Towards a quantitative understanding of the ΔI=1/2 rule

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Fermions and Extended Objects on the Lattice
Benasque, (25/02-02/03)/07

Outline

- The (many difficulties of studying the) $\Delta I=1/2$ rule:
  - Operator product expansion and long distance QCD effects.
  - Problems with pions and chirality.
  - Our strategy.
- Computational setup:
  - Low-energy description: p- vs ϵ-regime.
  - Chiral lattice fermions.
- Computational details:
  - Bare results and the approach to the chiral regime.
  - Matching to physics: renormalisation and chiral fits.
- Results and conclusions.

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$K \to \pi\pi$ decays in a nutshell

- Hamiltonian for the dynamics of K^0–$\bar{K}^0$system determined by hermiticity+CPT:

$$H = M - \tfrac{i}{2}\Gamma = \begin{pmatrix} A & p^2 \\ q^2 & A \end{pmatrix}$$

- If CP is conserved the eigenstates of the Hamiltonian are $|K_{1,2}\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle \pm |\bar{K}^0\rangle)$. CP violation in the SM leads to mixing:

$$|K_S\rangle = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(|K_1\rangle + \bar{\varepsilon}|K_2\rangle) \quad |K_L\rangle = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(|K_2\rangle + \bar{\varepsilon}|K_1\rangle) \qquad \bar{\varepsilon} = \frac{p-q}{p+q}$$

- CP violation parameters accessible via decay amplitudes into two pions:

$$-iT[K^0 \to (\pi\pi)_I] = A_I\, e^{i\delta_I} \qquad T[(\pi\pi)_I \to (\pi\pi)_I]_{l=0} = 2e^{i\delta_I}\sin\delta_I$$

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$$\varepsilon = \frac{T[K_L \to (\pi\pi)_0]}{T[K_S \to (\pi\pi)_0]} \simeq \bar{\varepsilon} + i\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}$$

$$\varepsilon' = \frac{\varepsilon}{\sqrt{2}}\left(\frac{T[K_L \to (\pi\pi)_2]}{T[K_L \to (\pi\pi)_0]} - \frac{T[K_S \to (\pi\pi)_2]}{T[K_S \to (\pi\pi)_0]}\right) \simeq \frac{1}{\sqrt{2}}e^{i(\delta_2-\delta_0+\pi/2)}\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

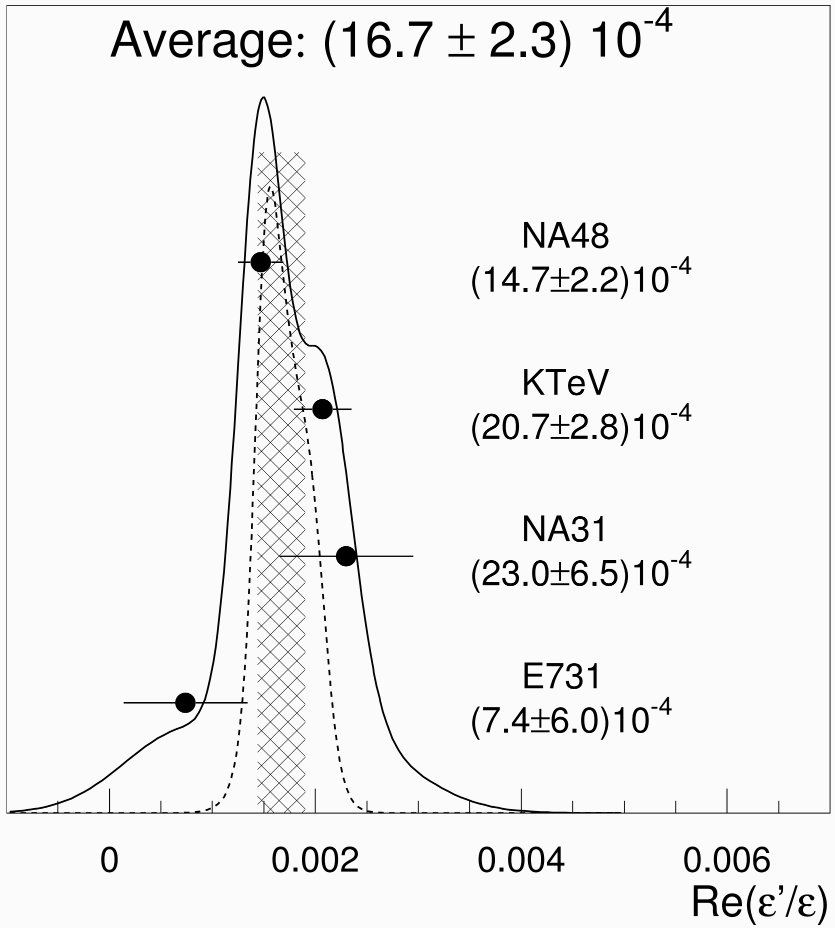
$K \to \pi\pi$ decays in a nutshell

Experiment:

$$\left| \frac{A_0}{A_2} \right| \simeq 22.1$$

$$|\varepsilon| = (2.282 \pm 0.017) \times 10^{-3}$$

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (16.7 \pm 2.3) \times 10^{-4}$$



The ΔI=1/2 rule for kaon decays

$$T(K \to (\pi\pi)_\alpha) = iA_\alpha e^{i\delta_\alpha}, \quad \alpha = 0,2 \qquad |A_0/A_2| = 22.1$$

- Bulk of enhancement in the SM must come from long-distance strong interaction effects ...

Gaillard & Lee, PRL 33 (1974) 108
Altarelli & Maiani, PLB 52 (1974) 351

- ... that have to be addressed non-perturbatively.

Cabibbo, Martinelli & Petronzio, NPB 244 (1984) 381
Brower, Maturana, Gavela & Gupta, PRL 53 (1984) 1318

- Lattice QCD studies hampered by no-go theorems on chiral fermions and multiparticle decays, almost no activity in the '90s.

- Theoretical breakthroughs in late '90s (mainly chiral lattice fermions) have led to a renewed interest and some "rough" lattice results.

CP-PACS & RBC Collaborations

- Still far from having an understanding of the mechanism(s) behind the enhancement.

Effective Weak Hamiltonian

$$\mathcal{A}(i \to f) \approx \langle f | H_{\mathrm{W}}^{\mathrm{eff}} | i \rangle$$

$$H_{\mathrm{W}}^{\mathrm{eff}} = \frac{G_{\mathrm{F}}}{\sqrt{2}} \sum_k f_k(V_{\mathrm{CKM}}) C_k(\mu / M_{\mathrm{W}}) \bar{O}_k(\mu)$$

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CKM parameters

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Wilson coefficients — high energy, NLO computation

Composite operators — low energy (hadronic) scales

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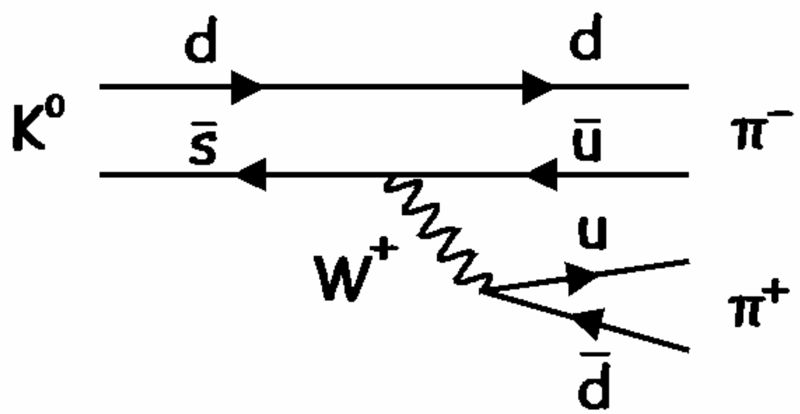

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With an active charm quark (CP-violating effects neglected):

$$H_{\mathrm{w}} = \frac{g_{\mathrm{w}}^2}{2M_{\mathrm{W}}^2} (V_{us})^* (V_{ud}) \sum_{\sigma=\pm} \{k_1^\sigma \mathcal{Q}_1^\sigma + k_2^\sigma \mathcal{Q}_2\}$$

$$\mathcal{Q}_1^{\pm} = (\bar{s}\gamma_\mu P_- u)(\bar{u}\gamma_\mu P_- d) \pm (\bar{s}\gamma_\mu P_- d)(\bar{u}\gamma_\mu P_- u) \; - \; [u \; \longrightarrow \; c]$$

$$\mathcal{Q}_2^{\pm} = (m_u^2 - m_c^2) \, \{m_d(\bar{s}P_+ d) \, + \, m_s(\bar{s}P_- d)\}$$

$\mathcal{Q}_1^{\pm}$ transform according to irreps of d=84 (+) and d=20 (-) of SU(4).
$\mathcal{Q}_2^{\pm}$ do not contribute to the physical K→ππ transition.

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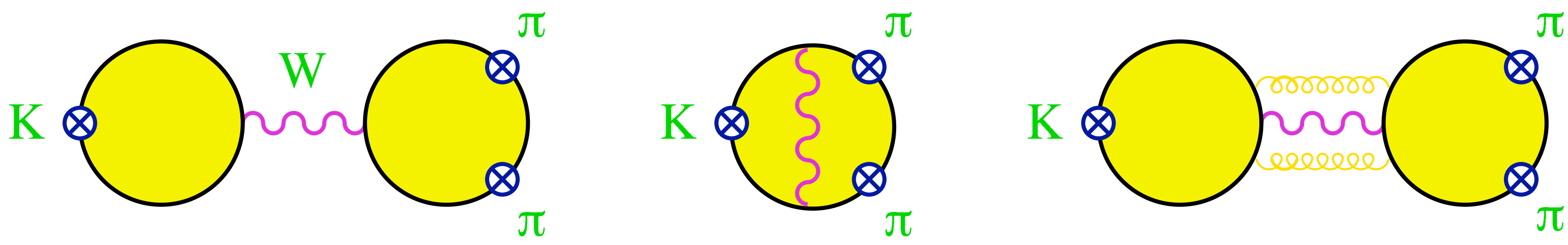
$$\left|\frac{A_0}{A_2}\right| = \frac{k_1^-(M_W)}{k_1^+(M_W)} \frac{\langle(\pi\pi)_{I=0}|\hat{Q}_1^-|K\rangle}{\langle(\pi\pi)_{I=2}|\hat{Q}_1^+|K\rangle} \qquad \frac{k_1^-(M_W)}{k_1^+(M_W)} = 2.8 \sim O(1)$$

Enhancement dominated by matrix elements of effective interaction vertices (long-distance regime of the strong interaction).

A flagrant failure of large N_c

$$H_{\mathrm{W}}^{\Delta S=1} \sim G_{\mathrm{F}} J_{\mathrm{W}}^{\mu} J_{\mathrm{W}}^{\mu}$$



$O(N_C^2)$ $O(N_C)$ $O(1)$

$$T(K^0 \to \pi^0\pi^0) \sim 0 \quad \Rightarrow \quad \left.\frac{A_0}{A_2}\right|_{N\to\infty} \sim \sqrt{2}$$

Fukugita et al. 1977; Chivukula, Flynn, Georgi 1986

Effective Weak Hamiltonian

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Enhancement dominated by matrix elements of effective interaction vertices (long-distance regime of the strong interaction).

Well, let's compute the matrix elements ...

The realm of no-go theorems

Maiani-Testa theorem: physical $A(i \to f_1 \ldots f_n)$ cannot be extracted from Euclidean lattice amplitudes in the infinite volume limit.

Maiani & Testa, PLB 245 (1990) 585

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Maiani & Testa, PLB 245 (1990) 585

Physical matrix elements can still be extracted by matching QCD at infinite and finite volume — but the volumes required are prohibitively large.

Lellouch & Lüscher, CMP 219 (2001) 31

Lin, Martinelli, Sachrajda & Testa, NPB 619 (2001) 467

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Bernard et al., PRD 32 (1985) 2343

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Absence of chiral symmetry induces operator mixing with (severely) power-divergent coefficients → it is very difficult to construct the renormalised $H_{\rm w}$.

Bochicchio et al., NPB 262 (1985) 331

Maiani, Martinelli, Rossi & Testa, NPB 289 (1987) 505

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Use regularisations with exact chiral symmetry, or with better chiral properties.

Capitani & Giusti, PRD 64 (2001) 014506
CP, Sint & Vladikas, JHEP 09 (2005) 069
Frezzotti & Rossi, JHEP 10 (2005) 070

A tale of various scales

M_W $\quad \mathcal{H}_{SM} \to \mathcal{H}_{\Delta S=1}^{N_f=4} = \sqrt{2} G_F V_{us}^* V_{ud} (k_+ Q_+ + k_- Q_-)$

$$Q_\pm \equiv [\bar{s}u]_{V-A}[\bar{u}d]_{V-A} \pm [\bar{s}d]_{V-A}[\bar{u}u]_{V-A} - (u \leftrightarrow c)$$

$SU(4)_L \times SU(4)_R$: $Q_+ \to (84,1) \quad Q_- \to (20,1)$

m_c $\quad \mathcal{H}_{\Delta S=1}^{N_f=4} \to \mathcal{H}_{\Delta S=1}^{N_f=3} = \sqrt{2} G_F V_{us}^* V_{ud} \sum_{\sigma=1,10} C_\sigma Q_\sigma$

$$Q_\sigma : \ldots, [\bar{s}d]_{V-A}[\bar{q}q]_{V+A}, \ldots$$

$SU(3)_L \times SU(3)_R$: $(27,1) \to A_2, A_0, (8,1) \to A_0$

Λ_χ $\quad \mathcal{H}_{\Delta S=1}^{N_f=3} \quad \to \quad \mathcal{H}_{\chi PT}^{N_f=3}$

A tale of various scales

The standard [?] lore:

- Resummation of $O(1/N)\log(\mu/M_W)$ up to $\mu > m_c$ gives a moderate enhancement.
- Charm threshold: $\mu < m_c \longrightarrow$ penguins.
- Penguin matrix elements can be large compared to that of left-left operators.

Still to be verified/discarded via an explicit computation ...

Shifman, Vainshtein, Zakharov 1977; Bardeen, Buras, Gerard 1986

Existing results for A_0, A_2?

- Use of χPT for weak decays already developed in the '80s.

  Georgi 84; Bernard et al. 85; Kambor et al. 91

- Exploratory lattice computations have obtained statistical signals for the relevant matrix elements in the quenched approximation, but suffer from uncontrolled systematic uncertainties.

  Kilcup, Pekurovsky 98; Blum et al. 01; Ali Khan et al. 01

  - Approximate chiral symmetry.
  - Charm integrated out: severe ultraviolet problems (effective Hamiltonian contains 10 operators, ultraviolet-divergent mixing even with exact chiral symmetry).
  - Large quark masses.

- Many works rely in models for low-energy strong interactions.

  See reviews in Bertolini et al. 00, Pallante et al. 01

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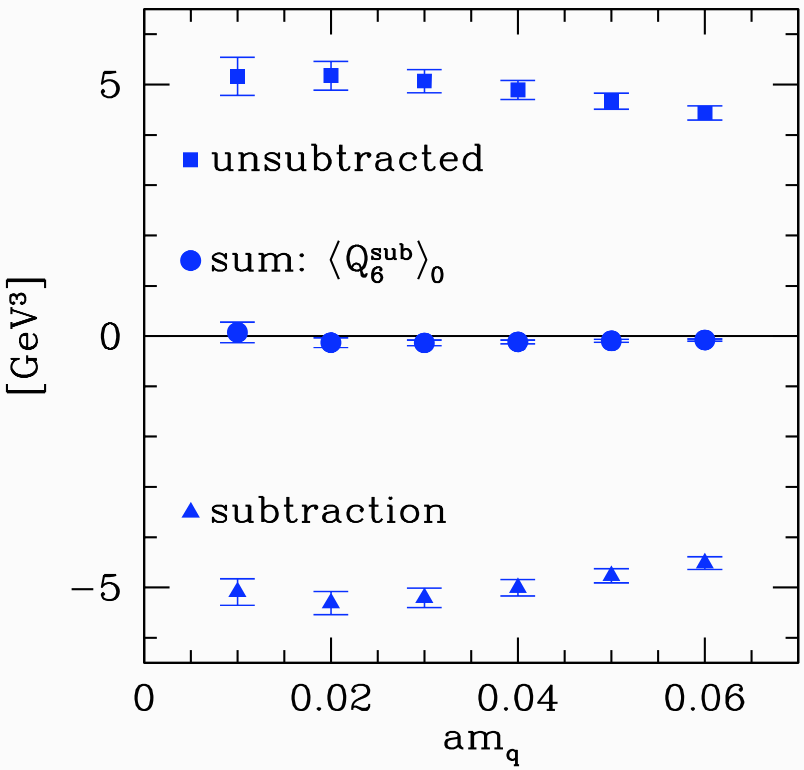

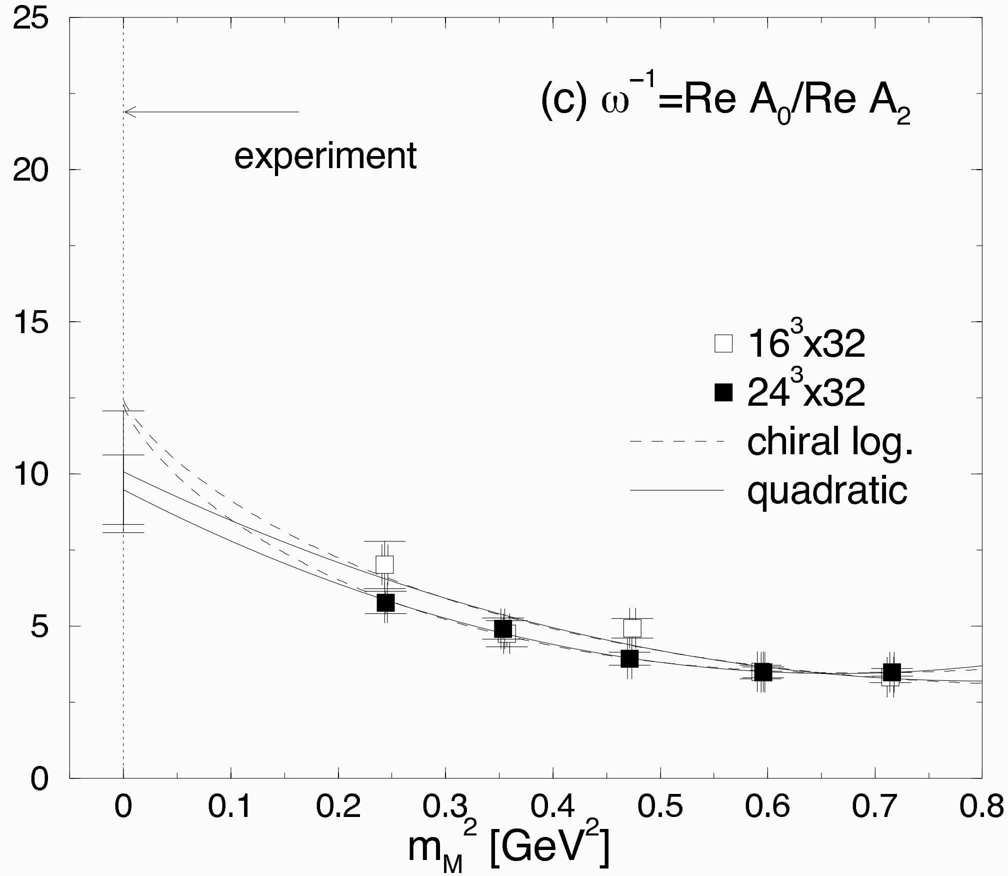

Lightest pion mass around 495 MeV.

CP-PACS Collaboration (Ali Khan et al.) 01

Our strategy to reveal the role of the charm

Disentangle several possible origins/contributions:

- Physics at the charm scale (via penguins).
- Physics at intrinsic QCD scale ~200-300 MeV.
- Final state interactions.
- All of the above (no dominating "mechanism").

Separate "intrinsic QCD" effects from physics at the charm scale:

Consider effective weak Hamiltonian with an active charm and study A_0, A_2 as a function of m_c.

$$m_u = m_d = m_s = m_c$$

$$\downarrow$$

$$m_u = m_d = m_s \ll m_c$$

Giusti, Hernández, Laine, Weisz & Wittig, JHEP 11 (2004) 016

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Effective low-energy description

Dynamics of Goldstone bosons @ LO:

$$\mathcal{L}_{\rm E} = \tfrac{1}{4}F^2 {\rm Tr}\left[\partial_\mu U \partial_\mu U^\dagger\right] - \tfrac{1}{2}\Sigma {\rm Tr}\left[U M^\dagger e^{i\theta/N_{\rm f}} + \text{ h.c.}\right]$$

$$U \in {\rm SU}(4)\,, \quad M = \text{ mass matrix}$$

Low-energy counterpart of the weak effective Hamiltonian @ LO:

$$\mathcal{H}_{\rm W}^{\chi{\rm PT}} = \frac{g_w^2}{2M_W^2}(V_{us})^*(V_{ud}) \sum_{\sigma=\pm} g_1^\sigma \left\{[\widehat{\mathcal{O}}_1^\sigma]_{suud} - [\widehat{\mathcal{O}}_1^\sigma]_{sccd}\right\}$$

$$[\widehat{\mathcal{O}}_1]_{\alpha\beta\gamma\delta} = \tfrac{1}{4}F^4 (U\partial_\mu U^\dagger)_{\gamma\alpha}(U\partial_\mu U^\dagger)_{\delta\beta}$$

Relation of LEC's to $K\to\pi\pi$ transition amplitudes @ LO in χPT:

$$\frac{A_0}{A_2} = \frac{1}{\sqrt{2}}\left(\frac{1}{2} + \frac{3}{2}\frac{g_1^-}{g_1^+}\right)$$

⇒ Determine LEC's using lattice QCD

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n.b.: $g_1^\pm\big|_{N\to\infty} = 1$

Matching QCD to the chiral expansion

$$R^{\pm}(x_0, y_0) = \frac{C^{\pm}(x_0, y_0)}{C(x_0)C(y_0)}$$

$$C^{\pm}(x_0, y_0) = \sum_{\mathbf{x},\mathbf{y}} \langle [J_0(x)]_{du} [Q_1^{\pm}(0)] [J_0(y)]_{us} \rangle$$

$$C(x_0) = \sum_{\mathbf{x}} \langle [J_0(x)]_{ds} [J_0(0)]_{sd} \rangle$$

QCD →

$$k_{\mathrm{RGI}}^{\pm} \left[\frac{Z^{\pm}}{Z_{\mathrm{A}}^{2}} \right]_{\mathrm{RGI}} R^{\pm} = g^{\pm} \mathcal{R}^{\pm}(m, V, \mathrm{LECs})$$

← **χPT**

$$\widehat{\mathcal{C}}^{\pm}(x_0, y_0) = \int \mathrm{d}^3x\, \mathrm{d}^3y\, \langle \mathcal{J}_0(x)\, \mathcal{O}_1^{\pm}(0) \mathcal{J}_0(y) \rangle$$

$$\mathcal{C}(x_0) = \int \mathrm{d}^3x\, \langle \mathcal{J}_0(x)\, \mathcal{J}_0(0) \rangle$$

$$\mathcal{R}^{\pm}(x_0, y_0) = \frac{\widehat{\mathcal{C}}^{\pm}(x_0, y_0)}{\mathcal{C}(x_0)\mathcal{C}(y_0)}$$

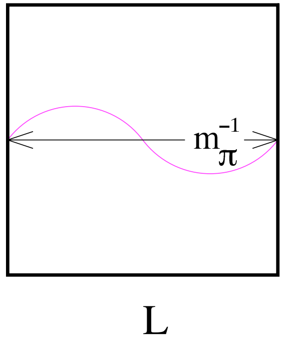
- p-regime: new LECs appear at NLO
- ϵ-regime: no additional $\Delta S{=}1$ interaction terms at $O(\epsilon^2)$ ⇒ enables matching at NLO!

ϵ- vs p-regime of Chiral Perturbation Theory

Gasser, Leutwyler 1987; Hansen 1990; Hansen, Leutwyler 1991

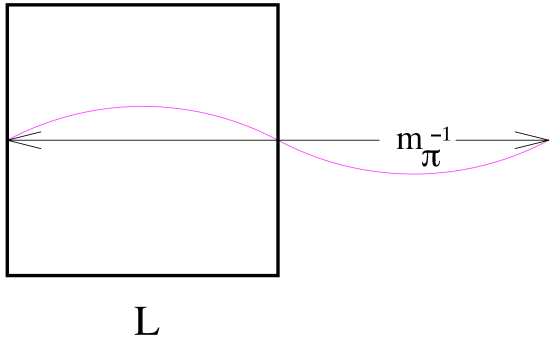
p-regime: $m\Sigma V \gg 1$

standard χPT in finite V: $m \sim p^2 \qquad L^{-1}, T^{-1} \sim p$



ϵ-regime: $m\Sigma V \lesssim 1$

reordering of the χ expansion: $m \sim p^4 \sim \epsilon^4 \qquad L^{-1}, T^{-1} \sim \epsilon$



➽ Constant field configurations (zero modes) are factored out and treated as collective variables.

➽ Gauge field topology and the low-lying spectrum of the Dirac operator play a crucial rôle in this regime.

➽ No additional interactions in the effective chiral theory at $O(\epsilon^2)$.

$R^{\pm}$ in the ϵ-regime

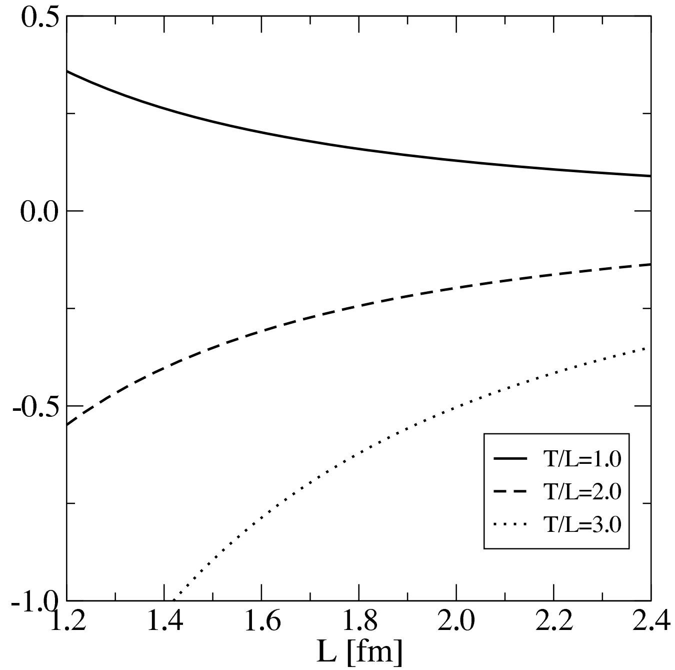
Hernández, Laine, 2003;
Giusti, Hernández, Laine, Weisz, Wittig 2004

$$2\,\mathcal{R}^{\pm}(x_0, y_0) = 1 \pm \frac{2}{(FL)^2}\left[\rho^{-1/2}\beta_1 - \rho k_{00}\right] = 1 \pm K$$

with $\rho \equiv T/L$ and β_1, k_{00} are shape coefficients of the box.

- same for all ν
- independent of x_0 and y_0
- same in (partially-)quenched theory
- no higher order weak or strong LECS

K



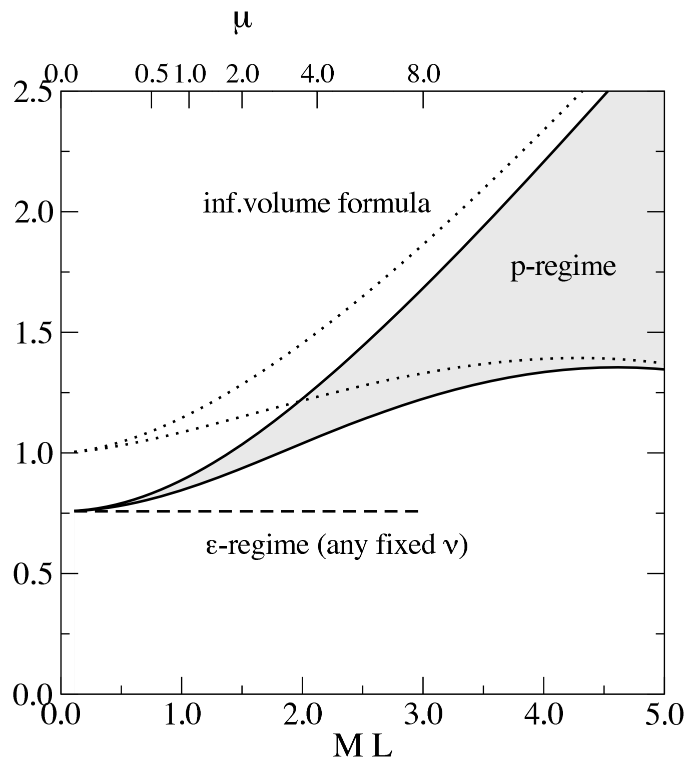
$R^{\pm}$ in the p-regime

Hernández, Laine 2006

For these observables the ϵ-regime and ∞-volume results can be smoothly reached from the p-regime expressions

$N_f = 3,\ L = 2\text{fm},\ T/L = 2,\ \Lambda_+ = 500 - 2000\text{MeV}$:

$2\mathcal{R}^+(-T/3, T/3)$



Deviations from the infinite volume expectation are significant for $ML \leq 5$

Lattice setup: chiral fermions

Breaking of chiral symmetry ⇒ less protection against mixing of composite operators under renormalisation.

- Power divergences in mixing with lower dimension operators.
- Mixing with operators in different chiral multiplets.

Bochicchio, Maiani, Martinelli, Rossi, Testa 1985
Maiani, Martinelli, Rossi, Testa 1987

Obvious alternatives: "better" Wilson regularisations (tmQCD-based) ...

CP, Sint, Vladikas 2005
Frezzotti, Rossi 2005

... or exact chiral symmetry ⇒ continuum-like renormalisation.

$$Q_1^{\pm} = \mathcal{Z}_{11}^{\pm} Q_1^{\pm,\mathrm{bare}} + \mathcal{Z}_{12}^{\pm} Q_2^{\pm,\mathrm{bare}}$$
$$Q_2^{\pm} = \mathcal{Z}_{21}^{\pm} Q_1^{\pm,\mathrm{bare}} + \mathcal{Z}_{22}^{\pm} Q_2^{\pm,\mathrm{bare}}$$

- exact chiral symmetry ⇒ arbitrarily low masses ⇒ ϵ-regime.

Lattice setup: chiral fermions

Ginsparg-Wilson fermions:

$$\gamma_5 D + D\gamma_5 = \bar{a}\, D\gamma_5 D\,, \qquad \bar{a} = \frac{a}{1+s}$$

Ginsparg, Wilson 1982

Kaplan; Hasenfratz, Laliena, Niedermayer; Neuberger; ...

Lattice QCD action enjoys an exact chiral symmetry:

$$\delta\psi = i\epsilon\hat{\gamma}_5\psi\,, \qquad \hat{\gamma}_5 = \gamma_5(\mathbf{1} - \bar{a}D)$$

$$\delta\bar{\psi} = i\epsilon\bar{\psi}\gamma_5$$

Lüscher 1998

Renormalisation and mixing patterns as in the formal continuum theory, provided:

$$\psi \;\rightarrow\; \tilde{\psi} = (\mathbf{1} - \tfrac{1}{2}\bar{a}D)\,\psi\,, \qquad \bar{\psi} \;\rightarrow\; \bar{\psi}$$

In particular, there is no dangerous mixing with lower dim. operators.

Lattice setup: chiral fermions

Ginsparg-Wilson fermions:

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Ginsparg, Wilson 1982

Kaplan; Neuberger; Hasenfratz, Laliena, Niedermayer; ...

Our choice: Neuberger-Dirac operator.

$$D_{\mathrm{N}} = \frac{1}{\bar{a}}\left\{1 - \frac{A}{(A^{\dagger}A)^{1/2}}\right\}, \qquad A = 1 - aD_{\mathrm{w}}$$

Neuberger 1997

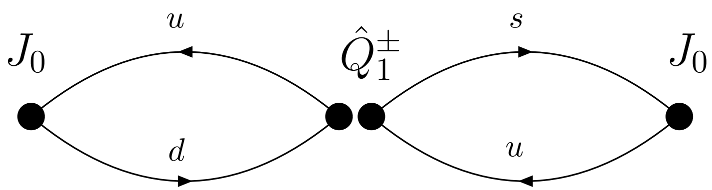
Numerical treatment challenging and expensive.

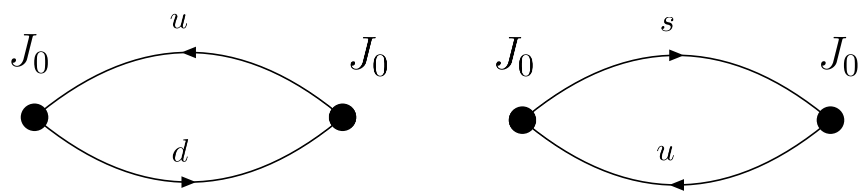
Giusti, Hoelbling, Lüscher, Wittig 2002

Outline

- The (many difficulties of studying the) $\Delta I = 1/2$ rule.
  - Operator product expansion and long distance QCD effects.
  - Problems with pions and chirality.
  - Our strategy.
- Computational setup.
  - Low-energy description: p- vs ϵ-regime.
  - Chiral lattice fermions.
- Computational details.
  - Bare results and the approach to the chiral regime.
  - Matching to physics: renormalisation and chiral fits.
- Results and conclusions.

Numerical simulations





$$R_{\pm}(x_0, y_0) = \frac{\sum_{\vec{x},\vec{y}} \langle J_0(x) \hat{Q}_1^{\pm}(0) J_0(y) \rangle}{\sum_{\vec{x}} \langle J_0(x) J_0(0) \rangle \cdot \sum_{\vec{y}} \langle J_0(0) J_0(y) \rangle} \propto g_1^{\pm}$$

Simulation parameters:

$\beta = 5.8485 \qquad \frac{V}{a^4} = 16^3 \cdot 32 \qquad a \approx 0.125 \text{ fm} \qquad V \approx 2^3 \cdot 4 \text{ fm}^4$

Quark masses: p-regime $m \sim m_s/2 - m_s/6$ O(200) cfgs
ϵ-regime $m \sim m_s/40, m_s/60$ O(800) cfgs

Quenched approximation.

Numerical simulations

- p-regime: ratios of correlation functions display plateaux at large enough separations ⇒ ratios of physical matrix elements.

- ϵ-regime: expansion performed at fixed topological charge, ratios of correlators fitted to constants and averaged over topological charges (no -dependence at (N)LO in χPT).

Simulation parameters:

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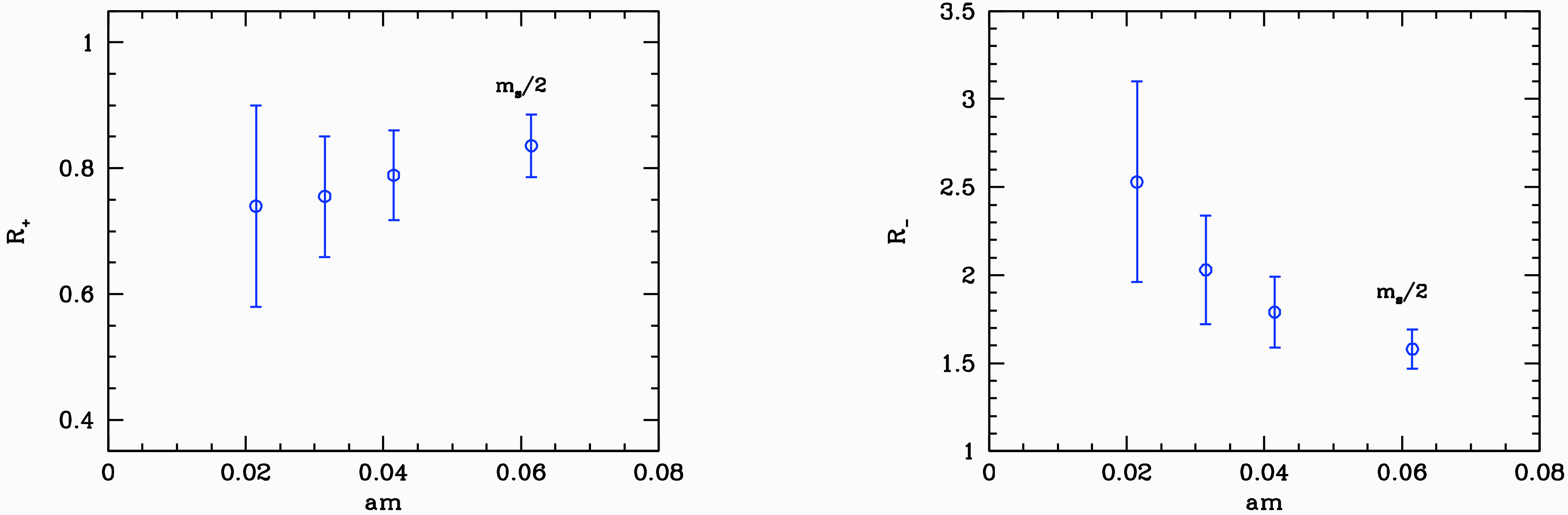

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Quenched approximation.

The origin of statistical fluctuations

$$S(x,y) = \frac{1}{V}\sum_k \frac{\eta_k(x) \otimes \eta_k(y)^\dagger}{\bar{\lambda}_k + m}, \qquad \bar{\lambda}_k = (1 - \tfrac{1}{2}\bar{a}m)\lambda_k$$

$m \gg 1/\Sigma V$

- Low-lying spectrum of D_m dense near m.
- Contributions from low modes averaged with same weight.

$m \ll 1/\Sigma V$

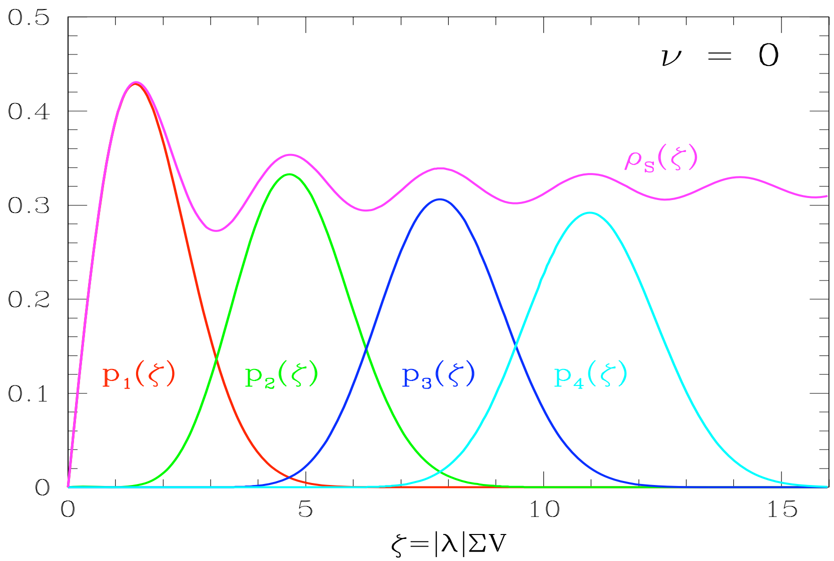
- Sizeable contribution of configurations with very small ev's.
- Strong dependence on the observable considered.
- Fluctuations reduced by unquenching.

$m \lesssim 1/\Sigma V$

- Low-lying spectrum of D_m discrete: $m \approx \Delta\lambda = 1/\Sigma V$
- A few low modes give sizeable contributions.
- “Bumpy” wave functions can induce large fluctuations.

The origin of statistical fluctuations

$$S(x,y) = \frac{1}{V}\sum_k \frac{\eta_k(x)\otimes\eta_k(y)^\dagger}{\bar{\lambda}_k + m}\,, \qquad \bar{\lambda}_k = (1-\tfrac{1}{2}\bar{a}m)\lambda_k$$



$$\langle\lambda_i\rangle_\nu = \frac{\mathcal{O}(1)}{\Sigma V}$$

$$\Delta\lambda = \lambda_{i+1} - \lambda_i \sim \frac{\mathcal{O}(1)}{\Sigma V} \geq m$$

- Low-lying spectrum of D_m discrete.
- “Bumpy” wave functions of few low-lying modes ⇒ large fluctuations.

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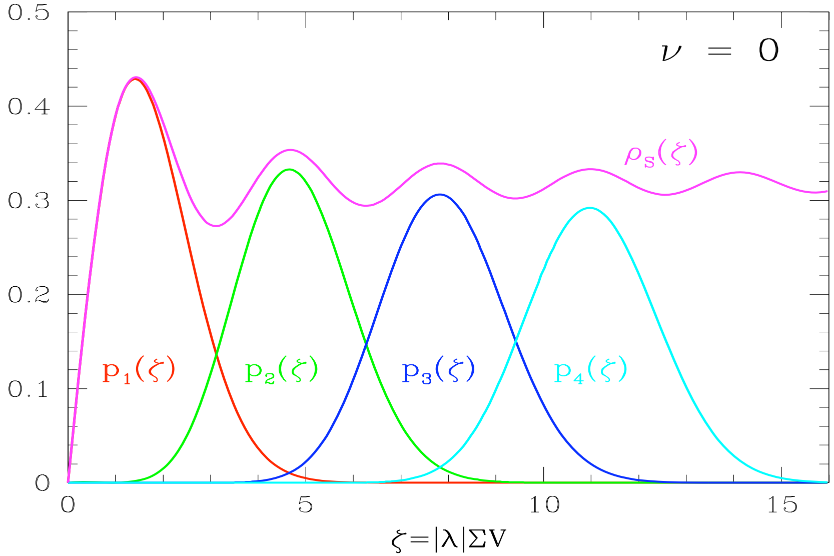

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Low-mode averaging: treat exactly a few low-modes — full translational invariance enforced in the most fluctuating contribution to the quark propagator.

Giusti, Hernández, Laine, Weisz, Wittig 2004
Giusti, Hernández, Laine, CP, Wennekers, Wittig 2005

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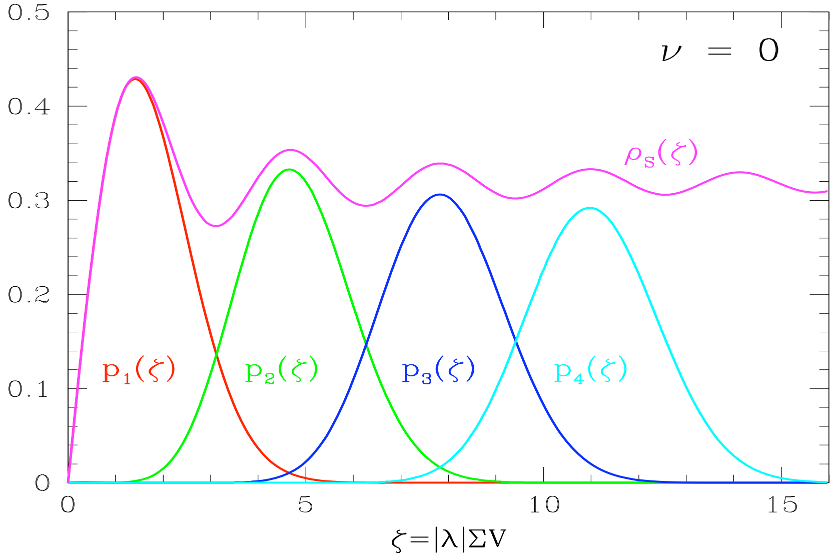

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- "Bumpy" wave functions of few low-lying modes ⇒ large fluctuations.

Alternative: extract physics from topological zero-mode wave functions.

Giusti, Hernández, Laine, Weisz, Wittig 2004
Hernández, Laine, CP, Torró, Wennekers, Wittig in progress

The origin of statistical fluctuations

$$P_- S(x,y) P_+ = P_- \left\{ \sum_{k=1}^{n} \frac{1}{\alpha_k} e_k(x) \otimes e_k(y)^\dagger + S^h(x,y) \right\} P_+$$

$$e_k = P_s u_k + P_{-s} D P_s u_k \,, \qquad P_s (D_m^\dagger D_m) P_s u_k = \alpha_k u_k + r_k$$

$$C_t(x_0) = C_t^{hh}(x_0) + C_t^{hl}(x_0) + C_t^{ll}(x_0)$$

$$C_t^{hh}(x_0) = -\sum_{\mathbf{x}} \langle \mathrm{Tr}[\gamma_0 P_- S^h(x,0)^\dagger \gamma_0 P_- S^h(x,0)] \rangle$$

$$C_t^{ll}(x_0) = -\frac{1}{V} \sum_{k,l=1}^{n} \sum_{y,z} \delta_{x_0, y_0 - z_0} \frac{1}{\alpha_k \alpha_l} \langle [e_k(y)^\dagger \gamma_0 P_- e_l(y)][e_k(z)^\dagger \gamma_0 P_- e_l(z)] \rangle$$

$$C_t^{hl}(x_0) = -\frac{1}{L^3} \sum_{k=1}^{n} \sum_{\mathbf{y},z} \delta_{x_0, y_0 - z_0} \frac{1}{\alpha_k} \langle e_k(y)^\dagger \gamma_0 P_- S^h(y,z) \gamma_0 P_- e_k(z) \rangle \; + \; [y \leftrightarrow z]$$

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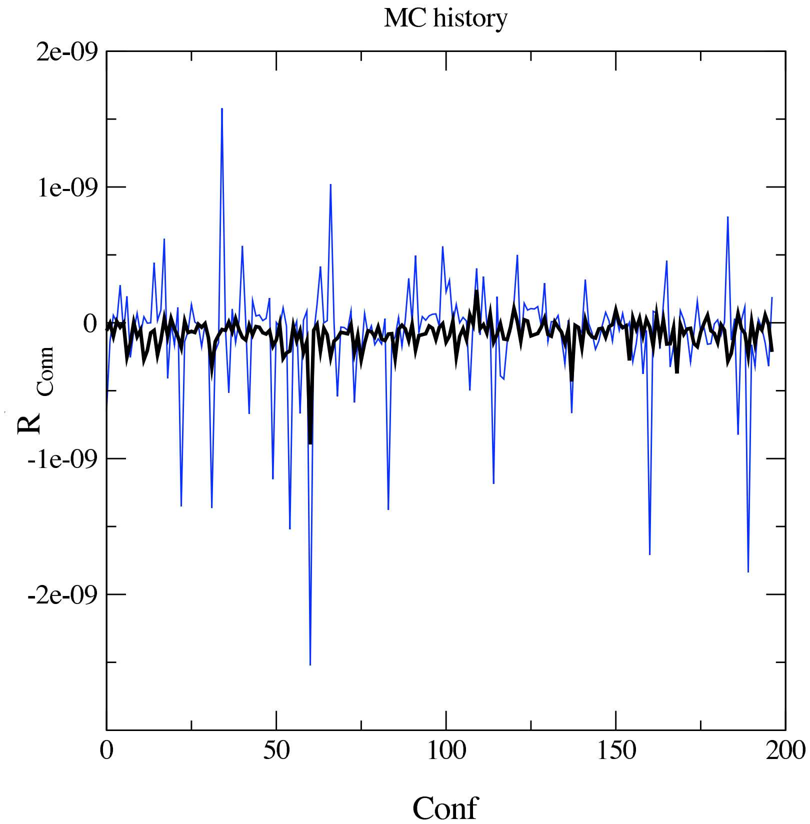

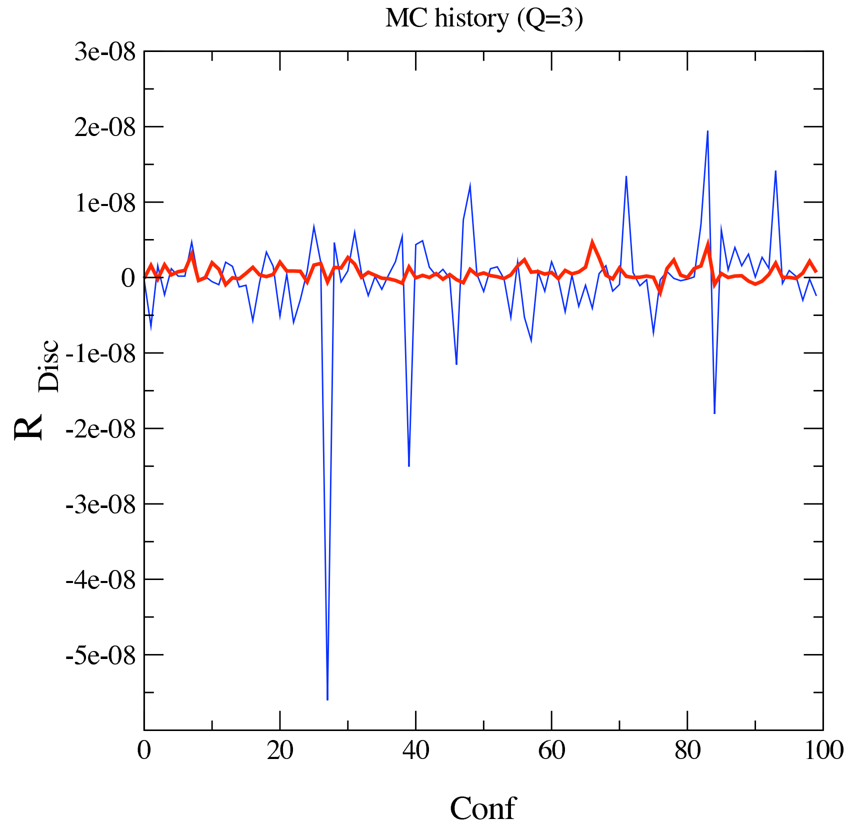

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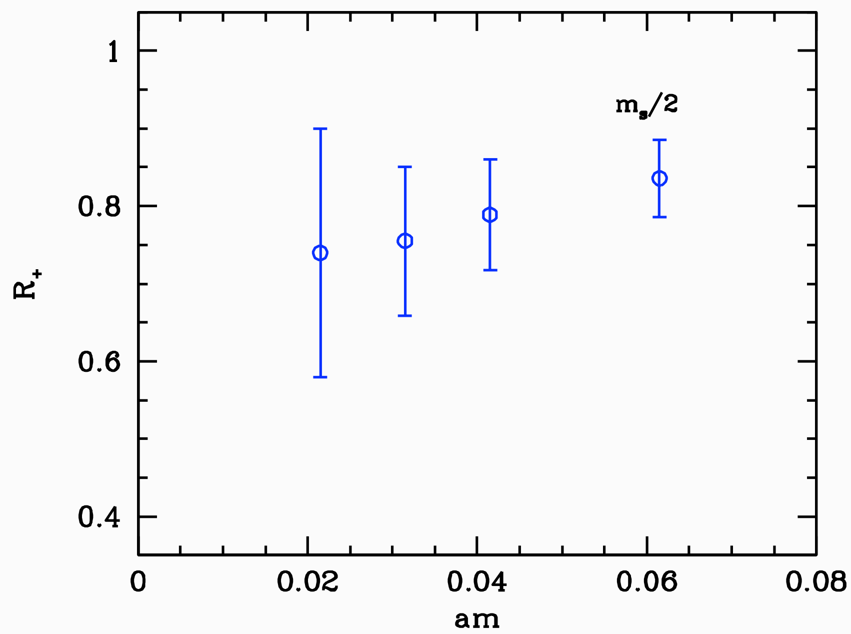
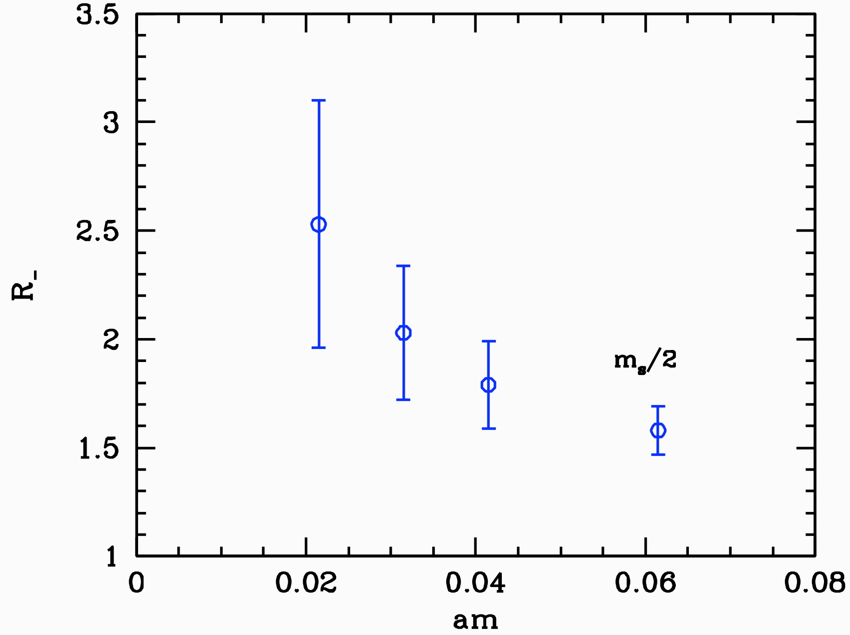
Numerical simulations: final (bare) results





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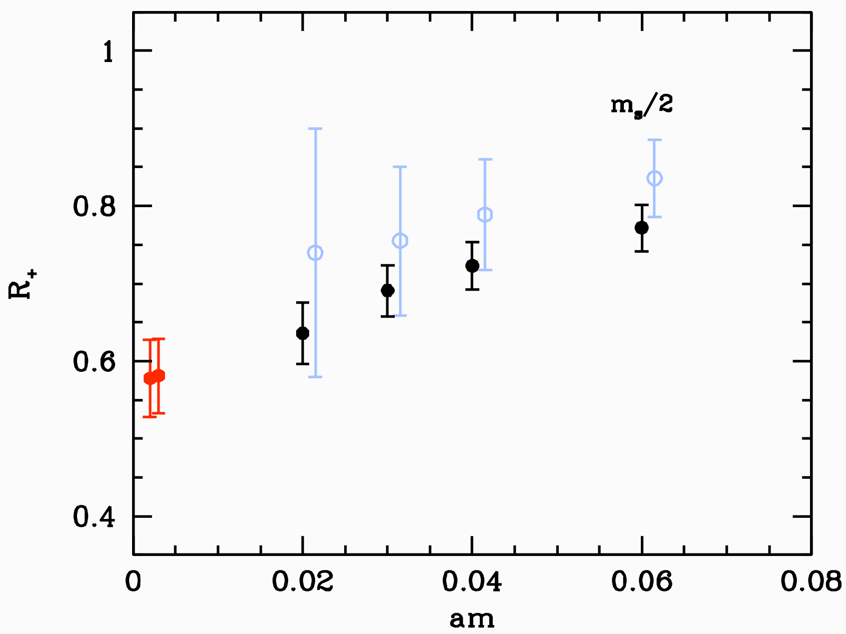

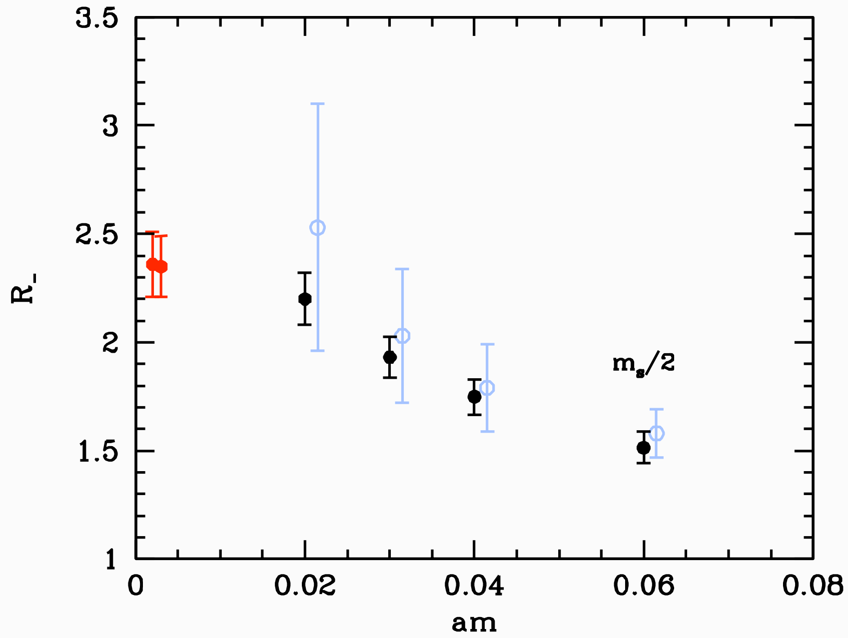

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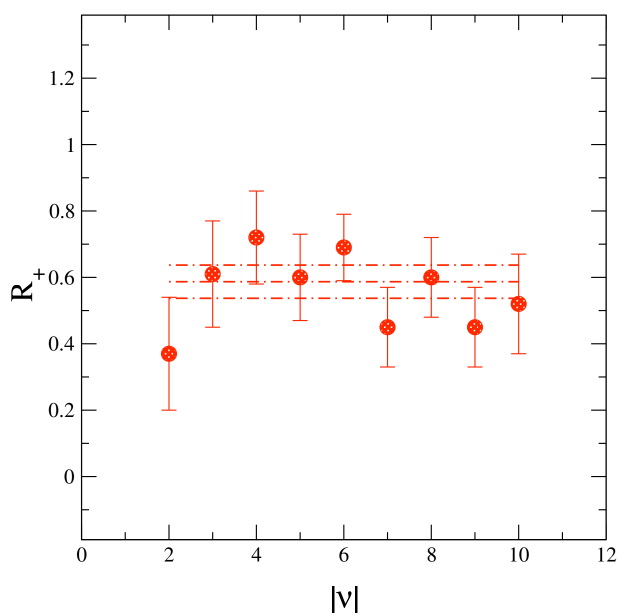
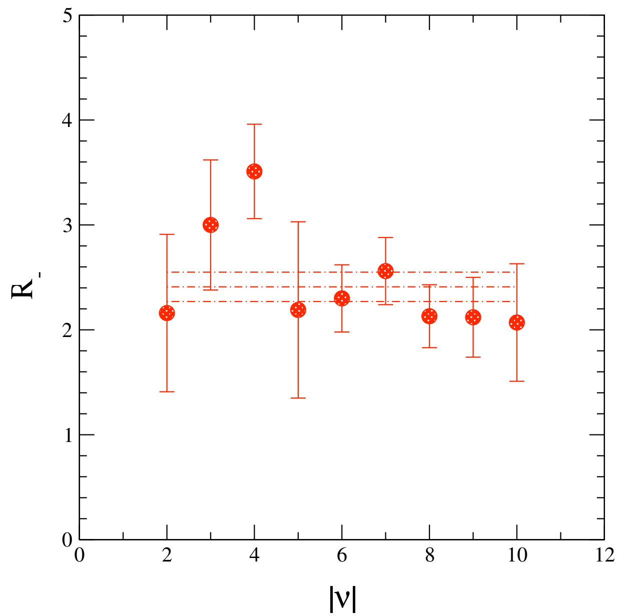
Quenched approximation.

Numerical simulations: final (bare) results

Expected ϵ-regime features — independence of $R^{\pm}$ on (x_0,y_0), m and ν — are all well reproduced by the data.





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Connecting bare results with physical amplitudes

$$g_1^{\pm}(1+K_{\pm}) = k_1^{\pm}(M_W/\Lambda_{\mathrm{QCD}})\frac{\hat{Z}^{\pm}(g_0)}{Z_{\mathrm{A}}^2(g_0)}R_{\pm}$$

- Chiral correction → LO/NLO chiPT.

Giusti, Hernández, Laine, Weisz, Wittig 2004
Hernández and Laine 2006

- Wilson coefficients → NLO QCD PT.

Ciuchini et al. 1998
Buras, Misiak, Urban 2000

- Renormalisation → QCD PT / nonperturbative result.

Dimopoulos et al. 2006

Renormalisation

Renormalisation factor $\hat{Z}^{\sigma}$ relates bare and RGI operators:

$$\hat{Z}^{\pm}(g_0) = c_S^{\pm}(\mu/\Lambda_{\rm QCD})\, Z_S^{\pm}(g_0, a\mu)$$

$$c_S^{\pm}(\mu/\Lambda_{\rm QCD}) = \left(2b_0\bar{g}^2(\mu)\right)^{\gamma_0^{\pm}/(2b_0)} \exp\left\{-\int_0^{\bar{g}(\mu)} {\rm d}g \left[\frac{\gamma^{\pm}(g)}{\beta(g)} + \frac{\gamma_0^{\pm}}{b_0 g}\right]\right\}$$

➸ NLO estimate in RI/MOM scheme available.

Capitani, Giusti 2000

➸ Nonperturbative renormalisation of chiral observables possible via a matching procedure to results obtained with Schrödinger Functional techniques.

Dimopoulos et al. 2006

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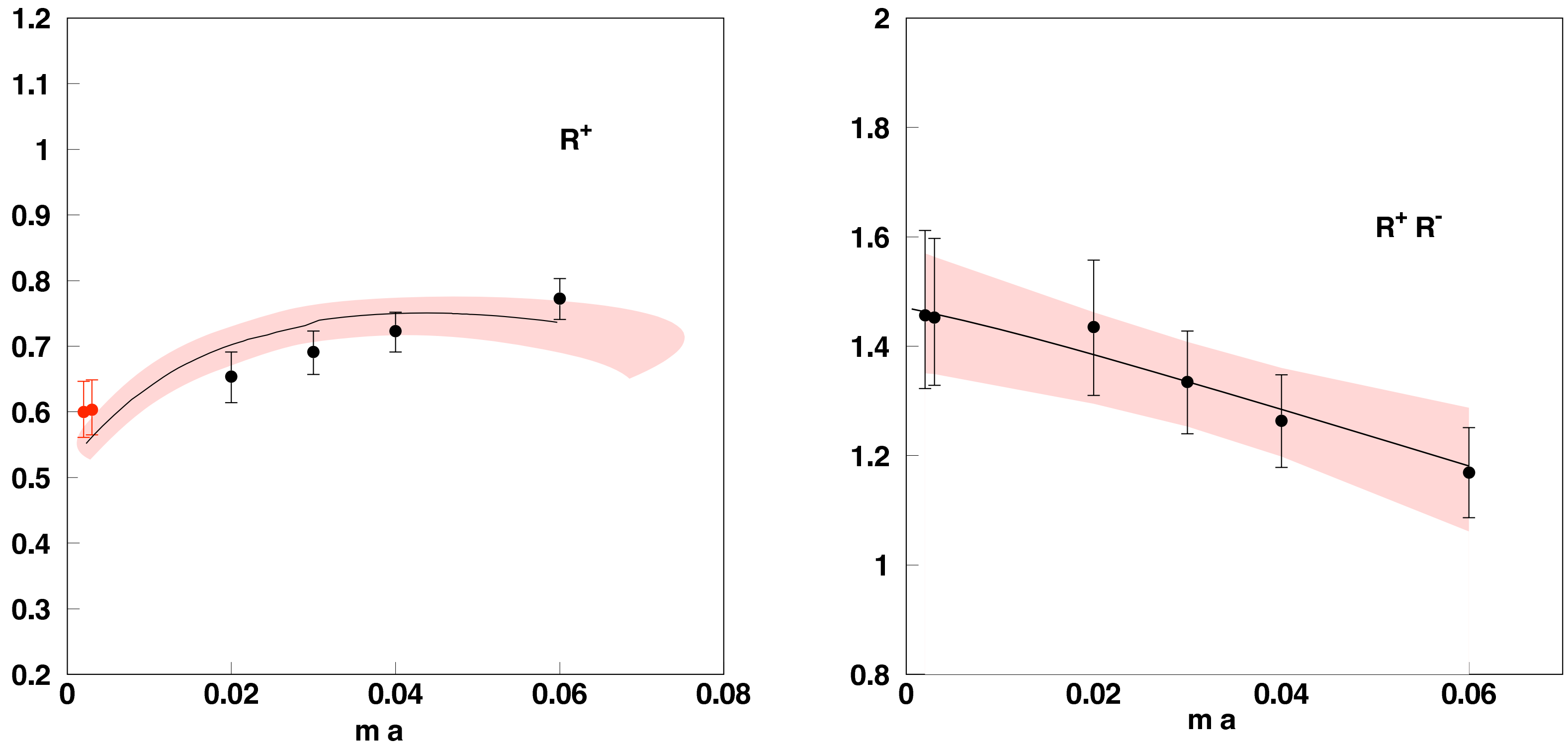
	bare P.T.	MFI P.T.	non-perturbative
$\hat{Z}_1^{+}/Z_{\rm A}^2$	1.242	1.193	1.15(12)
$\hat{Z}_1^{-}/Z_{\rm A}^2$	0.657	0.705	0.561(61)
$\hat{Z}_1^{-}/\hat{Z}_1^{+}$	0.525	0.582	0.584(62)

Fits to extract LECs

- Choose quantities with smaller mass corrections and statistical errors: R^+, R^+R^-

- Fit to NLO χPT to extract $g^{\pm}$ and $\Lambda^{\pm}$ (exploit smooth ϵ/p-regime transition).



Tension between ϵ- and p-regime may indicate non-negligible higher order corrections → systematic error included to account for this.

Outline

- The (many difficulties of studying the) $\Delta I=1/2$ rule.
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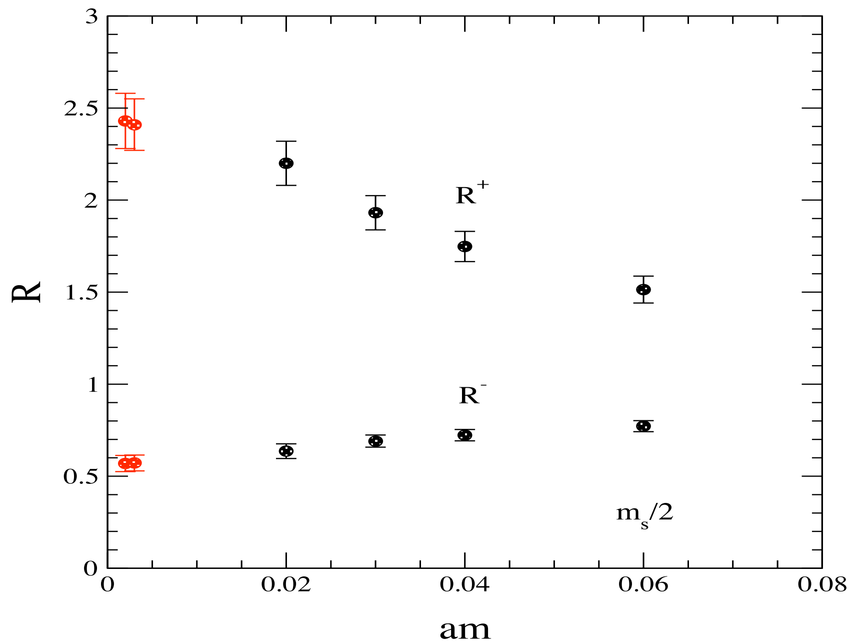
Results: $K \to \pi\pi$ amplitudes in the chiral limit

Giusti, Hernández, Laine, CP, Wennekers, Wittig 2006



	g^+	g^-
This work	$0.51(3)(5)(6)$	$2.6(1)(3)(3)$
"Exp"	~ 0.5	~ 10.4
Large N_c	1	1

- $\Delta I=3/2$ comes in the right ballpark (N.B.: charm effects enter only via quark loops).
- $\Delta I=1/2$ channel and amplitude ratio are a factor ~4 too small.
- Enhancement of the $\Delta I=1/2$ channel already present with an unphysically light charm quark (A_0/A_2 ~ 6): "pure no-penguin" effect.

Decoupling the charm quark

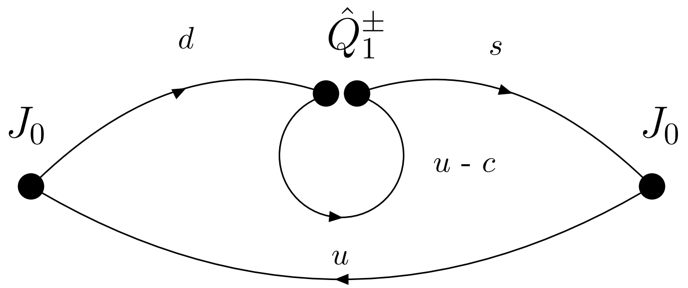
Giusti, Hernández, Koma, Koma,
Necco, CP, Wennekers, Wittig

- Matching between the SU(3)- and SU(4)-symmetric low-energy descriptions possible for non-degenerate but light charm masses: additional enhancement predicted to come out in the right direction.

- When the charm is heavy enough the usual SU(3) chiral expansion is recovered.

$$\bar{g}_1^{\pm} \to g_1^{\pm}(m_c) \qquad \bar{g}_2^{\pm} \to g_2^{\pm}(m_c)$$

- Numerical simulation requires the computation of penguin contractions, much more difficult to deal with. Hope is that low-mode averaging will tame the dominant statistical fluctuations.



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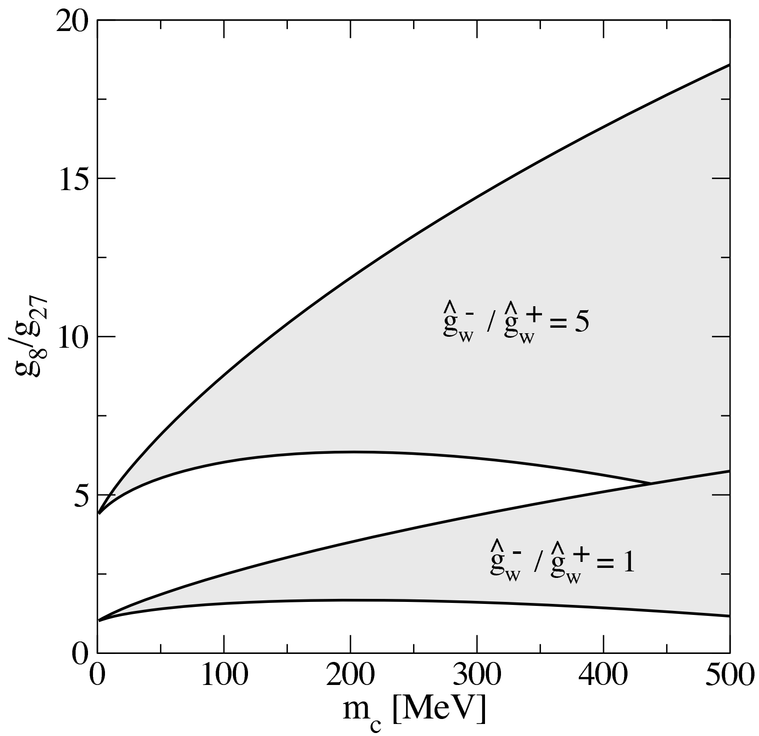
$$g_8(m_c) = \frac{1}{2}\left[\frac{1}{5}g^+\left(1 + 15\frac{M_c^2}{(4\pi F)^2}\ln\frac{\Lambda_\chi}{M_c}\right) + g^-\left(1 + 3\frac{M_c^2}{(4\pi F)^2}\ln\frac{\Lambda_\chi}{M_c}\right)\right]$$

$$g_{27}(m_c) = \frac{3}{5}g^+$$

- Logarithmic enhancement of octet.
- Many unknown LECs.

Hernández, Laine 2004-2006



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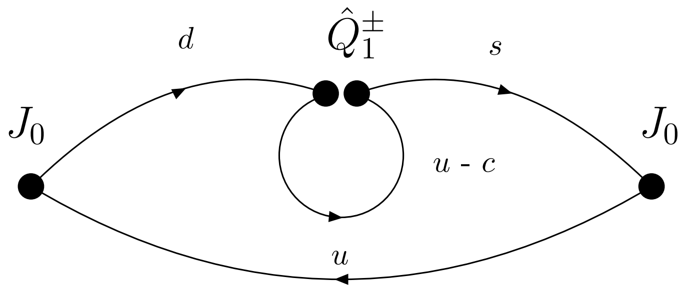

Conclusions and outlook

- SU(4) strategy allows to disentangle contributions to the $\Delta I{=}1/2$ enhancement with qualitatively different origins.
- First step: computation of low-energy couplings with an unphysically light charm quark (→ "pure QCD" contribution):
  - Chiral systematics under control via access to ϵ-regime. First time such light masses have been reached in numerical simulations.
  - UV effects under control (GW fermions, non-perturbative renormalisation).
- Computation of the couplings completed. Moderate enhancement found. Factor ~4 still missing. Main suspect: charm dependence of amplitudes.
- Next step: go to heavier charm masses and monitor amplitudes.
- Future: unquenching.