D-branes

1 Introduction

In the previous lecture we used supergravity to obtain partial information on non-perturbative states in string theory. We could rely on the existence and certain properties (tension, charge) of some of these p-brane states, when they satisfy some BPS condition.

In this lecture we propose a *microscopic* description, valid at weak coupling, for some of these solitoon (those we called Dp-branes), explicitly in terms of the underlying string theory. This description allows to recover the results we found in the supergravity approximation, and to describe several others (exactly in α'). Indeed, the study of D-branes from several viewpoints is one of the most active topics in string theory nowadays.

Let us emphasize that the microscopic description we are going to propose cannot be derived from out macroscopic description from the supergravity viewpoint. Rather, the microscopic description will show that the object we describe microscopically is a source of the supergravity fields with the same properties of the objects in the previous lecture.

2 General properties of D-branes

From the supergravity viewpoint, we introduced some solitonic solutions, the Dp-branes. They exist for p even in type IIA theory, for p odd in type IIB theory and for $p = 1, 5$ in type I. They are described by a gravitational background; fluctuations of the theory around the soliton solution are localized on the $(p + 1)$-dimensional volume of the soliton core.

The stringy description of Dp-branes, at weak coupling, is as follows.

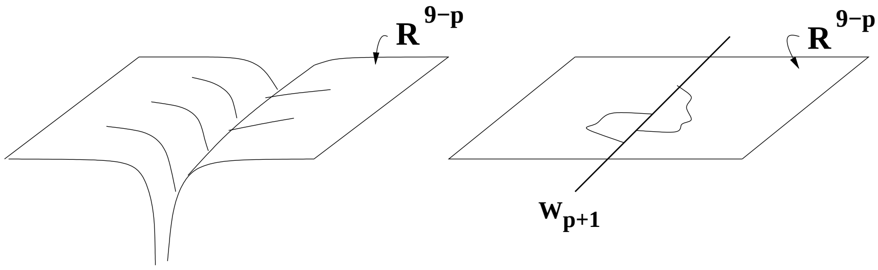

Figure 1: Fluctuations of the theory around a Dp-brane sugra solution can be described in stringy language as open strings with ends on a $(p+1)$-dimensional surface, located at the core of the topological defect.

They are described as $(p+1)$-dimensional planes W_{p+1} in flat space, with the prescription that the theory in its presence contains open strings, with endpoints on the $(p+1)$-dimensional plane W_{p+1}. See figure 1.

Equivalently, the fluctuations of the string theory around the topological defect are microscopically described as open strings ending on its $(p+1)$ volume.

A complementary point of view, relating the microscopic description with the supergravity solution, is that interactions of the $(p+1)$-dimensional plane with the closed string modes (via the open string modes on the brane) imply the plane is a source of the graviton, dilaton and RR fields, which creates a background as that described by the supergravity solution. See figure 3. Recall that the size of the throat in the supergravity solution is $g_s N \alpha'^{1/2}$, so this effect is bigger when g_s increases (and then the supergravity description is reliable, while for small g_s the stringy description is more precise).

That is, the object we have described as a $(p+1)$ plane on which open strings are allowed to end, has the correct properties to lead to a Dp-brane supergravity solution. The coupling to the closed string modes can be ob-

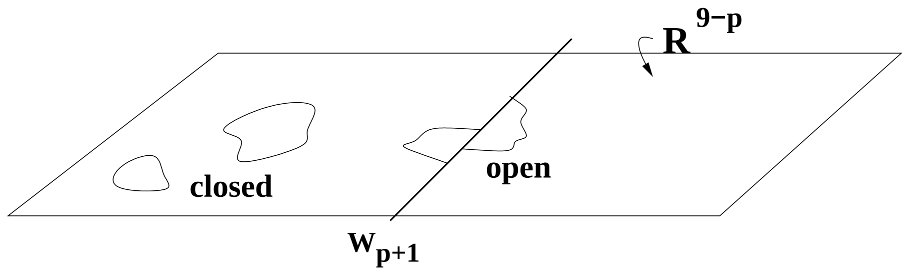

Figure 2: String theory in the presence of a Dp-brane. The closed string sector describes the fluctuations of the theory around the vacuum (gravitons, dilaton modes, etc), while the sector of open strings describes the spectrum of fluctuations of the soliton.

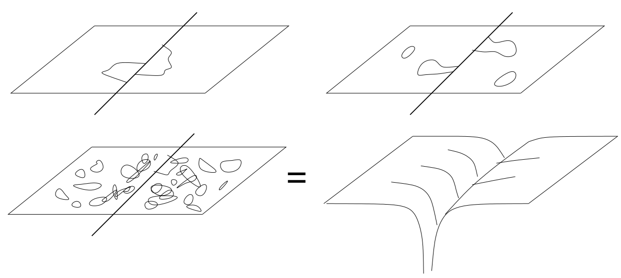

Figure 3: A Dp-brane interacts with closed strings via open strings, creating an effective background which describes the backreaction of the D-brane tension and charge on the configuration.

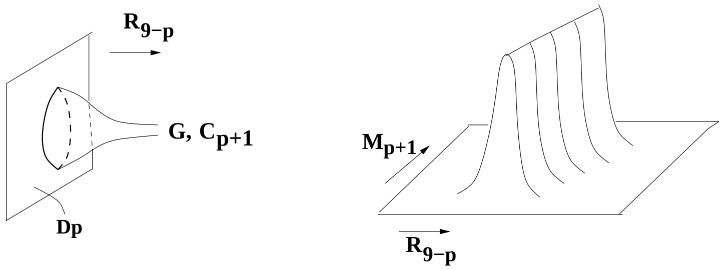

Figure 4: D-branes interact with closed string modes, and in particular couple to the bulk graviton and $(p+1)$-form fields, i.e. they have tension (of order $1/g_s$ in string units) and carry charge. Their backreaction on the background curves and deforms it into the p-brane solution seen in the supergravity regime.

tained from the disk diagram with a closed string insertion, see figure 4. In particular, it allows to obtain the tension and the charge under the RR $(p+1)$-form; they are of the order of $1/g_s$, since the Euler characteristic of the disk is $\xi = 1$. It is also possible to verify they satisfy the BPS condition; indeed, we will find below that they are supersymmetric states.

One could raise a number of objections against coupling this kind of open string sectors to a sector of closed strings.

i) The open string sector is not Poincare invariant. This is not a problem, since it is describing the fluctuations of the theory around a soliton state which breaks part of the Poincare invariance of the vacuum.

ii) The 2d worldsheet bosons associated to directions transverse to the D-brane, $X^i(\sigma,t)$ (and also the 2d fermions) obey Dirichlet boundary conditions

$$\partial_t X^i(\sigma,t)|_{\sigma=0,\ell} = 0 \qquad (1)$$

Are these boundary conditions consistent? Do we recover the same local 2d dynamics as for closed strings? In fact, we can check that Dirichlet boundary conditions do the job . Recall that the variation of the Polyakov action is

$$\begin{aligned}\delta S_P &= -\frac{1}{2\pi\alpha'}\int_\Sigma d^2\xi\, g^{ab}\partial_a X^\mu \partial_b \delta X^\mu = \\ &= -\frac{1}{2\pi\alpha'}\int_{-\infty}^{\infty} de\, (g^{ab}\delta X^\mu \partial_b X_\mu)|_{\sigma=0}^{\sigma=\ell} + \frac{1}{2\pi\alpha'}\int_\Sigma d^2\xi\, \delta X_\mu\, g^{ab}\partial_a\partial_b X^\mu \qquad (2)\end{aligned}$$

For Dirichlet boundary conditions, the corresponding endpoint is not allowed to move, so the allowed variations must satisfy $\delta X^i = 0$. Hence the boundary term for any coordinate drops, for Neumann or Dirichlet boundary conditions.

iii) In th lecture on open strings we saw that open strings allowed to end anywhere on spacetime cannot be consistently added to type IIB theory, due to RR tadpole cancellation conditions. In fact, this kind of configurations can be understood as type IIB theory in the presence of D9-branes, which are charged under C_{10} and lead to an inconsistency in the equations of motion. For configurations with lower-dimensional Dp-branes, $p < 9$, the corresponding RR form C_{p+1} does have a kinetic term and the equation of motion can be solved. RR charges are not dangerous if there are non-compact dimensions transverse to the D-brane. Intuitively, the fluxlines created by the D-brane charge can escape to infinity along the non-compact dimension. If there are no transverse directions, or they are compact, the flux cannot escape and one should require charge cancellation as a consistency condition.

3 World-volume spectra for type II D-branes

The fluctuations of the theory around the soliton background are described by open strings ending on the D-brane $(p + 1)$-dimensional world-volume. These modes describe the dynamics of the Dp-brane. For instance, zero mass oscillation modes of the open strings correspond to zero energy motions of the Dp-brane.

In this section we compute the spectrum of open strings ending on the Dp-brane. They give rise to fields propagating on the volume of the Dp-brane, and describe its dynamics. For concreteness we center on type IIB D-branes, which have even world-volume dimension.

3.1 A single Dp-brane

Consider a configuration given by a single Dp-brane with worldvolume spanning the directions X^μ, $\mu = 0, \ldots, p$ and transverse to the directions X^i, $i = p+1, \ldots, 9$. Consider an open string with both endpoints on the Dp-brane. Its worldsheet 2d theory is described by 2d bosons $X^\mu(\sigma, t)$, $\mu = 2, \ldots, p$ (in the light cone gauge) and $X^i(\sigma, t)$, $i = p+1, \ldots, 9$, and their 2d fermion partners. see fig 5.For directions along the brane volume, we have Neumann boundary conditions, while for directions transverse to it we have Dirichlet boundary conditions

$$\partial_\sigma X^\mu(\sigma, t)|_{\sigma=0,\ell} = 0 \quad ; \quad \partial_t X^i(\sigma, t)|_{\sigma=0,\ell} = 0 \quad ; \tag{3}$$

Using the mode expansions, of the form

$$X(\sigma, t) \quad = \quad x + w\sigma + \frac{p}{p^+}t + i\sqrt{\frac{\alpha'}{2}} \sum_\nu \frac{\alpha_\nu^i}{\nu} e^{-\pi i \nu (\sigma+t)/\ell} + i\sqrt{\frac{\alpha'}{2}} \sum_{\tilde{\nu}} \frac{\tilde{\alpha}_{\tilde{\nu}}^i}{\tilde{\nu}} e^{-\pi i \tilde{\nu} (\sigma+t)/\ell}$$

For X^μ we obtain

$$x^\mu, p^\mu \text{ allowed} \quad ; \quad w^\mu = 0 \quad ; \quad \nu = n \in \mathbf{Z} \quad ; \quad \alpha_{\mathbf{n}}^\mu = \tilde{\alpha}_{\mathbf{n}}^\mu \tag{4}$$

For X^i we obtain

$$x^i \text{ allowed} \quad ; \quad p^i = 0 \quad ; \quad w^i = 0 \quad ; \quad \nu = n \in \mathbf{Z} \quad ; \quad \alpha_{\mathbf{n}}^{\mathbf{i}} = -\tilde{\alpha}_{\mathbf{n}}^{\mathbf{i}} \tag{5}$$

For the NN directions we have the expansion familiar from the lesson on open strings. For the DD directions we have the expansion

$$X^i(\sigma, t) = x^i + \sqrt{2\alpha'} \sum_{n \neq 0} \frac{\alpha_n^i}{n} \sin(\pi n \sigma/\ell)\, e^{-\pi i\, nt/\ell} \tag{6}$$

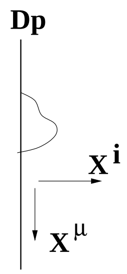

Figure 5: .

and similarly for fermions.

In total, we obtain integer modded bosonic oscillators α_n^μ, α_n^i, and fermionic oscillators $\psi_{n+\rho}^\mu$, $\psi_{n+\rho}^i$, with $\rho = 1/2, 0$ for NS or R fermions. Note also that these states have momentum only in directions along the volume of the D-brane, and not in those transverse to it. This implies that the correspondinng particles propagate only in the $(p+1)$-dimensional D-brane world-volume.

The spectrum is very similar to that of an open superstring sector, with the same states reinterpreted with respect to a lower-dimensional Lorentz group. In particular, at the massless level the states are

Sector	State	$SO(8)$ weight	$SO(p-1)$	$(p+1)$-dim field
NS	$\psi_{-1/2}^\mu \vert 0\rangle$	$(0,\ldots,0,\underline{\pm,\ldots,0})$	Vector	Gauge boson A_μ
	$\psi_{-1/2}^i \vert 0\rangle$	$(\underline{\pm,\ldots,0},0,\ldots,0)$	Scalar	$9-p$ real scalars ϕ^i
R	$A_a^+ \vert 0\rangle$	$\frac{1}{2}(\pm,\pm,\pm,\pm)$	spinor	$2^{(9-p)/2}$ ch. fermion λ_α
	$A_{a_1}^+ A_{a_2}^+ A_{a_3}^+ \vert 0\rangle$	# - = odd		

This corresponds to a $U(1)$ vector supermultiplet with respect to 16 supersymmetries in $(p+1)$dimension. This is also often described as the dimensional reduction of the $\mathcal{N}=1$ 10d vector multiplet. A prototypical example is provided by the spectrum on a D3-brane, which corresponds to a $U(1)$ vector multiplet of $\mathcal{N}=4$ susy in 4d, given by one gauge boson, six real scalars and four Majorana fermions.

In fact, supersymmetry extends to the complete open string spectrum, implying the property that the D-brane is a 1/2 BPS state. Indeed, it is possible to verify that the boundary conditions imposed for a D-brane on the open string sector relate the spacetime supercharges arising from the left and right-movers in 2d, so that the configuration is still invariant under 10d supersymmetry transformations with parameters ϵ_L, ϵ_R (which are 10d spinors) satisfying

$$\epsilon_L = \Gamma^0 \ldots \Gamma^p \epsilon_R \tag{7}$$

Using the above microscopic description, and knowing how to quantize open string sectors, it is possible to compute explicitly the tension and charge of a Dp-brane. The standard technique is to evaluate the annulus amplitude, namely the one-loop vacuum amplitude for open strings with both ends on a D-brane, and go to the factorization limit where the amplitude splits into the square of the disk. The disk provides the the coupling between the D-brane and the NSNS fields, like the graviton, and the RR fields (i.e. the D-brane tension and RR charge). The computation is pictorially sketched in figure 6 and gives the result (see section 13.3 in [1]

$$T_p^2 = \frac{\pi}{\kappa^2}(4\pi^2\alpha')^{3-p} \quad ; \quad Q_p = T_p/g_s \tag{8}$$

3.2 Effective action

The $(9-p)$ real scalars in the volume of the Dp-brane are the goldstone bosons associated to translational symmetries of the vacuum, broken by the

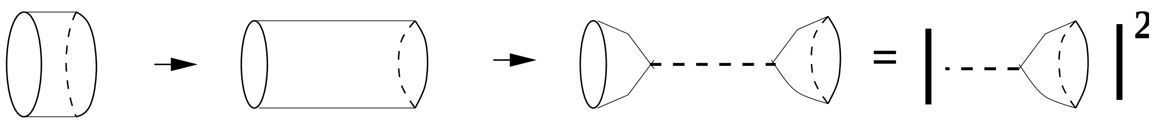

Figure 6: The D-brane charge and tension arise from a disk diagram with insertions, which can be obtained from factorization of the annulus diagram.

presence of the soliton [1]. This implies that the vevs of these scalars provide the location of the Dp-brane in transverse space $\mathbf{R^{9-p}}$. It also implies that non-trivial profiles for these scalar fields (that is, configurations with x^μ-dependent backgrounds for these scalars) correspond to fluctuations of the embedding of the D-brane worldvolume on spacetime, see fig 7. Namely $\phi^i(x^\mu)$ describes the embedding of the D-brane volume in spacetime. Therefore the effective action for the massless open string modes on the D-brane worldvolume corresponds to an effective action for the D-brane, controlling its dynamics.

There are two strategies to obtain this effective action, which are conceptually analogous to the computation of effective actions for closed string sectors. The first is to compute scattering amplitudes in string theory and to cook up an action that reproduces them. The second is to couple a general background of the massless fields to the 2d worldsheet theory, and to demand conformal invariance (both locally on the 2d worldsheet and on the boundary conditions for general backgrounds); the conformla invariance constraints can then be interpreted as equations of motion for the spacetime fields, arising from some effective action, see [2].

The resulting effective action has several pieces. One of them is the Dirac-

[1]Similarly, the fermions can be regarded as the goldstone fermions associated to supersymmetries of the vacuum, broken by the presence of the D-brane.

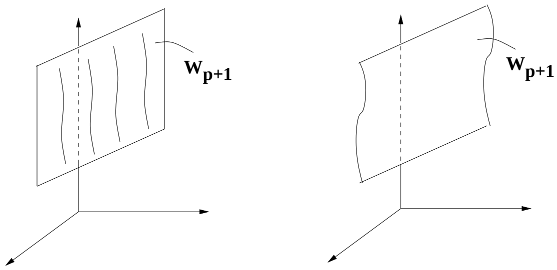

Figure 7: A nontrivial configuration for one of the worldvolume translational zero modes corresponds to a non-trivial embedding of the soliton worldvolume in spacetime.

Born-Infeld action, which has the form

$$S_{Dp} = -T_p \int_{W_{p+1}} d^{p+1}x^{\mu}\, \left(-\det(G + B + 2\pi\alpha' F)\right)^{1/2} \tag{9}$$

where $G_{\mu\nu} = \partial_\mu \phi^i \partial_\nu \phi^j\, G_{ij}$ is the metric induced on the D-brane worldvolume [2], and similarly $B_{\mu\nu}$ is the induces 2-form. These terms introduce the dependence of the action on the embedding fields $\phi^i(x^\mu)$. Finally $F_{\mu\nu}$ is the field strength of the worldvolume gauge field.

The Dirac-Born-Infeld action carries the information about the coupling of the D-brane to the NSNS field. The Dirac-Born-Infeld action is α' exact in terms not involving derivatives of the field strength. Neglecting the dependence on the field strength, it reduces to the D-brane tension times the D-brane volume $\int(\det G)^{1/2})$. At low energies, i.e. neglecting the α' corrections, it reduces to a kinetic term for the scalars plus the $(p+1)$-dimensional Maxwell action for the worldvolume $U(1)$, with gauge coupling

[2] We have implicitly fixed the worldvolume reparametrization invariance to fix a 'static gauge'. The scalars associated to these gauge degrees of freedom do not appear in the light-cone spectrum.

given by $g^2_{U(1)} = g_s$. Of course the above action should include superpartner fermions, etc, but we skip their discussion.

A second piece of the effective action is the Wess-Zumino terms, of the form

$$S_{WZ} = -Q_p \int_{W_{p+1}} \mathcal{C} \wedge \mathrm{ch}(F)\, \hat{A}(R) \tag{10}$$

where $\mathcal{C} = C_{p+1} + C_{p-1} + C_{p-3} + \ldots$ is a formal sum of the RR forms of the theory, and $\mathrm{ch}(F)$ is the Chern character of the worldvolume gauge bundle on the D-brane volume

$$\mathrm{ch}(F) = \exp(\frac{F}{2\pi}) = 1 + \frac{1}{2\pi}\mathrm{tr}\, F + \frac{1}{8\pi^2}\mathrm{tr}\, F^2 + \ldots \tag{11}$$

and $\hat{A}(R)$ is the A-roof genus, characterizing the tangent bundle of the D-brane world-volume $\hat{A}(R) = 1 - \mathrm{tr}\, R^2/(2\pi^2)$. Integration is implictly defined to pick up the degree $(p+1)$ pieces in the formal expansion in wedge products. Hence we get terms like

$$\begin{aligned} S_{WZ} = \; & \textstyle\int_{W_{p+1}} -Q_p \Big(\int_{W_{p+1}} C_{p+1} + \frac{1}{2\pi} \int_{W_{p+1}} C_{p-1} \wedge \mathrm{tr}\, F + \\ & + \textstyle\frac{1}{8\pi^2} \int_{W_{p+1}} C_{p-3} \wedge (\mathrm{tr}\, F^2 - \mathrm{tr}\, R^2) + \ldots \Big) \end{aligned} \tag{12}$$

A very important property of this term is that it is topological, independent of the metric or on the particular field representatives in a given topological sector. This is related to the fact that these terms carry the information about the RR charges of the D-brane configuration.

3.3 Stack of coincident Dp-branes

As a consequence of the BPS property, the interaction between several parallel Dp-branes exactly vanishes. This can be understood from a cancellation of the attractive interaction due to exchange of NSNS fields (like the graviton) and the repulsive interaction due to exchange of RR fields. It can also

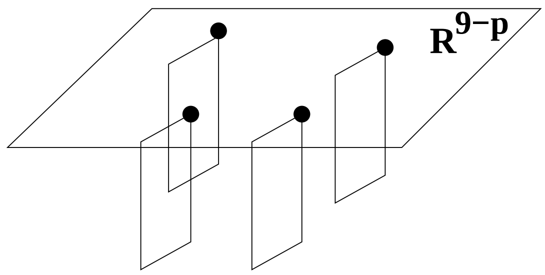

Figure 8: .

be understood from the fact that the supersymmetry transformations unbroken by a D-brane depend only on the directions it spans, so several parallel D-branes preserve the same supersymmetries (7).

We would like to consider the spectrum of open strings in a configuration of n parallel Dp-branes, labelled $a = 1, \ldots, n$, spanning the directions x^μ, $\mu = 0, \ldots, p$, and sitting at the locations $x^i = x^i_a$ in the $(9-p)$ transverse directions. See figure 8.

There are in this situation n^2 open string sectors, labelled ab, corresponding to open strings starting at the a^{th} D-brane and ending at the b^{th} D-brane. It is important to recall that we are working with oriented open strings (whose closed string sector is type II theory, which is oriented). For each of these n^2 sectors, the boundary conditions are NN for the 2d bosons $X^\mu(\sigma, t)$ (and fermions partners) and DD for the $X^i(\sigma, t)$ (and fermion partners). Namely, for an ab string we have

$$\begin{gathered} \partial_\sigma X^\mu(\sigma,t)|_{\sigma=0,\ell} = 0 \\ X^i(\sigma,t)|_{\sigma=0} = x^i_a \quad ; \quad X^i(\sigma,t)|_{\sigma=\ell} = x^i_b \end{gathered} \tag{13}$$

The mode expansion reads

$$X^i(\sigma,t) = x_a^i + \frac{x_b^i - x_a^i}{\ell}\sigma + i\sqrt{\frac{\alpha'}{2}}\sum_{\nu}\frac{\alpha_n^i}{n}\,e^{-\pi i n\,(\sigma+t)/\ell} + i\sqrt{\frac{\alpha'}{2}}\sum_{\tilde{n}}\frac{\tilde{\alpha}_{\tilde{n}}^i}{\tilde{n}}\,e^{-\pi i \tilde{n}\,(\sigma+t)/\ell}$$

The moddings etc works as in the case of just one Dp-brane. The spacetime mass formula is similar to the usual one for open strings, with and additional contribution arising from the winding term; we have

$$M^2 = \left(\sum_{i=p+1}^{9}\frac{x_b^i - x_a^i}{2\pi\alpha'}\right)^2 + \frac{1}{\alpha'}(N_B + N_F + E_0) \tag{14}$$

with $E_0 = -1/2, 0$ in the NS, R sectors.

This leads to the same kind of massless states as above, for each of the n^2 ab sectors. Namely, we obtain a total of n^2 gauge bosons, $(9-p)$ times n^2 real scalars and $2^{(9-p)/2}$ times n^2 chiral fermions in $p+1$ dimensions. It is not difficult to realize that the aa strings lead to massless states, no matter what the x_a^i are, and produce a gauge group $U(1)^n$, each $U(1)$ propagating on the volume of each D-brane. On the other hand the ab states are generically massive, with mass squared proportional to $\sum_i (x_a^i - x_b^i)^2$, and have charges $(+1,-1)$ under $U(1)_a \times U(1)_b$.

When some, say k, of the location of the D-branes in transverse space $\mathbf{R^{9-p}}$ coincide, the corresponding ab states become massless. In this situation, with additional massless vector bosons, we expect the world-volume gauge group to enhance beyond $U(1)^k$. The charges of the ab gauge bosons under the aa gauge symmetries correspond to the non-zero roots of the gauge group, which is easily checked to be $U(k)$. Hence, for k coincident Dp-branes the massless open string sector yields a $U(k)$ vector multiplet with respect to the 16 unbroken supersymmetries. In other words, in the configuration of k coincident D-branes the corresponding states are described by a $k \times k$ matrix, which represents their wavefunction with respect to Chan-Paton factors.

That is, Chan-Paton factors receive a geometric interpretation as encoding on which branes the string is starting and ending.

Changing continuously the locations of the D-branes away from each other corresponds to turning on a vev for the diagonal components of the scalar fields on the D-branes. This produces a Higgs effect breaking the enhanced $U(k)$ gauge symmetry, generically to the Cartan subalgebra $U(1)^k$. This is in agreement with the interpretation of these scalars as coordinates of the D-branes in transverse space. In this respect, it is amusing (and possibly a very profound property of the nature of spacetime in string theory) that these coordinates become matrices (and therefore non-commutative) at distances of the order of the string scale (where the scalars in ab open string sectors become light).

We conclude by mentioning that the effective action for world-volume massless fields in coincident D-branes should be a non-abelian generalization of the above. This is not exactly known for the Dirac-Born-Infeld piece, due to ambiguities in the precise gauge trace structure prescription. In any event, at low energies the action reduces to non-abelian Yang-Mills interactions with coupling $g_{YM}^2 = g_s$.

3.4 Comments

We conclude the discussion of type II D-branes with some comments:

• Although we have centered on type IIB D-branes, the same kind of results hold for type IIA D-branes, namely the worldvolume massless fields gather in vector multiplets with respect to the 16 unbroken susys, and their dynamics is described by the Dirac-Born-Infeld plus Wess-Zumino action.

• Spacetime supersymmetric D-branes exist only for p odd in type IIB and p even in type IIA. For the reverse dimensions, no GSO projection can be introduced in the open string sector (in a way consistent with open-closed duality and the GSO in the closed sector). However, there exist non-supersymmetric D-branes with p odd in type IIA and p even in type IIB. They are non-supersymmetric, contains worldvolume tachyons, and are unstable against decay. We may study them in the lecture on stable non-BPS states in string theory.

• Recall that type IIA and IIB theories are T-dual once we compactify on $\mathbf{S^1}$. The action of T-duality of D-brane states is easy to obtain, since T-duality acts on open string boundary conditions by exchanging Dirichlet and Neumann boundary conditions (see lecture on T-duality for type I). This implies the mapping

IIB on $\mathbf{S^1}$ of radius R	IIA on $\mathbf{S^1}$ of radius $1/R$
wrapped D$(2k+1)$	unwrapped D$(2k)$
unwrapped D$(2k+1)$	wrapped D$(2k)$

D-brane states moreover form a multiplet under the perturbative T-duality groups in compactifications on $\mathbf{T^d}$. For instance, consider type II compactified on $\mathbf{T^6}$, which has a T-duality group $SO(6,6;\mathbf{Z})$, and consider 4d particle-like D-brane states. Type IIB theory contains 4d particle-like states arising from D1-branes wrapped in one of the internal $\mathbf{T^6}$ directions (6 states), from D3-branes wrapped in three internal directions (20 states) and from D5-branes wrapped in five internal directions (6 states). In total, we have 32 states, transforming in the spinor representation 32 of $SO(6,6;\mathbf{Z})$. (Similarly, for type IIA we obtain 32 states from 1 D0-brane state, 15 D2-brane states, 15 D4-brane states adn 1 D6-brane state). These states, together with the perturbative states (momentum, winding) and other non-D-brane non-perturbative states (NS5-brane states, KK monopoles) fill out multiplets of the U-duality group $E_7(\mathbf{Z})$ as described in previous lecture.

• We would like to make a small remark on some D-branes which can be defined using the microscopic stringy description, and which were not encountered in the supergravity discussion.

- The type IIB D7-brane and the type IIA D8-branes change the asymptotic metric of spacetime, which is not flat, hence are not nicely described as asymptotically flat supergravity branes. The D7-brane is magnetically charged under the type IIB RR scalar a, which suffers a shift (monodromy) $a \to a+1$ in going around a D7-brane. This is consistent because the scalar is periodic, or equivalentely, because this transformation is an exact symmetry of IIB theory (in fact, in a subgroup of $SL(2, \mathbf{Z})$.

- Type IIA D8-brane is formally magnetically charged with respect to a (-1)-form. This simply means that it acts as a domain wall for a RR 0-form (the 'field strength') of type IIA theory, which is the cosmological constant, or mass parameter of massive IIA theory (Romans theory [3]).

- The type IIB D9-brane cannot be thought as a BPS non-perturbative state of type IIB theory, since it is charged under the RR 10-form and generates a tadpole rendering the theory inconsistent. Supersymmetric D9-branes only exist in the presence of O9-plane in type I theory, and in this situation they are present already in the vacuum, they are not an excited state of the theory. In the lecture on non-BPS states we will discuss excited states of type IIB theory with D9 - anti-D9 -brane pairs. These are excited states, but are not supersymmetric.

- Finally, the type IIB D(-1)-brane, which can be defined in the theory with spacetime euclidean signatures, is a sort of instanton, localized boht in space and in time. It is formally electrically charge under the type IIB RR scalar a, hence the instanton action is weighted by e^{ia}, so a acts as a theta parameter for type IIB theory.

4 D-branes in type I theory

4.1 Type I in terms of D-branes

Type I contains a sector of open strings already in its vacuum, with endpoints allowed to be anywhere in 10d spacetime. So in a sense it contains a set of (spacetime filling) D9-branes in the vacuum, on which these open strings end. These D-branes should thus not be regarded as excitated states above the vacuum, but part of it, since the theory is inconsistent without them. Nevertheless, these vacuum D9-branes are mathematically identical to the D-branes studied above, so it is useful to use the same language to describe them.

Indeed, both kinds of branes are in a sense related, as we described at the end of the lesson on T-duality for type I string theory. Recall that type I theory has one O9-plane (set of points fixed under the orientifold action Ω), and 32 D9-branes. Compactifying on $\mathbf{S^1}$ and performing a T-duality along it we obtain type I' theory, which is type IIA theory modded out by ΩR, with $R: x^9 \to -x^9$. It contains two O8 planes sitting at $x^9 = 0, \pi R$, and 32 D8-branes located at points in $\mathbf{S^1}$, which are part of the vacuum. However, taking the limit of infinite radius, keeping the D8-branes at a finite distance, the O8-planes go off to infinity and we are left with type IIA theory in flat 10d, with D8-branes. In this setup the D8-branes should be regarded as excitations over the type IIA vacuum.

The BPS D-branes of type I theory are the D5-brane and the D1-brane. In this section we obtain their world-volume modes, by quantizing the open string sectors of the configuration. Notice that other D-branes of type IIB theory, like the D3- or the D7-brane are projected out by the Ω projection and do not exist as BPS D-branes in type I theory.

4.2 Type I D5-brane

A useful reference for this section should be [4].

In principle, the computation of the world-volume spectrum for type I D5-branes is similar to that of type IIB D5-branes, with two new ingredients

i) In addition to the sector of open strings with both endpoints on the D5-branes, there is a sector of open strings with one end on the D5-branes and the other end on the vacuum D9-branes.

ii) We need to impose the orientifold projection on the open string spectrum, since we are working with unoriented open strings

To deal with i), let us start first forgetting the Ω projection, and consider a system of k coincident D5-branes and N D9-branes in the oriented theory. The geometry of the directions spanned/transverse to the branes is depicted by lines/crosses as follows

	0	1	2	3	4	5	6	7	8	9
D9	-	-	-	-	-	-	-	-	-	-
D5	-	-	-	-	-	-	$\times$	$\times$	$\times$	$\times$

The geometry is shown in figure 9. As we will see, the configuration preserves 8 supersymmetries, i.e. the equivalent of $\mathcal{N}=1$ 6d supersymmetry. This is the familiar criterion that some susy is preserved when the number of DN directions is a multiple of four.

In the sector of 55 strings, we obtain a $U(k)$ vector multiplet of 6d $\mathcal{N}=(1,1)$ supersymmetry, containing $U(k)$ gauge bosons, four real adjoint scalars, and two 6d Weyl fermions. In terms of 6d $\mathcal{N}=1$ supersymmetry, they correspond to a $U(k)$ vector multiplet (gauge boson plus one Weyl fermion) and an adjoint hypermultiplet (Weyl fermion plus four scalars).

In the sector of strings starting at the D5-branes and ending on the D9-branes (59 sector), the open strings have NN boundary conditions on the

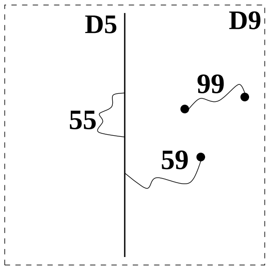

Figure 9: Open string sectors in type I theory in the presence of a D5-brane.

direction 2345 and DN conditions on 6789. The NN directions work as usual. For $X^i(\sigma,t)$, $i=6789$, we have

$$\partial_\sigma X^i(\sigma,t)|_{\sigma=0}=0 \quad ; \quad \partial_t X^i(\sigma,t)|_{\sigma=\ell}=0 \tag{15}$$

Using the mode expansions for the DN directions, we obtain that the center of mass x^i is fixed at the location of the D5-brane; that momentum and winding are not allowed $p^i=0$, $w^i=0$; and that oscillator modding is shifted by 1/2 with respect to their usual values, namely 2d bosons have modes $\alpha^i_{n+1/2}$ and 2d fermions have modes $\psi^i_{n+\rho+1/2}$, with $\rho=1/2,0$ for NS, R.

The mass formula for 59 states is

$$\alpha' M^2 = N_B + N_F \tag{16}$$

since $E_0=0$ both in the NS and R sectors. In the NS sector, there are four fermion zero modes, along 6789, hence the massless groundstate is degenerate. Splitting the zero modes in creation and annihilation operators, and constructing the representaion of the zero mode Clifford algebra as usual, the GSO projection selects the massless groundstates

State $\quad SO(4)_{6789}$

$$
\begin{array}{cc}
|0\rangle & (-\tfrac{1}{2},-\tfrac{1}{2}) \\
A^+_{a_1}A^+_{a_2}|0\rangle & (\tfrac{1}{2},\tfrac{1}{2})
\end{array}
\tag{17}
$$

$$
\tag{18}
$$

where the $SO(4)$ is the unbroken rotation group in 6789. These states are scalars under the 6d little group $SO(4)$. In the R sector, we have four fermion zero modes along 2345, hence the massless groundstate is degenerate. Splitting the zero modes in creation and annihilation operators, and constructing the representaion of the zero mode Clifford algebra as usual, the GSO projection selects the massless groundstates

$$
\begin{array}{cc}
\text{State} & SO(4)_{2345} \\
|0\rangle & (-\tfrac{1}{2},-\tfrac{1}{2}) \\
A^+_{a_1}A^+_{a_2}|0\rangle & (\tfrac{1}{2},\tfrac{1}{2})
\end{array}
\tag{19}
$$

$$
\tag{20}
$$

these states are spinors under the 6d $SO(4)$ Lorentz little group. Gathering states from the 59 and 95 sectors (the latter are similar), we obtain one hypermultiplet of 6d $\mathcal{N}=1$ supersymmetry. Noticing that the states carry D5- and D9- Chan-Paton labels, encoding on which D5- and on which D9-brane their endpoints lie, we realize the 6d $\mathcal{N}=1$ hypermultiplet transforms in the bi-fundamental representation (N,k) under the D9- and D5-brane world-volume gauge groups.

Let us now address ii) and impose the orientifold projection. To make a long story short, let us simply say that we need to specify the action of Ω on the D9- and D5-brane Chan-Paton indices, via $N\times N$ and $k\times k$ matrices $\gamma_{\Omega,5}$, $\gamma_{\Omega,9}$, and that consistency requires $N=32$ and [5]

$$
\gamma_{\Omega,9}=\mathbf{1}_{32}quad;\quad \gamma_{\Omega,5}=\begin{pmatrix} 0 & \mathbf{1}_{k/2} \\ -\mathbf{1}_{k/2} & 0 \end{pmatrix}
\tag{21}
$$

Note that consistency requires k to be even.

The projections go as follows. In the 99 sector, all fields suffer a projection

$$\lambda = -\gamma_{\Omega,9}\lambda^T\gamma_{\Omega,9}^{-1} \tag{22}$$

and the surviving spectrum is the 10d $\mathcal{N}=1$ $SO(32)$ vector multiplet.

In the 55 sector, the Ω action on oscillators along DD and NN directions differ by a sign. This follows from the definition of the action of Ω as $X^\Omega(\sigma,t) = X(-\sigma,t)$, and the mode expansions

$$\begin{aligned} X^\mu(\sigma,t) &= \ldots + \sum_{n\neq 0}\frac{\alpha_n^\mu}{n}\cos(\pi n\sigma/\ell)\, e^{-\pi n t/\ell} \\ X^i(\sigma,t) &= \ldots + \sum_{n\neq 0}\frac{\alpha_n^i}{n}\sin(\pi n\sigma/\ell)\, e^{-\pi n t/\ell} \end{aligned} \tag{23}$$

The Ω projections are different for the 6d $\mathcal{N}=1$ hyper and vector multiplets. The surviving states must satisfy the conditions

$$\begin{aligned} \lambda = -\gamma_{\Omega,5}\lambda^T\gamma_{\Omega,5}^{-1} & \qquad \text{vect.mult.} \\ \lambda = \gamma_{\Omega,5}\lambda^T\gamma_{\Omega,5}^{-1} & \qquad \text{hypermult.} \end{aligned} \tag{24}$$

leading to a 6d $\mathcal{N}=1$ $USp(k)$ vector multiplet, and one hypermultiplet in the two-index antisymmetric representation (which is reducible into a singlet and a representation of dimension $k(k-1)/2-1$).

Finally, the 59 sector is mapped to the 95 sector by Ω, so it is enough to keep the degrees of freedom in the 59 sector and not perform any projection. This leads to one half hypermultiplet of 6d $\mathcal{N}=1$ susy in the representation $(k,32)$ under $USp(k)_{55}$ and $SO(32)_{99}$. A half-hypermultiplet contains two real scalars and one Weyl fermion satisfying a reality condition, and only exists for multiplets in pseudo-real representation of the gauge group.

Some comments are in order

- The complete spectrum on the D5-brane worldvolume is

$$USp(k) \ \mathcal{N}=1 \text{ vector multiplet}$$
$$\mathcal{N}=1 \text{ hypermultiplet in } \begin{array}{|c|}\hline \\ \hline \\ \hline\end{array} + \tfrac{1}{2}(\Box; 32)$$

This 6d theory is chiral and miraculously free of anomlies. Again, another strong check of the self-consistency of string theory.

• As discussed in [4], a D5-brane provides the limit of zero size instantons in the D9-brane world-volume gauge theory. In fact, using the WZ couplings in the D9-brane theory, and instanton is charged under the RR 6-form C_6, exactly as a D5-brane.

$$\frac{1}{8\pi^2}\int_{10d} C_6 \wedge \operatorname{tr} F^2 \to k \int_{6d} C_6 \tag{25}$$

where $k = \frac{1}{8\pi^2}\int \operatorname{tr} F^2$ is the instanton number. Instanton have a bosonic zero mode which parametrizes their size. In the limit of zero size, the instanton is pointlike in four dimensions and is exactly described by a D5-brane.

4.3 Type I D1-brane

One can perform a similar computation of the world-volume massless spectrum for D1-branes. We consider a configuration of N D9-branes and k coincident D1-branes, the geometry is described by

	0	1	2	3	4	5	6	7	8	9
D9	-	-	-	-	-	-	-	-	-	-
D1	-	-	×	×	×	×	×	×	×	×

The configuration preserves 8 supersymmetries, more specifically $\mathcal{N} = (0,8)$ susy in the 2d volume of the D1-brane.

Before the orientifold projection, the 99 massless sector leads to the 10d $\mathcal{N}=1$ $U(N)$ vector multiplet; the 11 massless sector lead to the 2d $\mathcal{N}=(8,8)$ $U(k)$ vector multiplet. In the 19+91 sector, we have DN boundary conditions along the 8 light-cone directions; the moddings of oscillators are as for the DN directions discussed above, and the mass formula for 19 states is

$$\alpha' M^2 = N_B + N_F + E_0 \tag{26}$$

with $E_0 = 1/2, 0$ for the NS, R sectors. IN the NS sector, all states are massive. Massless states only arise from the R sector groundstate, which is unique since there are no fermion zero modes. The 19 and 91 groundstates behave as a 2d spinor, and transform in the representation (k, N) under the D1- and D9-brane gauge groups.

Let us now impose the orientifold projection. In this case, consistency requires

$$\gamma_{\Omega,9} = \mathbf{1}_{32} quad; \quad \gamma_{\Omega,1} = \mathbf{1}_k \tag{27}$$

The projections go as follows. In the 99 sector, all fields suffer a projection

$$\lambda = -\gamma_{\Omega,9} \lambda^T \gamma_{\Omega,9}^{-1} \tag{28}$$

and the surviving spectrum is the 10d $\mathcal{N} = 1$ $SO(32)$ vector multiplet.

In the 11 sector, the Ω action on oscillators along DD and NN directions differ by a sign. The Ω projections are different for the 2d $\mathcal{N} = (0, 8)$ vector multiplet (2d gauge bosons plus 8 left-moving 2d chiral fermions) and the 2d $\mathcal{N} = (0, 8)$ chiral multiplet (8 real scalars plus 8 2d chiral right-moving fermions). In fact we have

$$\begin{aligned} \lambda = -\gamma_{\Omega,1} \lambda^T \gamma_{\Omega,1}^{-1} \qquad & \text{vect.mult.} \\ \lambda = \gamma_{\Omega,1} \lambda^T \gamma_{\Omega,1}^{-1} \qquad & \text{ch.mult.} \end{aligned} \tag{29}$$

leading to a 2d $\mathcal{N} = (0, 8)$ $SO(k)$ vector multiplet and a 2d $\mathcal{N} = (0, 8)$ chiral multiplet in the two-index symmetric representation (which is reducible into a singlet and a representation of dimension $k(k+1)/2 - 1$).

Finally, the 19 sector is mapped to the 91 sector by Ω, so it is enough to keep the degrees of freedom in the 19 sector and not perform any projection. This leads to one 2d chiral (left-moving) spinor, with just one component, in the representation $(k, 32)$ under $SO(k)_{11}$ and $SO(32)_{99}$. This is sometimes called a Fermi multiplet of 2d $\mathcal{N} = (0, 8)$ susy.

Some comments are in order

• The complete spectrum on the D1-brane worldvolume is

$$\begin{array}{r} SO(k)\ \mathcal{N} = (0,8) \text{ vector multiplet (gauge boson plus 8 left fermions)} \\ \mathcal{N} = (0,8) \text{ chiral multiplet (8 scalars plus 8 right fermions)} \\ \mathcal{N} = (0,8) \text{ Fermi multiplet (8 left fermions) in } (\square; 32) \end{array}$$

This 2d theory is chiral and miraculously free of anomlies. Yet another strong check of the self-consistency of string theory.

• As discussed in later lectures, this content will provide support for the interesting duality conjecture for the strong coupling regime of type I theory.

5 Final comments

We have shown that a detailed treatment of Dp-branes is possible from our microscopic description. It allows to rederive the results from the supergravity analysis of solitons, and to obtain new results, like the detailed worldvolume theories, the appearance of enhanced gauge symmetries, etc.

Many other interesting phenomena appear in configurations with D-branes. For instance the existence of bound states of D-branes of different dimensions, configurations where D-branes end on D-branes, the D-brane dielectric effect, etc. D-branes properties is one of the hot topics in todays string theory. In the following lectures we will become familiar with some of them.

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