Motivation

1 Standard Model and beyond

1.1 Our Model of Elementary Particles and Interactions

Our description of particles and interactions treats strong-electroweak interactions and gravitational interactions in a very different way.

• Electromagnetic, weak and strong interactions are described by a **quantum gauge field theory**. Interactions are mediated by gauge vector bosons, associated with the gauge group

$$SU(3)_c \times SU(2)_W \times U(1)_Y \tag{1}$$

While matter is described by left-handed Weyl fermions in the following representation of the gauge group

$$\begin{array}{ll} 3\,[\,(3,2)_{1/6} + (\overline{3},1)_{1/3} + (\overline{3},1)_{-2/3} + & Q_L\,,\,U\,,\,D \\ +\,(1,2)_{-1/2} + (1,1)_1\,] + 3(1,1)_0 & E\,,\,L\,,\,\nu_R \end{array} \tag{2}$$

where the subscript denotes $U(1)_Y$ charge (hypercharge), and where we have also included right-handed neutrinos (although they have not been observed experimentally).

An important property of these fermions is their chirality (this is at the heart of parity violation in the Standard Model). There are no left-handed Weyl fermions with conjugate quantum numbers (if there would be, we could rewrite the pair as a left-handed and a right-handed Weyl fermion, both with equal quantum numbers; this is called a vector-like pair, and does not violate parity, it is non-chiral).

Our description considers all these objects to be pointlike. This assumption works as far as the model has been tested experimentally, i.e. up to energies about 1 TeV.

In order to break the electroweak symmetry $SU(2)_W \times U(1)_Y$ down to the $U(1)$ of electromagnetism, the model contains a Higgs sector, given by a complex scalar ϕ with quantum numbers

$$(2,1)_{-1/2} \tag{3}$$

The theory contains a scale M_W, which is the scale of pontaneous breaking of the symmetry [1]. It is fixed by the vacuum expectation value $<\phi>$ acquired by the scalar, as determined by a potential of the form

$$V(\phi) \,=\, -m^2\,\phi^*\phi \,+\, \lambda\,(\phi^*\phi)^2 \tag{4}$$

The electroweak scale is then

$$M_W \simeq <\phi> \simeq \frac{m}{\sqrt{\lambda}} \simeq 10^2\,\mathrm{GeV} \tag{5}$$

Chirality of the fermions forbid writing a Dirac mass term for them. The only way for them to get a mass is via coupling to the Higgs multiplet via Yukawa couplings schematically of the form

$$Q_L\,U\,\phi \quad ; \quad Q_L\,D\,\phi^* \quad ; \quad L\,E\,\phi \tag{6}$$

so the scale of fermion masses is linked to the scale of electroweak symmetry breaking.

This theory is well defined at the quantum mechanical level, it is unitary, renormalizable (leaving the issue of 'triviality' of the Higgs sector aside), etc...

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[1] To be fair, there is also a further scale in the model, the QCD scale around 1 GeV, which is understood in terms of dimensional transmutation, i.e. it is the energy at which the $SU(3)$ coupling constant becomes strong.

- On the other hand, the gravitational interactions are described by the classical theory of general relativity. Interactions are encoded in the spacetime metric $G_{\mu\nu}$ via the principle of diffeomorphism (or coordinate reparametrization) invariance of the physics. This leads to an action of the form

$$S_{grav} = M_P^2 \int_{X_4} R\sqrt{-G}\, d^4x \tag{7}$$

with a typical scale of

$$M_P \simeq 10^{19}\,\text{GeV} \tag{8}$$

Four-dimensional Einstein theory has been tested experimentally to be good description of the gravitational interactions down to length scales of about 10^{-7} m.

Since the interaction contains an explicit dimensionful coupling, it is difficult to make sense of the theory at the quantum level. They theory is non-renormalizable, it presents loss of unitarity at loop levels, it cannot be quantized in the usual fashion, it is not well defined in the ultraviolet.

The modern viewpoint is that Einstein theory should be regarded as an effective field theory, which is a good approximation at energies below M_P (or some other cutoff scale at which four-dimensional classical Einstein theory ceases to be valid). There should exist an underlying, quantum mechanically well-defined, theory which exists for all ranges of energy, and reduces to classical Einstein at low energies, below the cutoff scale. Such a theory would be called an ultraviolet completion of Einstein theory (which by itself is ill-defined in the ultraviolet).

1.2 Theoretical questions raised by this description

There are many such questions, and have led to a great creative effort by the high energy physics (and general relativity) communities. To be fair, most

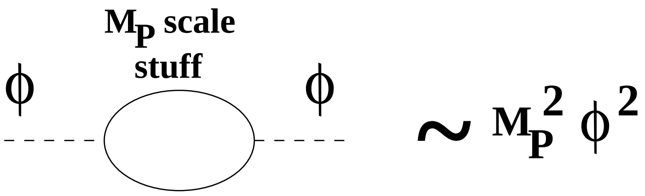

Figure 1: Quantum corrections to the Higgs mass due to Planck scale stuff.

of them have not been successfully answered, so the quest for solutions goes on. These are some of these questions

- The description is completely schizophrenic! We would like to make gravitational interactions consistent at the quantum mechanical level. Can this really be done? and how?
- Are all interactions described together in a unified setup? Or do they remain as intrinsecally different, up to arbitrary energies? Is there a microscopic quantum theory that underlies the gravitational and the Standard Model gauge interactions? Is there a more modest description which at least unifies the gauge interactions of the Standard Model (leaving for the moment gravity aside)?
- Why are there two different scales, M_W and M_P? Why are there so widely separated? Are they related in any way, and if so, which?
- Why M_W, which is fixed by the mass of the Higgs scalar, is not modified by quantum loops of stuff related to physics at the scale M_P? Power counting would suggest that the natural value of these corrections is of order M_P^2, which would then push the electroweak scale up to the Planck scale.
- Are there other scales between M_W and M_P? or is there just a big desert in energies in between? (there are some suggestions of intermediate masses, for instance from the see-saw mechanism for neutrino masses, which

points to new physics at an energy scale of 10^{12} GeV).

- Why the gauge sector is precisely as it is? Why three gauge factors, why these fermion representations, why three families? How are these features determined from an underlying microscopic theory that includes gravity?

- Are global symmetries of the Standard Model exact symmetries of the underlying theory? Or just accidental symmetries? Is baryon number really conserved? Why is the proton stable, and if not what new physics mediates its decay?

- Why are there four dimensions? Is it true that there are just four dimensions? Does this follow from any consistency condition of the theory supposedly underlying gauge and gravitational interactions?

- ..., ..., ... ?

1.3 Some proposals for physics beyond the Standard Model

These and other similar questions lie at the origin of many of the ideas of physics beyond the Standard Model. Let us review some of them (keeping in mind that they do not exclude each other, and mixed scenarios are often the most attractive). For a review along similar lines, see e.g. [1].

1.3.1 Grand Unification Theories (GUTs)

See for instance [2, 3].

In this setup the Standard Model gauge group is a low-energy remnant of a larger gauge group. This group G_{GUT} is usually taken to be simple (contains only one factor) like $SU(5)$, $SO(10)$, or E_6, and so unifies all low-energy gauge interactions into a unique kind. The GUT group is broken spontanously by a Higgs mechanism (different form that of the Standard

Model, of course) at a large scale M_{GUT}, of about 10^{16}-10^{17} GeV.

This idea leads to a partial explanation of the fermion family gauge quantum numbers, since the different fermions are also unified into a smaller number of representations of G_{GUT}. For $SU(5)$ a Standard Model family fits into a representation $10 + \overline{5}$; for $SO(10)$ it fit within an irreducible representation, the 16.

A disadvantage is that the breaking of G_{GUT} down to the Standard Model group requires a complicated scalar Higgs sector. In minimal $SU(5)$ theories, the GUT-Higgs belongs to a 24-dimensional representation; $SO(10)$ is even more involved.

Additional interesting features of these theories are

- Extra gauge interactions in G_{GUT} mediate processes of proton decay (violate baryon number), which are suppressed by inverse powers of M_{GUT}. The rough proton lifetime in these models is around 10^{32} years, which is close to the experimental lower bounds. In fact, some models like minimal $SU(5)$ are already experimentally ruled out because they predict a too fast proton decay.
- If we assume no new physics between M_W and M_{GUT} (desert hypothesis), the Standard Model gauge couplings run with scale towards a unified value at a scale around 10^{16} GeV. This may suggest that the different low-energy interactions are unified at high energies.

Besides these nice features, it is fair to say that grand unified theories do not address the fundamental problem of gravity at the quantum level, or the relation between gravity and the other interactions.

1.3.2 Supersymmetry (susy)

See graduate course by A. Casas, also review like e.g. [4]

Supersymmetry is a global symmetry that relates bosonic and fermionic degrees of freedom in a theory. Infinitesimal supersymmetry transformations are associated so (super)generators (also called supercharges), which are operators whose algebra is defined in terms of anticommutation (rather than commutation) relations (these are the so-called superalgebras, and generate supergroups). The minimal supersymmetry in four dimensions (so-called $D = 4$ $N = 1$ supersymmetry is generated by a set of such fermionic operators Q_α, which transform as a left-handed Weyl spinor under the 4d Lorentz group. The supersymmetry algebra is

$$\{Q_\alpha, Q_\beta\} \; = \; (\sigma^\mu)_{\alpha\beta}\, P_\mu \tag{9}$$

where $\sigma^\mu = (\mathbf{1}_2, \sigma^i)$ are Pauli matrices, and P_μ is the four-momentum operator.

A simple realization of supersymmetry transformations is: consider a four-dimensional Weyl fermion ψ^α and a complex scalar ϕ, and realize Q_α acting as

$$\begin{aligned} Q_\alpha \phi &= \psi_\alpha \\ Q_\beta \psi_\alpha &= i(\sigma^\mu)_{\alpha\beta} \partial_\mu \phi \end{aligned} \tag{10}$$

The algebra closes on these fields, so the (super)representation (also called supermultiplet) contains a 4d Weyl fermion and a complex scalar. Such multiplet is known as the chiral multiplet. Another popular multiplet of $N = 1$ susy) is the vector multiplet, which contains a four-dimensional massless vector boson and a 4d Weyl fermion (the latter is often re-written as a 4d Majorana fermion).

There exist superalgebras generated by more supercharges, they are called extended supersymmetries. The N-extended supersymmetry is generated by supercharges Q_α^a with $a = 1, \ldots, N$. Any supersymmetry with $N > 1$ is inconsistent with chiral fermions (any multiplet contains fermions with both

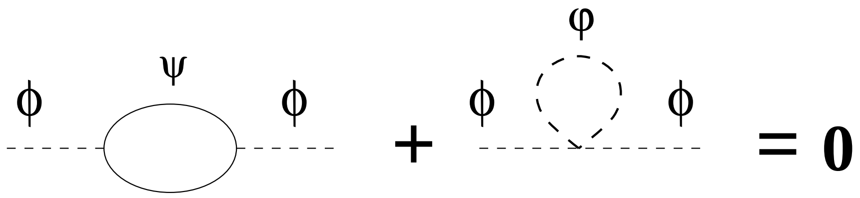

Figure 2: Fermionic and bosonic loop corrections to the higgs mass cancel in a supersymmetric theory.

chiralities, i.e. is vector-like), so such theories have limited phenomenological applications and we will skip them here.

The reason why susy may be of phenomenological interest is that it relates scalars (like the Higgs) with chiral fermions, and the symmetry requires them to have equal mass. The mass of a chiral fermion is forced to be zero by chirality, so the mass of a scalar like the Higgs is protected against getting large $O(M_P)$ corrections, so supersymmetry stabilizes M_W against M_P.

Diagrammatically, any corrections to the Higgs mass due to fermions in the theory are cancelled against corrections to the Higgs mass due to their boson superpartners. There is a non-renormalization theorem of certain couplings in the lagrangian (like scalar masses) which guarantees this to any order in perturbation theory.

SUSY commutes with gauge symmetries. So in trying to build a supersymmetric version of the standard model the simplest possibility is to add superpartners to all observed particles: fermion superpartners (gauginos) for gauge bosons to promote them to vector multiplets; boson superpartners (squarks and sleptons) for the quark and leptons, to promote them to chiral multiplets; and fermion superpartner (higgssino) for the scalar Higgs (for technical reasons, like anomaly cancellation, a second Higgs chiral multiplet must be included). Interactions are dictated by gauge symmetry and supersymmetry. Such model is known as the minimal supersymmetric standard model (MSSM).

However, superpartners have not been observed in Nature, so it is clear that they are not mass-degenerate with usual matter. Supersymmetry is not an exact symmetry of Nature and must be broken. The most successful way to do so, without spoiling the absence of quadratic corrections to the Higgs mass is explicit breaking. That is, to introduce explicitly non-supersymmetric terms of a certain kind (so-called soft terms) in the MSSM lagrangian. These terms render superpartners more massive than standard model fields. Cancellation of loop contributions to the Higgs mass is not exact, but is not quadratically dependent on M_P, only logarithmically. In order to retain 10^2 GeV as a natural scale, superpartner mass scale (supersymmetry breaking scale in the MSSM) should be around 1 TeV or so.

The MSSM is a theoretically well motivated proposal for physics beyond the Standard Model, it is concrete enough and experimentally accessible. It addresses the question of the relation between M_W and M_P. On the other hand, it leaves many others of our questions unanswered.

1.3.3 Supergravity (sugra)

See for instance [5].

It is natural to consider theories where supersymmetry is realized as a local gauge symmetry. Given the susy algebra (10), this means that the four-momentum operator P_μ, which generates global translations, is also promoted to a gauge generator. Local translations are equivalent to coordinate reparametrization (or diffeomorphism) invariance

$$x^\mu \to x^\mu + \xi(x) \tag{11}$$

so the resulting theories are generalizations of general relativity, and hence contain gravity. They are called supergravities.

A very important 4d $N = 1$ supermultiplet is the gravity multiplet, which contains a spin-2 graviton $G_{\mu\nu}$ and its spin-3/2 superpartner (gravitino) ψ_{α}^{μ} (also called Rarita-Schwinger field) . Other multiplets are like in global susy, the chiral and vector multiplets. The sugra lagrangian is basically obtained from the global susy one by adding the Einstein term for the graviton, a kinetic term for the gravitino, and coupling the graviton to the susy theory stress-enery tensor,and coupling the gravitino to the susy theory supercurrent (current associated to the supersymmetry).

In applications to phenomenology, a nice feature of supergravity is that spontaneous breaking of local supersymmetry becomes, in the limit of energies much below M_P, explicit breaking of global supersymmetry by soft terms. A popular scenario is to construct models with a MSSM sector (visible sector), a second sector (hidden sector) decoupled from the MSSM (except by gravitational interactions) and which breaks local supersymmetry at a scale of $M_{hidden} = 10^{12}$ GeV. Transmission of supersymmetry breaking to the visible sector is manifest at a lower scale M_{hidden}/M_P of around 1 TeV, i.e. the right superpartner mass scale.

Supergravity is a nice and inspiring idea, which attempts to incorporate gravity. However, it does not make gravity consistent at the quantum level, supergravity is neither finite nor renormalizable, so it does not provide an ultraviolet completion of Einstein theory.

1.3.4 Extra dimensions

There are many scenarios which propose that spacetime has more than four dimensions, the addibional ones being unobservable because they are compact and of very small size. We briefly mention two ideas, which differ by whether the usual Standard Model matter is able to propagate in the new dimensions or not. Again, mixed scenarios are often very popular and interesting.

- **Kaluza-Klein idea**

Kaluza-Klein theories propose the appearance of four-dimensional gauge bosons as components of the metric tensor in a higher-dimensional spacetime. The prototypical example is provided by considering a 5d spacetime with topology $M_4 \times S^1$ and endowed with a 5d metric G_{MN}, $M, N = 1, \ldots, 5$. From the viewpoint of the low-energy four-dimensional theory (at energies much lower than the compactification scale $M_c = 1/R$, with R the circle radius) he 5d metric decomposes as

$$\begin{array}{llll} G_{MN} \to \;\; G_{\mu\nu} & \mu,\nu = 0,\ldots,3 & G_{\mu\nu} & \text{4d graviton} \\ & \;\; G_{\mu 4} & A_\mu & \text{4d gauge boson} \\ & \;\; G_{44} & \phi & \text{4d scalar (modulus)} \end{array} \tag{12}$$

We obtain a 4d metric tensor, a 4d massless vector boson and a 4d massless scalar. Moreover, diffeomorphism invariance in the fifth dimension implies gauge invariance of the interactions of the 4d vector boson (so it is a $U(1)$ gauge boson).

The idea generalizes to d extra dimensions. Take (4+d)-dimensional spacetime of the form $M_4 \times X_d$. The metric in $(4+d)$ dimensions gives rise to a 4d metric and to gauge bosons associated to a gauge group which is the isometry group of X_d. Specifically, let k_a^M be a set of Killing vectors in X_d; the 4d gauge bosons are obtained as $A_\mu^a = G_{\mu N} k_a^N$.

The Kaluza-Klein idea is beautiful, but it is difficult to use for phenomenology. It is not easy to construct manifolds with isometry group that of the Standard Model. Moreover, a generic difficulty first pointed out by Witten (see [6]) is how to obtain chiral 4d fermions in this setup. For this

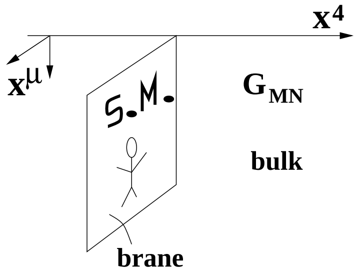

Figure 3: Schematic picture of the brane-world idea.

to be possible one needs to include elementary gauge fields already in the higher-dimensional theory, so much of the beauty of the idea is lost.

On top of that, although the idea involves gravity, it still suffers from quantum inconsistencies, so it does not provide an ultraviolet completion of Einstein theory, consistent at the quantum level.

- **Brane-world idea**

This is a recent proposal (see e.g. [7]), building on the idea of extra dimensions, but with an interesting new ingredient. It is based on the observation that it is conceivable that extra dimensions exist, but that the Standard Model fields do not propagate on them, and that only gravity does. In modern jargon, the Standard Model is said to live on a 'brane' (generalization of a membrane embedded in a higher dimensional spacetime), while gravity propagates in the 'bulk' of spacetime.

In such a scenario, Standard Model physics is four-dimensional up to energies around the TeV, even if the extra dimensions have sizes larger than $(\mathrm{TeV})^{-1}$. The best experiments able to probe the extra dimensions are measurements of deviations from four-dimensional Newton's law in Cavendish experiments, to put a bound at the length scale at which gravity starts being five- or higher-dimensional. The present bound implies that extra dimensions should be smaller than 0.1 mm. This energy scale is surprisingly small, still we do not detect these extra dimensions.

This scenario allows for an alternative interpretation of the four-dimensional Planck scale. Starting with a fundamental Planck scale M_d in the $(4+d)$ dimensional theory, the 4d Planck scale is

$$M_P^2 \,=\, (M_d)^{d+2} V_{X_d} \tag{13}$$

where V_{X_d} is the volume of the internal manifold. The scenario allows for a low value of the fundamental $(4+d)$ Planck scale, keeping a large 4d M_P by taking a large volume compactification. In usual Kaluza-Klein, such large volumes would imply light Kaluza-Klein excitation of Standard Model fields, in conflict with experiment. In the brane-world scenario, such fields do not propagate in the bulk so they do not have Kaluza-Klein replicas. In certain models, it is possible to set $M_{4+d} \simeq$ TeV, obtaining $M_P \simeq 10^{19}$ GeV as a derived quantity, due to a choice of large volume for the internal manifold. Is is therefore a possible alternative explanation for the hierarchy between M_W and M_P.

Again, it is fair to emphasize that this setup does not provide a ultraviolet completion of Einstain gravity, gravity is treated classically. Moreover, it is not clear to start with that a quantum field theory on a slice of full spacetime can be consistently defined at the quantum level.

1.4 String theory as a theory beyond the Standard Model

String theory is also a proposal for physics beyond the Standard Model. It differs from the above in that it addresses precisely the toughest of all issues: it provides a quantum mechanically well-defined theory underlying gauge and gravitational interactions. Hence it provides an ultraviolet completion of Einstein theory, which is finite order by order in perturbation theory. Einstein theory is recovered as a low-energy effective theory for energies below a typical scale, the string scale M_s. That is the beautiful feature of string theory.

Moreover, string theory incorporates gauge interactions, and is able to lead to four-dimensional theories with chiral fermions. In addition, string theory incorporates many of the ingredients of the previous proposals beyond the standard model, now embedded in a consistent and well-defined framework, and leading to physical theories very similar to the Standard Model at energies below a typical scale of the theory (the string scale M_s).

Finally, string theory contains physical phenomena which are new and quite different from expectations from other proposals beyond the standard model. As a theory of quantum gravity, it has the potential to give us some insight into questions like the nature of spacetime, the black hole information paradox. As a theory underlying gauge interactions, it has the potential to explain what is the origin of the number of families in theories like the Standard Model, how do chiral fermions arise, etc...

String theory is an extremely rich structure, from the mathematical, theoretical and phenomenological viewpoints. It is certainly worth being studied in a graduate course in high energy physics!

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