Overview of string theory beyond perturbation theory

1 The problem

The prescription we have given to compute amplitudes in string theory in perturbation theory is well-defined and consistent. However, it is not the complete string theory, there are indications that there is plenty of non-perturbative structure missed by the prescription we have given.

Making an analogy with point particle physics, the perturbative prescription we have given is equivalent to giving the propagators for the different particles, and giving a set of interaction vertices. With both ingredients one can build the Feynmann diagrams of the theory and recover the complete perturbative expansion.

On the other hand, we know that in point particle physics there are plenty of non-perturbative effects (like non-perturbative states (solitons), instanton effects, etc) which are obtained only when we compute non-perturbatively (e.g. using lattice methods) the path integral over spacetime field configurations, using the spactime action of the theory.

Now in string theory we do NOT have a spacetime action for the spacetime fields configurations (we just have a worldsheet action, which is the analog of the worldline action in point particle physics, clearly not the same as a spacetime field action). Therefore we do not have a well-defined prescription to compute non-perturbatively the path integral over spacetime field configurations, and it is very likely that we are missing plenty of non-perturbative physics.

There exists an approach to string theory, dubbed string field theory, which introduces a string field $\Psi[X^\mu(\sigma,t)]$, which is a functional of the string configuration function $X^\mu(\sigma,t)$. It can be thought of as the spacetime wavefunction providing the quantum amplitude for a state to correspond to a string configuration given by $X^\mu(\sigma,t)$. Expanding in oscillator modes, the string field splits as an infinite set of spacetime (point particle) fields, each corresponding to a string oscillator mode (i.e. to a spacetime particle).

Subsequently, it is possible to build a spacetime action for the string field, such that the perturbative expansion reproduces exactly the perturbative string theory amplitudes computed with the above prescription.

On the other hand, one would expect that string field theory also encodes information about string theory beyond perturbation theory. For some reason, this last hope has not been quite fulfilled. String field theory is technically very involved, so not many solutions to the string field equations are known. In particular, string field theory has been unable to provide information about some string theory non-perturbative states found via other indirect methods (p-branes, D-branes) [1], so it is not clear that string field theory is the right tool to address non-perturbative dynamics in string theory (or else, perhaps is not the tool that we know how to handle). We will not discuss string field theory in these course.

In this lecture we discuss several other indirect methods which have uncovered part of the non-perturbative structure of string theory (although not to a complete microscopic definition of it).

One may wonder why, if there is no complete definition of string theory beyond perturbation theory, we still claim that it is a consistent, finite, theory of gravity at the quantum level, etc. This was only checked with the perturbative description. A related objection is why to bother about

[1]Nevertheless, string field theory has led to important results in the context of open string tachyon condensations, see [1]

non-perturbative effects, and simply state that our theory is *defined* by the perturbative prescription. The objections are reasonable.

The reason why we need non-perturbative effects, and why we believe that they do not spoil (but rather improve) the good properties of string theory, is that there exist some very special, very singular, situations where perturbative string theory would break down, and certain computable non-perturbative effects make the physics non-singular and well-behaved.

So, our present understanding is that in smooth situations, the non-perturbative sectors do not spoil the good properties of perturbative string theory, they merely induce some small corrections. In other singular situations, however, the perturbative prescription would break down, and it is precisely the non-perturbative sector that saves the situation. We will see several examples of this phenomenon.

2 Non-perturbative states in string theory

A basic non-perturbative effect in string theory is the existence of states which are not seen in perturbation theory. That is, they do not appear in the Hilbert space of the quantized string. They are not modes of the fundamental string, so are not stringy in nature. They are more similar to solitons in field theories of point particles, which we now briefly review.

2.1 Non-perturbative states in field theory

An excellent discussion can be found in [2]. See also [3].

A soliton in a (to start with, classical) field theory is a finite energy solution to the equations of motion which is localized in some spatial dimensions, and is static in time. For instance, it the solution is localized (i.e. vanishes or goes to the trivial vacuum solution quickly outside a sphere of

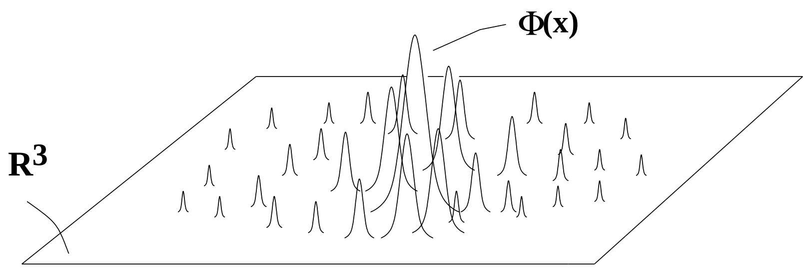

Figure 1: Artistic view of a soliton in a field theory.

characteristic size R (the size of the soliton)) in three spatial directions in a four-dimensional field theory, then the solition looks like a 'fat' particle propagating in time. See picture 1.

There are explicit examples of such solitions. The simplest is the 't Hooft - Polyakov monopole [10], which we describe briefly.

The 't Hooft - Polyakov monopole

Consider the Georgi-Glashow model. It is an $SO(3)$ (or $SU(2)$) gauge field theory in four dimensions, with a complex scalar field (Higgs) charged in the adjoint representation (**3** of $SO(3)$). We denote it by $\vec{\phi}$, with the vector notation refering to the internal $SO(3)$. Let us take the scalar potential to have a minimum at $|\vec{\phi}|^2 = v^2$ [2]

[2]In many situations, for instance in supersymmetric models, the scalar potential is identically zero, and the vev for $\vec{\phi}$ is undetermined. Any vev defines a possible vacuum of the theory, the set of all possible vevs (up to gauge transformations) is called the moduli space of the theory. Notice that the name 'moduli' is associated to fields with no potentials, either in the string theory context (like the dilaton, or the compactficiation radii moduli) or in the field theory context. For each vev condition $|\vec{\phi}|^2 = v^2$ one may repeat the argument below.

The action is roughly speaking

$$S_{GG} \;=\; \int d^4x\, \frac{1}{g^2}\,[\,\mathrm{tr}\; F^a_{\mu\nu} F^{\mu\nu}_a \;+\; D_\mu\vec{\phi}\cdot D^\mu\vec{\phi}\,] \;+\; V(\phi) \tag{1}$$

with

$$D_\mu \phi_i \;=\; \partial_\mu \phi_a + A^a_\mu\, (T_a)_{ij} \phi_j \tag{2}$$

Different vacua $|\vec{\phi}|^2 \;=\; v^2$ are related by $SO(3)$, so we may pick $\vec{\phi} \;=\; (v,0,0)$. The gauge group is spontaneously broken to $SO(2)$, equivalentely $U(1)$. This is the structure of the vacuum. Perturbative states of the theory are obtained by expanding the fields around the vacuum configuration, and contain the massive Higgs field, the massive vector bosons, etc. These generate different states in the quantum theory.

Now there also exist some finite energy configurations, which are therefore states in the quantum theory, which do not correspond to the above perturbative states. Consider a configuration where asymptotically in space $\mathbf{R^3}$ the field $\vec{\phi}(x)$ points (in the internal $SO(3)$) in the direction specified by the location $\vec{x}$ (in the space $\mathbf{R^3}$ $SO(3)$. Namely, for very large $r = |\vec{x}|$

$$\begin{aligned} \phi^a(\vec{x},t) \;&\to\; \frac{v}{r}\, x^a \;+ \mathcal{O}(1/r^2) \\ A^a(\vec{x},t) \;&\to\; \frac{1}{r^2}\, x^a \;+ \mathcal{O}(1/r^2) \end{aligned} \tag{3}$$

This is the so-called hedgehdoge configuration, shown in figure 2.

Since asymptotically $|\vec{\phi}| \;\to\; v$, the potential energy vanishes at infinity. The kinetic energy also vanishes asymptotically because we choose a gauge background which makes the covariant derivative vanish. Statitc solutions (solitons) with those asymptotics exist, and therefore have finite energy. They represent lumps of energy localized in the three spatial directions, i.e. particle-like states.

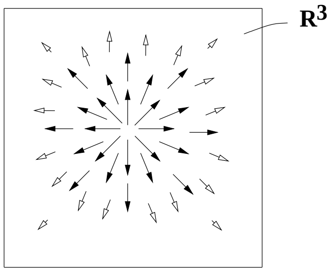

Figure 2: Picture of the hedgehodge configuration for the Higgs field in the Georgi-Glashow model.

Their main properties are: their mass (energy of the configuration) is of the order of v/g^2, and so they are very heavy at weak coupling, and non-perturbative in nature. They are magnetically charged under the surviving $U(1)$ gauge group, i.e. taking the gauge field configuration in the soliton background, and integrating the field strength of the $U(1)$ part $F = F^a \phi^a$ around a large $\mathbf{S^2}$ in $\mathbf{R^3}$ we get

$$\int_{\mathbf{S^2}} F = 1 \tag{4}$$

These solitons are therefore called magnetic monopoles (in fact, magnetic monopoles in more realistic models, like grand unified theories, are constructed similarly). Since the charge they carry arises from the topology of the background (notice that the quantity (4) is topological, it is independent of the spacetime metric), they are also called topological defects.

Notice that if we had started with a higher dimensional theory, say in $D+1$ dimensions, one can still pick a particular $\mathbf{R^3}$ and construct the above soliton background. It is still localized in three dimensions, but the configuration is now Poincare invariant under the spectator $D-2$ dimensions. The soliton now represents an extended object with $D-3$ spatial dimensions. It is still charged magnetically with respect to the unbroken $U(1)$. The volume swept out by the soliton core as it moves in time is called the soliton world-volume (generalizing the ideas of worldlines and worldsheets).

Collective coordinates

It is interesting to see what the theory looks like around the soliton background. This is done by expanding the fields as background plus fluctuations, and substituting into the field theory action to obtain a field theory for the fluctuation fields. An interesting subset of fluctuations are zero modes, which correspond to fluctuations which are massless in the background of the soliton. They parametrize changes in the fields which do not change the energy of the soliton.

For instance, it is clear that applying translations $\phi_{x_0}(x) = \phi(x - x_0)$, one can construct solitons centered not at $\vec{x} = 0$ but at any $\vec{x} = \vec{x}_0$. The difference between two configurations $Y^i = \phi_0$ and $\phi_{\delta x^i}$ is a zero mode fluctuation. Notice tht both configurations are equal almost everywhere, so the fluctuation is localized on the volume of the soliton [3]. So, it can be roughly written as a field depending on the $p+1$ worldvolume coordinates (for a soliton with p spatial extended dimensions) $Y^i(x^0, \ldots, x^p)$, with $i = p+1, \ldots, D+1$. See picture 3 below.

In fact, the zero mode fluctuations describe dynamics of the soliton (and not dynamics of the underlying vacuum), they are sometimes called collective coordinates of the configuration. Very often they are associated to symmetries of the vacuum which are broken by the presence of the soliton (just like the above translational symmetries). So these massless fluctuations can be understood as Goldstone bosons of the symmetries broken in the soliton

[3] Beyond those three translational collective coordinates, there is a fourth one associated to gauge transformations which do not vanish at infinity and therefore related different configurations which are not gauge equivalent. We will skip this mode in our discussion.

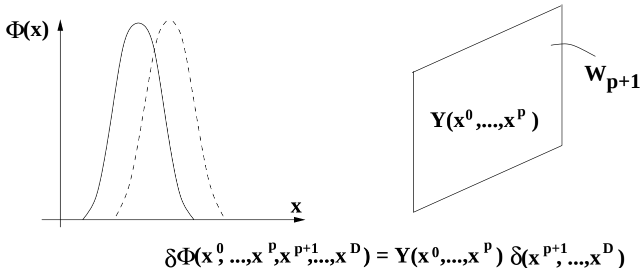

Figure 3: Picture of the zero modes of a soliton.

background.

Their vevs parametrize the possible configurations of the soliton background with the same energy; i.e. the set of soliton solutions of the same kind, e.g. location of solition worldvolume

$$\langle\, Y^i(x^0, \ldots, x^p)\,\rangle = a^i \tag{5}$$

The set of such vevs, the set of soliton configurations, is called the moduli space of solitons of that particluar kind (magnetic monopole moduli space in this case). Non-trivial configurations for these fields $Y^i(x^0, \ldots, x^p)$ describe excitations of the solition background; for instance a non-trivial profile for some of the translational zero modes corresponds to a non-flat soliton worldvolume (an energetically costly configuration). See picture 4

It is possible to write down a worldvolume effective action for these worldvolume fields, which describes the dynamics of the soliton. We will not do so for the field theory example, but we will come back to this point when we look at non-perturbative states in string theory.

Beyond the classical approximation, the quantum behaviour of the soliton is obtained by expanding the classical theory around the soliton background, and quantizing the fluctuations. Concerning the subsector of the zero modes,

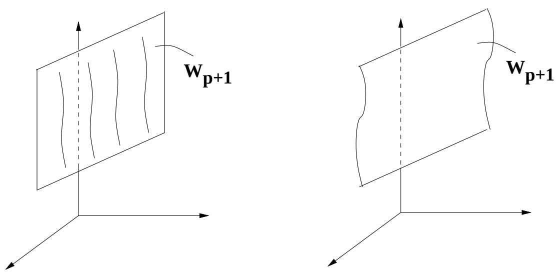

Figure 4: A nontrivial configuration for one of the worldvolume translational zero modes corresponds to a non-trivial embedding of the soliton worldvolume in spacetime.

this corresponds to promoting the worldvolume field theory to a quantum field theory in $p+1$ dimensions. And corresponds to quantizing the soliton state.

Many of these properties will have analogs in non-perturbative states in string theory, and that is why we discussed them in some detail.

2.2 Non-perturbative p-brane states in string theory

In order to try to find similar non-perturbative states in string theory, the only spacetime action that we can use to find spacetime field configurations is the low-energy effective action for the light modes of string theory (the graviton, dilaton, antisymmetric tensor fields, etc). It is important to realize that this is only the low-energy approximation to string theory, and it is questionable if any solution to its equation of motion is really a solution of full string theory. This issue will be settled for a particular class of solutions, as we will see below.

The approach is remarkably successful. Taking the different low-energy effective actions for the different superstrings (which correspond to different

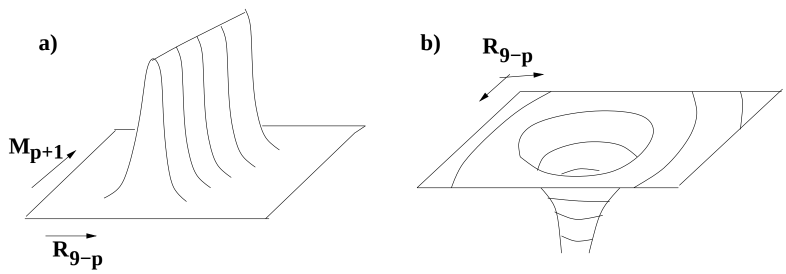

Figure 5: Two pictures of the p-brane as a lump of energy. The second picture shows only the transverse directions, where the p-brane looks like point-like.

ten-dimensional supergravity theories), it is possible to find finite energy solutions (which are of a special kind (1/2 BPS) see below) to the equations of motion, which look like lumps of energy localized in some directions and extended in p spatial directions. They are known as p-branes; they have Poincare invariance in $p+1$ dimensions, and the core of the non-perturbative lump is called the p-brane world-volume. See 5 for a picture

To give one example, the supergravity solution for a 3-brane (with N units of charge) in type IIB theory is given by

$$\begin{aligned} ds^2 &= f(r)^{-1/2}\,[(dx^0)^2 + \ldots + (dx^3)^2] \,+\, f(r)^{1/2}\,[(dx^4)^2 + \ldots + (dx^9)^2] \\ f(r) &= 1 + \frac{4\pi g_s \alpha'^2 N}{r^4} \quad ; \quad r = [(x^4)^2 + \ldots + (x^9)^2] \\ F_5 &\simeq d(Vol)_{\mathbf{S^5}} \end{aligned} \tag{6}$$

where the field strength 5-form is proportional to the volume form of the angular 5-sphere in the transverse six-dimensional space.

The main properties of these solutions are

- For a given string theory, there exist p-brane solutions for values of p for which there exists a $(p+1)$-form field in the (perturbative) massless spectrum of the string. See table 1

- The energy per unit volume of these branes is of order $1/g_s$ or $1/g_s^2$ in string units $M_s = 1$. So they are intrinsecally non-perturbative

- p-branes are charged electrically under the $(p+1)$-forms; conversely, they are charge magnetically under the dual $(7-p)$-forms, namely

$$\int_{\mathbf{S^{8-p}}} H_{8-p} = 1 \tag{7}$$

  where H_{8-p} is the field strength for the $(7-p)$-form, and we integrate over a $(8-p)$-sphere in the transverse $\mathbf{R^{9-p}}$.

- The solutions are invariant under half of the supersymmetric transformations of the vacuum theory. The solutions are said to be 1/2 BPS. This is the key property that makes these solutions special, and reliable beyond the supergravity approximation.

- We will not discuss these theories in detail, but the worldvolume field theories for these p-branes are known. They contain $9-p$ real scalar fields, Goldstone bosons of the broken translational symmetries, and some fermions, which can be understood as Goldstinos of the supersymmetries broken by the background. These (and other) fields group together in multiplets of the unbroken supersymmetries, and define a supersrymmetric field theory in $p+1$ dimensions.

We turn to the issue of why the existence of these non-perturbative states should be trusted in the full string theory. After all, we found them as solutions of a truncated theory, the supergravity effective action describing the $\alpha' = 0$ regime.

The key feature is that BPS states are remarkably stable under smooth deformations of the theory (like for instance, turning on α' i.e. including more

String theory	Branes	$(p+1)$-form	Tension
Type IIA	F1, NS5	B_2, $\hat{B}_6$	$\simeq 1/g_s^2$
	D0, D2, D4, D6, D8	C_1, C_3 $\hat{C}_5$, $\hat{C}_7$	$\simeq 1/g_s$
Type IIB	F1, NS5	B_2, $\hat{B}_6$	$\simeq 1/g_s^2$
	D(-1), D1, D3, D5, D7	a, $\tilde{B}_2$, C_4, $\hat{C}_6$, $\hat{C}_8$	$\simeq 1/g_s$
Heterotic	F1, NS5	B_2, $\hat{B}_6$	$\simeq 1/g_s^2$
Type I	D1, D5	$\tilde{B}_2$, $\hat{C}_6$	$\simeq 1/g_s$

Table 1: Partial list of the spectrum of p-branes in the different string theories.

and more stringy corrections until we eventually reach full string theory). The argument proceeds through various steps

i) Recall how one builds supersymmetric multiplets of states in a supersymmetric theory. One separates the supergenerators of the theory, in two sets (creators and annihilators), and defines the ground state of the multiplet as annihilated by annihilators. The rest of the multiplet is obtaine by applying creators to the ground state and using the algebra.

A 1/2 BPS state is invariant under half of the supersymmetries, so the ground state of the supermultiplet is annihilated by the creator operators of the corresponding susys. This means that this kind of multiplet contains half the number of states as a generic multiplet. Consequently, multiplets are called short and long, according to the number of states they contain.

To give a toy description, consider four supercharges, separated as two annihilators Q_1, Q_2 and their adjoints the creators $Q_1^\dagger$, $Q_2^\dagger$. A generic multiplet, constructed form a ground state $|st.|rangle$ satisfying $Q_i|st.\rangle = 0$, is given by

$$|st.\rangle \quad , \quad Q_1^\dagger|st.\rangle \quad , \quad Q_2^\dagger|st.\rangle \quad , \quad Q_1^\dagger Q_2^\dagger|st.\rangle \tag{8}$$

A 1/2 BPS multiplet is built out of a ground state which in addition satisfies $Q_2^\dagger|st.\rangle$, so the multiplet contains

$$|st.\rangle \quad , \quad Q_1^\dagger|st.\rangle \tag{9}$$

Namely contains half the number of states.

ii) Since the number of states in short and long multiplets is different, it is not possible that a BPS state becomes non-BPS upon a continuous change of parameters of the system. In particular, BPS states remain BPS upon turning on α'.

iii) The supersymmetry algebra in the presence of p-form charges is modified by the inclusion of central charges $Z(\phi)$ (operators that commute with all supergenerators and the hamiltonian, and appear in the susy algebra). They are related to the charges of the configurations, and are known functions of the moduli. The susy algebra looks like

$$\{Q_\alpha^A, Q_{\dot\alpha}^B\} = \delta_{AB}\,(\sigma^\mu)_{\alpha\dot\alpha}P_\mu + Z_{\alpha\dot\alpha}^{AB}(\phi) \tag{10}$$

Applying the algebra to the ground state of the BPS multiplet for the choice of Q^B that annihilates it, the left hand side gives zero. On the right hand side, in the rest frame of the brane, the momentum operator looks like $(M, 0, \ldots, 0)$ with M the mass or tension of the object, while Z gives its charge. Roughly speaking we get a relation $M = Q$, namely the tension of the BPS object is determined in terms of its charge.

iv) Since charges are quantized, they cannot change as we change parameters continuously. Since BPS states remain BPS upon such changes, their tension remains determined by their charges, so it is possible to determine them exactly even after all α' corrections are included.

This concludes the argument. If we find a BPS state in the supergravity approximation and compute its properties (charge, tension), there will exist a BPS state (a stringy improved version of the original one) with the same properties in the full string theory. The tension of the object is determine from its charge as dictated by the central extension of the susy algebra, so they can be reliably followed as moduli change (for instance, as the coupling gets strong).

BPS states are a subsector of the theory which is protected by supersymmetry, so it can be reliably studied in some simpler approximation schemes, like low-energy effective supergravity.

2.3 Duality in string theory

2.3.1 p-brane democracy

We start this section by pointing out a remarkable fact. Some of the p-branes that we have discussed above carry the same charges as the string, namely they have electric coupling to the (NS-NS) 2-form in the massless sector, just like string. In fact, the corresponding supergravity solution corresponds to the background created by a macroscopic, infinitely extended, string. But which is not essentially different from the basic string of the theory. For this reason, such 1-branes are known as fundamental string solutions and denoted F1-branes.

The fact that the fundamental string arises, in this sense, in the same way as other p-branes, suggests the idea that perhaps all p-brane solutions should be treated on an equal footing. This is also suggested by the fact that different brane solutions are often related by symmetries in supergravity, called U-duality symmetries (a discrete subgroup of which is realized in full-fledged string theory. This idea that different branes are on an equal footing is called p-brane democracy [4].

Of course, we have learned that in perturbation theory the fundamental string is more fundamental than any other object in the theory. In particular, a large part of the spectrum of the theory is obtained by quantizing the oscillation modes of the fundamental string. The p-brane democracy idea proposes that this is just and artifact of the perturbative description.

The idea is that there is a unique underlying theory with a bunch of BPS states. As one moves to a particular limit (like weak coupling) some of these states look more fundamental than others, and the light spectrum in that limit can be computed by quantizing these fundamental objects. In particular, it is conceivable that there exist other limits where other BPS states are fundamental and are more useful to describe the physics of the system.

This is the picture underlying the proposal of string duality.

2.3.2 String duality

Indeed this idea is realized in many string configurations. The simplest case is that of the ten-dimensional superstrings. There exists a perturbative limit where the theory is described in terms of weakly interacting strings and one recovers the perturbation theory we have described in previous lectures. As one moves to the non-perturbative regime, the different branes look really democratical. In the limit of infinite coupling the theory again simplifies and becomes a weakly interacting theory, but where the fundamental degrees of freedom correspond to originally non-perturbative states. The situation is shown in picture 6. Notice that the tensions of the objects can be realiably followed as a function of the moduli (the dilaton vev, string coupling) thanks to the fact that these states are BPS.

Thus, roughly speaking, the strong coupling limit of a string theory can be described as a weak coupling limit of a dual string theory (which may be or not of the same kind). Perturbative and non-perturbative states are reshuf-

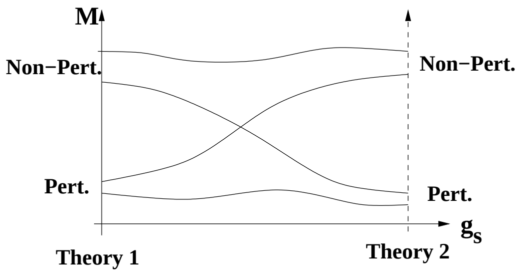

Figure 6: As a modulus (the dilaton vev) is changed, the original weakly coupled string theory becomes strongly interacting, and at infinite coupling it can be described as a weakly interacting *dual* theory. Perturbative and non-perturbative states are reshuffled in this interpolation.

fled as one changes the vev of the dilaton modulus to interpolate between them. We will see explicit examples below

We now explain the dual theories describing the strong coupling regime of the ten-dimensional superstrings. The original reference for these results is [5]

2.3.3 Duality for ten-dimensional superstrings

Type IIB self-duality

The limit of strong coupling of type IIB string theory is described by a different type IIB string theory, with weak coupling. The string couplings in the two theories are related by

$$(g_s)_1 = 1/(g_s)_2 \tag{11}$$

The basic mapping of branes are as follows

Type IIB	$\leftrightarrow$	Type IIB
F1, NS5		D1, D5
D3		D3

The mapping of massless fields is easy as well, roughly speaking

Type IIB	$\leftrightarrow$	Type IIB
$\tau = a + ie^{-\phi}$		$-1/\tau$
G_{MN}		G_{MN}
B_2		$\tilde{B}_2$
$\tilde{B}_2$		B_2
C_4		C_4

The transformation $g_s \to 1/g_s$ is a transformation that maps type IIB string theory to itself. In particular, it is a subgroup of an exact $SL(2, \mathbf{Z})$ symmetry of type IIB theory. This symmetry group is a particular case of U-duality, which encodes duality properties of the theories upon compactification, and can be used to find dual description in other limits. See [6].

$SO(32)$ heterotic - Type I duality

The strong coupling limit of the $SO(32)$ heterotic string is described by a dual weakly coupled type I theory, and viceversa. The mapping of branes is

$SO(32)$ Heterotic	$\leftrightarrow$	Type I
F1, NS5		D1, D5

The mapping of fields is: the string coupling is inverted, the 2-forms are exchanged, the metric and the $SO(32)$ gauge fields are invariant.

Notice that the relation implies a mapping between the low-energy supergravity theories, written in terms of heterotic and type I variables. This is possible because both sugra theories are $d = 10$ $N = 1$ sugra coupled to $SO(32)$ gauge multiplets.

Type IIA - M-theory duality

As the coupling constant of type IIA theory gets stronger, the strong coupling limit is not described by a dual string theory, but rather in terms of a far more mysterious theory called M-theory. The argument is as follows.

Type IIA theory contains non-perturbative particle-like D0-branes, with masses given by k/g_s, where k is the D0-brane charge under C_1. In the strong coupling limit, all these states are becoming massless, so the strong coupling limit is a theory with an infinite tower of states becoming massless.

The idea is to propose that type IIA theory has a dual description as an 11d theory compactified on a circle, with radius related to the string coupling as $R = g_s$. The states with mass k/g_s correspond in the dual picture to the Kaluza-Klein replicas of the 11d graviton multiplet. Type IIA theory at extreme strong coupling corresponds to the decompactification limit of this theory.

There is a supergravity theory in 11d which under compactification on a circle reduces to $d = 10$ $N = 2$ non-chiral supergravity. It contains and 11d gravition, a 3-form C_3 (and its dual $\tilde{C}_6$), plus gravitino etc superpartners. In particular, it does not contain a dilaton field, so it does not have a coupling constant. This theory is however ill-defined in the UV (non-renormalizable, etc), so should be regarded as an effective description of an underlying quantum theory, which for the moment is completely unknown . So the natural proposal is that the strong coupling limit of type IIA theory corresponds to a quantum theory, called M-theory, whose low energy limit is given by 11d supergravity.

This is a nice result, and explains the role of 11d sugra in string theory (previously this sugra was unrelated to string theory, in contrast with its 10d cousins). Understanding the microscopic degrees of freedom of M-theor, the theory underlying 11d sugra, in one of the main challenges in string theory today.

M-theory also contains p-brane states, which are found as BPS solutions to 11d sugra, which therefore must exist in the full theory (since they are BPS). They correspond to a 2-brane and a 5-brane, denoted M2-, M5-branes, resp. The mapping of fields between Type IIA and M-theory is

M-theory	$\leftrightarrow$	Type IIA
G_{MN}	$\rightarrow$	$G_{\mu\nu}$
		$A_\mu = G_{\mu,10}$
		$\phi = G_{10,10}$
C_{MNP}	$\rightarrow$	$B_{\mu\nu} = C_{\mu\nu,10}$
		$C_{\mu\nu\rho}$

On the other hand, Type IIA D0-branes are KK replicas of the 11d fields, the D2-brane is an M2-brane transverse to the M-theory $\mathbf{S^1}$, the F1 is an M2 wrapped on the $\mathbf{S^1}$, the D4 is an M5 wrapped on $\mathbf{S^1}$, the NS5 is an unwrapped M5. Finally the D6-brane corresponds to a purely gravitational background in M-theory known as Taub-NUT metric.

$E_8 \times E_8$ heterotic - Horava-Witten duality

The strong coupling limit of the $E_8 \times E_8$ heterotic is also not a string theory, but is related to a compactification of M-theory. Heterotic theory has less supersymmetry than M-theory, so we need to break some of the supersymmetry in the compactification. The compactification is taken to be not on a circle $\mathbf{S^1}$, but on the quotient of a circle by the $\mathbf{Z_2}$ symmetry corresponding to reflection with respect to one of its diameters, and simultaneously mapping C_3 to $-C_3$. This is equivalent to compactification on an interval, see picture 7 This compactification of M-theory is known as Horava-Witten theory [7].

The $E_8 \times E_8$ heterotic at string coupling g_s is proposed to be equivalent to the compactification of M-theory on the interval of radius $R = g_s$. Again, the heterotic strong coupling limit corresponds to the decompactification limit.

The mapping of fields is as follows. The $N = 1$ $d = 1$ supergravity multiplet of the heterotic theory is mapped to the sector of 11d supergravity which is invariant under the $\mathbf{Z_2}$ symmetry. On the other hand, the E_8 gauge

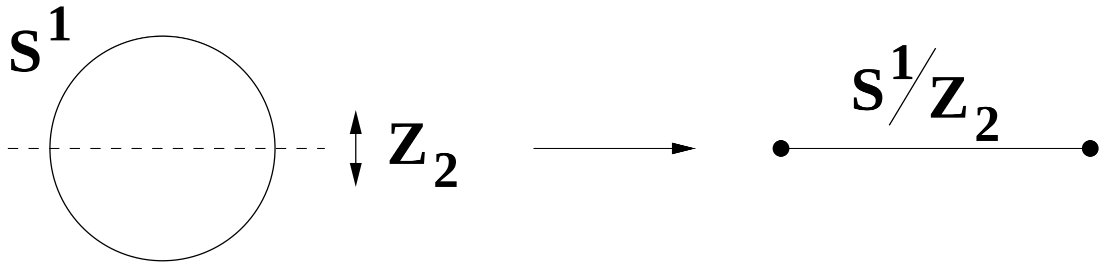

Figure 7: The quotient of a circle by a reflection under a diameter is an interval $I = \mathbf{S^1}/\mathbf{Z_2}$.

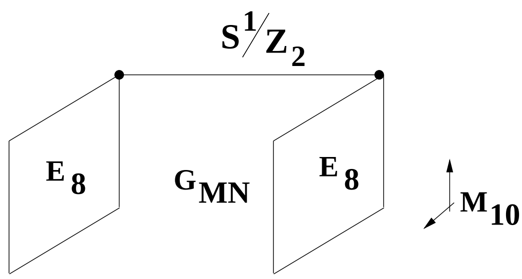

Figure 8: The strong coupling description of $E_8 \times E_8$ heterotic involves the compatification of M-theory on a space with two 10d boundaries. Gravity propagates in 11d, while gauge interactions are localized on the 10d subpaces at the boundaries.

multiplets must necessarily arise at the fixed points of the $\mathbf{Z_2}$ action, so they are localized at the ten-dimensional boundaries of the spacetime $M_{10} \times I$. Each E_8 gauge multiplets propagates at one of the boundary points of I times M_{10}, and does not propagate in the M-theory direction. This is our first example of gauge interactions localized on a submanifold of spacetime. see figure 8.

The duality web

As one compactifies the 10d theories, more moduli appear, associated to the geometry of the compactification space. Then there are more limits that can be taken, for instance, strong coupling and small radii, with fixed ratios. In this situation more duality relations appear; These dualities involve non-perturbative as well as perturbative dualities, like T-duality. To give just one example, compactification of M-theory on a two-torus is dual or equivalent to compactification of type IIB theory on a circle, etc. This can be understood by taking M-theory reducing to IIA on a circle, then reducing on a second circle, and T-dualizing to type IIB theory.

Different compactifications of the different superstrings and M-theory are related by an intricate duality web. We will not describe any more dualities in this lecture. But they suggest a nice picture that we would like to discuss

The picture that emerges is that in a sense there is a unique theory, which describes all kinds of extended BPS objects, and which in different limits reduces to perturbatives string theories (where strings are the fundamental objects) or to other more exotic theories (like M-thoery, which is not a string theory). This picture has become popular in the pictorial representation 9. By abuse of language, the underlying theory is often called M-theory as well.

Surprisingly enough, string theory is NOT just a theory of strings!! It is a huge challenge to really understand what string theory is about, once we are far from any perturbative regime.

3 D-branes

We conclude this lecture with a brief review of a very simple description of some p-brane states in type II and type I theories, the Dp-branes.

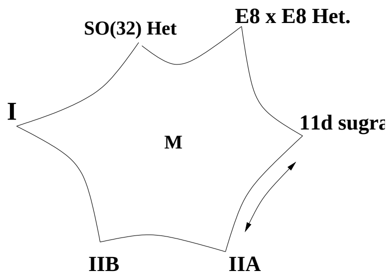

Figure 9: Map of the moduli space of the underlying theory and its different known limits.

3.1 What are D-branes

Given a p-brane state, one is interested in the spectrum of the theory when expanded around this state. In general, this can be computed only in the supergravity approximation, by expanding the sugra fields in background plus fluctuations and computing the action for fluctuations by substitution in the sugra action. This is extremely involved, and moreover suffers from plenty of corrections.

The remarkable insight by Polchinski [8] is that he gave a completely stringy proposal to obtain the spectrum of fluctuations of string theory around certain p-brane states, the Dp-branes mentioned above. In fact, it is a stringy definition of such p-brane states.

The proposal is to replace the p-brane soliton core by a $(p+1)$ dimensional hypersurface in flat space. The fluctuations of the theory around the p-brane background correspond to open strings with ends on this hypersurface. The spectrum of fluctuations of the theory around the p-brane background can be obtained by simply quantizing such open strings. The hyperplane is known as Dp-brane. The situation is shown in figure 10.

Notice that the Dp-brane, as a state, is non-perturbative, it does not

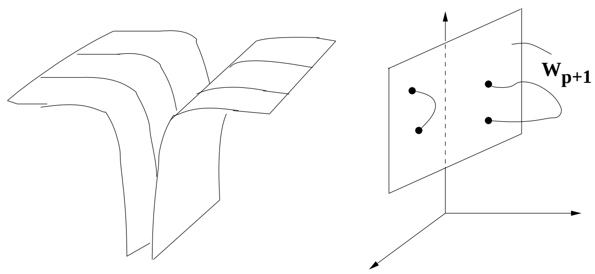

Figure 10: Fluctuations of the theory around a Dp-brane sugra solution can be described in stringy language as open strings with ends on a $(p+1)$-dimensional surface, located at the core of the topological defect.

appear as an oscillator state of the string. On the other hand, what we have provided is a stringy description of the spectrum of fluctuations of the theory around the p-brane state, in terms of oscillation modes of open strings with ends on the Dp-brane worldvolume.

Properties

This surprising proposal works. The Dp-brane interacts with closed string via diagrams with the topology of a disk, as in figure 11.

In particular, they can be seen to carry tension and charge, which matches the tension and charge of the p-brane solutions in supergravity. This suggests that the Dp-branes described as subspaces where open strings can end is a stringy version of the fat p-brane solutions of supergravity. The back-reaction of the Dp-brane on the flat background curves and modifies it to the full sugra solution.

Moreover, it can be seen that the Dp-branes described in this way break half of the supersymmetries, so they are BPS states of the theory.

It is important to notice that NOT all p-branes in string theory are Dp-branes. For the NS5-branes and others, there is no simple stringy description

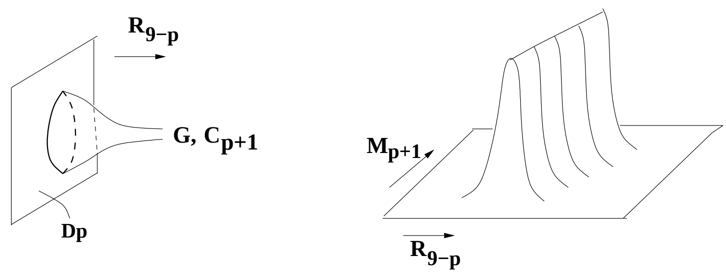

Figure 11: D-branes interact with closed string modes, and in particular couple to the bulk graviton and $(p+1)$-form fields, i.e. they have tension (of order $1/g_s$ in string units) and carry charge. Their backreaction on the background curves and deforms it into the p-brane solution seen in the supergravity regime.

for their spectrum of fluctuations. So the study of the dynamics of these objects is much more complicated than for D-branes.

It is also important to realize that NOT all superstring theories contain D-branes. Namely, the p-branes in heterotic string theories are not Dp-branes, so there are no D-branes in heterotic theories. Type IIB theory contains D$(2p+1)$-branes, while IIA contains D$2p$-branes, and type I contains D1, D5 and D9-branes.

3.2 Worldvolume theory

The quantization of open strings leads to a stringy tower of modes. The lightest of these are massless and correspond to the zero modes of the topological defect as introduced above. Consider a Dp-brane with $(p+1)$-dimensional worldvolume extended along the directons $x^0, \ldots, x^p$, in flat 10d spacetime. Consider an open string with both endpoints on the Dp-brane. The lightest oscillation states of this string correspond to gauge bosons, A_μ, $9-p$ scalars Y^i (Goldstone bosons of the translational symmetries of the vacuum, broken by the Dp-brane), and some fermions λ^a (Goldstinos of the supersymmetries of the vacuum which are broken by the Dp-brane). Notice that since the open string endpoint must be on the D-brane worldvolume, these fields are naturally localized on the D-brane worldvolume. They define a $(p+1)$-dimensional field theory, which describes the dynamics of the Dp-brane. For instance, for a D3-brane in type IIB theory, the massless modes of an open string with ends on the D3-brane correspond to a $U(1)$ vector boson, six real scalar fields, and four Majorana fermions, all neutral under the $U(1)$ group.

An important feature of Dp-branes (and p-branes) in general, is that the BPS property implies that several parallel Dp-branes of the same kind do not suffer net attraction or repulsion. The equality of tension and charge for BPS branes guarantees that the gravitational attraction is cancelled by the repulsion due to their equal charges. So it is possible to consider configurations with several parallel Dp-branes at arbitrary points in the transverse space.

In particular, several of these Dp-branes may coincide at the same point. This is an interesting configuration, so let us consider n coincident Dp-branes in flat 10d space. Without going into much details, it is possible to understand that now there are n^2 possible open strings, depending on on which brane the string is starting (out of the n possible ones) and on which it is ending (out of the n possible ones). It is important to recall that we work with oriented open strings. The situation is shown in figure 12. The spectrum in each sector is similar, so the total open string sector, for D3-branes for instance, contains n^2 4d gauge bosons, which can be seen to organize into an $U(n)$ gauge group, six 4d real scalars, with transform in the adjoint representation (of dimension n^2), and four 4d Majorana fermions, also in the adjoint.

If the D-branes are slightly separated, the stretching of the open string

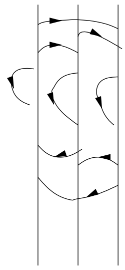

Figure 12: Open string stretched within a stack of 3 overlapping D-branes. They are shown as separated for the sake of clarity.

means that some of the fields are slightly massive, with mass given by the string tension times the D-brane separations. The above modes are massless for overlapping D-branes, and have small masses $\ll M_s$ if the inter-D-brane distance is much smaller than the string length.

The interpretation of these modes is trickier than for just one brane. In general, we may say that the eigenvalues of the scalars vevs (which are matrices in the adjoint) correspond to the positions of the D-branes in transverse space. However, there is an intriguing underlying matrix structure, which leads some researchers to the idea that spacetime positions, coordinates, should become matrices at length scales much smaller than the string length. This idea underlies some of the most advanced proposals to understands string theory, M-theory, and the structure of spacetime, like the M(atrix) theory proposal [11].

The effective action for light modes of the open strings can be obtained by computing their scattering amplitudes using the rules in the previous sections, and cooking up an effective action reproducing them. Alternatively, one can consider turning on a background for these fields (for instance, for the D-brane gauge fields), writing a 2d action for the worldsheet in the presence of these fields, and imposing that the worldsheet theory is conformally invariant. The coupling of gauge fields to the worldsheet is described by adding to the usual Polyakov action the boundary action

$$S_{bdry} = \int_{\partial\Sigma} d\xi^a \partial_a X^\mu(\sigma,t) A_\mu(X(\sigma,t)) \tag{12}$$

where $\partial\Sigma$ is the boundary of the wordsheet Σ. It amounts to taking the 1-form A_1 on the D-brane worldvolume, and integrating it along the 1d worldsheet boundary, i.e. $S_{bdry} = \int_{\partial\Sigma} A_1$. This shows that the string endpoints are charged with respect to the worldvolume 1-form gauge field.

By either method, one obtains a $(p+1)$-dimensional effective action for the worldvolume massless modes, which looks like (a supersymmetrization with respect to the 16 unbroken supercharges, in type II D-branes)

$$S_{Dp} \;=\; T_{Dp} \int \, d^{p+1}x \, [\, -det(G + B + \alpha' F)\,]^{1/2} \tag{13}$$

plus some topological terms (Wess-Zumino terms) which will not interest us for the moment. This is the so called Dirac-Born-Infeld action (DBI). Here G and B are the induced metric and 2-form induced on the worldvolume from the 10d ones, and F is the worldvolume field strenght. The leading order of this action is just the string tension times the worldvolume volume; next order in F is the Yang-Mills action for the worldvolume gauge bosons [4] So the vector bosons A_μ are indeed gauge bosons.

So this is a second situation where we find that gauge interactions can be consistently localized to subspaces of spacetime, while gravity propagates in full spacetime. These gauge interactions are therefore qualitatively different from those in heterotic string theory.

A last comment is that considering a non-trivial background for the worldvolume scalar fields $Y^i(x^0,\ldots,x^p)$ amounts to considering a curved Dp-brane

[4] In fact the DBI action is valid just for $U(1)$, the generalization to the non-abelian case is not known.

worldvolume. Dp-brane can therefore do all kinds of things, like wrap a non-trivial cycle in a topologically non-trivial spacetime (for example, wrap around a circle in the internal space in a $M_4 \times T^6$ compactification).

3.3 D-branes in string theory

Here we review some of the main applications where D-branes are important in string theory

3.3.1 Theories with open strings

Some string theories, like type I, contain open strings already in their vacuum state. D-branes have become so useful and popular, that now any theory with open strings is rephrased in D-brane language. Using the above rules, the space where open strings are allowed to end IS a D-brane, which is present in the vacuum of the theory (so in the present context should not be regarded as a soliton-like excited state!).

For instance, type I theory contains open strings already in its vacuum, so contains a number of D-branes in its vacuum. Since the endpoints of type I open string can be anywhere in 10d space, the D-branes in the vacuum of type I theory have a 10d worldvolume, which fills 10d spacetime completely, namely they are D9-branes. The gauge bosons in type I theory can be regarded as the gauge bosons on the worldvolume of these D-branes. There are 32 D9-branes in type I theory, so the gauge group in the open string sector would be $U(32)$, but the fact that the open strings are unoriented reduces the group to $SO(32)$. We will construct this theory in more detail in subsequent lectures.

3.3.2 Non-perturbative effects and D-branes

Effects of non-perturbative states in string theory can be very important. Here we would like to review a situation where the perturbative description of string theory breaks down and give singular answers for some quantities; happily, non-perturbative effects come to the rescue precisely in this situation and make physics of string theory smooth.

Strominger's conifold

In the study of the compactification of type IIB theory on Calabi-Yau spaces, one realizes that the effective action becomes singular at a point in the moduli space of Calabi-Yau geometries. This means that the perturbative prescription for computing amplitudes is giving some infinite answers, which appear as a singular behaviour in the dependence of the string action on moduli vevs.

This seemingly ill behaviour of string theory puzzled experts for many years. The issue was solved in a beautiful paper [12], which realized there is a non-perturbative state playing a key role in this situation.

It can be seen that the singular behaviour appears precisely at the point in moduli space where one submanifold of the Calabi-Yau, a 3-cycle, degenerates to zero size. The geometry of the Calabi-Yau near this 3-cycle can be locally described by the set of points in $\mathbf{C}^4$ satisfying the equation

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 = \epsilon \tag{14}$$

and ϵ is the vev of a modulus field in 4d, which controls the size of the 3-cycle (for instance, if ϵ is real, the above CY contais a 3-sphere of radius $\epsilon^{1/2}$, obtained by restricting to real z_i). This geometry is called the conifold singularity, and is very popular in the string theory community (it is the most generic singularity in Calabi-Yau spaces).

Strominger's insight was to realize that there exist a non-perturbative state which corresponds to a D3-brane wrapped on this 3-sphere, so which looks like a particle-like state in 4d. Its mass is the D3-brane tension times the 3-sphere volume

$$M_{D3} = T_{D3} V_{\mathbf{S}^3} \tag{15}$$

so the particle is becoming massess as $\epsilon \to 0$. Therefore, the dynamics of this state is exteremly relevant, precisely at the point at which the perturbative effective action is becoming singular. Strominger moreover provided quantitative arguments showing that including the additional light state into the effective action makes it smooth and well behaved. And integrating it out in the smooth effective action leads to the singularity observed using just the perturbative prescription.

In fact, the theory has 4d $N = 2$ susy, so its action is completely determined once the spectrum is known. The relevant piece of the spectrum is an $N = 2$ $U(1)$ vector multiplet, whose gauge boson arises from the IIB 4-form with three indices along the 3-cycle; and one $N = 2$ hypermultiplet, given by the D3-brane state, charged under the vector multiplet. The effective action is just an $N = 2$ $U(1)$ gauge field theory with one charged hypermultiplet. Completely standard and completely smooth!

Notice that the result is present no matter how small the string coupling is. Here non-perturbative effects are crucial even in the perturbative regime.

Notice also that the result is amazing from the string theory perspective. Here we have a light particle, which is not describe as an oscillation mode of the string. It is however natural from the viewpoint of non-perturbative string theory, where objects with different string or brane nature are on an equal footing.

There are many other examples of this kind of behaviour. As usual, string theory is clever enough to give finite answers even in the most singular

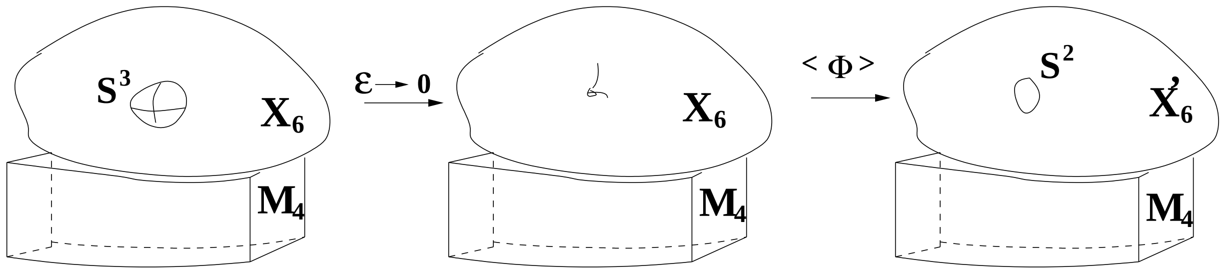

Figure 13: Tuning a modulus in the Calabi-Yau geometry, a 3-cycle shrinks and the geometry develops a conifold singularity.

situations. The theory has an incredible amount of self-consistency.

Topology change

Further investigation of the conifold non-perturbative states led to a fantastic effect [13]. Non-perturbative states can mediate phase transitions where the topology of the internal space (and so, of spacetime, changes). Taking a Calabi-Yau with two conifold singularities (with homologically related $\mathbf{S^3}$'s), and shrinking the corresponding 3-cycles, one finds that at the singular point in moduli space the low energy field theory is $N = 2$ $U(1)$ gauge theory with two charged massless hypermultiplets, Φ_a. This theory has a Higgs branch, where these hypermultiplets (which have non-perturbative origin!) acquire and expectation value along a flat direction of the scalar potential. The flat direction is parametrized by a field with no potential, a modulus. It has a geometric interpretation, which corresponds to parametrizing the size of 2-spheres which resolve the conifold singularities. This is schematically shown in fig 13.

In the process of sending $\epsilon \to 0$ and going to the Higgs branch the topology has changed, we have replaced an $\mathbf{S^3}$ by and $\mathbf{S^2}$. The transition is codified in a picture like 14

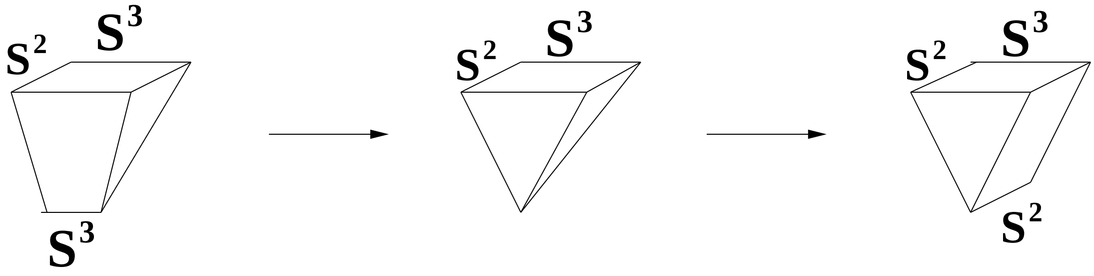

Figure 14: Topology change in the neighbourhood of a conifold singularity. Starting with a finite size $\mathbf{S^3}$ we tune a modulus to shrink it; at this stage massless state appear; a vev for them parametrizes growing an $\mathbf{S^2}$ out of the conifold singualrity.

This fact is remarkably important. The fact that string theory can smoothly interpolate between compactification spaces of different topology means that the choice of compactification manifold is in a sense dynamical, and determined by vevs of dynamical fields of the theory. All moduli spaces of different compactifications are connected into a huge universal moduli space.

3.4 D-branes as probes of spacetime

As already mentioned, vevs of worldvolume massless scalar fields correspond to coordinates of the brane in transveser space. This means that the moduli space of vacua of the field theory on the volume of a D-brane is the geometry of the space transverse to the D-brane. In this sense, spacetime can be considered a concept derived from more fundamental entities, like the field theory on the D-branes. This proposal generlizes to more general and less supersymmetric situations (like D-branes at singularities [14]).

This idea lies at the heart of some proposals like M(atrix) theory, which attempts at providing a microscopic definition of 11d M-theory [11]. The fundamental concept in M(atrix) theory is the worldvolume (worldline) action on a bunch of n type IIA D0-branes, in the limit $n \to \infty$. This is given by the dimensional reduction to $0+1$ dimensions of $d = 10$ $N = 1$ $U(n)$ super Yang-Mills.

In this approach, spacetime is obtained as the moduli space of the D0-brane gauge theory. Moreover, it is possible to reproduce supergravity interactions between objects by considering the dynamics of the $0+1$ gauge theory on configurations with slowly varying backgrounds for scalar fields (i.e. wavepackets slowly moving in spacetime). The arbitrariness in the number of D0-branes allows to explore arbitrarily high momentum in the M-theory dimension, and to recover 11d physics of M-theory.

Other applications of D-branes as probes includes throwing D-branes to diverse singularities of spacetime to see whether string theory can make sense of them. This approach has been successful in some cases, and has led to the understanding of certain naked singus in spacetime [15].

3.5 D-branes and gauge field theories

It is possible to take a low-energy limit in string theory in the presence of D-branes, which keeps all physical quantities of the worldvolume gauge field theory finite. In this limit the dynamics reduces to a quantum gauge field theory in $p+1$ dimensions, with gravity decoupled from it. Knowledge about perturbative and non-perturbative dynamics of string theory and D-branes can be used to explore or reproduce the dynamics of quantum gauge field theories. There are several examples of this, let us review two prototypical cases.

Montonen-Olive duality

One can use dualities of string theory to derive dualities in quantum gauge field theories. For instance, consider the 4d $N = 1$ supersymmetric $U(n)$ gauge theory obtained in the low-energy limit on a stack of n overlapping Type D3-branes. Gauge bosons and superspartners are obtained from open strings stretched between the different D3-branes. The gauge coupling is fixed by the string coupling $(g_{YM})^2 = g_s$.

Type IIB theory has a dual description in terms of another type IIB theory with string coupling $1/g_s$. In the dual theory, our configuration is given by n D3-branes, so it is a $U(n)$ gauge theory but now with gauge coupling $g'_{YM} = 1/g_{YM}$. The original perturbative states, open strings between the original D3-branes, are mapped to D1-branes stretched between D3-branes; it is possible to see that they correspond to 'tHooft Polyakov monopoles of the dual theory.

Hence $N = 4$ $U(n)$ super Yang-Mills has a strong-weak duality relating the theory with coupling g_{YM} and $1/g_{YM}$, and exchanging fundamental and solitonic degrees of freedom. This duality had been previously proposed from purely field theoretical considerations [16], but we see here that it follows easily from the conjectured self-duality of type IIB string theory.

AdS/CFT correspondence (Maldecena conjecture)

We have proposed two different descriptions for the same object, the Dp-brane; one in terms of a solution to the sugra equations of motion, the other in terms of open strings ending on a $(p + 1)$-dimensional hyperplane. In principle both describe the same dynamics.

The Maldecan conjecture proposes to take a low energy limit in these two descriptions and match the result. On one side, we recover 4d $N = 4$ super Yang-Mills, decoupled from gravity; on the supergravity side, the 3-brane solution becomes and $\mathrm{AdS}_5 \times S^5$ geometry. So the proposal by Maldacena [17] is that $N = 4$ $U(n)$ super Yang-Mills is completely equivalent to type IIB string theory in $\mathrm{AdS}_5 \times S^5$.

This is a striking statement, that a string theory is completely equivalent to a gauge field theory! In fact, a subtle features makes this statement less striking. String theory on the curved space $AdS_5 \times S^5$ does not have an exactly solvable worldsheet theory, so we can study it only in the supergravity approximation, valid for small curvatures. This regime corresponds, in the language of the dual field theory, to the limit of large $\lambda = g_{YM}^2 N$, this is a strongly coupled regime; λ is known as the 't Hooft coupling, and 't Hooft indeed proposed that in the large λ regime gauge field theory should be described as a string theory [18]. Hence the tractable regime in string theory is mapped to an untractable regime in gauge theory (because of the strong coupling). On the other hand, the tractable regime in gauge theory (small N) maps to string theory in spaces with string scale curvatures, which is completely untractable. So no paradox arises in relating a gauge field theory and a full-fledged string theory.

This conjecture has led to many important insights into gauge field theories in the large N limit, using the dual supergravity as a computational tool. In cases with less susy than $N = 4$ one can show at a qualitative level some features of strongly coupled gauge theories like confinement, chiral symmetry breaking, etc.

4 Our world as a brane-world model

We conclude this discussion by mentioning what applications all these non-perturbative objects may have in constructing phenomenological models of our world. The main motivation is that branes provide us with a mechanism to generate non-abelian gauge symmetries very different from that in heterotic theory. In particular, it is possible to localize gauge interactions in a subspace of spacetime, while gravity is still able to feel full spacetime.

The brane world idea is that it may be possible to construct string/M theory models where all or some of the particles of the standard model are part of the gauge sector of some branes, and hence are unable to propagate in some directions transverse to the brane. On the other hand, gravity would still be able to propagate on such directions.

There are basically two scenarios where this can be realized in string theory.

Horava-Witten phenomenology

The first is the Horava-Witten theory, which already before compactification has E_8 gauge interactions localized on 10d subspaces in an 11d world.

In order to build a phenomenological model, one may operate in a manner similar to that in the weakly coupled heterotic. Namely, compactify six of the ten dimensions in a Calabi-Yau manifold, endowed with some internal background for some of the E_8 gauge bosons. This configuration leads to 4d gravitational interactions and gauge interactions (with a gauge group determined by the internal gauge background), plus several families of charged chiral fermions.

Most of the phenomneology is similar to that in weakly coupled heterotic theory, except for hte choice of fundamental scale. As we discuss later on, the existence of one direction transverse to all gauge interactions allows to lower the fundamental scale below the 4d Planck scale. A nice choice in this context is to take the fundamental scale (11d Planck length to be around the gut scale 10^{16} GeV). This scenario was proposed in [19], and explored in many subsequent papers.

D-brane worlds

This possibility has been considered in [9] and many subsequent papers. It corresponds to considering compactifications of type II or type I theories on say a Calabi-Yau manifold X_6, with D-branes spanning four-dimensional

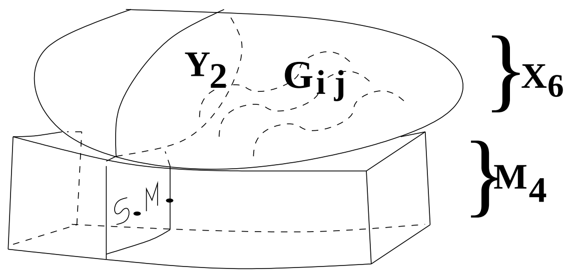

Figure 15: Schematic picture of a brane-world construction, with the Standard Model localized on the volume of e.g D5-branes with worldvolume $M_4 \times Y_2$, with Y_2 a compact submanifold of X_6.

Minkowski space and wrapped on a submanifold of X_6.

The simplest possibility would be to consider the standard model to be embedded in the volume of a D3-brane sitting at a point in X_6. Other possibilities would be to consider it to be embedded in a D5-brane whose worldovlume spans 4d Minkowski space and wraps a 2-cycle in X_6. The situation is shown in fig 15. In general Dp-brane leads to a 4d gauge sector if it wraps a $(p-3)$-dimensional submanifold Σ of X_6.

In principle, compactification in X_6 leads to 4d gravity; on the other hand, the gauge sector on the D-brane is also compactified on Σ and leads to 4d gauge sector. One has to work rather hard to construct configurations of D-branes whose open string sector leads to something like the standard model, but this has been achieved in several ways. We will skip these details here.

This kind of construction allows to build models where the fundamental string scale is not of the order of the 4d Planck mass, and can in fact be much lower (in order to be consistent with experiment, it cannot be lower than a few TeV. The largeness of M_P can be generated if the compactification manifold is very large, so that gravity gets diluted. On the other hand, we should keep the internal directions along the brane of small to avoid too light KK replicas of Standard Model particles ($M_c \leq$ TeV along directions in Y_2 in the picture). However, constraints on the size of the directions in X_6 transverse to the brane (which are felt only gravitaionaly) are very mild, and such size can be as large as 0.1 mm.

More quantitatively, before compactification gravitational and gauge interactions are described by an effective action

$$\int d^{10}x \, \frac{M_s^8}{g_s^2} \, R_{10d} + \int d^{p+1}x \, \frac{M_s^{p-3}}{g_s} \, F_{(p+1)d}^2 \tag{16}$$

where the powers of g_s follow from the Euler characteristic of the worldsheet which produces the propagator of gravitons (sphere) and gauge bosons (disk), while the powers of M_s are fixed by dimensional analysis.

Upon compactification, the 4d action picks us a volume factor, as we saw in the discussion of KK compactification, and reads

$$\int d^4x \, \frac{M_s^8 V_{X_6}}{g_s^2} \, R_{4d} + \int d^4x \, \frac{M_s^{p-3} V_\Sigma}{g_s} \, F_{4d}^2 \tag{17}$$

This allows to read off the 4d Planck mass and gauge coupling, which are experimentally measured.

$$\begin{aligned} M_P^2 &= \frac{M_s^8 V_{X_6}}{g_s^2} \simeq 10^{19}\,\mathrm{GeV} \\ 1/g_{YM}^2 &= \frac{M_s^{p-3} V_\Sigma}{g_s} \simeq 0.1 \end{aligned} \tag{18}$$

If the geometry is factorizable, we can split $V_{X_6} = V_\Sigma V_{trans}$, with V_{trans} the transverse volume. One therefore obtains

$$M_P^2 \, g_{YM}^2 = \frac{M_s^{11-p} V_{trans}}{g_s} \tag{19}$$

This shows that it is possible to generate a large Planck mass in 4d with a low string scale, by simply increasing the volume transverse to the brane.

This allows to rephrase the hierarchy problem in geometric terms. The fundamental string scale could be close to the weak scale, around a few TeV, and the 4d Planck scale could be a derived scale arising from a large transversal volume.

It is important however, that having a low string scale is a possibility, not a necessity, in the brane world picture. However, it is an exciting possibility to provide new realizations of theories similar to our standard model within the framework of string theory.

Whether it is heterotic string theory or a brane-world scenario the way in which string theory is realized in Nature (if any of these mechanisms, there may be other ways not known to us for the moment), it is matter of experiment for coming generations of experiments. For the moment, we should be happy enough with the possibility of realizing such rich theories into a beatiful structure such as string theory.

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