02

Cosmo 101

Abundance of Relativistic Species

DAVID G. CERDEÑO
2022

Universidad Autónoma de Madrid

Some basics on Dark Matter Production

Dark matter was present in the Early Universe and it is present now, however, there are many different mechanisms to account for its correct abundance

- Thermal production (freeze-out)
- Out of equilibrium production (freeze-in)
- Late decays of unstable exotics
- Vacuum misalignment (axions)
- Asymmetry

Cosmology 101

Friedmann-Lemaître-Robertson-Walker (FLRW) metric for a homogeneous and isotropic universe that is expanding (or contracting)

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) = g_{\mu\nu}dx^\mu dx^\nu$$

k = curvature

Components of the metric

$$g_{tt} = 1$$

$$g_{rr} = \frac{-a^2(t)}{1-kr^2}$$

$$g_{\theta\theta} = -r^2 a^2(t)$$

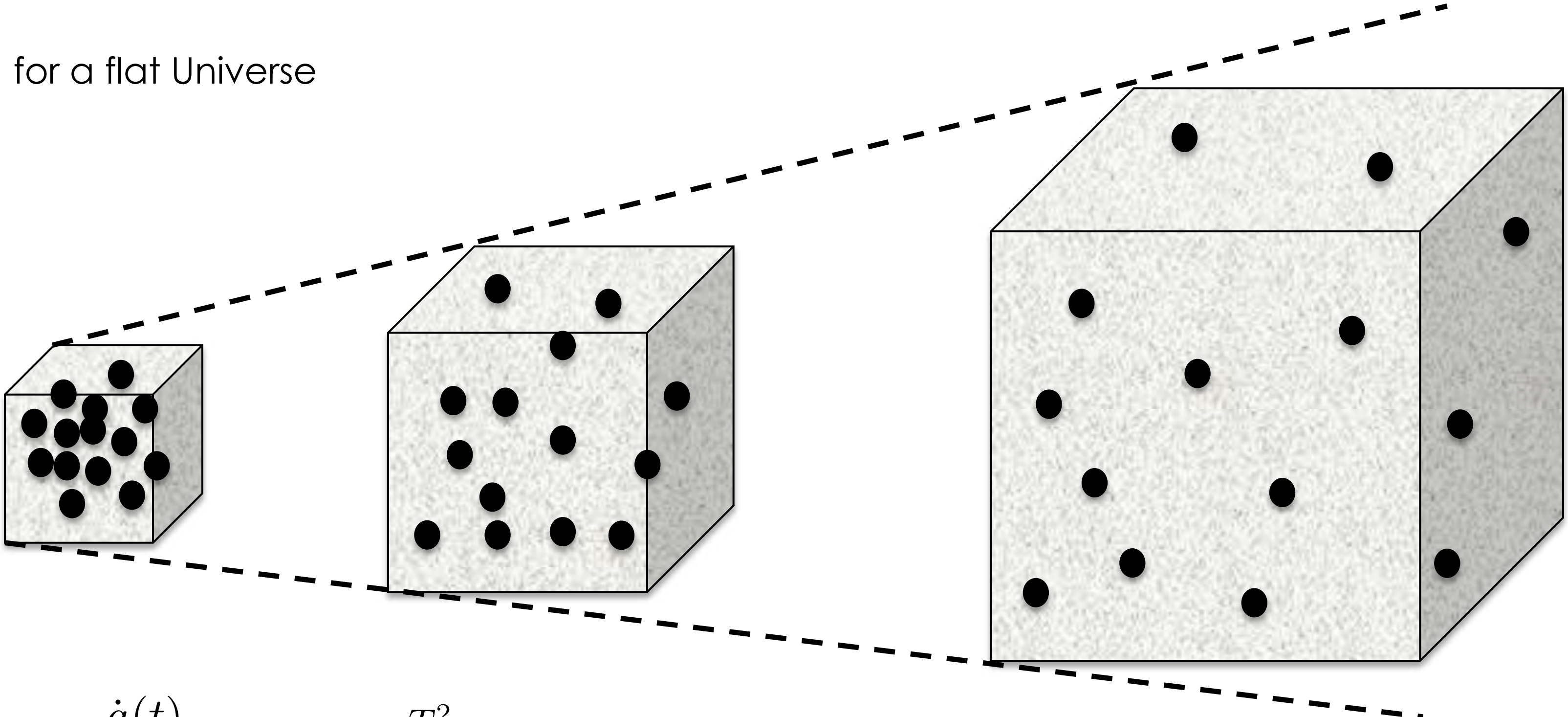
$$g_{\phi\phi} = -r^2 \sin^2\theta a^2(t)$$

a(t) is the scale parameter

WIMP dilution

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)$$

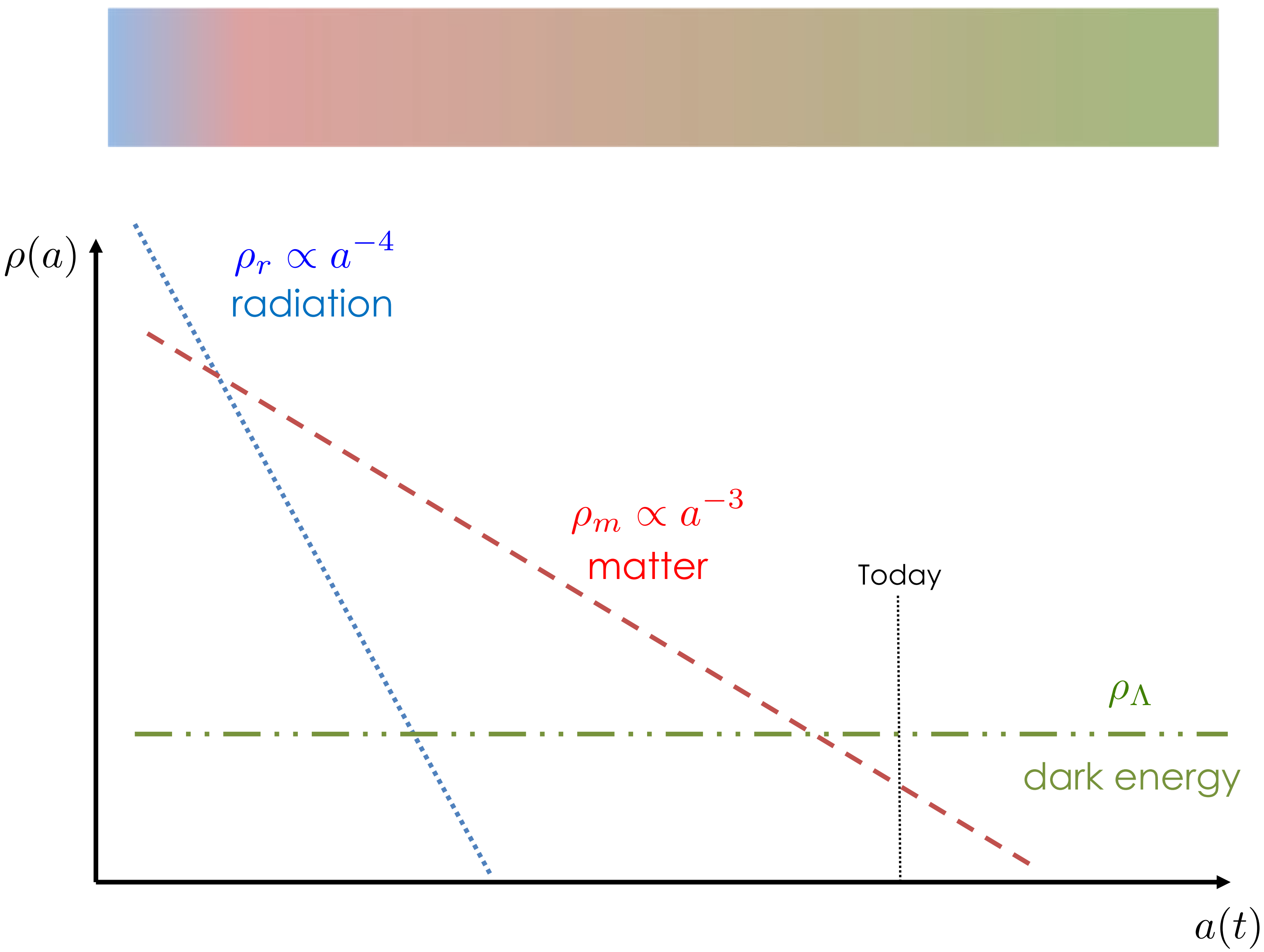
k=0 for a flat Universe



$$H = \frac{\dot{a}(t)}{a(t)} = 1.66\, g_*^{1/2}\frac{T^2}{M_P}$$

$$M_P = 1.22 \times 10^{19}\ \mathrm{GeV}$$

Event	time t	redshift z	temperature T
Inflation	10^{-34} s (?)	–	–
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling	1 s	6×10^9	1 MeV
Electron-positron annihilation	6 s	2×10^9	500 keV
Big Bang nucleosynthesis	3 min	4×10^8	100 keV
Matter-radiation equality	60 kyr	3400	0.75 eV
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV
Reionization	100–400 Myr	11–30	2.6–7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.8 Gyr	0	0.24 meV



$\rho(a)$
$\rho_r \propto a^{-4}$
radiation
$\rho_m \propto a^{-3}$
matter
Today
ρ_Λ
dark energy
$a(t)$

A system of particles in kinetic equilibrium has a phase space occupancy f given by the Bose-Einstein or Fermi-Dirac distributions at temperature T:

$$f(\mathbf{p}) = \frac{1}{e^{\frac{E-\mu}{T}} \pm 1}$$

The phase space distribution allows one to compute the associated number density n, energy density ρ and pressure p for a dilute and weakly-interacting gas of particles with g internal degrees of freedom:

$$\begin{aligned} n &= \frac{g}{(2\pi)^3} \int f(\mathbf{p})\, d^3p, \\ \rho &= \frac{g}{(2\pi)^3} \int E(\mathbf{p})\, f(\mathbf{p})\, d^3p, \\ p &= \frac{g}{(2\pi)^3} \int \frac{|\mathbf{p}|^2}{3E(\mathbf{p})} f(\mathbf{p})\, d^3p. \end{aligned}$$

Relativistic particles

$$T \gg m \qquad E \sim |\mathbf{p}|$$

$$\boxed{\begin{aligned} n_b &= \frac{g}{\pi^2}\zeta(3)T^3 & \rho_b &= \frac{\pi^2}{30} g T^4 \\ n_f &= \frac{3}{4}\frac{g}{\pi^2}\zeta(3)T^3 & \rho_f &= \frac{7}{8}\frac{\pi^2}{30} g T^4 \end{aligned}}$$

Non-Relativistic particles

$$T \ll m, \qquad E = (|\mathbf{p}|^2 + m^2)^{1/2} = m\left(1 + \frac{|\mathbf{p}|^2}{m^2}\right)^{1/2} \simeq m + \frac{|\mathbf{p}|^2}{2m}$$

$$\boxed{n \simeq g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}}$$

It is customary to define the Yield (equivalent to the number density but in a comoving volume) in terms of the entropy density (which scales as $a^3(t)$)

$$Y = \frac{n}{s} \qquad s = \frac{2\pi^2}{45} g_{*s} T^3$$

For relativistic particles, we have

$$n = \frac{g_{eff}}{\pi^2}\zeta(3)T^3 \longrightarrow Y_{eq} = \frac{45}{2\pi^4}\zeta(3)\frac{g_{eff}}{g_{*s}}$$

For non-relativistic particles, we have

$$n = g_{eff}\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T} \longrightarrow Y_{eq} = \frac{45}{2\pi^4}\left(\frac{\pi}{8}\right)^{1/2}\frac{g_{eff}}{g_{*s}}\left(\frac{m}{T}\right)^{3/2} e^{-m/T}$$

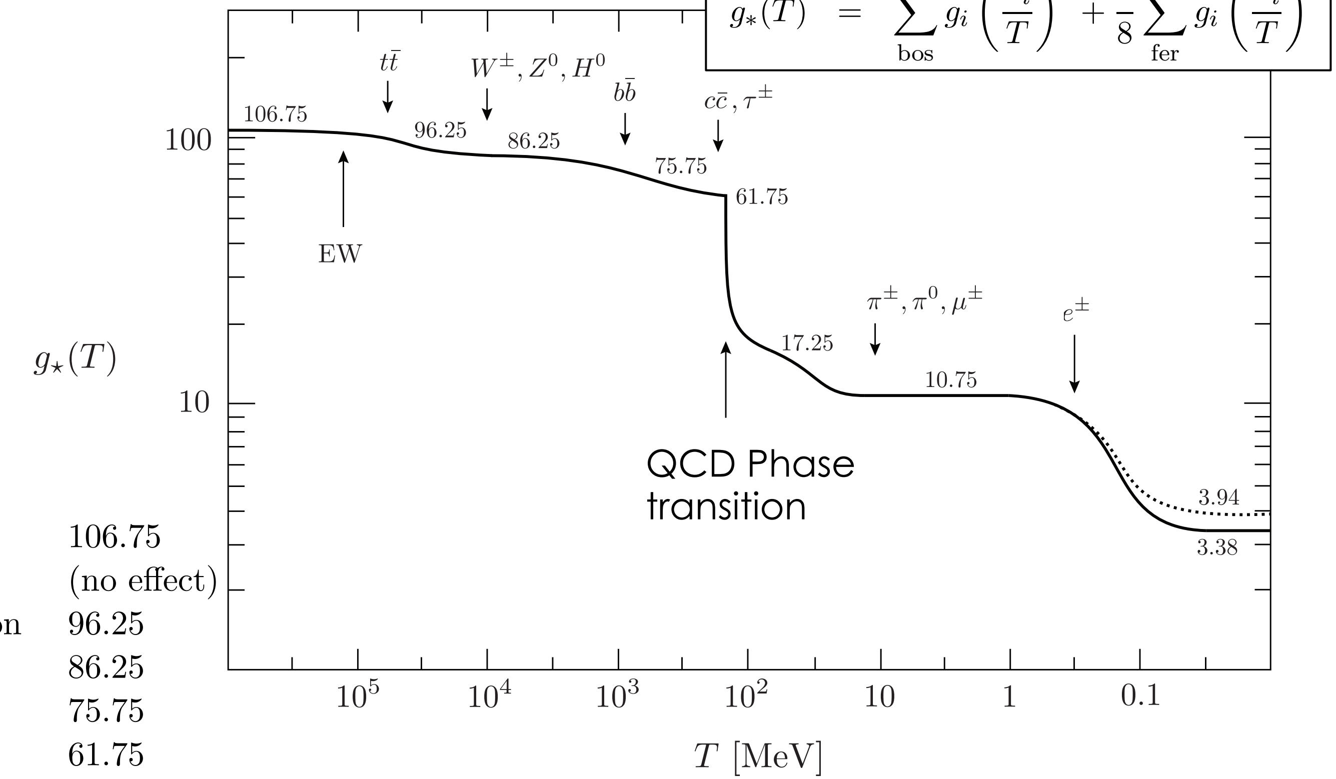
Number of relativistic degrees of freedom in the Standard Model

Quarks	t	$174.2 \pm 3.3\mathrm{GeV}$	$\bar{t}$	spin=$\frac{1}{2}$	$g = 2 \cdot 2 \cdot 3 = 12$	
	b	$4.20 \pm 0.07\mathrm{GeV}$	$\bar{b}$	3 colors		
	c	$1.25 \pm 0.09\mathrm{GeV}$	$\bar{c}$			
	s	$95 \pm 25\mathrm{MeV}$	$\bar{s}$			
	d	3–7MeV	$\bar{d}$			
	u	1.5–3.0MeV	$\bar{u}$			
						72
Gluons	8 massless bosons			spin=1	$g = 2$	16
Leptons	τ^-	$1777.0 \pm 0.3\mathrm{MeV}$	τ^+	spin=$\frac{1}{2}$	$g = 2 \cdot 2 = 4$	
	μ^-	105.658MeV	μ^+			
	e^-	510.999keV	e^+			
						12
	ν_τ	$< 18.2\mathrm{MeV}$	$\bar{\nu}_\tau$	spin=$\frac{1}{2}$	$g = 2$	
	ν_μ	$< 190\mathrm{keV}$	$\bar{\nu}_\mu$			
	ν_e	< 2 eV	$\bar{\nu}_e$			
						6
Electroweak gauge bosons	W^+	$80.403 \pm 0.029\mathrm{GeV}$		spin=1	$g = 3$	
	W^-	$80.403 \pm 0.029\mathrm{GeV}$				
	Z^0	$91.1876 \pm 0.0021\mathrm{GeV}$				
	γ	$0 \quad (< 6 \times 10^{-17}\mathrm{eV})$			$g = 2$	
						11
Higgs boson (SM)	H^0	*125.5* GeV		spin=0	$g = 1$	1

$$g_f = 72 + 12 + 6 = 90$$
$$g_b = 16 + 11 + 1 = 28$$

Number of relativistic degrees of freedom in the Standard Model

$$g_*(T) = \sum_{\text{bos}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{\text{fer}} g_i \left(\frac{T_i}{T}\right)^4$$



$T \sim 200$ GeV	all present	106.75	
$T \sim 100$ GeV	EW transition	(no effect)	
$T < 170$ GeV	top annihilation	96.25	
$T < 80$ GeV	$W^\pm$, Z^0, H^0	86.25	
$T < 4$ GeV	bottom	75.75	
$T < 1$ GeV	charm, τ^-	61.75	
$T \sim 150$ MeV	QCD transition	17.25	(u,d,g$\rightarrow \pi^{\pm,0}$, $37 \rightarrow 3$)
$T < 100$ MeV	$\pi^\pm$, π^0, μ^-	10.75	$e^\pm$, ν, $\bar{\nu}$, γ left
$T < 500$ keV	e^- annihilation	(7.25)	$2 + 5.25(4/11)^{4/3} = 3.36$

QCD Phase transition $T \sim 150 \text{ MeV}, t \sim 20 \; \mu\text{s}.$

The temperature and thus the quark energies have fallen so that the quarks lose their asymptotic freedom

There are no more free quarks and gluons; the quark-gluon plasma has become a hadron gas

The lightest baryons are the nucleons: the proton and the neutron. The lightest mesons are the pions

all except pions are nonrelativistic below the QCD phase transition temperature.

Thus the only particle species left in large numbers are the pions (g=3), muons (4), electrons (4), neutrinos (2x3), and the photons (2).

$g_*=17.25$

Freeze-out of relativistic species

Since neutrinos decouple while they are still relativistic, their yield reads

$$Y_{eq} \approx 0.278 \, \frac{g_{eff}}{g_{*s}} \, . \tag{2.41}$$

Neutrinos decouple at a few MeV, when the species that were still relativistic are $e^{\pm}$, γ, ν and $\bar{\nu}$. Thus, the number of relativistic degrees of freedom is $g_* = g_{*s} = 10.75$. For one neutrino family, the effective number of degrees of freedom is $g_{eff} = 3g/4 = 3/2$. Using these values, the relic density today an be written as

$$\begin{aligned} \Omega h^2 &= \frac{\sum_i m_{\nu_i} Y_\infty s_0 h^2}{\rho_c} \\ &\approx \frac{\sum_i m_{\nu_i}}{91\,\mathrm{eV}} \, . \end{aligned} \tag{2.42}$$