Twisʇəd Four-Fermion Physics

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Plan

- Why
- B_K
- B_B
- $K \to \pi(\pi)$ transitions
- Conclusions

WME's with Wilson fermions

Difficulties induced by the breaking of chiral symmetry:

- Chiral limit not defined at finite lattice spacing ⇒ it is difficult to go chiral in a systematically controlled way.
- Additional/worsened mixings under renormalisation, with dire consequences for d=6 operators.
- Numerical instabilities (exceptional configurations/bad algorithmic performance) related to unphysical zero modes of the Dirac operator (➛SAP?).

Power of tmQCD: chiral symmetry can be broken in a "smarter" way.

Price: (partial) soft breaking of flavour symmetries — and P,T.

Renormalisation of $O^{\Delta S=2}$ with Wilson fermions

$$O^{\Delta S=2} = \underbrace{[(\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu d) + (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu\gamma_5 d)]}_{O_{\mathrm{VV+AA}}} - \underbrace{[(\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu d) + (\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu\gamma_5 d)]}_{O_{\mathrm{VA+AV}}}$$

$$\bar{O}_{\mathrm{VV+AA}}(\mu) = \lim_{a\to 0} Z_{\mathrm{VV+AA}}(g_0^2, a\mu)\left[O_{\mathrm{VV+AA}}(a) + \sum_{k=1}^{4} \Delta_k(g_0^2) O_k(a)\right]$$

Induced by chiral symmetry breaking

Observation: the parity-odd part renormalises multiplicatively, as it is protected by discrete symmetries.

Matrix element without extra mixings

- Use an axial WI to relate three-point functions of the parity-even operator to four-point functions of the parity-odd operator.

  Bećirević et al., PLB 487 (2000) 74

- Use a tmQCD regularisation which maps three-point functions of the VV+AA operator to three-point functions of the VA+AV operator.
  - ($\pi/2$)-twisted light doublet quark and untwisted s quark.

    Frezzotti, Grassi, Sint and Weisz, JHEP 08 (2001) 058
  - ($\pm\pi/2$)-twisted valence quarks on ($\pi/2$)-twisted sea.

    Frezzotti and Rossi, JHEP 10 (2004) 070
  - ($\pi/4$)-twisted s-d doublet.

    Dimopoulos et al., hep-lat/0409026

tmQCD bonus: push safely towards lighter quark masses.

Alpha approaches to B_K: a bit of comparison

	Alpha ($\pi/2$)	Alpha ($\pi/4$)
ready for unquenching	✔	✘*
O(a) improvement	✘	✘
quenched: physical M_K at m_s=m_d	✘	✔
quenched computation cost	x2	x1

Alpha precision computation of B_K in qQCD

Dimopoulos et al., in preparation

Bring all systematics (but for quenching) under control:

- tmQCD used to kill operator mixing and avoid exceptional configurations.
- Two different regularisations → good control of the CL.
- SF nonperturbative renormalisation.

Guagnelli et al., to appear

Palombi, C.P. and Sint, to appear

- N.B.: action is O(a) improved, but four-fermion operator is not ⇒ expected approach to the continuum limit linear in a.

Alpha precision computation of B_K: layout

- Schrödinger Functional framework, nonperturbatively O(a) improved action in the bulk, one-loop O(a) improvement on SF boundaries.

- Computations carried out in the $SU(3)_V$-symmetric limit ($m_s = m_d$). Effect of $m_s \neq m_d$ investigated.

- Finite volume effects checked to be under control.

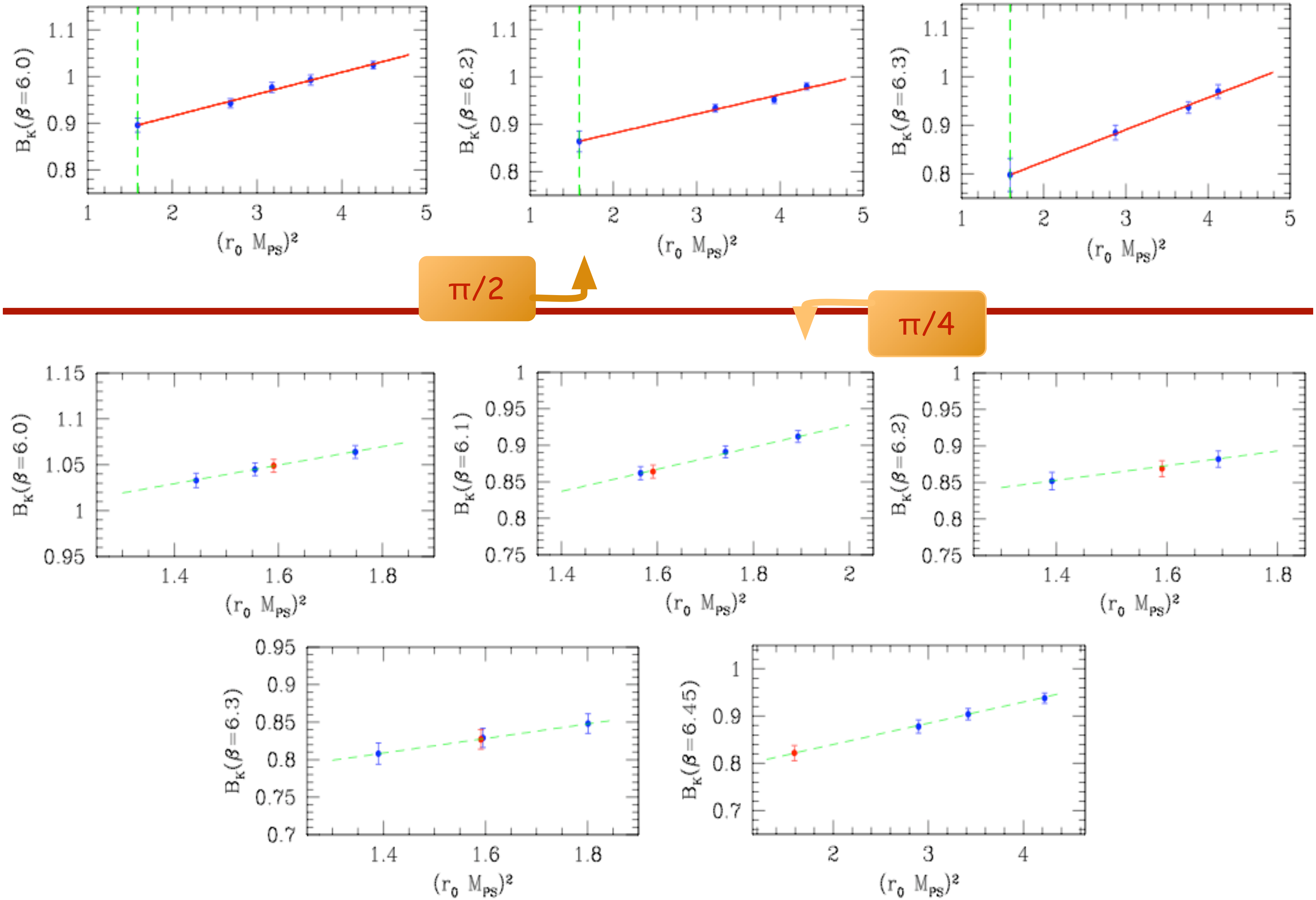
- No sign of corrections to M_{PS}^2 dependence when extrapolation in the pseudoscalar mass is necessary.

Simulation points ($\pi/4$):

β	L/r_0	$(\kappa,a\mu)$	$r_0\, M_{PS}$
6.0	4.47	(0.134739,0.010412)	1.322(5)
		(0.134795,0.009142)	1.247(5)
		(0.134828,0.008937)	1.201(5)
6.1	3.79	(0.135152,0.008100)	1.376(6)
		(0.135190,0.007200)	1.320(6)
		(0.135235,0.0061500)	1.251(6)
6.2	4.33	(0.135477,0.007595)	1.301(7)
		(0.135539,0.006125)	1.180(7)
6.3	3.76	(0.135509,0.007600)	1.342(9)
		(0.135546,0.006700)	1.263(9)
		(0.135584,0.005800)	1.179(9)
6.45	3.08	(0.135105,0.014590)	2.055(14)
		(0.135218,0.011850)	1.849(11)
		(0.135293,0.010020)	1.702(13)

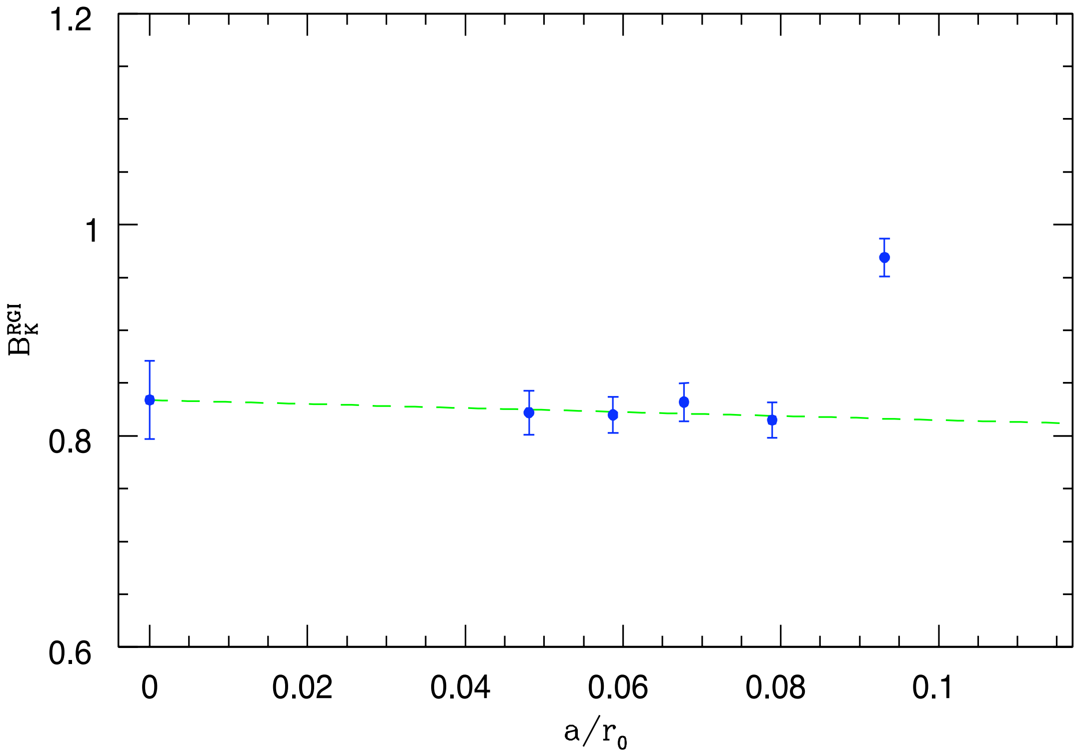
Simulation points (π/2) [N.B.: independent simulations for each mass]:

β	L/r_0	κ_s	$(\kappa_d, a\mu_d)$	$r_0\, M_{PS}$
6.0	2.98	0.1335	(0.135169,0.031860)	2.091(6)
		0.1338	(0.135178,0.031520)	1.906(6)
		0.1340	(0.135183,0.027080)	1.782(6)
		0.1342	(0.135187,0.022610)	1.639(6)
6.2	3.25	0.1346	(0.135780,0.028324)	2.078(6)
		0.1347	(0.135783,0.025985)	1.982(7)
		0.1349	(0.135787,0.021290)	1.796(7)
6.3	2.82	0.1348	(0.135771,0.023639)	2.030(9)
		0.1349	(0.135773,0.021254)	1.916(9)
		0.1351	(0.135776,0.016467)	1.672(10)

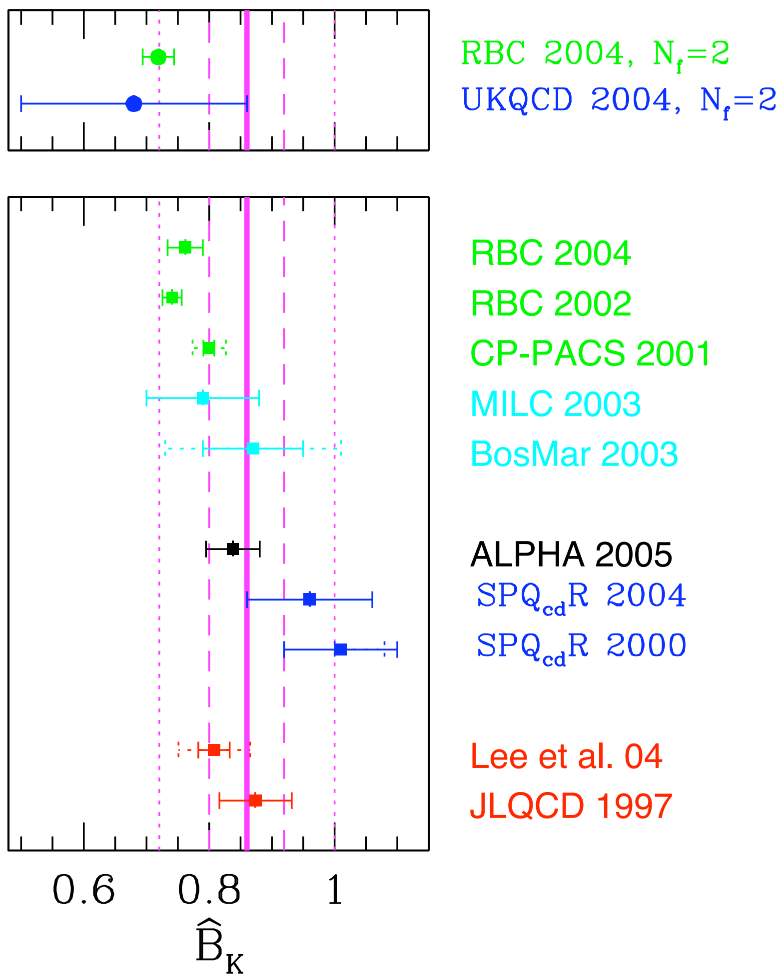


B_K(β=6.0)
B_K(β=6.2)
B_K(β=6.3)
(r_0 M_PS)²
π/2
π/4
B_K(β=6.0)
B_K(β=6.1)
B_K(β=6.2)
B_K(β=6.3)
B_K(β=6.45)
(r_0 M_PS)²

Preliminary Alpha result vs. rest of the world



$$B_K^{RGI} = 0.834(37)$$



Frezzotti-Rossi approach to B_K

Frezzotti and Rossi, JHEP 10 (2004) 070

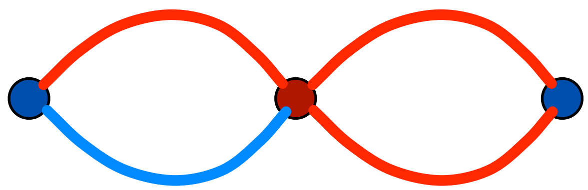
A different logic:

- Use only fully twisted quarks to preserve automatic O(a) improvement.
- Set up N_f=4 tmQCD for sea quarks with a non-diagonal twist (⇒ fermion determinant is always real).
- Set up a valence quark sector with Osterwalder-Seiler action (aka singlet chiral rotations allowed in the twist).

Key ingredient: tune the structure of the valence sector such that a correlation function can be constructed which:
1. Provides the observable of interest in the continuum limit.
2. Exploits maximally the symmetry structure ⇒ simplify renormalisation.

B_K: use 6 valence quarks (only 4 enter the relevant 3-point correlation function). Adjust signs of OS Wilson term as:

$$d = + \qquad s = + \qquad d' = + \qquad s' = -$$



- Spurionic (aka CPS) symmetry protects from mixing ⇒ "chiral" renormalisation properties.
- Two different (by $O(a^2)$) "kaons" running in the diagram.
- No additional complications related to renormalisation (mass-independent scheme assumed).

Remark: argument can be recast eliminating d'.

Renormalisation of $O^{\Delta B=2}$

Difficulties to fit all scales into a lattice computation → specific treatment of heavy quarks.

A favourite: HQET.

$$O_{\rm VV+AA}(\mu) = C_{\rm L}(m_b,\mu) O^{\rm HQET}_{\rm VV+AA}(\mu) + C_{\rm S}(m_b,\mu) O^{\rm HQET}_{\rm SS+PP}(\mu) + \mathcal{O}(1/m_b)$$

- Standard Wilson relativistic fermions: the four operators in the basis mix.
- Chiral relativistic fermions: the VV+AA and SS+PP operators renormalise multiplicatively (same Z).

Bećirević and Reyes, NPB (PS) 129 (2004) 435

Fully twisted relativistic fermion → VV+AA renormalises multiplicatively.

Della Morte, hep-lat/0409012

The argument can be generalised to the rest of the operator basis: in twisted variables, the relevant operators renormalise as if chiral relativistic fermions were employed (CPS in action!).

Starting point for a full nonperturbative renormalisation of the operator basis.

Alpha, work in progress

Technique applied in a quenched computation of bare matrix elements for B_{B_s} by the Orsay group.

Bećirević, private communication

Status of Orsay computation:

- Bare matrix element computed at three values of β, very good scaling properties → tentative CL extrapolation.
- Very small statistical errors thanks to HYP massaging of links.
- Excellent agreement with computation performed with overlap relativistic fermions at β=6.0.
- *Very* preliminary CL estimate:

$$\bar{B}_{B_s}(m_b) = 0.937(9)(\text{systematics})$$

- TO DO: control CL better, NP renormalisation, NP matching to QCD ...

Very encouraging results — improvement in the control of systematics in the computation of this observable is badly needed.

ΔS=1 transitions: K→ππ

The weak Hamiltonian contains several operators, strong cancellations between different contributions → very difficult problem.

CP-conserving approximation: above the charm threshold and neglecting top quark effects the LO effective Hamiltonian contains only two operators:

$$O^{\pm} = \left(\bar{s}\gamma_{\mu}^{L}d\right)\left(\bar{u}\gamma_{\mu}^{L}u\right) \pm \left(\bar{s}\gamma_{\mu}^{L}u\right)\left(\bar{u}\gamma_{\mu}^{L}d\right) - [u \leftrightarrow c]$$

Most striking dynamical feature: ΔI=1/2 rule.

$$A_I e^{i\delta_I} = \mathcal{A}(K^0 \to (\pi\pi)_I) \qquad \left|\frac{A_0}{A_2}\right| \approx 22$$

$K \to \pi\pi$ amplitudes on the lattice

- Final state interactions: physical amplitudes can only be extracted from Euclidean correlation functions if the volume is large enough.

  Maiani and Testa, PLB 245 (1990) 585

  Lellouch and Lüscher, CMP 219 (2000) 31

  Lin, Martinelli, Sachrajda and Testa, NPB 619 (2001) 467

  - Alternative: use χPT to relate $K \to \pi$ to $K \to \pi\pi$ amplitudes.

    Bernard et al., PRD 32 (1985) 313

- Operator mixing under renormalisation takes place even if chiral symmetry is present, but lost of chiral symmetry brings in extra mixing and diverging coefficients.

Renormalisation of $O^{\pm}$ with Wilson fermions

$$O^{\pm} = O^{\pm}_{\mathrm{VV+AA}} - O^{\pm}_{\mathrm{VA+AV}}$$

Parity-even part: K→π
Parity-odd part: K→ππ

$$\bar{O}^{\pm}_{\mathrm{VA+AV}}(\mu) = \lim_{a\to 0} Z^{\pm}_{\mathrm{VA+AV}}(g_0^2, a\mu)\left[O^{\pm}_{\mathrm{VA+AV}}(a) + \frac{c^{\pm}_P(g_0^2, am)}{a}(m_c - m_u)(m_s - m_d)\bar{s}\gamma_5 d\right]$$

$$\bar{O}^{\pm}_{\mathrm{VV+AA}}(\mu) = \lim_{a\to 0} Z^{\pm}_{\mathrm{VV+AA}}(g_0^2, a\mu)\left[O^{\pm}_{\mathrm{VV+AA}}(a) + \frac{c^{\pm}_S(g_0^2, am)}{a^2}(m_c - m_u)\bar{s}d + \right.$$
$$\left. + \sum_{k=1}^{4} \Delta^{\pm}_k(g_0^2) O_k(a) + c^{\pm}_{\sigma}(m_c - m_u)\bar{s}\sigma_{\mu\nu}F_{\mu\nu}d\right]$$

Observation: the parity-odd part exhibits better renormalisation properties, as it is protected by discrete symmetries.

Improving renormalisation properties of $O^{\pm}$

Use a tmQCD regularisation which maps three-point functions of the VV+AA operator to three-point functions of the VA+AV operator — preserving — or improving — its renormalisation properties.

- Four-flavour tmQCD.

  C.P., Sint and Vladikas, JHEP 09 (2004) 069

- Custom $(\pm\pi/2)$-twisted valence sector on $(\pi/2)$-twisted sea.

  Frezzotti and Rossi, JHEP 10 (2004) 070

N_f=4 tmQCD for the ΔI=1/2 rule

C.P., Sint and Vladikas, JHEP 09 (2004) 069

Two <u>different</u> twist angles introduced for the (u,d) and (s,c) doublets:

$$S_F^{\mathrm{ph}} = a^4 \sum_x (\bar{u}, \bar{d})(x) \Bigg[\frac{1}{2} \sum_\mu \gamma_\mu (\nabla_\mu + \nabla_\mu^*) + \begin{pmatrix} M_l & 0 \\ 0 & M_l \end{pmatrix} + $$

$$+ e^{-i\alpha\gamma_5\tau_3} \left(-\frac{ar}{2} \sum_\mu \nabla_\mu^* \nabla_\mu + M_{cr} \right) \Bigg] \begin{pmatrix} u \\ d \end{pmatrix} (x) +$$

$$+ a^4 \sum_x (\bar{s}, \bar{c})(x) \Bigg[\frac{1}{2} \sum_\mu \gamma_\mu (\nabla_\mu + \nabla_\mu^*) + \begin{pmatrix} M_s & 0 \\ 0 & M_c \end{pmatrix} + $$

$$+ e^{-i\beta\gamma_5\tau_3} \left(-\frac{ar}{2} \sum_\mu \nabla_\mu^* \nabla_\mu + M_{cr} \right) \Bigg] \begin{pmatrix} s \\ c \end{pmatrix} (x)$$

In the twisted basis:

$$S_F^{\rm tm} = a^4 \sum_x \left\{ (\bar{u}, \bar{d})(x) \left[D_W + m_l + i\mu_l \gamma_5 \tau_3 \right] \begin{pmatrix} u \\ d \end{pmatrix} (x) + \right.$$

$$\left. + \, \bar{s}(x) \left[D_W + m_s + i\mu_s \gamma_5 \right] s(x) \; + \; \bar{c}(x) \left[D_W + m_c + i\mu_c \gamma_5 \right] c(x) \right\}$$

$$m_l = M_l \cos\alpha \qquad \mu_l = M_l \sin\alpha$$

$$m_s = M_s \cos\beta \qquad \mu_s = M_s \sin\beta$$

$$m_c = M_c \cos\beta \qquad \mu_c = -M_c \sin\beta$$

Remarks:

- Renormalisation proceeds as in the N_f=2 case. Subtleties due to $M_s \neq M_c$ do not pose particular difficulties.

- The fermion determinant is complex for $M_s \neq M_c$, $\beta \neq 0$ → practical problem for dynamical simulations.
  - N.B.: fermion determinant real if a more general axial rotation is introduced.

    Frezzotti and Rossi, JHEP 10 (2004) 070

- Mapping between renormalised correlation functions proceeds as usual.

- O(a) improvement of the action in practical cases requires only the clover term.
  - N.B.: in the fully dynamical case there is an additional counterterm $(\mu_s + \mu_c) F\tilde{F}$.

Renormalisation of $O^{\pm}$

$$\langle \pi | \bar{O}^{\pm}_{\mathrm{VV+AA}}(\mu) | K \rangle_{\mathrm{QCD}} = -i \langle \pi | \bar{O}^{\pm}_{\mathrm{VA+AV}}(\mu) | K \rangle_{\alpha+\beta=\pi}$$

$$\langle \pi\pi | \bar{O}^{\pm}_{\mathrm{VA+AV}}(\mu) | K \rangle_{\mathrm{QCD}} = \langle \pi\pi | \bar{O}^{\pm}_{\mathrm{VA+AV}}(\mu) | K \rangle_{\alpha+\beta=0}$$

Renormalisation pattern of VA+AV:

$$\bar{O}^{\pm}_{\mathrm{VA+AV}}(\mu) = \lim_{a \to 0} Z^{\pm}(g_0^2; a\mu) \left[O^{\pm}_{\mathrm{VA+AV}} + c_P^{\pm}(g_0^2, M) \bar{s}\gamma_5 d + c_S^{\pm}(g_0^2, M) \bar{s} d \right]$$

$$c_P^{\pm} = \frac{c_P^{\pm\,(1)}(g_0^2)}{a}(m_c - m_u)(m_s - m_d) + \frac{c_P^{\pm\,(2)}(g_0^2)}{a}(\mu_c - \mu_u)(\mu_s - \mu_d) + \mathcal{O}(aM^3)$$

$$c_S^{\pm} = \frac{c_S^{\pm\,(1)}(g_0^2)}{a}(m_c - m_u)(\mu_s - \mu_d) + \frac{c_S^{\pm\,(2)}(g_0^2)}{a}(\mu_c - \mu_u)(m_s - m_d) + \mathcal{O}(aM^3)$$

The divergence in K→π matrix elements is only linear.

Low-energy constants from K→π matrix elements

$$\mathcal{A}(K^+(\mathbf{p}) \to \pi^+(\mathbf{q})) = \sqrt{2}\, G_F\, V_{ud} V_{us}^*\, F^2 \left\{ \left(g_8 + \frac{2}{3} g_{27} \right) p \cdot q + 2 M_K^2 g_8' \right\}$$

$$\mathcal{A}(K^0(\mathbf{p}) \to \pi^0(\mathbf{q})) = G_F\, V_{ud} V_{us}^*\, F^2 \left\{ (g_8 - g_{27}) p \cdot q + 2 M_K^2 g_8' \right\}$$

Simplest framework: both doublets maximally twisted.

Remark: flavour symmetries broken by twisting isospin decomposition of operators not defined in the regularised theory → study both $K^+ \to \pi^+$ and $K^0 \to \pi^0$ amplitudes.

K→π extracted e.g. from the three-point functions:

$$\langle (P_{du})_R(x)\, (O^{\pm}_{\mathrm{VA+AV}})_R(y)\, (S_{us})_R(z) \rangle_{(\pi/2,\pi/2)}$$

$$\langle (A_0^3)_R(x)\, (O^{\pm}_{\mathrm{VA+AV}})_R(y)\, (S_{ds})_R(z) \rangle_{(\pi/2,\pi/2)}$$

$c_P^{\pm}$ determined by imposing restoration of parity up to O(a):

$$\langle (O^{\pm}_{\mathrm{VA+AV}})_R(x)\ (P_{ds})_R(y)\rangle_{(\pi/2,\pi/2)} = 0 = \langle O^{\pm}_{\mathrm{VA+AV}}(x)\ P_{ds}(y)\rangle_{(\pi/2,\pi/2)}$$
$$+c_P^{\pm}\langle P_{sd}(x)\ P_{ds}(y)\rangle_{(\pi/2,\pi/2)}$$
$$+c_S^{\pm}\langle S_{sd}(x)\ P_{ds}(y)\rangle_{(\pi/2,\pi/2)}$$

Pure O(a):

$$\langle (S_{sd})_R(x)\ (P_{ds})_R(y)\rangle_{(\pi/2,\pi/2)} \stackrel{\text{C.L.}}{=} \langle (S_{sd})_R(x)\ (P_{ds})_R(y)\rangle_{(0,0)} = 0$$

$c_S^{\pm}$ determined by fixing the unphysical LEC g'_8, e.g.:

$$\langle \pi(\mathbf{q}=\mathbf{0})|(O^{\pm})_R|K(\mathbf{p}=\mathbf{0})\rangle_{\mathrm{QCD}} = 0$$

Getting rid of the power divergence

Observation: the subtraction of P arises from parity breaking, and is O(1) [= (1/a) x a]. Is it possible to pair the two subtractions into just one with a finite coefficient?

[Study sources of parity breaking ...]

O(a) improved action and quark masses

$\Downarrow$

1. It is enough to subtract S_{sd}.
2. The subtraction coefficient is finite.

[Assumption: gap between scalar and pseudoscalar channels broad enough.]

"Chiral" renormalisation pattern

Frezzotti-Rossi approach to K→ππ

Frezzotti and Rossi, JHEP 10 (2004) 070

Same logic as for B_K: set up valence sector which allows to construct a three- (four-) point function with optimal renormalisation properties.

It is possible to kill all undesired mixings via spurionic symmetries ⇒ same renormalisation pattern as if chiral fermions were used.

$$\bar{O}^{\pm}_{\rm VV+AA}(\mu) = \lim_{a\to 0} Z^{\pm}_{\rm VV+AA}(g_0^2, a\mu)\Big[O^{\pm}_{\rm VV+AA}(a) + c^{\pm}_S(g_0^2, am)(m_c^2 - m_u^2)(m_s + m_d)\bar{s}d +$$

$$+ac^{\pm}_P(g_0^2, am)(m_c^2 - m_u^2)(m_s^2 - m_d^2)\bar{s}\gamma_5 d\Big]$$

[N.B.: necessary to preserve O(a) improvement!]

Ten valence quark flavours needed:

K→ππ

K→π

$$d = + \qquad s = \pm$$

$$u = + \qquad u' = - \qquad u'' = + \qquad u''' = -$$

$$c = + \qquad c' = - \qquad c'' = + \qquad c''' = -$$

Remarks:

- Fermion determinant always real.
- Replication of valence quarks can be reduced in practice.
- Determination of mixing coefficients/matching for LEC's can be worked out pretty much in the same way as in the PSV framework.
- Quark masses entering subtractions have to be treated carefully.

tmQCD vs Overlap

[Cf. P. Hernández/A. Shindler's session]

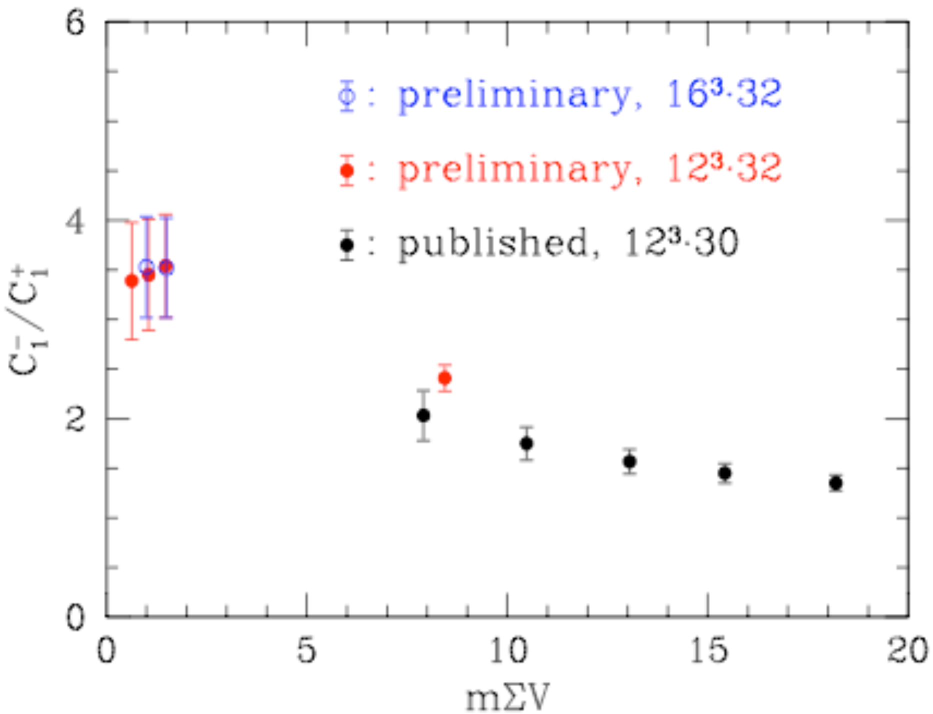
Overlap regularisations have a large number of advantages, particularly relevant in this context:

- Continuum-like renormalisation. (tmQCD almost there.)
- Chiral limit well defined in the regularised theory → no lower limit on the quark masses that can be reached (essential for matching to χPT).
- Cutoff effects are $O(a^2)$ and checked to be small.
- No numerical instabilities due to unphysical quasi-zero modes of the Dirac operator (same for tmQCD).

... and an important disadvantage: they remain very expensive, and currently are not a viable alternative for dynamical simulations.

Example of what tmQCD (most likely) cannot do: matching to χPT in the ϵ-regime:

Giusti, Hernández, C.P., Wittig, Wennekers, in preparation



N.B.: very expensive computation, albeit quenched: large volume, low-mode averaging, ... (cf. P. Hernández's talk).

Crucial issue: can tmQCD go down to M_{PS}~300 MeV and below without suffering from disruptive cutoff effects?

See M. Papinutto; R. Lewis, C. Michael, A. Shindler

Only in that case it will allow safe matching to χPT in the p-regime and hence be a competitive alternative to the overlap!

Conclusions

- tmQCD has led to remarkable progress in studies of matrix elements of four-fermion operators with Wilson-type quarks.
- Power of the approach demonstrated in practice: qualitative improvement in (preliminary) results for B_K and B_{B_s} with respect to previous Wilson computations.
- Work needed to understand the relationship between different tmQCD regularisations (particularly in the quenched case).
- It is very important to understand to what extent the chiral regime can be accessed with good control over cutoff effects.