Probing the mixed regime of ChiPT with mixed actions

I. Introduction and spectral observables

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CLS based

Themes

Mixed actions: use different regularisations for sea and valence quarks to optimal effect.

$\Rightarrow$ chiral symmetry / renormalisation

Mixed chiral regimes: explore low-energy QCD with quark masses in different kinematical regimes.

$\Rightarrow$ systematic uncertainties in LECs

Outline

- Mixed-action approach to Lattice QCD.

- (continuum) Chiral Perturbation Theory.
  - ChiPT and finite volume regimes.
  - Random Matrix Theory.
  - ChiPT in a mixed regime.

- Results for spectral observables in N_f=2 QCD.
  - Topology.
  - Low-energy couplings: chiral condensate and L's.

Mixed action

Best of two worlds? “Cheap” sea sector with broken chiral symmetry, valence quarks with exact chiral symmetry.

Bär, Rupak, Shoresh 02-03

Valence: Neuberger fermions.

Sea: non-perturbatively O(a) improved Wilson fermions (N_f=2 CLS configurations).

sophisticated Neuberger fermion setup:
Giusti et al. 2003-2006

Mixed action — pros and cons

✓ Direct access to chiral limit in valence sector.

- (optimistic:) Greater possibilities than other regularisations.
- (pragmatic:) Handle on systematics.

  finite volume/mixed regimes, saturation of correlators with topological zero modes, ...

- Wilson sea samples all topological sectors, Neuberger valence leads to well-defined topological charge.

✓ Simplified renormalisation → phenomenology.

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✓ Simplified renormalisation → phenomenology.

– Neuberger fermions significantly more expensive than e.g. Wilson.

– Different cutoff dependence in valence and sea + partial quenching ⇒ potentially large lattice artifacts.

Golterman, Izubuchi, Shamir PRD 71 (2005)
Dürr et al./Aubin, Laiho, van de Water PoS LAT2007
Cichy, Herdoíza, Jansen 2011

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this talk: use Neuberger-Dirac operator as probe for chiral symmetry breaking (topology, spectral quantities).

Chiral Perturbation Theory

Subtle dichotomy:

- Guide mass extrapolations, circumvent difficult problems (weak decays).
- Test ChiPT: first-principles LECs, check predictions for m,V dependence.

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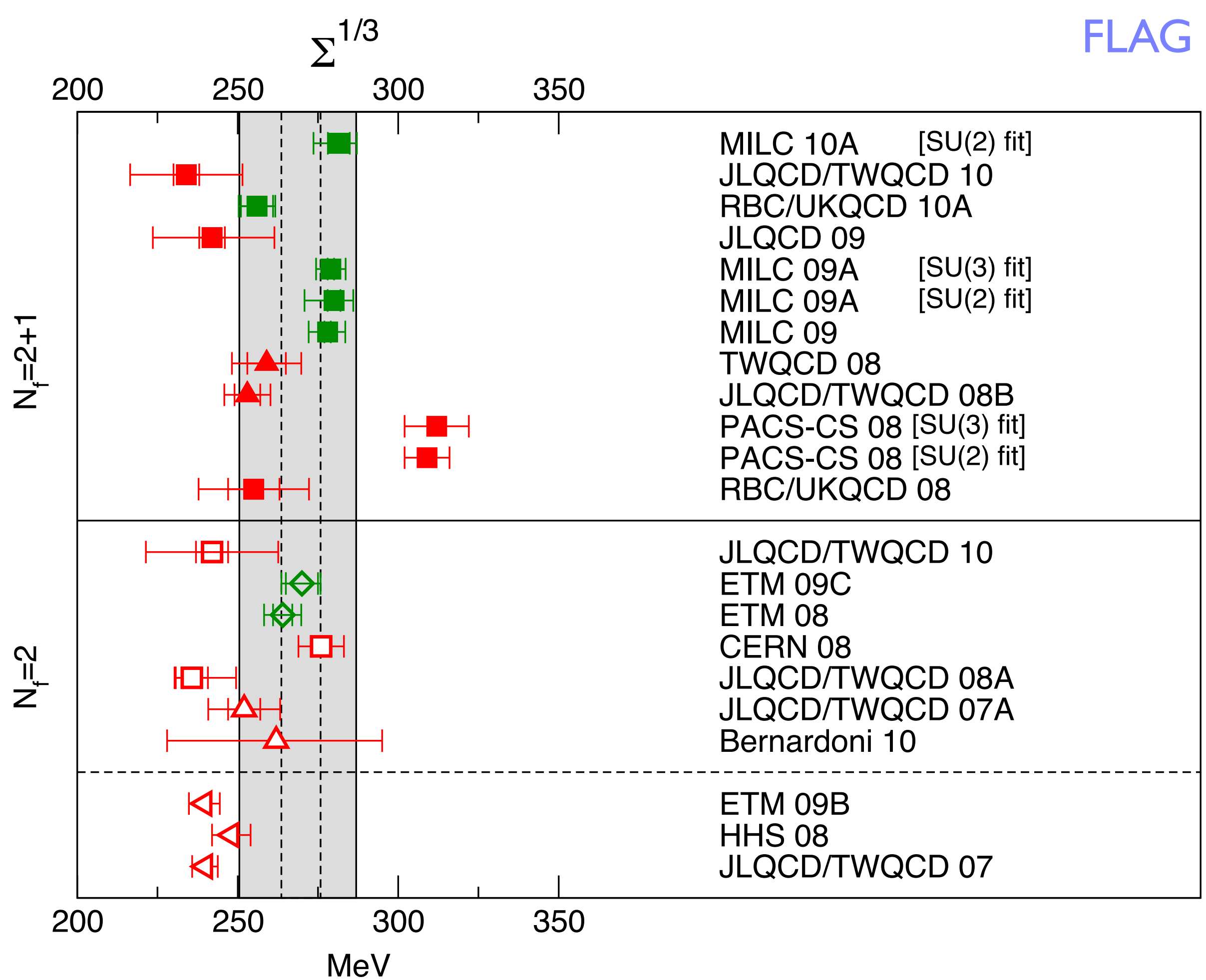
check systematic uncertainties!



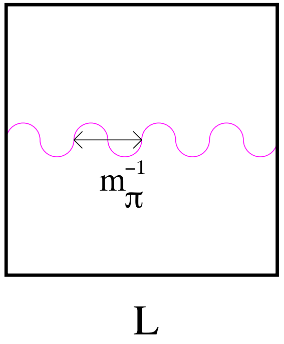
Finite volume scaling and chiral regimes

Gasser, Leutwyler 1987;
Hansen 1990; Hansen, Leutwyler 1991

p-regime: $\lim_{m \to 0} [m_\pi L \gg 1, 4\pi F \gg L^{-1}]$

standard χPT in finite V: $m \sim p^2 \qquad L^{-1}, T^{-1} \sim p$

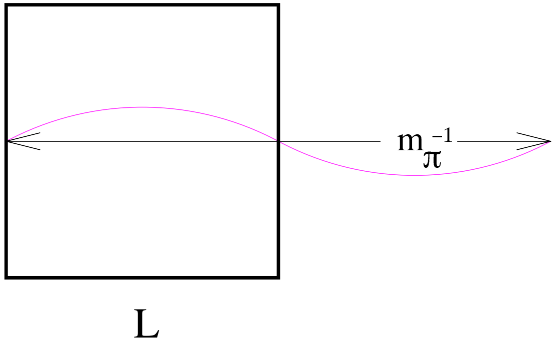
mass, volume effects relevant



ε-regime: $\lim_{m \to 0} [m \Sigma V \sim \mathcal{O}(1), 4\pi F \gg L^{-1}]$

reordering of the χ expansion: $m \sim p^4 \sim \epsilon^4 \qquad L^{-1}, T^{-1} \sim \epsilon$

volume effects relevant, <u>mass effects suppressed</u>



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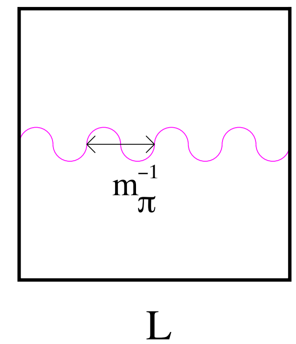

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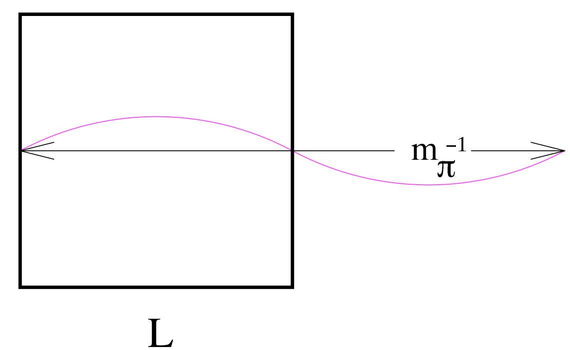

Low energy constants universal: check systematics of matching to QCD

Finite volume scaling and chiral regimes

Gasser, Leutwyler 1987;
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NLO: $\chi \equiv MU \quad \mathcal{L}_\mu \equiv i\partial_\mu U U^\dagger, \quad \mathcal{W}_{\mu\nu} = 2(\partial_\mu \mathcal{L}_\nu + \partial_\nu \mathcal{L}_\mu); \quad (\Delta_{ij})_{ab} = \delta_{ai}\delta_{bj}$

			p-regime	ϵ-regime
Gasser, Leutwyler	$\mathcal{L}_{QCD}$	$L_4 \langle D_\mu U^\dagger D^\mu U\rangle \langle U^\dagger \chi + \chi^\dagger U\rangle$		$\times$
		$L_5 \langle D_\mu U^\dagger D^\mu U \left(U^\dagger \chi + \chi^\dagger U\right)\rangle$		$\times$
		$L_6 \langle U^\dagger \chi + \chi^\dagger U\rangle^2$		$\times$
		$L_8 \langle \chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi\rangle$		$\times$
Kambor, Missimer, Wyler	$\mathcal{H}_{weak}^{SU(4)}$	$D_2^\pm t_{ij,kl}^\pm \langle \Delta_{ij}(\chi - \chi^\dagger)\rangle\langle \Delta_{kl}(\chi - \chi^\dagger)\rangle$		$\times$
		$D_4^\pm t_{ij,kl}^\pm \langle \Delta_{ij}\mathcal{L}_\mu\rangle\langle \Delta_{kl}\left\{\mathcal{L}^\mu, (\chi + \chi^\dagger)\right\}\rangle$		$\times$
		$D_7^\pm t_{ij,kl}^\pm \langle \Delta_{ij}\mathcal{L}_\mu\rangle\langle \Delta_{kl}\mathcal{L}_\mu\rangle\langle(\chi + \chi^\dagger)\rangle$		$\times$
		$D_{20}^\pm t_{ij,kl}^\pm \langle \Delta_{ij}\mathcal{L}_\mu\rangle\langle \Delta_{kl}\partial_\nu \mathcal{W}_{\mu\nu}\rangle$		$\times$
		$D_{24}^\pm t_{ij,kl}^\pm \langle \Delta_{ij}\mathcal{W}_{\mu\nu}\rangle\langle \Delta_{kl}\mathcal{W}_{\mu\nu}\rangle$		$\times$

ChiPT in the ε-regime

Pion zero-momentum modes become non-perturbative.

Gasser, Leutwyler 1987;
Hansen 1990; Hansen, Leutwyler 1991

$$U = U_0\, e^{2i\xi(x)/F} \qquad \int \mathrm{d}^4x \xi(x) = 0$$

$$Z = \int_{\mathrm{SU}(N_f)} \mathrm{d}U_0 \int \mathrm{d}^4x\, J(\xi) e^{-S(U_0,\xi)}$$

"Exact" factorisation at LO:

$$S^{\mathrm{LO}}(U_0,\xi) = \int \mathrm{d}^4x\, \mathrm{Tr}[\partial_\mu\xi\partial_\mu\xi] - \frac{\Sigma V}{2}\mathrm{Tr}\left[MU_0 + U_0^\dagger M\right]$$

Integrals over zero-mode manifold can be done exactly via master integral.

$$\mathcal{Z}(N_\mathrm{f}, M, \theta) = \int_{\mathrm{SU}(N_\mathrm{f})} \mathrm{d}U_0\, \exp\left\{\frac{\Sigma V}{2}\mathrm{Tr}\left[e^{i\theta/N_\mathrm{f}} MU_0 \,+\, \mathrm{h.c.}\right]\right\}$$

$$\mathcal{Z}_\nu(N_\mathrm{f}, \mu) = \int \frac{\mathrm{d}\theta}{2\pi} e^{-i\nu\theta}\, \mathcal{Z}^{(0)} = \int_{\mathrm{U}(N_\mathrm{f})} \mathrm{d}U_0\, \det(U_0)^\nu\, e^{\frac{\mu}{2}\mathrm{Tr}[U_0+U_0^\dagger]} = \det_{ij}\, I_{i-j+\nu}(\mu)$$

Brower, Rossi, Tan 1981

ChiPT in the ε-regime: the role of topology

Correlation functions in the ε-regime depend on topology.

Leutwyler, Smilga 1992

$$D^{-1} = \sum_k \frac{\phi_k(x) \otimes \phi_k(y)^\dagger}{mV}$$

1/m poles in quark propagator

$$\rho_\nu \sim \lambda^{2(|\nu|+N_f)+1}$$

zero modes are repelled

Consider averages in fixed topological sectors: topological charge becomes scaling variable.

p-regime: match dependence on m, L

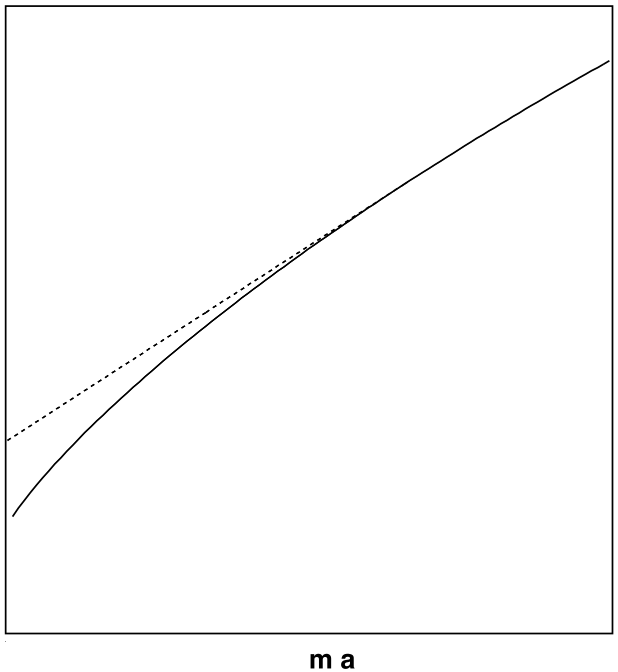
ε-regime: match dependence on $L, |\nu|$

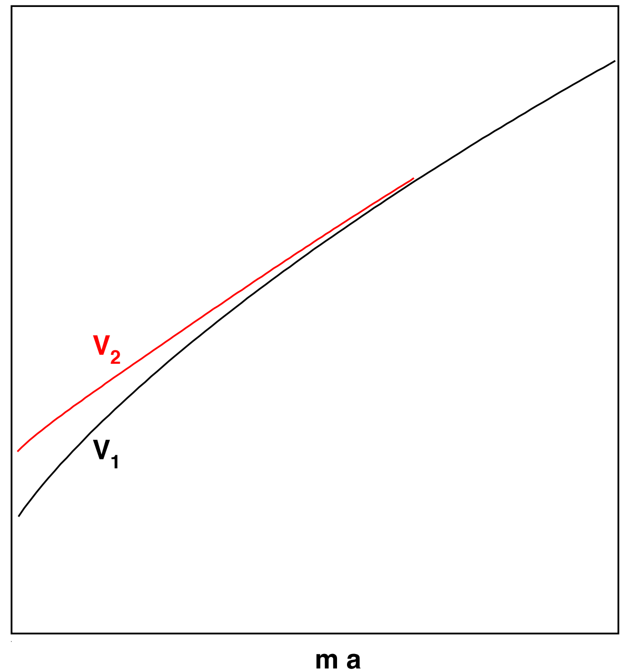
p-regime vs. ε-regime

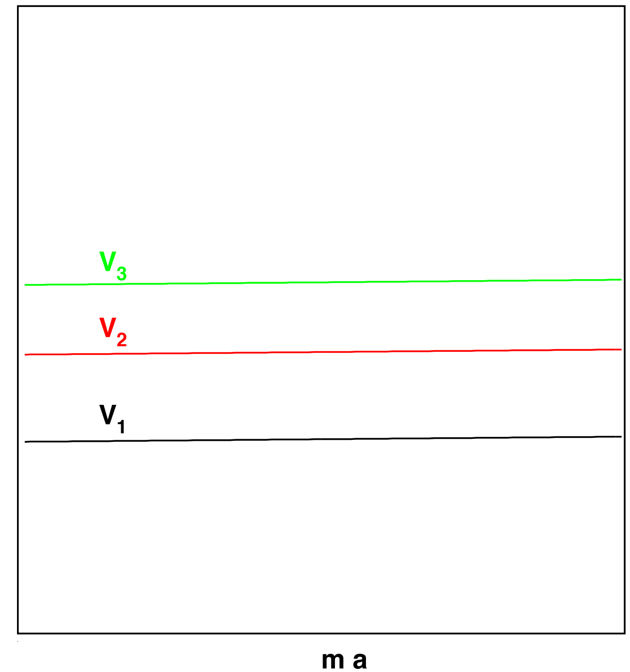
$V = \infty$

$M_\pi L \geq \mathcal{O}(1)$

$m\Sigma V \leq \mathcal{O}(1)$







$$C_\infty^{NLO}(m, \underbrace{\Sigma, F}_{LO}, \underbrace{L_i}_{NLO}, \ldots)$$

$$C_p^{NLO}(m, V, \underbrace{\Sigma, F}_{LO}, \underbrace{L_i}_{NLO}, \ldots)$$

$$C_\epsilon^{NLO}(m, V, \underbrace{\Sigma, F}_{LO}, \ldots)$$

ε-regime cleanest for LO LECs, p-regime probes NLO LECs (mass dependence).

p-regime vs. ε-regime

p-regime: check mass dependence

$$M_\pi^2 = M^2\left[1 + \frac{M^2}{32\pi^2 F^2}\ln\left(\frac{M^2}{\Lambda_3^2}\right) + \mathcal{O}(p^4)\right]$$

$$F_\pi = F\left[1 - \frac{M^2}{16\pi^2 F^2}\ln\left(\frac{M^2}{\Lambda_4^2}\right) + \mathcal{O}(p^4)\right]$$

$$M^2 \equiv \frac{2\Sigma m}{F^2}$$

$$\bar{l}_k \equiv \ln\left(\frac{\Lambda_k^2}{M^2}\right)\Bigg|_{M=139.6\ \mathrm{MeV}}$$

ε-regime: check volume, topology dependence

$$C_P(t) = \frac{1}{L^3}\int \mathrm{d}^3x\langle P(x)P(0)\rangle = \Sigma^2\left[a_P + \frac{T}{F^2L^3}b_P h_1(t/T) + \mathcal{O}(\varepsilon^4)\right]$$

$$C_A(t) = \frac{1}{L^3}\int \mathrm{d}^3x\langle A_0(x)A_0(0)\rangle = \frac{F^2}{V}\left[a_A + \frac{T}{F^2L^3}b_A h_1(t/T) + \mathcal{O}(\varepsilon^4)\right]$$

$$h_1(x) = \tfrac{1}{2}\left[(x-\tfrac{1}{2})^2 - \tfrac{1}{12}\right]$$

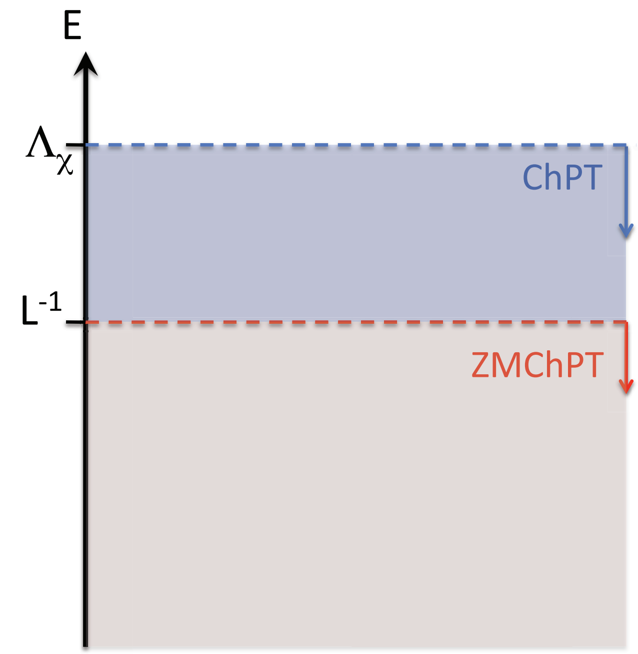
$a_{A,P}, b_{A,P}$ functions of L, T, ν

Zero mode domination — or, Random Matrix Theory

ε-regime: gap between zero ($E \sim M$) and non-zero ($E \sim 2\pi/L$) momentum modes.

⇒ possible to obtain an effective description of ChiPT by integrating out non-zero modes *within* ChiPT.

Shuryak, Verbaarschot, Zahed 93-94



$$[Z^{\nu}_{\rm ZMChPT}]_{\rm LO} = k \int_{U(N_f)} dU_0 \; \det(U_0)^{\nu} \; e^{\frac{\mu}{2}{\rm Tr}[U_0+U_0^{\dagger}]} = Z_{\rm RMT}(N_f,\mu,\nu)$$

N.B.: ZMChPT matches same RMT at NLO, all corrections absorbed in $\mu_{\rm eff}$.

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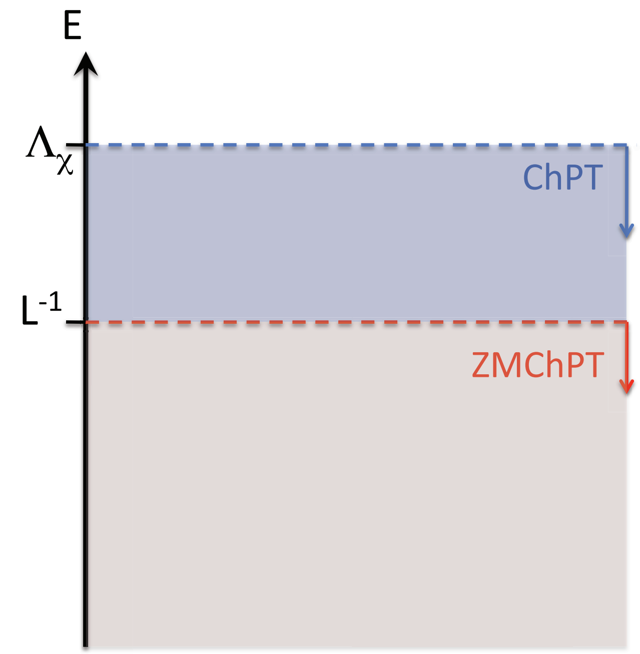

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$$[Z^{\nu}_{\mathrm{ZMChPT}}]_{\mathrm{NLO}} = k' \int_{U(N_f)} dU_0 \; \det(U_0)^{\nu} \; e^{\frac{\mu_{eff}}{2}\mathrm{Tr}[U_0 + U_0^{\dagger}]} = Z_{\mathrm{RMT}}(N_f, \mu_{eff}, \nu)$$

Zero mode domination — or, Random Matrix Theory

Corresponding RMT is a Gaussian chiral unitary model. Computations easy.
→ Dirac spectral properties in QCD (in appropriate finite volume chiral regime).

$$Z_{\mathrm{RMT}} = \int \mathrm{d}W e^{-\frac{N}{2}\mathrm{Tr}[W^\dagger W]} \prod_{i=1}^{N_f} \det(\hat{D} + \hat{m})$$

exact at large *N*

$$\hat{D} = \begin{pmatrix} 0 & W \\ -W^\dagger & 0 \end{pmatrix}, \qquad W \sim (N + |\nu|) \times N$$

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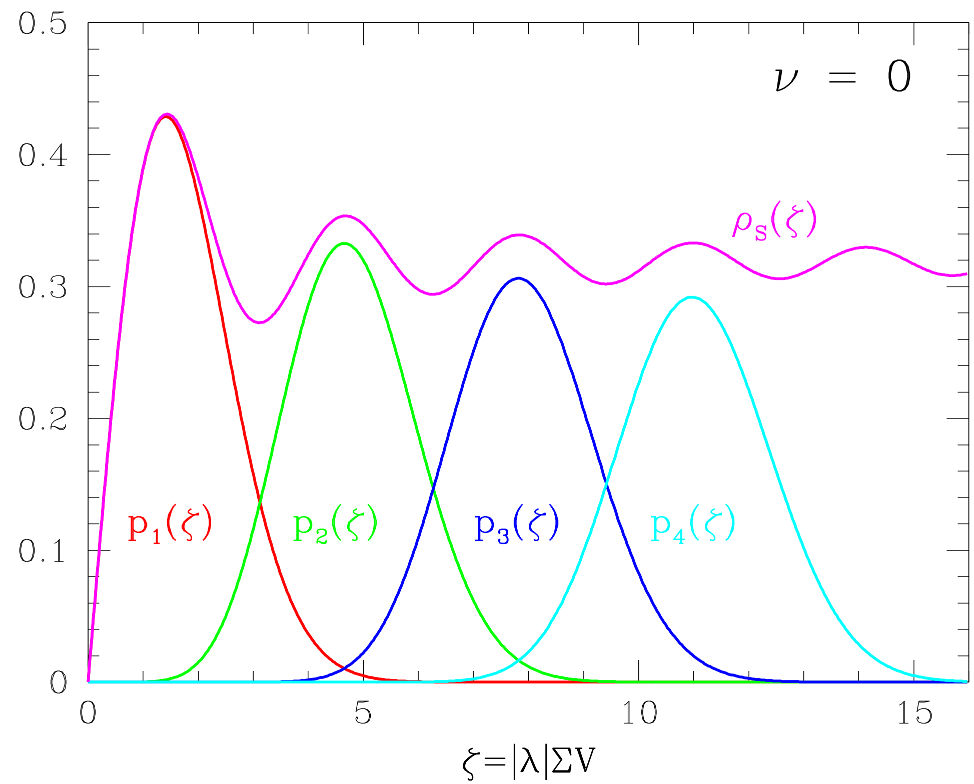
Celebrated example: microscopic spectral density of Dirac operator.

$$\langle \zeta_k \rangle^{\nu}_{\mathrm{RMT}} = \Sigma V \langle \lambda_k \rangle^{\nu}_{\mathrm{QCD}}$$



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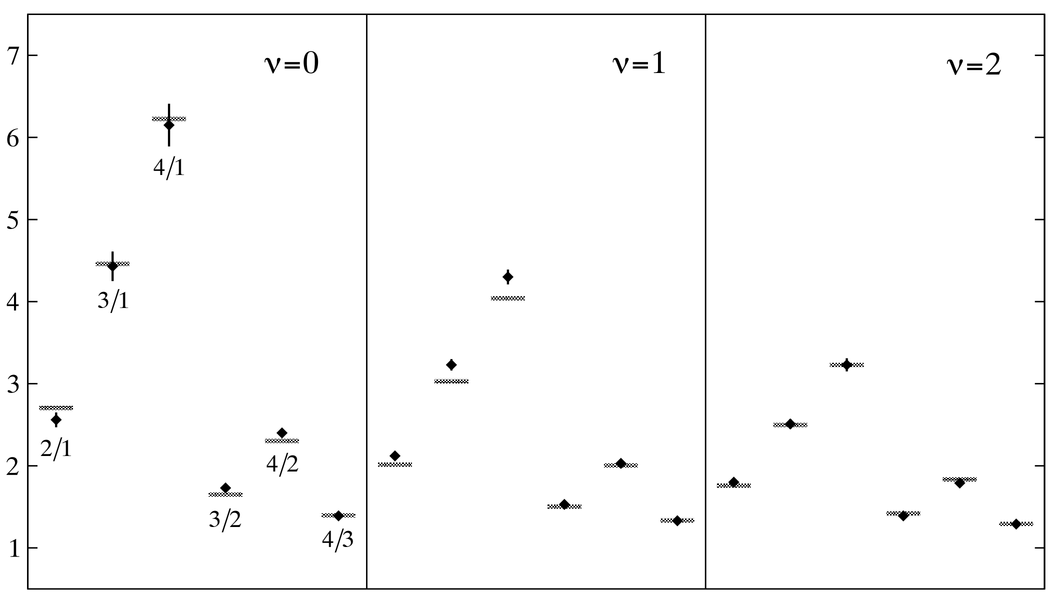
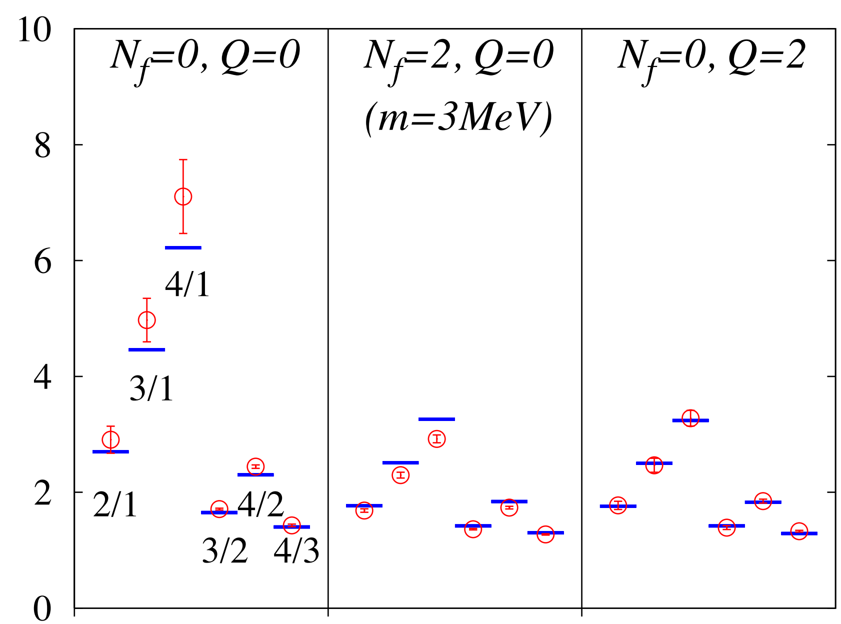


Giusti, Lüscher, Weisz, Wittig 2003



JLQCD+TWQCD, 2007

Mixed chiral regimes

Bernardoni, Hernández 07; Daamgard, Fukaya 07; BHDF 08;
Bernardoni, Hernández, Necco 09

$$m_l \Sigma V \leq 1 \qquad m_s \Sigma V \gg 1$$

Our specific setup: valence sector in the ε-regime, sea sector in the p-regime. (⇒ partial quenching).

Factorisation of perturbative and non-perturbative modes modified as:

$$U = \begin{pmatrix} U_0 & 0 \\ 0 & I_s \end{pmatrix} \exp\left(\frac{2i\xi}{F}\right), \quad \int d^4x \, \mathrm{tr}\left[\xi T^a\right] = 0, T^a \in Algebra(SU(N_v))$$

Power counting rules: $m_l \sim m_s^2 \sim p^4 \sim V^{-1}$. All 2p functions computed to NLO.

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Example: two-point function of left-handed current:

$$\mathrm{Tr}[T^a T^b]\,\mathcal{C}(x_0) = \int \mathrm{d}^3x \left\langle \mathcal{J}_0^a(x)\mathcal{J}_0^b(0)\right\rangle_\nu$$

For ε-regime valence quarks, scales order as $M_{vv}^2\,,\; L^{-2} \;\leq\; M_{ss}^2\,,\; M_{sv}^2 \;\ll\; (4\pi F)^2$

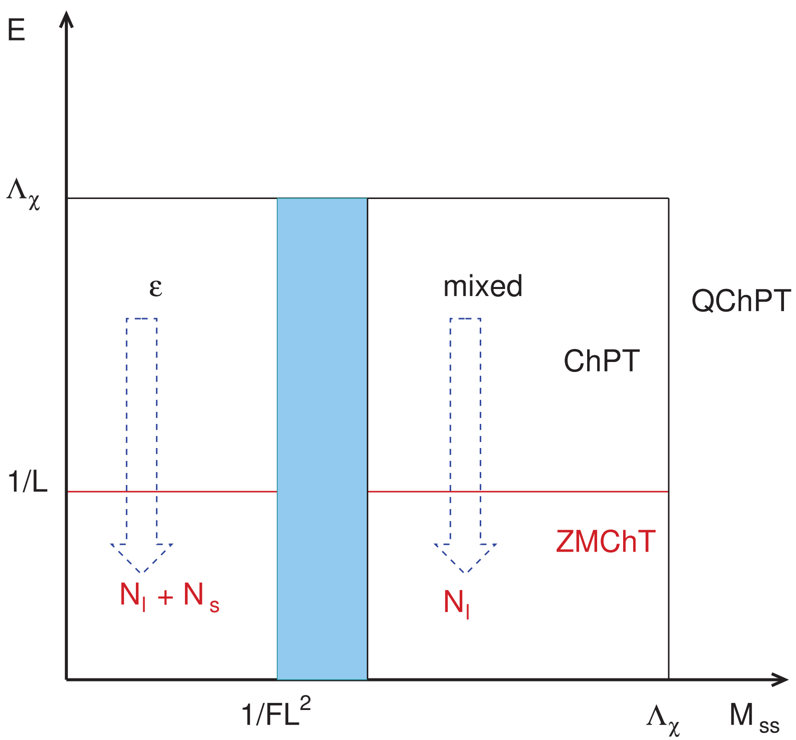
Heavier (sea) quarks behave as decoupling particles, sea meson mass dependence appears as "renormalisation" of N_f=2 LECs in ε-regime quenched expressions.

$$\mathcal{C}(x_0) = -\frac{\tilde{F}^2}{4T} - 2m\Sigma_\nu T H_1\left(\frac{x_0}{T}\right)$$

$$\tilde{F}^2 = F^2\left\{1 - \frac{N_s}{F^2}\left[\frac{M_{ss}^2}{16\pi^2}\log\left(\frac{M_{ss}}{\mu}\right) - 8L_4 M_{ss}^2\right]\right\}$$

Mixed chiral regimes: RMT

ZMChiPT can be derived along same lines.



Goldstones with "heavy" flavours treated like non-zero momentum modes $(M_{ss} \geq L^{-1})$ and integrated out.

RMT with N_l flavours, LECs of theory with $N_s + N_l$ flavours.

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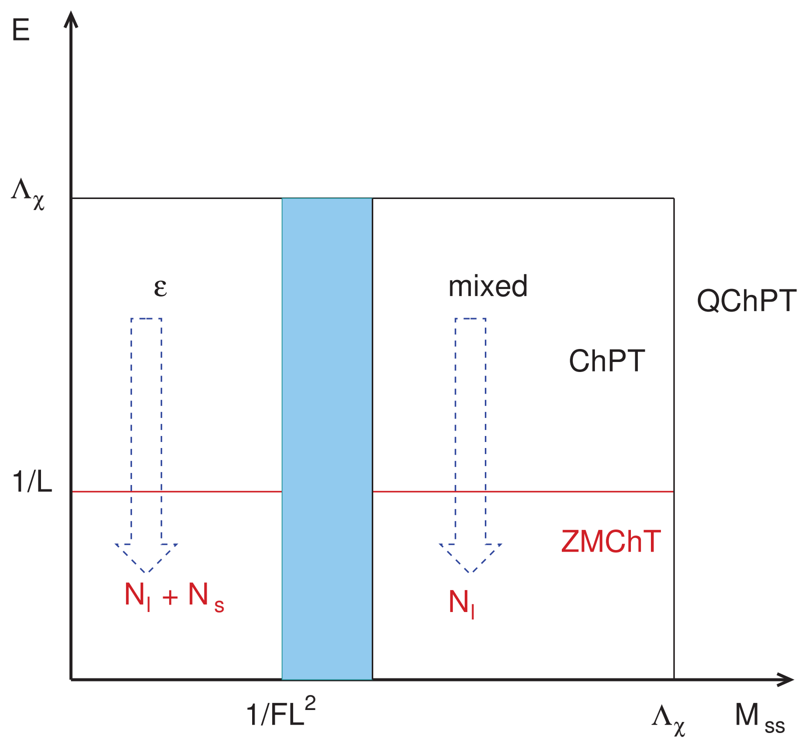

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$$\langle \lambda_k \rangle_{\mathrm{QCD},N_\mathrm{f}=2} \, \Sigma_{\mathrm{eff}}(M_{ss})V \; = \; \langle \zeta_k \rangle_{\mathrm{RMT},N_\mathrm{f}=0}$$

Mixed chiral regimes: RMT

Topological susceptibility and chiral condensate at NLO can be worked out in standard fashion:

$$\langle \nu^2 \rangle_{\rm NLO} = \frac{m_s \Sigma V}{N_s}\left\{1 - \frac{N_s^2-1}{N_s}\left[\frac{M_{ss}^2}{16\pi^2 F^2}\log\left(\frac{M_{ss}^2}{\mu^2}\right) + g_1(M_{ss},L,T) + \frac{16 M_{ss}^2}{F^2}\left(L_8^r(\mu) + N_s L_6^r(\mu) + N_s L_7^r(\mu)\right)\right]\right\}$$

Mao et al 09; Aoki et al 09; Bernardoni et al 10

$$\lim_{N_l \to 0} \Sigma_{\rm eff}(M_{ss}) = \Sigma\left\{1 + \frac{M_{ss}^2}{F^2}\left[\frac{\beta_2}{N_s} + \frac{\log(\mu V^{1/4})}{8\pi^2 N_s} + 16 N_s L_6^r(\mu) - \frac{N_s}{4\pi^2}\log\left(\frac{M_{ss}}{\sqrt{2}\mu}\right)\right] - \frac{\beta_1}{N_s F^2 \sqrt{V}} - \frac{N_s}{F^2} + g_1(M_{ss}/\sqrt{2},L,T)\right\}$$

Bernardoni et al.

N_f=2 QCD in partially quenched mixed regime

Bernardoni, Garron, Hernández, Necco, CP PRD 83 (2011) 054503

Mixed action: non-perturbatively $O(a)$ improved Wilson fermions for sea, Neuberger fermions for valence.

$\beta = 5.3,\ c_{\rm sw} = 1.90952,\ V/a^4 = 48 \times 24^3$			
label	κ	aM_{ss}	$N_{\rm cfg}$
D_4	0.13620	0.1695(14)	156
D_5	0.13625	0.1499(15)	169
D_6	0.13635	0.1183(37)	246 (D_{6a}: 159, D_{6b}: 87)

$a \simeq 0.078$ fm (K, K^*), 0.070 fm (Ω, r_0), 0.065 (F_K)

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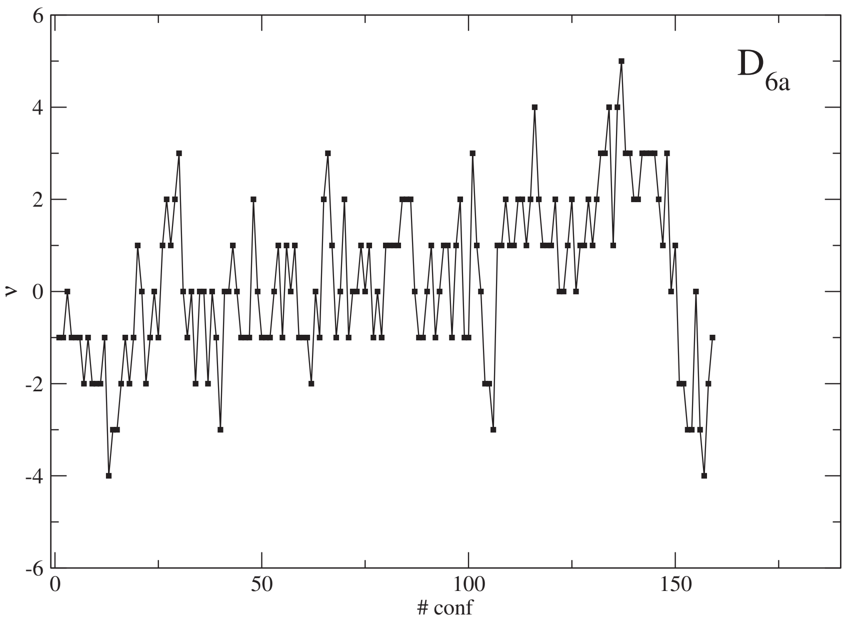
$a \simeq 0.078\ \text{fm}\ (K, K^*)$, $0.070\ \text{fm}\ (\Omega, r_0)$, $0.065\ (F_K)$

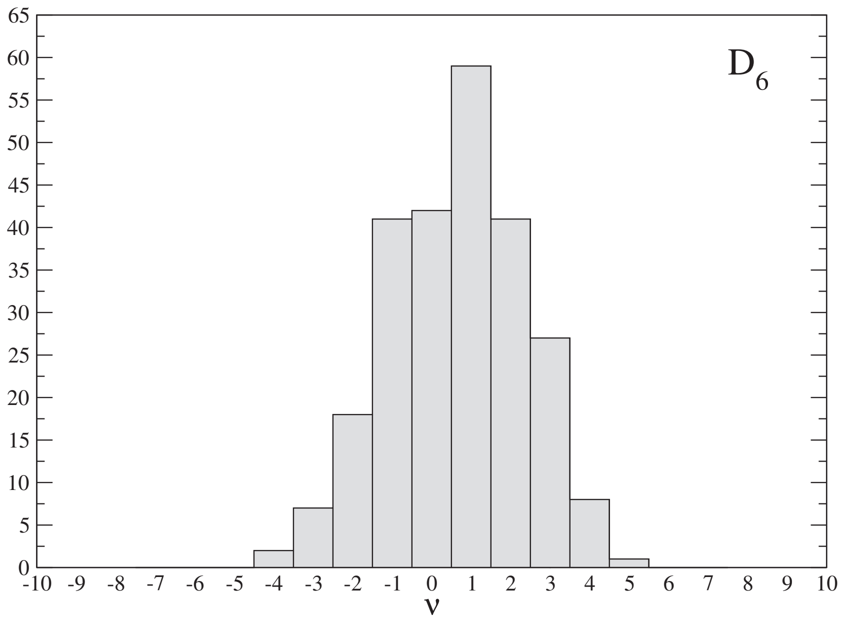
$\longrightarrow$ $M_{ss} \simeq 477,\ 422,\ 333\ \text{MeV}$

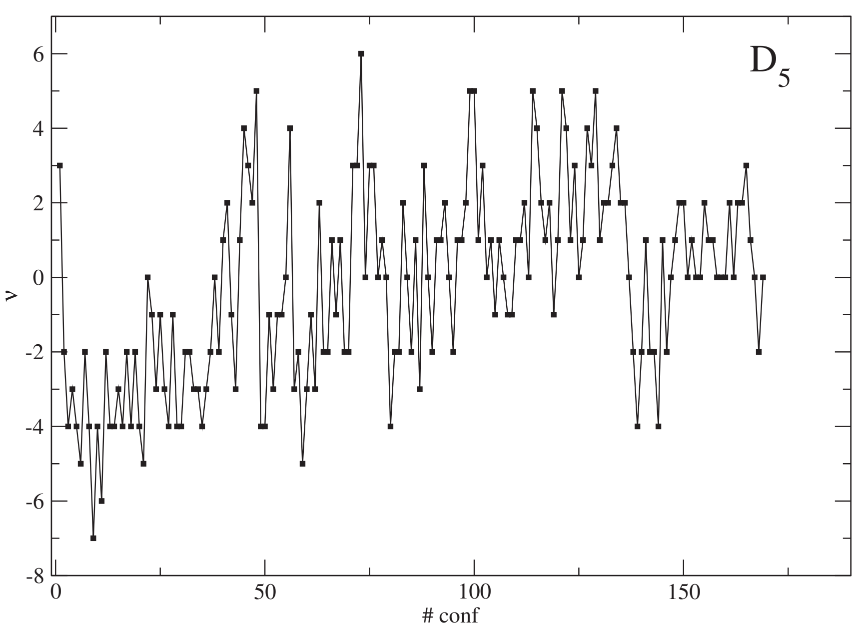
Observables: topological charge, low-lying eigenvalues of Dirac operator.

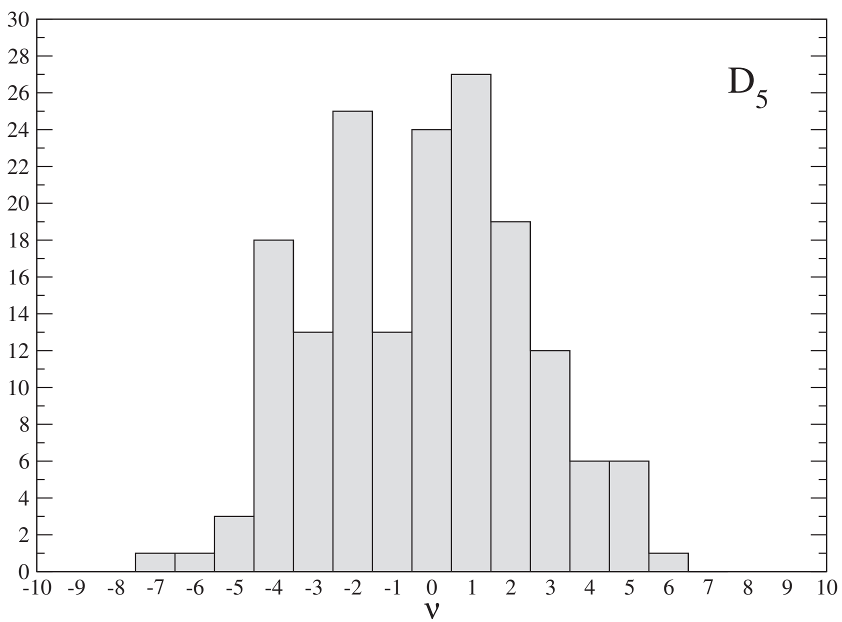
Aim (this talk): Study scaling with sea quark mass for topological susceptibility and chiral condensate, test matching to quenched RMT.

Topological susceptibility



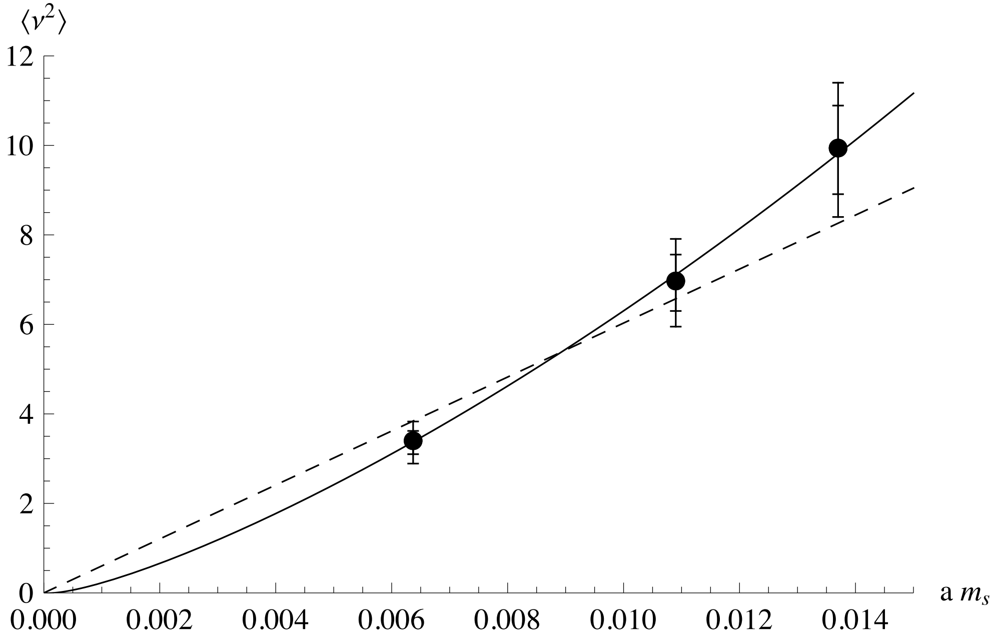






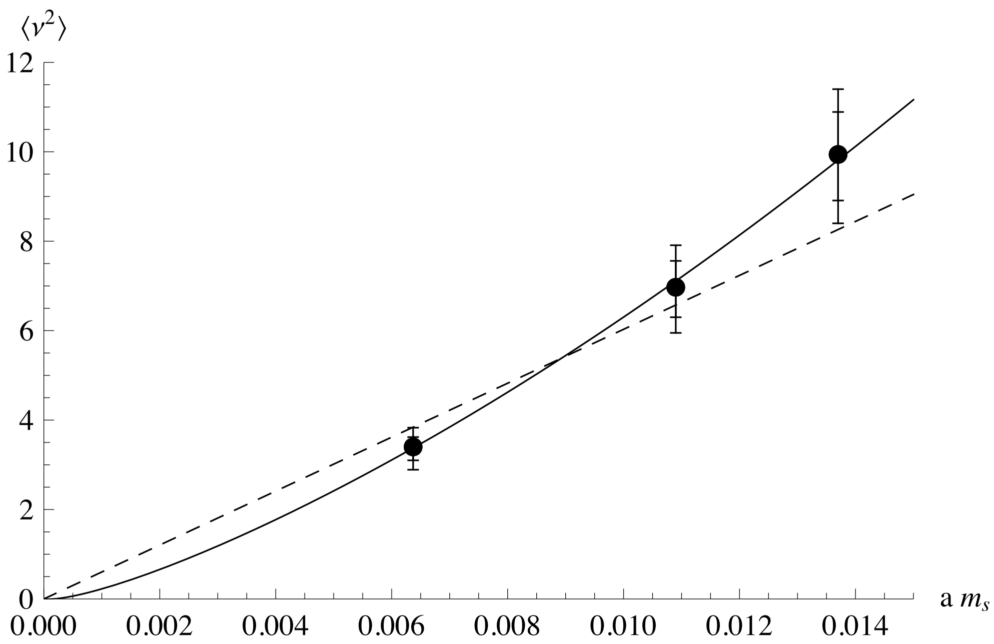
Topological susceptibility

$$\langle \nu^2 \rangle_{\rm NLO} = \frac{m_s \Sigma V}{N_s} \left\{ 1 - \frac{N_s^2 - 1}{N_s} \left[\frac{M_{ss}^2}{16\pi^2 F^2} \log\left(\frac{M_{ss}^2}{\mu^2} \right) + g_1(M_{ss}, L, T) + \frac{16 M_{ss}^2}{F^2} \left(L_8^r(\mu) + N_s L_6^r(\mu) + N_s L_7^r(\mu) \right) \right] \right\}$$



Topological susceptibility

$$\langle \nu^2 \rangle_{\mathrm{NLO}} = \frac{m_s \Sigma V}{N_s} \left\{ 1 - \frac{N_s^2 - 1}{N_s} \left[\frac{M_{ss}^2}{16\pi^2 F^2} \log\left(\frac{M_{ss}^2}{\mu^2} \right) + g_1(M_{ss}, L, T) + \frac{16 M_{ss}^2}{F^2} \left(L_8^r(\mu) + N_s L_6^r(\mu) + N_s L_7^r(\mu) \right) \right] \right\}$$



$$M_{ss}^2 = \frac{2 m_s \Sigma}{F^2}$$

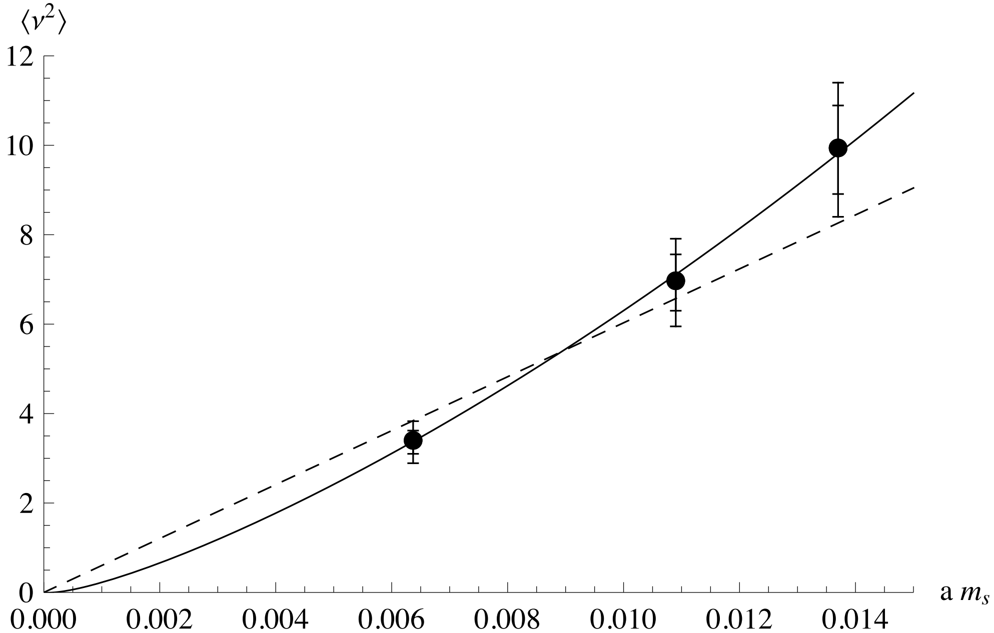
$$\left(Z_{\mathrm{S}}^{\overline{\mathrm{MS}}} \right)^{-1} m \Big|_{M_\pi^{\mathrm{ref}}}^{\mathrm{overlap}} = m^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) \Big|_{M_\pi^{\mathrm{ref}}}^{\mathrm{Wilson}}$$

$$Z_{\mathrm{S}}^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 1.84(10)$$

cf. talk by S. Necco

Topological susceptibility

$$\langle \nu^2 \rangle_{\rm NLO} = \frac{m_s \Sigma V}{N_s} \left\{ 1 - \frac{N_s^2 - 1}{N_s} \left[\frac{M_{ss}^2}{16\pi^2 F^2} \log\left(\frac{M_{ss}^2}{\mu^2} \right) + g_1(M_{ss}, L, T) + \frac{16 M_{ss}^2}{F^2} \left(L_8^r(\mu) + N_s L_6^r(\mu) + N_s L_7^r(\mu) \right) \right] \right\}$$



$$M_{ss}^2 = \frac{2 m_s \Sigma}{F^2}$$

$$a = 0.070 \text{ fm}$$

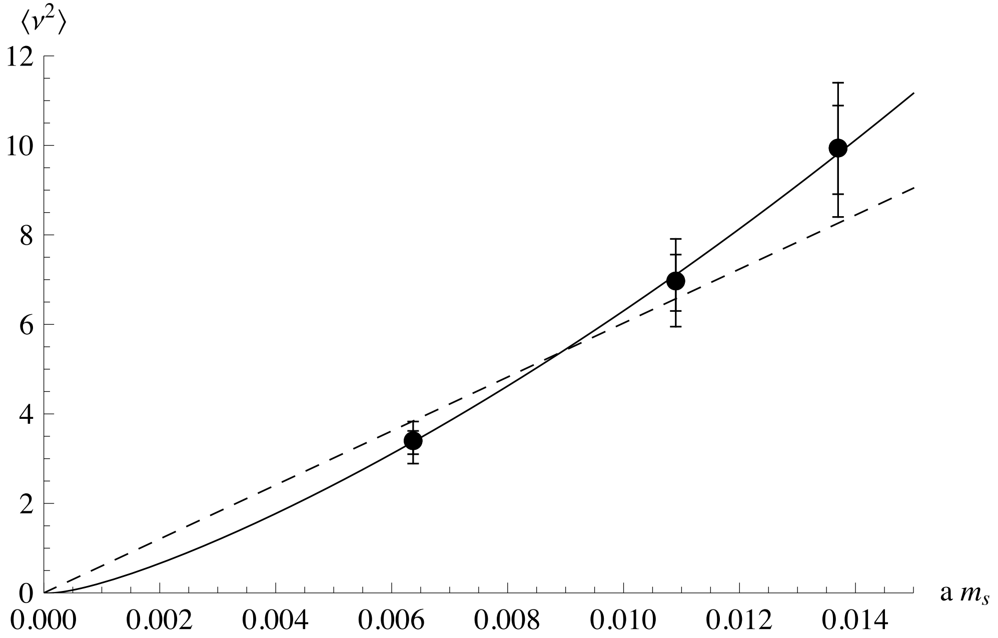
$$F = 90 \pm 10 \text{ MeV}$$

NLO: $\Sigma^{\overline{\rm MS}}(2 \text{ GeV}) = \left[287^{(35)(5)}_{(36)(7)} \text{ MeV} \right]^3$ $\quad [L_8^r + 2(L_6^r + L_7^r)]\,(M_\rho) = 0.0018(30)$

cf. LO: $\Sigma^{\rm MS}(2 \text{ GeV}) = [344(10) \text{ MeV}]^3$

Topological susceptibility

$$\langle \nu^2 \rangle_{\rm NLO} = \frac{m_s \Sigma V}{N_s} \left\{ 1 - \frac{N_s^2 - 1}{N_s} \left[\frac{M_{ss}^2}{16\pi^2 F^2} \log\left(\frac{M_{ss}^2}{\mu^2} \right) + g_1(M_{ss}, L, T) + \frac{16 M_{ss}^2}{F^2} \left(L_8^r(\mu) + N_s L_6^r(\mu) + N_s L_7^r(\mu) \right) \right] \right\}$$



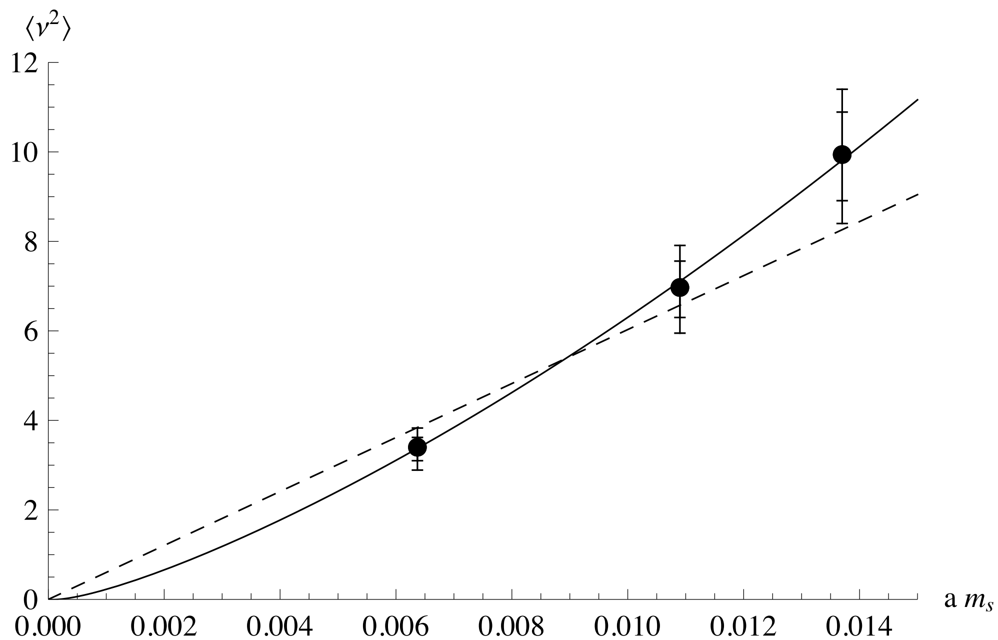
NLO: $\Sigma^{\overline{\rm MS}}(2\ {\rm GeV}) = \left[287^{(35)(5)}_{(36)(7)}\ {\rm MeV} \right]^3$

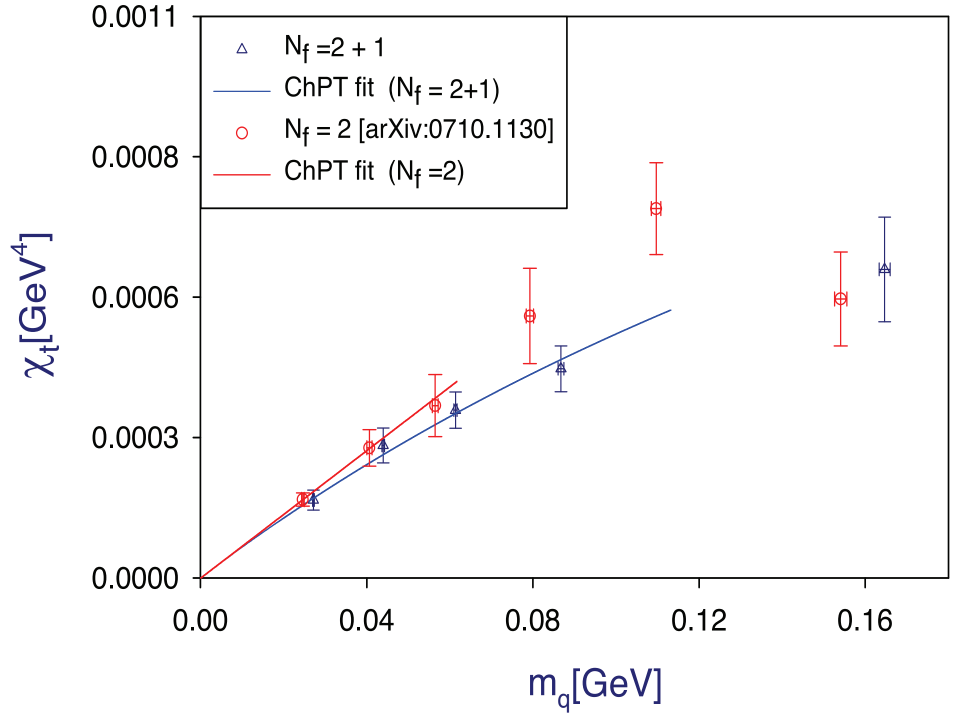
$\Sigma^{\rm MS}(2\ {\rm GeV}) = [269(18)\ {\rm MeV}]^3$

FLAG

Topological susceptibility

$$\langle \nu^2 \rangle_{\mathrm{NLO}} = \frac{m_s \Sigma V}{N_s} \left\{ 1 - \frac{N_s^2 - 1}{N_s} \left[\frac{M_{ss}^2}{16\pi^2 F^2} \log\left(\frac{M_{ss}^2}{\mu^2} \right) + g_1(M_{ss}, L, T) + \frac{16 M_{ss}^2}{F^2} \left(L_8^r(\mu) + N_s L_6^r(\mu) + N_s L_7^r(\mu) \right) \right] \right\}$$





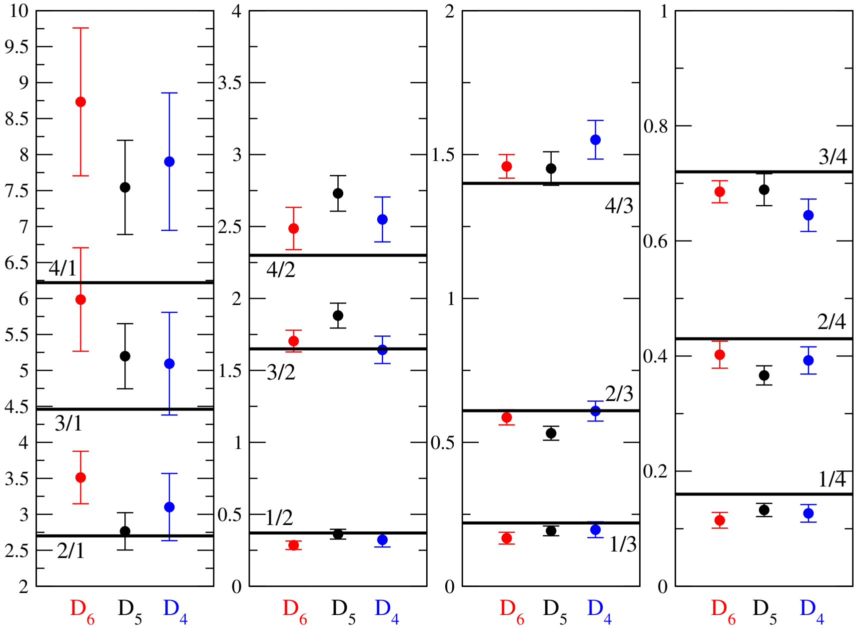
NLO: $\Sigma^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = \left[287^{(35)(5)}_{(36)(7)}\ \mathrm{MeV} \right]^3$

$\Sigma^{\mathrm{MS}}(2\ \mathrm{GeV}) = [249(4)(2)\ \mathrm{MeV}]^3$

Chiu et al. 08

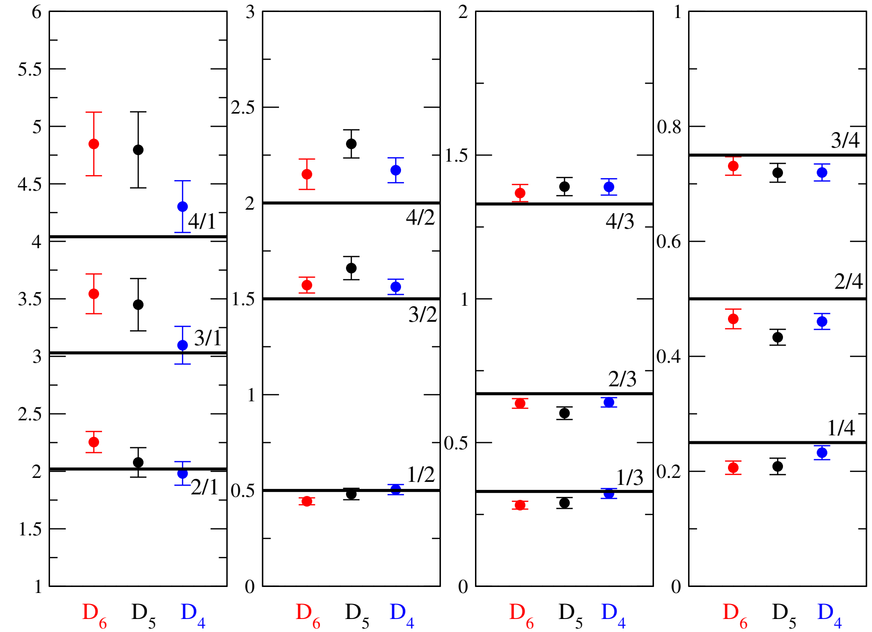
Eigenvalue ratios

ν=0

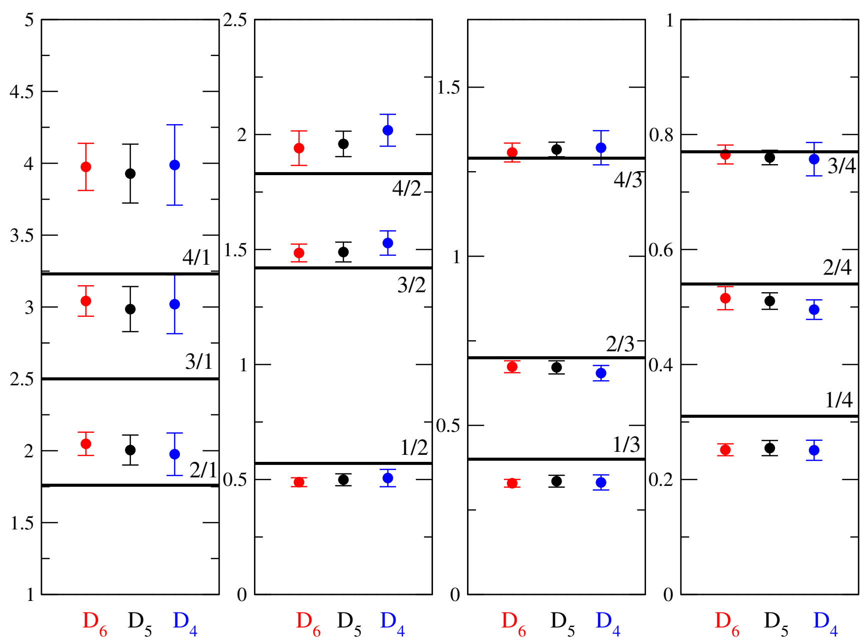


$$\langle \lambda_k \rangle_{\mathrm{QCD},N_f=2}\, \Sigma_{\mathrm{eff}}(M_{ss})V \;=\; \langle \zeta_k \rangle_{\mathrm{RMT},N_f=0}$$

|ν|=1

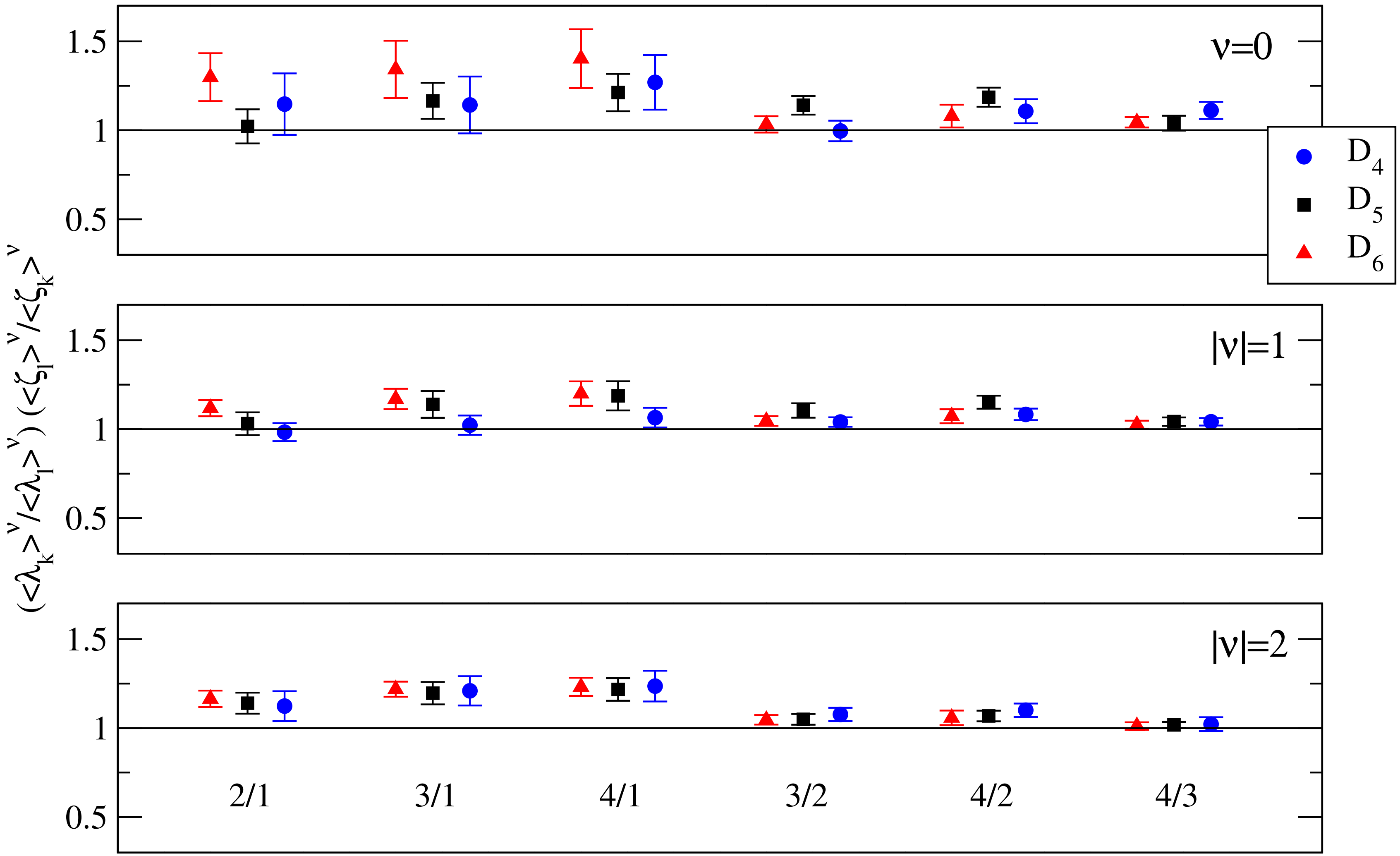


|ν|=2



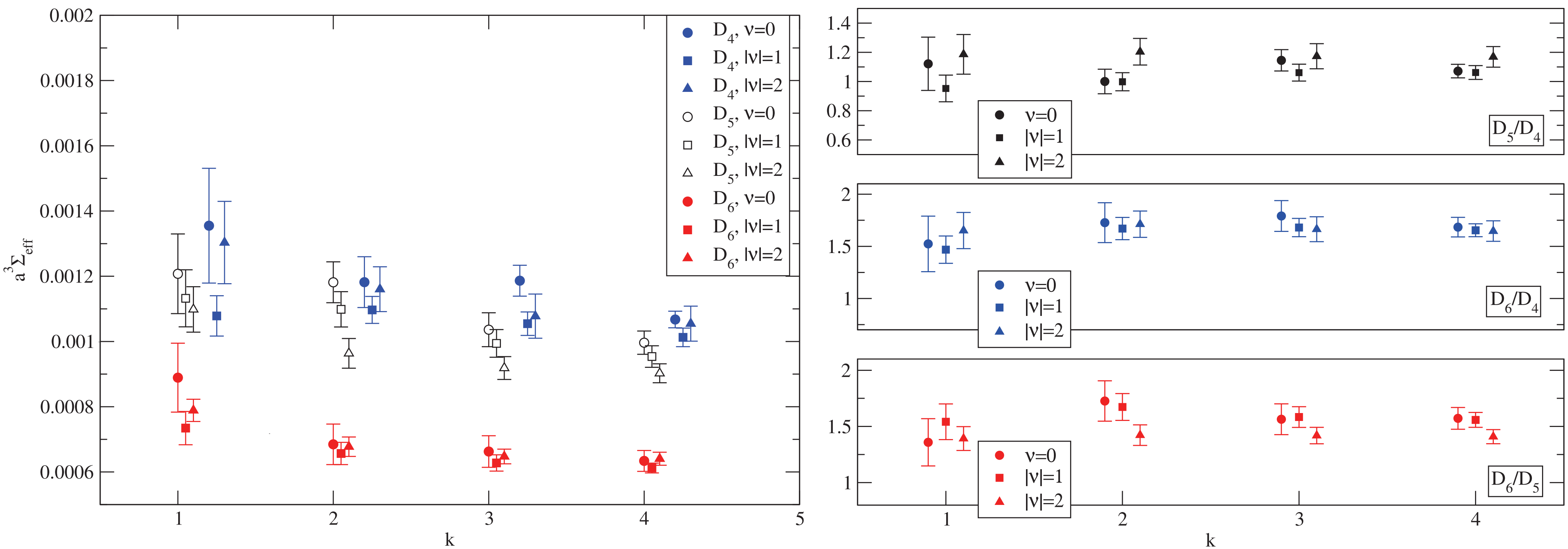
Eigenvalue ratios

$$\langle \lambda_k \rangle_{\mathrm{QCD},N_f=2}\, \Sigma_{\mathrm{eff}}(M_{ss}) V \;=\; \langle \zeta_k \rangle_{\mathrm{RMT},N_f=0}$$



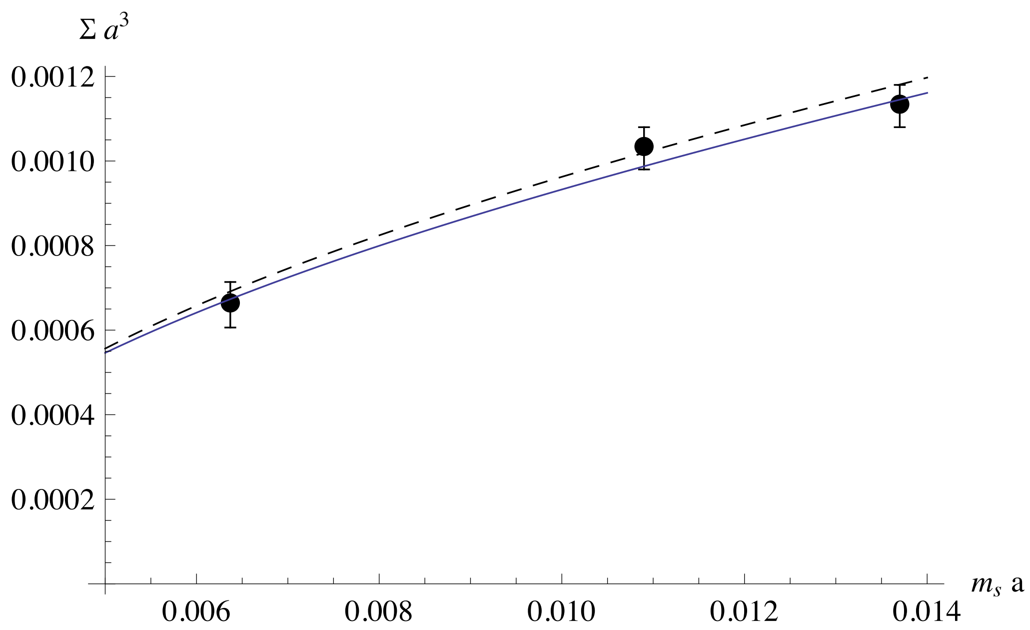
Chiral condensate from spectrum

$$\lim_{N_l \to 0} \Sigma_{\rm eff}(M_{ss}) = \Sigma\Big\{1 + \frac{M_{ss}^2}{F^2}\Big[\frac{\beta_2}{N_s} + \frac{\log(\mu V^{1/4})}{8\pi^2 N_s} + 16 N_s L_6^r(\mu) - \frac{N_s}{4\pi^2}\log\Big(\frac{M_{ss}}{\sqrt{2}\mu}\Big)\Big] - \frac{\beta_1}{N_s F^2\sqrt{V}} - \frac{N_s}{F^2} + g_1(M_{ss}/\sqrt{2}, L, T)\Big\}$$



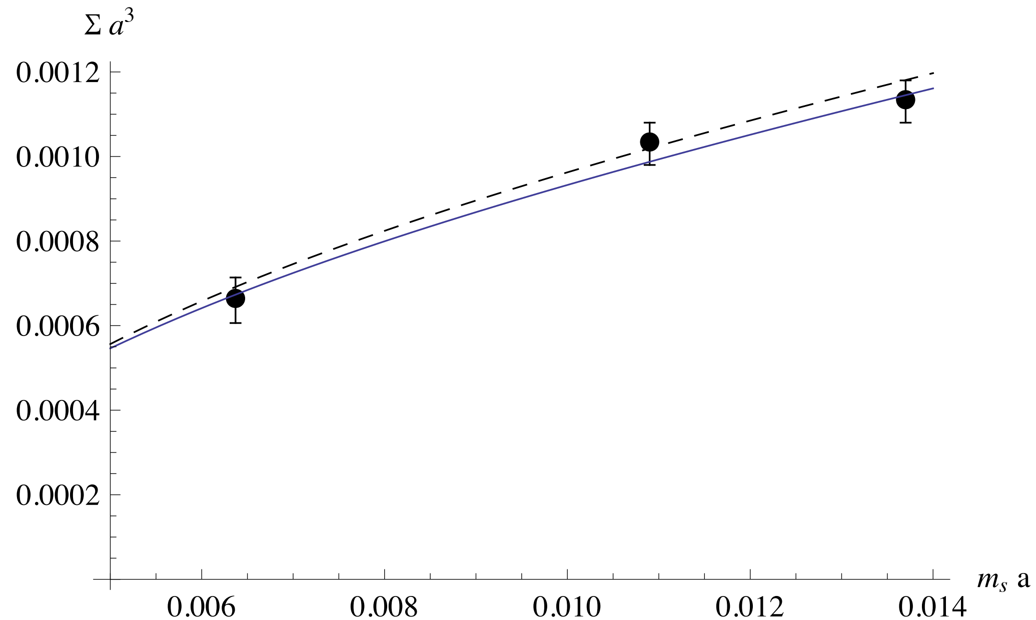
Chiral condensate from spectrum

$$\lim_{N_l \to 0} \Sigma_{\rm eff}(M_{ss}) = \Sigma \left\{ 1 + \frac{M_{ss}^2}{F^2} \left[\frac{\beta_2}{N_s} + \frac{\log(\mu V^{1/4})}{8\pi^2 N_s} + 16 N_s L_6^r(\mu) - \frac{N_s}{4\pi^2} \log\left(\frac{M_{ss}}{\sqrt{2}\mu} \right) \right] - \frac{\beta_1}{N_s F^2 \sqrt{V}} - \frac{N_s}{F^2} + g_1(M_{ss}/\sqrt{2}, L, T) \right\}$$



Chiral condensate from spectrum

$$\lim_{N_l\to 0}\Sigma_{\rm eff}(M_{ss}) = \Sigma\left\{1+\frac{M_{ss}^2}{F^2}\left[\frac{\beta_2}{N_s}+\frac{\log(\mu V^{1/4})}{8\pi^2 N_s}+16N_s L_6^r(\mu)-\frac{N_s}{4\pi^2}\log\left(\frac{M_{ss}}{\sqrt{2}\mu}\right)\right]\right.$$
$$\left.-\frac{\beta_1}{N_s F^2\sqrt{V}}-\frac{N_s}{F^2}+g_1(M_{ss}/\sqrt{2},L,T)\right\}$$



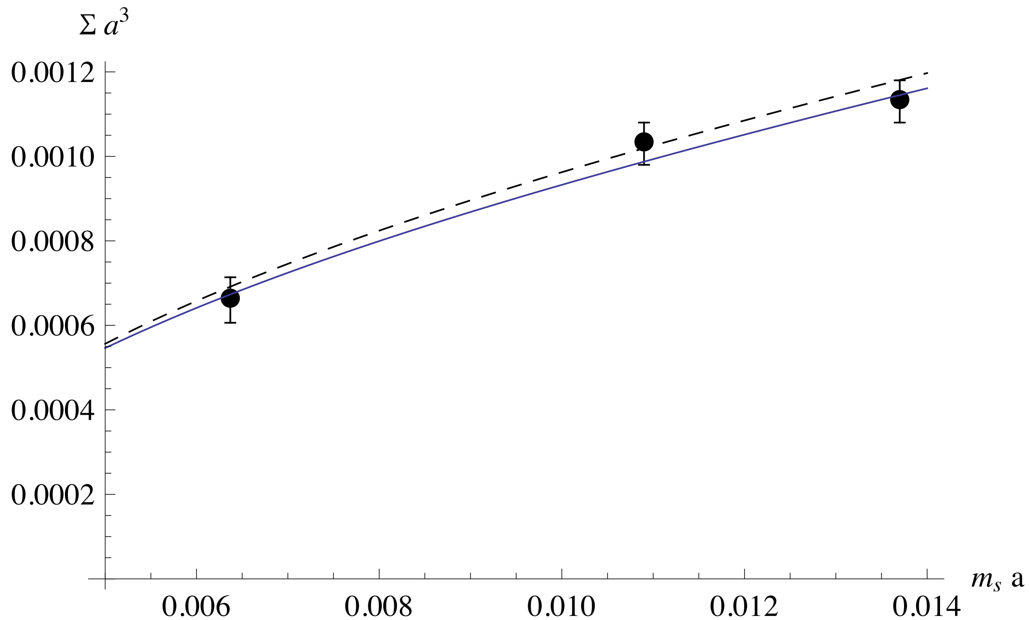
$$M_{ss}^2=\frac{2m_s\Sigma}{F^2}$$

$$a = 0.070 \text{ fm}$$
$$F = 90\pm 10 \text{ MeV}$$

NLO: $\Sigma^{\overline{\rm MS}}(2\text{ GeV}) = \left[280^{(14)(4)}_{(16)(5)}\text{ MeV}\right]^3$ $\quad L_6^r(M_\rho)=0.0010(7)$

Chiral condensate from spectrum

$$\lim_{N_l \to 0} \Sigma_{\mathrm{eff}}(M_{ss}) = \Sigma\left\{1 + \frac{M_{ss}^2}{F^2}\left[\frac{\beta_2}{N_s} + \frac{\log(\mu V^{1/4})}{8\pi^2 N_s} + 16 N_s L_6^r(\mu) - \frac{N_s}{4\pi^2}\log\left(\frac{M_{ss}}{\sqrt{2}\mu}\right)\right] - \frac{\beta_1}{N_s F^2\sqrt{V}} - \frac{N_s}{F^2} + g_1(M_{ss}/\sqrt{2}, L, T)\right\}$$



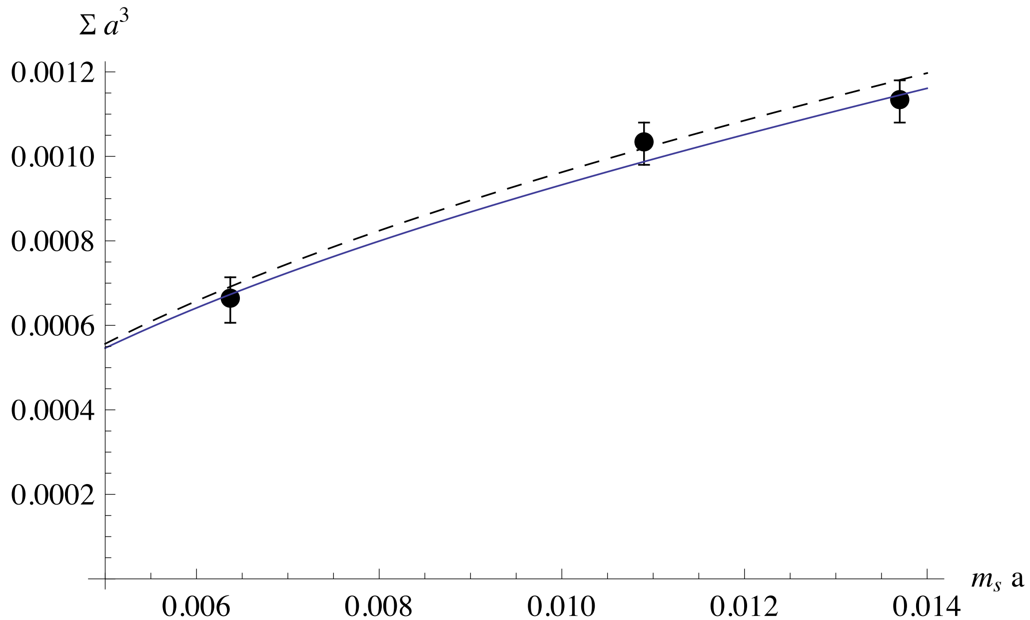
NLO: $\Sigma^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = \left[280^{(14)(4)}_{(16)(5)}\ \mathrm{MeV}\right]^3$

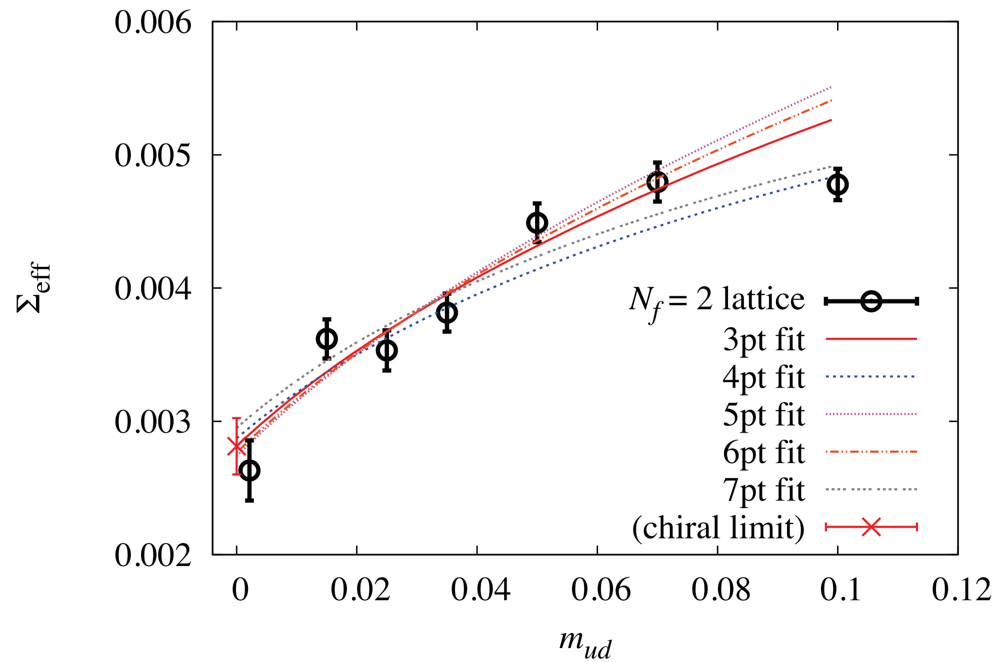
$\Sigma^{\mathrm{MS}}(2\ \mathrm{GeV}) = [269(18)\ \mathrm{MeV}]^3$

FLAG

Chiral condensate from spectrum

$$\lim_{N_l \to 0} \Sigma_{\text{eff}}(M_{ss}) = \Sigma \Bigg\{ 1 + \frac{M_{ss}^2}{F^2} \left[\frac{\beta_2}{N_s} + \frac{\log(\mu V^{1/4})}{8\pi^2 N_s} + 16 N_s L_6^r(\mu) - \frac{N_s}{4\pi^2} \log\left(\frac{M_{ss}}{\sqrt{2}\mu} \right) \right] - \frac{\beta_1}{N_s F^2 \sqrt{V}} - \frac{N_s}{F^2} + g_1(M_{ss}/\sqrt{2}, L, T) \Bigg\}$$





NLO: $\Sigma^{\overline{\text{MS}}}(2\text{ GeV}) = \left[280^{(14)(4)}_{(16)(5)}\text{ MeV} \right]^3$

$\Sigma^{\text{MS}}(2\text{ GeV}) = [242(05)(20)\text{ MeV}]^3$

Fukaya et al. 11

Summary and Outlook

- First results seem to validate mixed-regime approach, results for LECs in comparable precision ballpark as other setups.

- Check finite size scaling, cutoff effects.

- Get *F*, other *L*'s from analysis of two-point functions (talk by S. Necco).

- Provide solid estimation of valence-sea relative cutoff effects.

- Move to phenomenology: weak LECs for $K\to\pi\pi$ decays, understanding the role of the charm quark in $\Delta I=1/2$ rule.

Giusti, Hernández, Laine, CP, Torró, Weisz, Wennekers, Wittig 04-08
work in progress

BACKUP

Scale setting uncertainty

Different results for scale setting in our lattices:

$$K, K^* \;\rightarrow\; a \simeq 0.078\ \mathrm{fm}$$ Del Debbio et al. 06

$$r_0, \Omega \;\rightarrow\; a \simeq 0.070\ \mathrm{fm}$$ Donnellan et al. 11; Brandt et al. Lat10

$$F_K \;\rightarrow\; a \simeq 0.065\ \mathrm{fm}$$ Marinkovic Lat11 (cf. R. Sommer's talk)

Impact on LECs:

topology

a	$\Sigma^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})^{1/3}$	$[L_8^r + 2(L_6^r + L_7^r)](M_\rho)$
0.078	$262^{(33)(4)}_{(34)(5)}$ MeV	0.0023(43)
0.070	$287^{(35)(5)}_{(36)(7)}$ MeV	0.0023(43)
0.065	n/a	n/a

RMT

a	$\Sigma^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})^{1/3}$	$L_6^r(M_\rho)$
0.078	$255^{(12)(1)}_{(13)(4)}$ MeV	0.0015(10)
0.070	$280^{(13)(4)}_{(14)(5)}$ MeV	0.0010(6)
0.065	$291^{(12)(7)}_{(12)(9)}$ MeV	0.0005(4)