Griest, Kamionkowski '90)

Special cases

- The low-temperature expansion for the annihilation cross section

$$\sigma_A v = a + \frac{b}{x}$$

is not valid in some cases:

Resonant annihilation

Thresholds

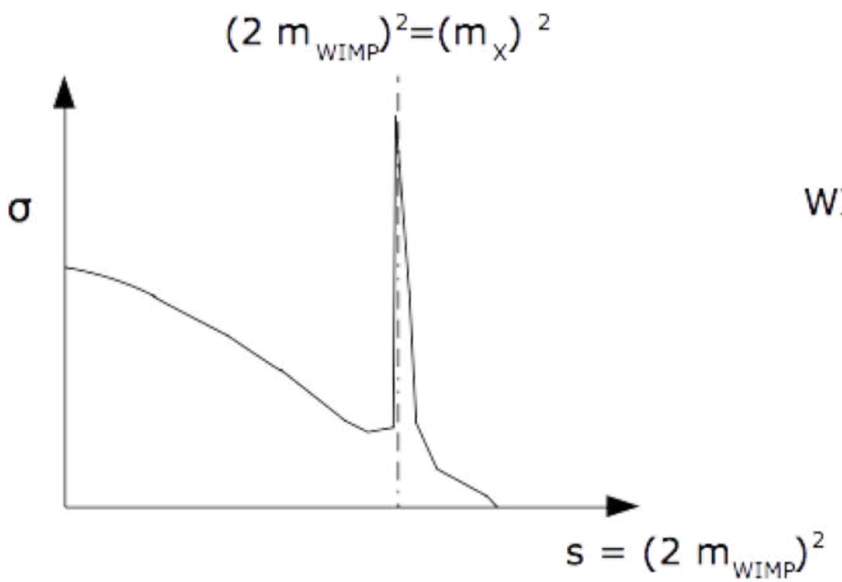
(Gondolo, Gelmini '91)

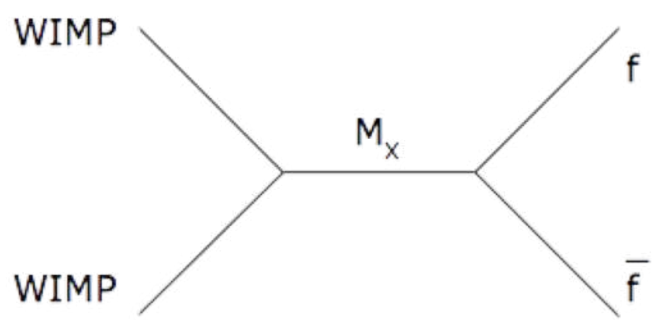
Coannihilations with other particles close in mass

(Griest, Seckel '91)

Special cases

- Resonant annihilation:





The resonant increase in the cross section implies a sharp decrease in the relic abundance.

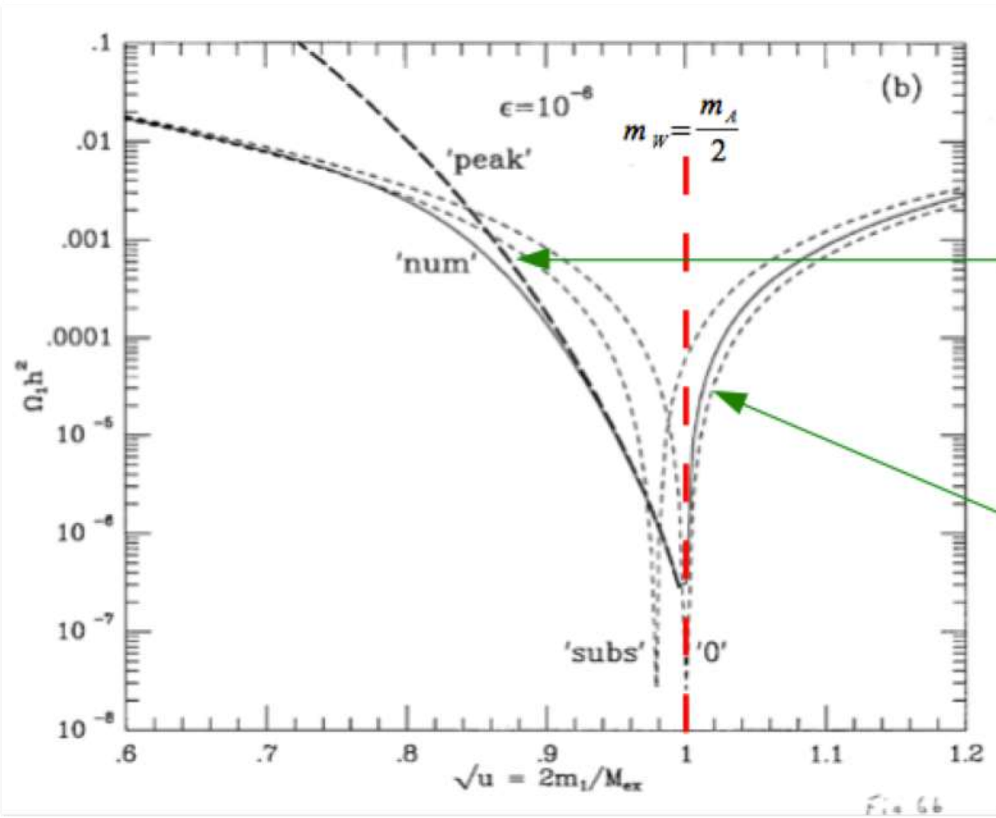
General expression for thermal average of annihilation cross section

(Gondolo, Gelmini '91)

Special cases

- Resonant annihilation

The annihilation cross section is significantly increased in the pole of the propagator.



As a consequence, the relic density decreases rapidly.

Thermal motion allows resonant annihilation when

$$m_W < \frac{m_A}{2}$$

This is not possible for

$$m_W > \frac{m_A}{2}$$

(Griest, Seckel '91)

The WIMP paradigm is extremely convenient

- It is “easy” to fit in BSM models (both minimal and complete)

  - Or it can be tuned with coannihilation or resonance effects

- It gives us hopes that DM can be observed in direct or indirect searches

- … or produced at the LHC

However WIMPs (may*) have not been observed yet

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Freeze-in Dark Matter

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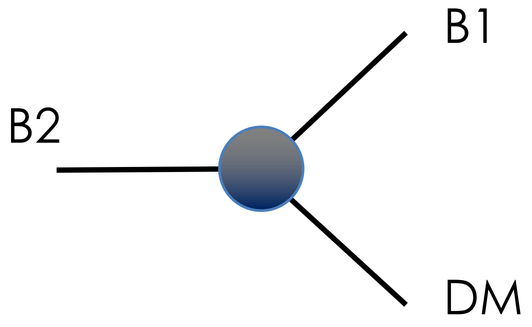
Freeze-in paradigm

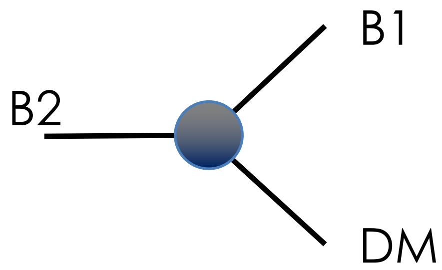
If the DM-SM coupling is extremely small, the annihilation rate is insufficient for thermal equilibrium.

However, annihilations or decays of particles in the bath can produce DM particles (that are out of equilibrium)

See e.g. Hall et al. 0911.1120



One can solve the associated Boltzmann equation



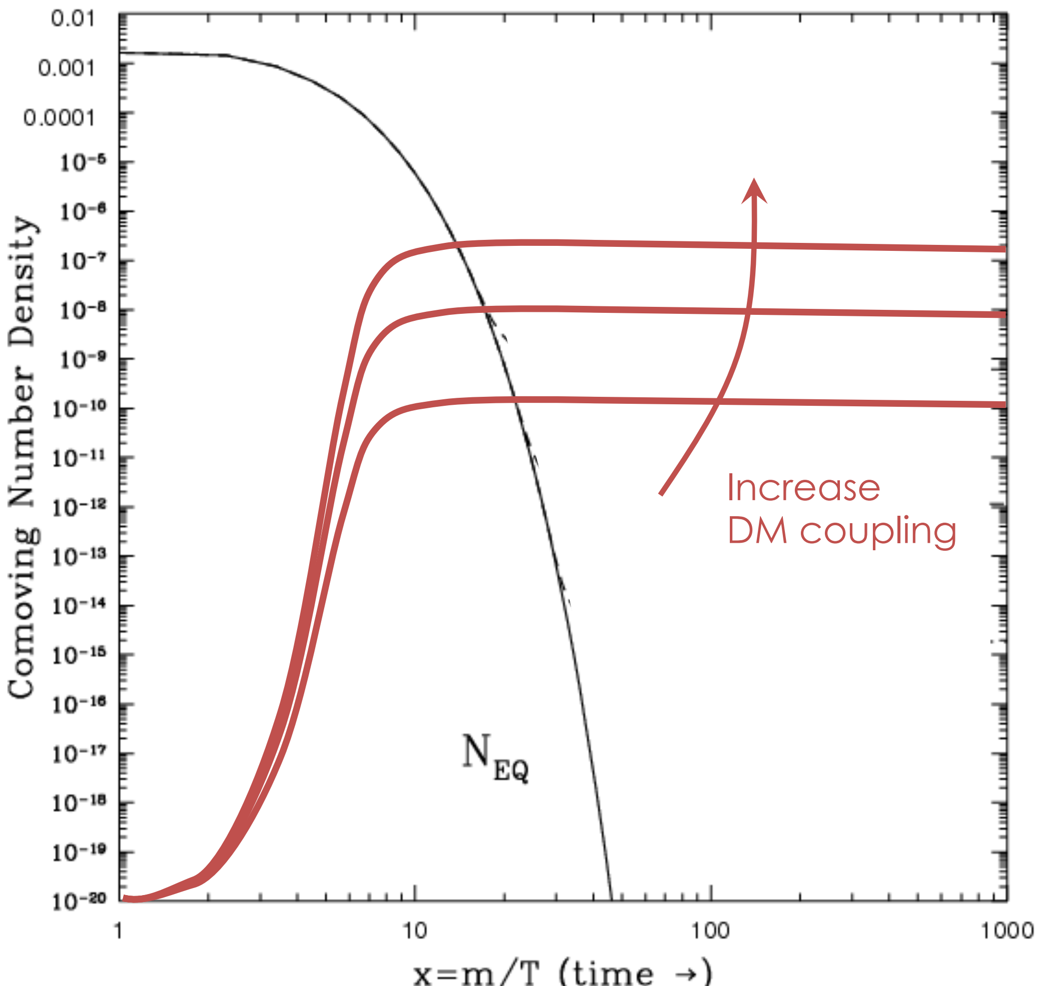
$$\frac{dn}{dt} + 3Hn = \frac{g}{(2\pi)^3} \int \frac{C[f]}{E} d^3\mathbf{p}$$

$$\begin{aligned} &= \int d\Pi_{B_1} d\Pi_{B_2} d\Pi_\chi \, (2\pi)^4 \delta^4 \left(p_{B_2} - p_{B_1} - p_\chi\right) \times \\ &\quad \left[\left|\mathcal{M}_{B_2 \to B_1 \chi}\right|^2 \, f_{B_2} (1 \pm f_{B_1})(1 \pm f_\chi) - \left|\mathcal{M}_{B_1 \chi \to B_2}\right|^2 \, f_{B_1} f_\chi (1 \pm f_{B_2}) \right] \\ &= \int d\Pi_{B2} \Gamma_{B2} 2 g_{B2} m_{B2} f_{B2} \, . \end{aligned} \tag{2.64}$$

$$Y = \frac{45 g_{B_2} M_p \Gamma_{B_2}}{4\pi^4 (1.66) m_{B_2}^2 g_*^S \sqrt{g_*}} \int_{x_{min}}^{\infty} K_1(x) \, x^3 \, dx$$

The abundance of **FIMPs** builds up and eventually stabilises

$$Y \approx \frac{135\, g_{B_2}}{8\pi^3 (1.66) g_*^S \sqrt{g_*}} \left(\frac{\Gamma_{B2} M_p}{m_{B_2}^2} \right)$$



The relic density is proportional to the SM particle decay rate.

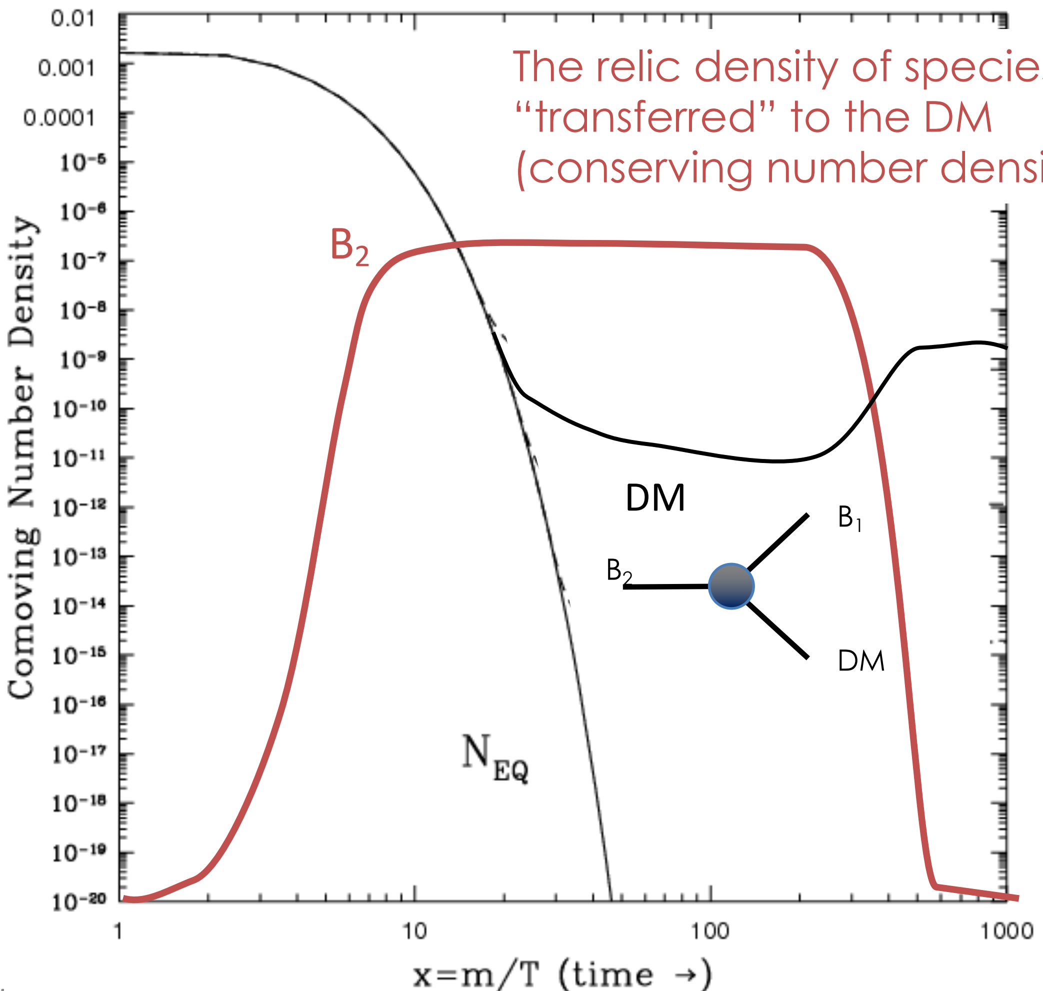
Depending on the actual process, the correct relic abundance can be obtained for

$$\lambda \approx 10^{-13}$$

One can also have “mixed” scenarios in which a particle that has decoupled decays (late) into the DM

The relic density of species B_2 is “transferred” to the DM (conserving number density)

$$\Omega_{DM} h^2 = \Omega_{B_2} h^2 \frac{m_{DM}}{m_{B_2}}$$



The DM therefore can have “thermal” and “non-thermal” contributions