understanding the anatomy of the $\Delta I = 1/2$ rule

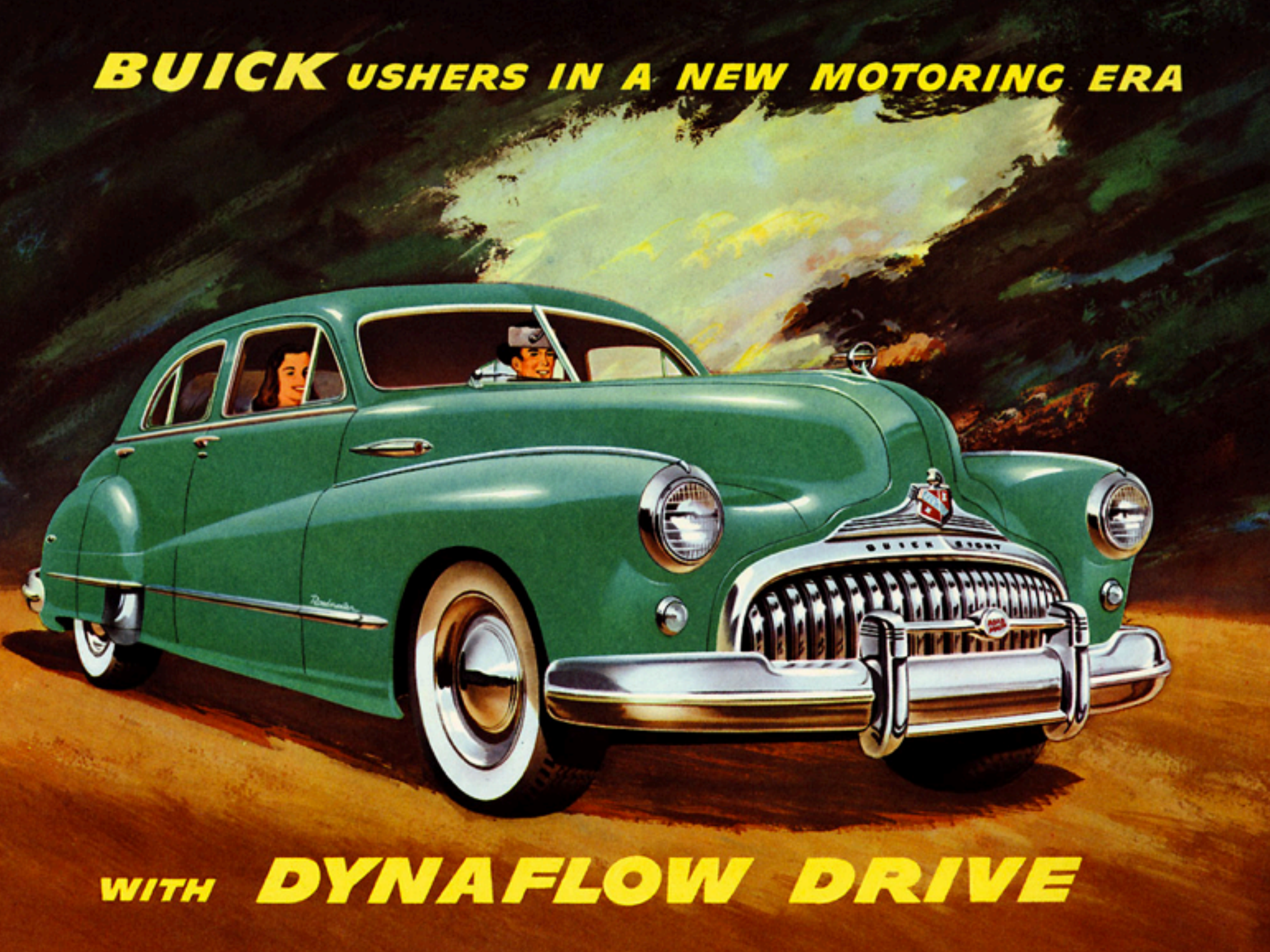
Carlos Pena

in collaboration with:
E Endress
L Giusti
P Hernández
M Laine
J Wennekers
H Wittig

EXCELENCIA SEVERO OCHOA

[PRL 98 (2007) 082003]
[JHEP 0805 (2008) 043]
[PRD 90 (2014) 9 094504]
[arXiv:1402.0831]

in memoriam Jan Wennekers 1978-2009


BUICK USHERS IN A NEW MOTORING ERA
WITH DYNAFLOW DRIVE

neutral kaon decay

VOLUME 13, NUMBER 4 PHYSICAL REVIEW LETTERS 27 JULY 1964

EVIDENCE FOR THE 2π DECAY OF THE K_2^0 MESON*†

J. H. Christenson, J. W. Cronin,‡ V. L. Fitch,‡ and R. Turlay§
Princeton University, Princeton, New Jersey
(Received 10 July 1964)

This Letter reports the results of experimental studies designed to search for the 2π decay of the K_2^0 meson. Several previous experiments have served[1,2] to set an upper limit of 1/300 for the fraction of K_2^0's which decay into two charged pions. The present experiment, using spark chamber techniques, proposed to extend this limit.

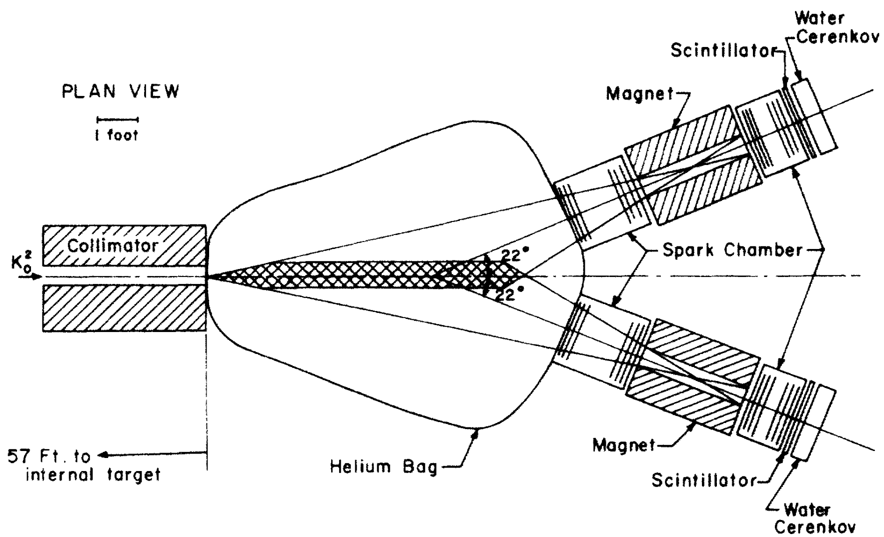
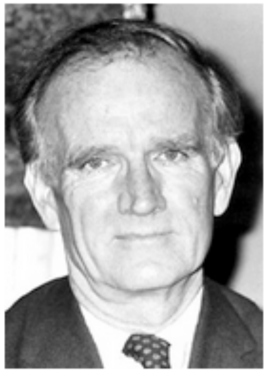


FIG. 1. Plan view of the detector arrangement.

The Nobel Prize in Physics 1980

James Watson Cronin

Prize share: 1/2



Val Logsdon Fitch

Prize share: 1/2

The Nobel Prize in Physics 1980 was awarded jointly to James Watson Cronin and Val Logsdon Fitch *"for the discovery of violations of fundamental symmetry principles in the decay of neutral K-mesons"*

neutral kaon decay

K^0_S

$I(J^P) = \frac{1}{2}(0^-)$

Mean life $\tau = (0.8954 \pm 0.0004) \times 10^{-10}$ s (S = 1.1) Assuming *CPT*

Mean life $\tau = (0.89564 \pm 0.00033) \times 10^{-10}$ s Not assuming *CPT*

K^0_S DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Hadronic modes			
$\pi^0\pi^0$	(30.69 ± 0.05) %		209
$\pi^+\pi^-$	(69.20 ± 0.05) %		206

K^0_L

$I(J^P) = \frac{1}{2}(0^-)$

$m_{K_L} - m_{K_S}$

$= (0.5293 \pm 0.0009) \times 10^{10}\ \hbar\ s^{-1}$ (S = 1.3) Assuming *CPT*

$= (3.484 \pm 0.006) \times 10^{-12}$ MeV Assuming *CPT*

$= (0.5289 \pm 0.0010) \times 10^{10}\ \hbar\ s^{-1}$ Not assuming *CPT*

Mean life $\tau = (5.116 \pm 0.021) \times 10^{-8}$ s (S = 1.1)

K^0_L DECAY MODES			Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Semileptonic modes					
$\pi^\pm e^\mp \nu_e$		[o]	(40.55 ± 0.11) %	S=1.7	229
$\pi^\pm \mu^\mp \nu_\mu$		[o]	(27.04 ± 0.07) %	S=1.1	216
Hadronic modes, including Charge conjugation×Parity Violating (*CPV*) modes					
$3\pi^0$			(19.52 ± 0.12) %	S=1.6	139
$\pi^+\pi^-\pi^0$			(12.54 ± 0.05) %		133
$\pi^+\pi^-$	*CPV*	[q]	$(1.967 \pm 0.010) \times 10^{-3}$	S=1.5	206
$\pi^0\pi^0$	*CPV*		$(8.64 \pm 0.06) \times 10^{-4}$	S=1.8	209

neutral kaon decay

Hamiltonian for K^0–$\bar{K}^0$ system determined by hermiticity + CPT

$$H = M - \tfrac{i}{2}\Gamma = \begin{pmatrix} A & p^2 \\ q^2 & A \end{pmatrix}$$

eigenstates of Hamiltonian are $|K_{1,2}\rangle = \tfrac{1}{\sqrt{2}}(|K^0\rangle \pm |\bar{K}^0\rangle)$ if CP conserved ($p = q = 0$). CP violation in SM leads to mixing:

$$|K_\mathrm{S}\rangle = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(|K_1\rangle + \bar{\varepsilon}|K_2\rangle)\,, \quad |K_\mathrm{L}\rangle = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(|K_2\rangle + \bar{\varepsilon}|K_1\rangle)\,, \quad \bar{\varepsilon} = \frac{p-q}{p+q}$$

CP-violation parameters accessible via decay amplitudes into two pions

$$-iT[K^0 \to (\pi\pi)_I] = A_i e^{i\delta_I} \qquad T[(\pi\pi)_I \to (\pi\pi)_I]_{l=0} = 2e^{i\delta_I}\sin\delta_I$$

$$\varepsilon = \frac{T[K_\mathrm{L} \to (\pi\pi)_0]}{T[K_\mathrm{S} \to (\pi\pi)_0]} \simeq \bar{\varepsilon} + i\,\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}$$

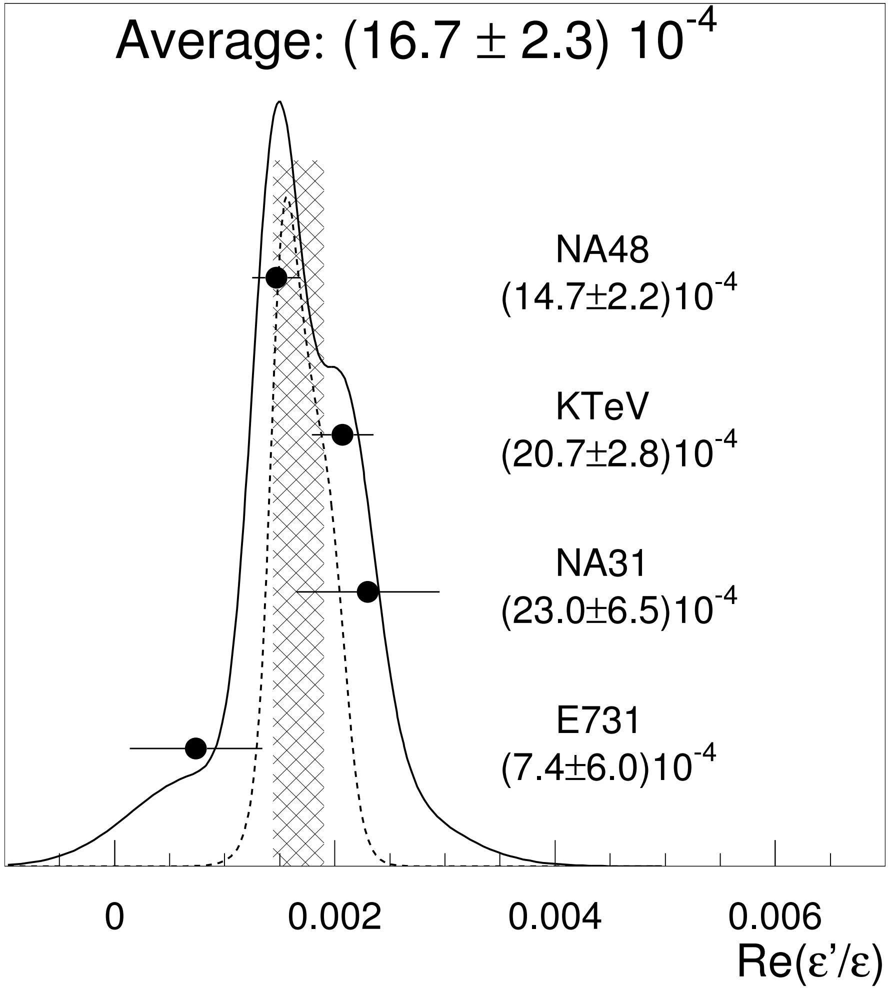
$$\varepsilon' = \frac{\varepsilon}{\sqrt{2}}\left(\frac{T[K_\mathrm{L} \to (\pi\pi)_2]}{T[K_\mathrm{L} \to (\pi\pi)_0]} - \frac{T[K_\mathrm{S} \to (\pi\pi)_2]}{T[K_\mathrm{S} \to (\pi\pi)_0]}\right) \simeq \frac{1}{\sqrt{2}}\,e^{i(\delta_2-\delta_0+\pi/2)}\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

experiment:

$$|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}$$

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (1.65 \pm 0.26) \times 10^{-3}$$

$$\left|\frac{A_0}{A_2}\right| = 22.35$$



neutral kaon decay

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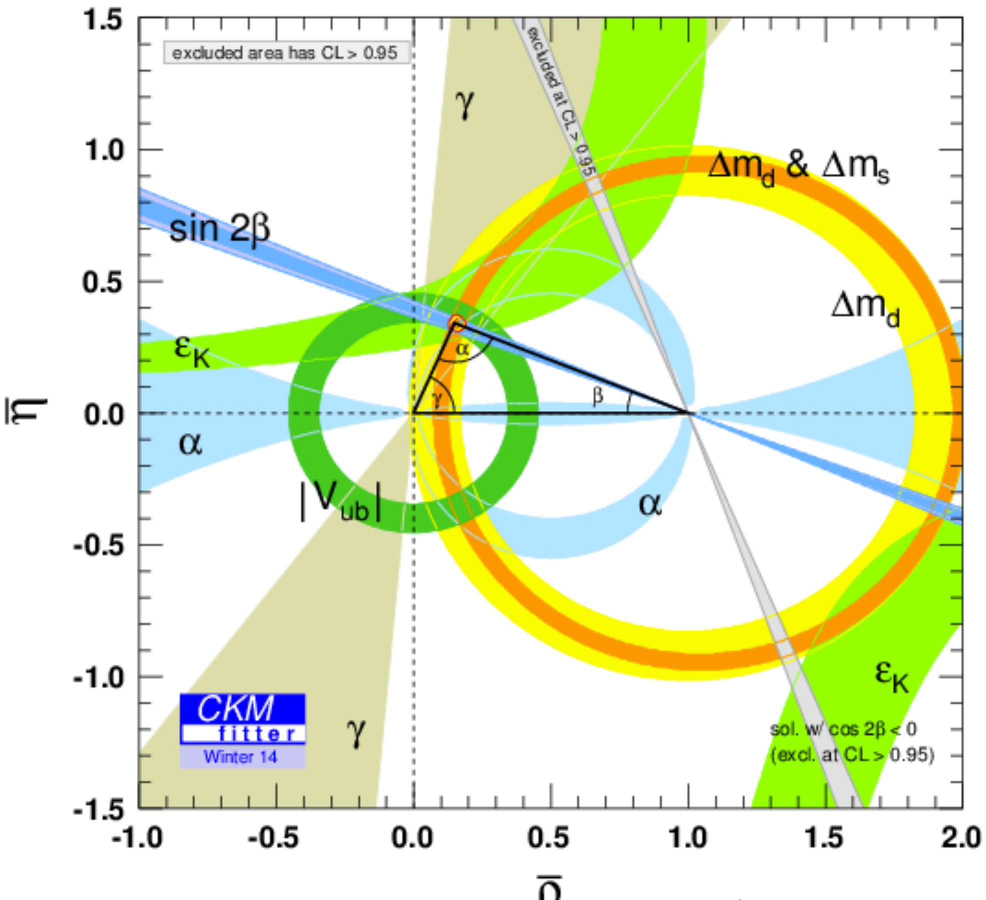

[CKMFitter 2014]

neutral kaon decay

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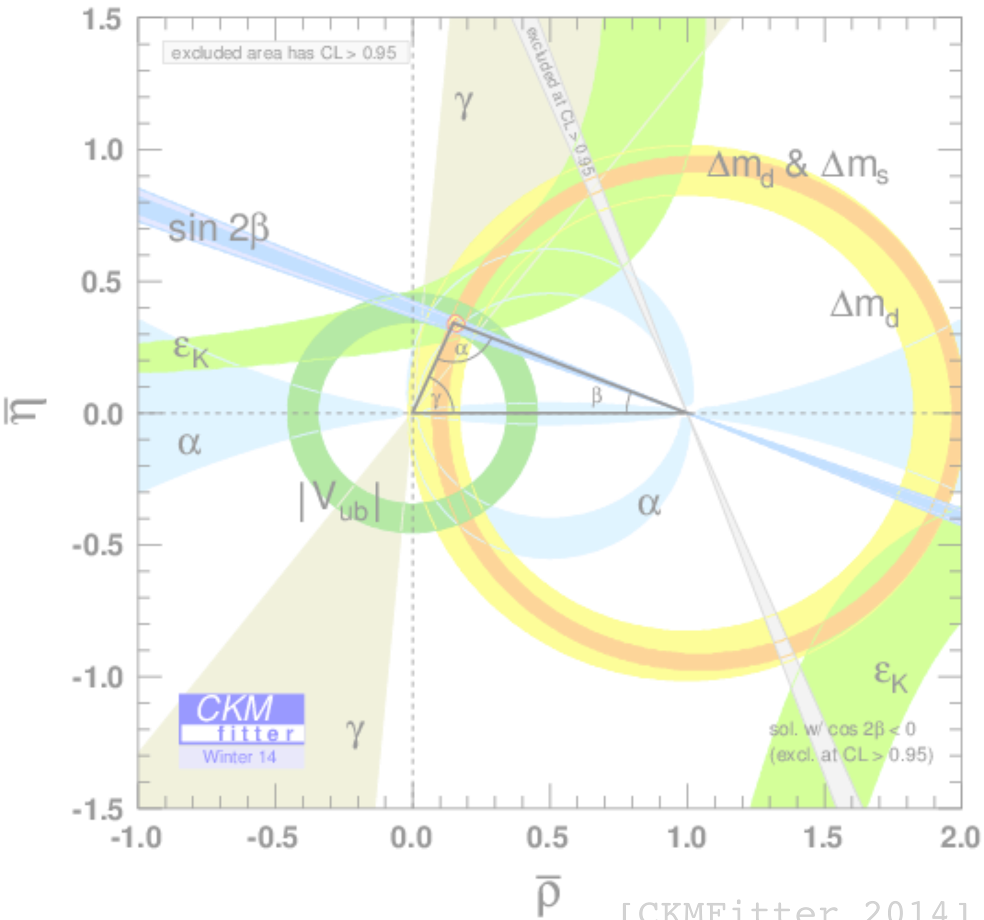

[CKMFitter 2014]

(similar observations in baryon sector e.g. $\Lambda\,/\,\Sigma \to N\pi$, heavy meson decay, ...)

[fully?] satisfactory understanding of result within SM framework lacking for 40 years

outline

- computing kaon decay amplitudes
  - EW effective Hamiltonian analysis
  - why is it so difficult?
  - status
- understanding the anatomy: strategy
  - disentangling scales
  - low-energy effective description and the role of chiral symmetry
  - can’s and cannot’s
- some results
  - long-distance effects in GIM limit
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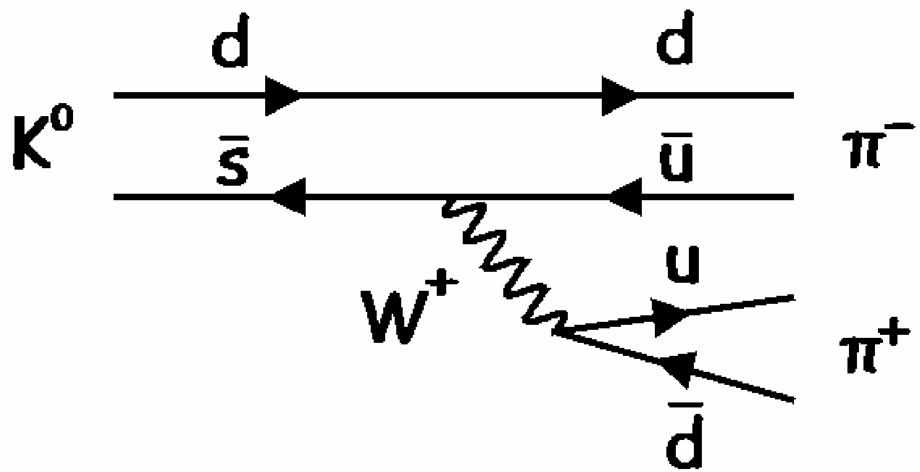
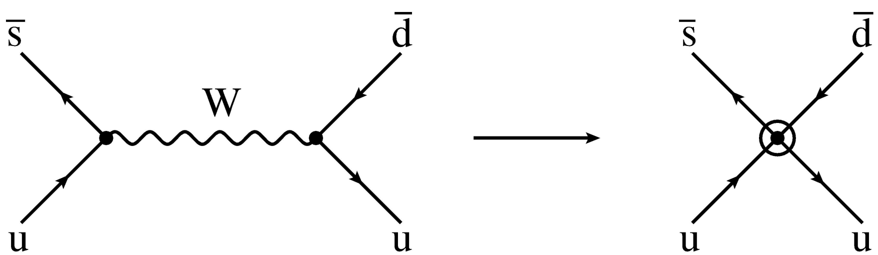
effective weak Hamiltonian





$$T[K \to \pi\pi] \approx \langle \pi\pi | \mathcal{H}_{\mathrm{w}}^{\mathrm{eff}} | K \rangle + \mathcal{O}\left(\frac{p^2}{M_W^2}\right)$$

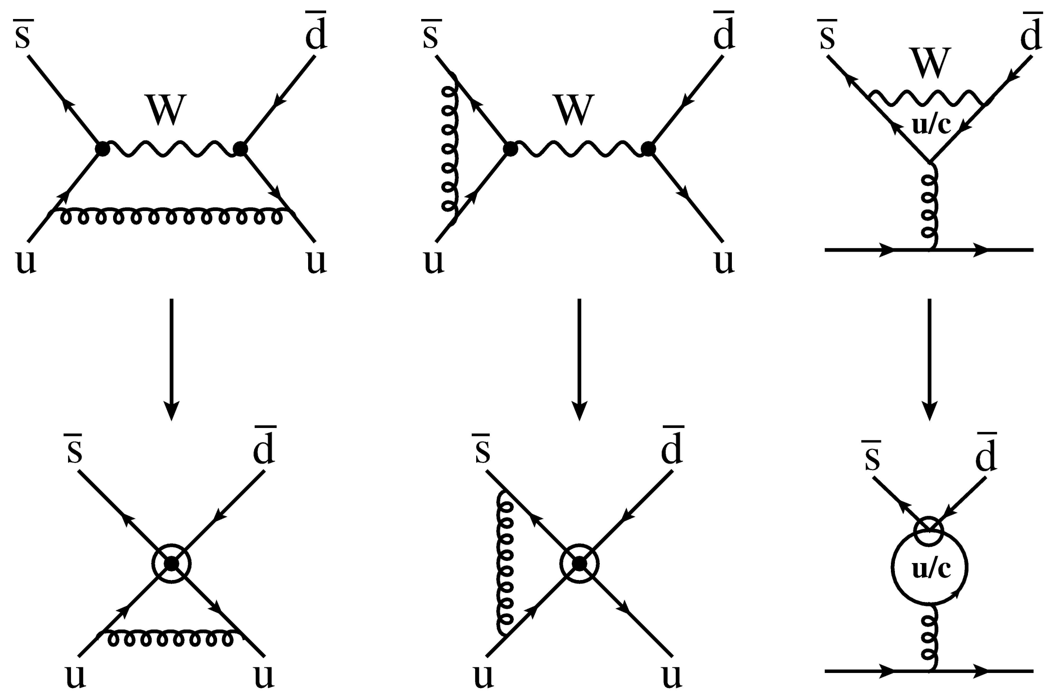
$$\mathcal{H}_{\mathrm{w}}^{\mathrm{eff}} = \frac{G_{\mathrm{F}}}{\sqrt{2}} \sum_k f_k(V_{\mathrm{CKM}})\, C_k(\mu/M_W)\, \bar{O}_k(\mu)$$

Wilson coefficients
(short-distance physics)

four-quark operators
(long-distance physics)

effective weak Hamiltonian

CP-violation effects neglected ($\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*} \sim 10^{-3}$), keep active charm quark:



$$\mathcal{H}_{\rm w}^{\rm eff} = \frac{g_{\rm w}^2}{2M_W^2} V_{us}^* V_{ud} \sum_{\sigma=\pm} \{k_1^\sigma Q_1^\sigma + k_2^\sigma Q_2^\sigma\}$$

$$Q_1^\pm = (\bar{s}_{\rm L}\gamma_\mu u_{\rm L})(\bar{u}_{\rm L}\gamma_\mu d_{\rm L}) \pm (\bar{s}_{\rm L}\gamma_\mu d_{\rm L})(\bar{u}_{\rm L}\gamma_\mu u_{\rm L}) - [u \leftrightarrow c]$$

$$Q_2^\pm = (m_u^2 - m_c^2)\{m_d(\bar{s}_{\rm L} d_{\rm R}) + m_s(\bar{s}_{\rm R} d_{\rm L})\}$$

effective weak Hamiltonian

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$$Q_2^\pm = (m_u^2 - m_c^2)\{m_d(\bar{s}_{\rm L} d_{\rm R}) + m_s(\bar{s}_{\rm R} d_{\rm L})\}$$

(do not contribute to physical $K \to \pi\pi$ transitions)

$$\left|\frac{A_0}{A_2}\right| = \frac{k_1^-(M_W)}{k_1^+(M_W)} \frac{\langle(\pi\pi)_{I=0}|Q_1^-|K\rangle}{\langle(\pi\pi)_{I=2}|Q_1^+|K\rangle} \qquad \frac{k_1^-(M_W)}{k_1^+(M_W)} \simeq 2.8$$

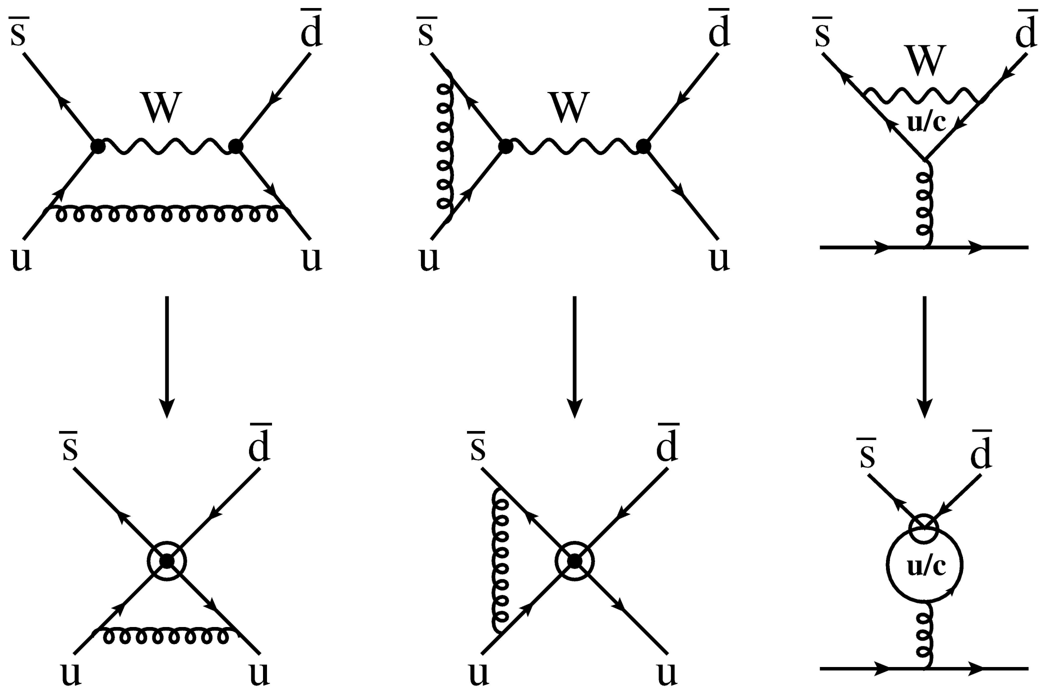
- bulk of effect should come from long-distance QCD contribution

[Gaillard, Lee; Altarelli, Maiani 1974]

- reliable non-perturbative determination mandatory

[Cabibbo, Martinelli, Petronzio; Brower, Maturana, Gavela, Gupta 1984]

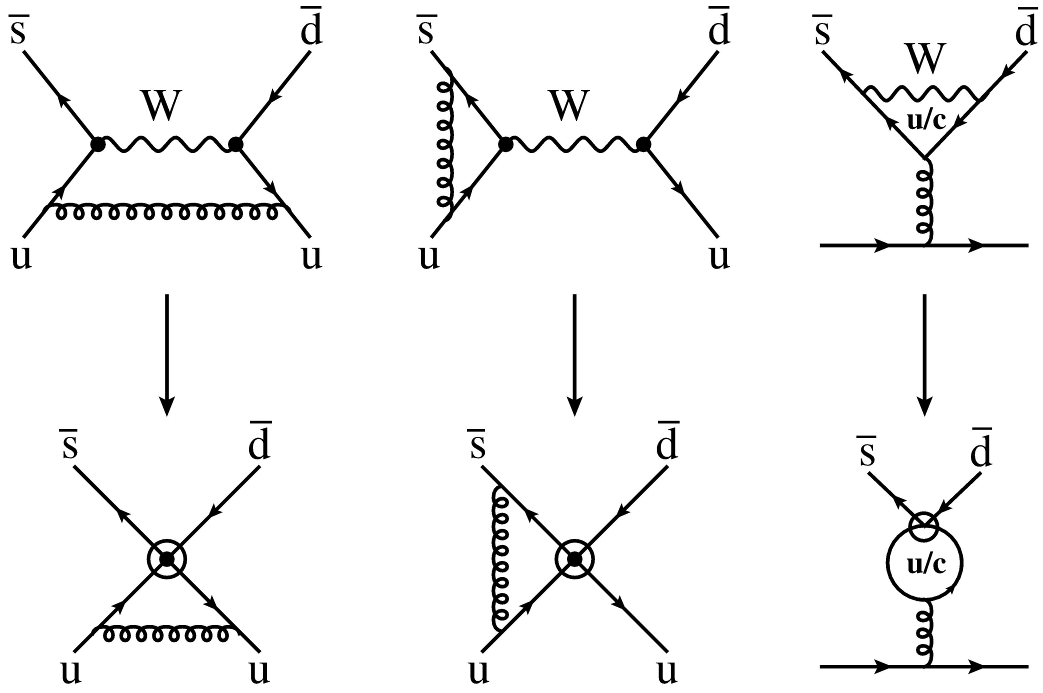
effective weak Hamiltonian: integrating out the charm



$$\mathcal{H}_{\mathrm{w}}^{\mathrm{eff}} = \frac{g_{\mathrm{w}}^2}{2M_W^2} V_{ud}V_{us}^* \sum_{i=1}^{10} \left[z_i + \tau y_i\right] Q_i \qquad \tau = -\frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*}$$

- several four-fermion operators contribute (2 current-current, 4 QCD penguins, 4 EW penguins)
- missing GIM mechanism leads to quadratic divergences in penguin operators (cf. log divergences if charm active)
- suggests enhancement mechanism due to peculiar role of charm scale

effective weak Hamiltonian: integrating out the charm



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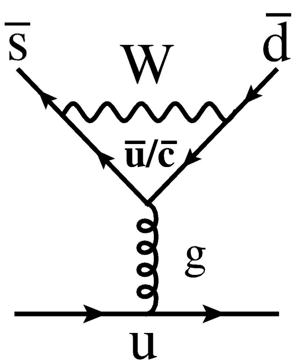


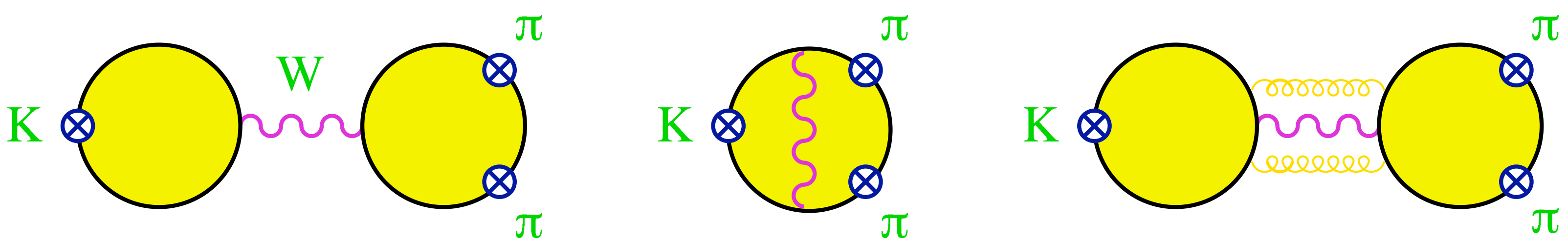
$$\sim \frac{m_c^2 - m_u^2}{\mu^2}\,, \quad m_c \ll \mu \ll M_W$$

$$\sim \ln \frac{m_c^2}{\mu^2}\,, \quad \mu \lesssim m_c$$

[Shifman, Vainshtein, Zakharov 1975-77]

large *N*?

$$\mathcal{H}_{\rm w}^{\rm eff} \sim G_{\rm F} J_{\rm w}^{\mu} J_{\rm w}^{\mu}$$



$$O(N_C^2) \qquad O(N_C) \qquad O(1)$$

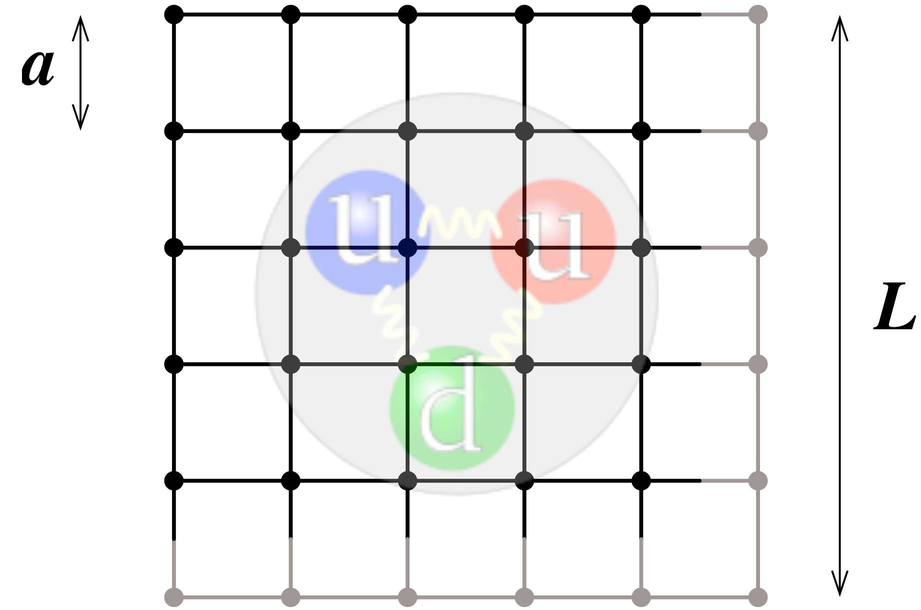
$$T[K^0 \to \pi^0\pi^0] \sim 0 \quad \Rightarrow \quad \left|\frac{A_0}{A_2}\right|_{N\to\infty} \sim \sqrt{2}$$

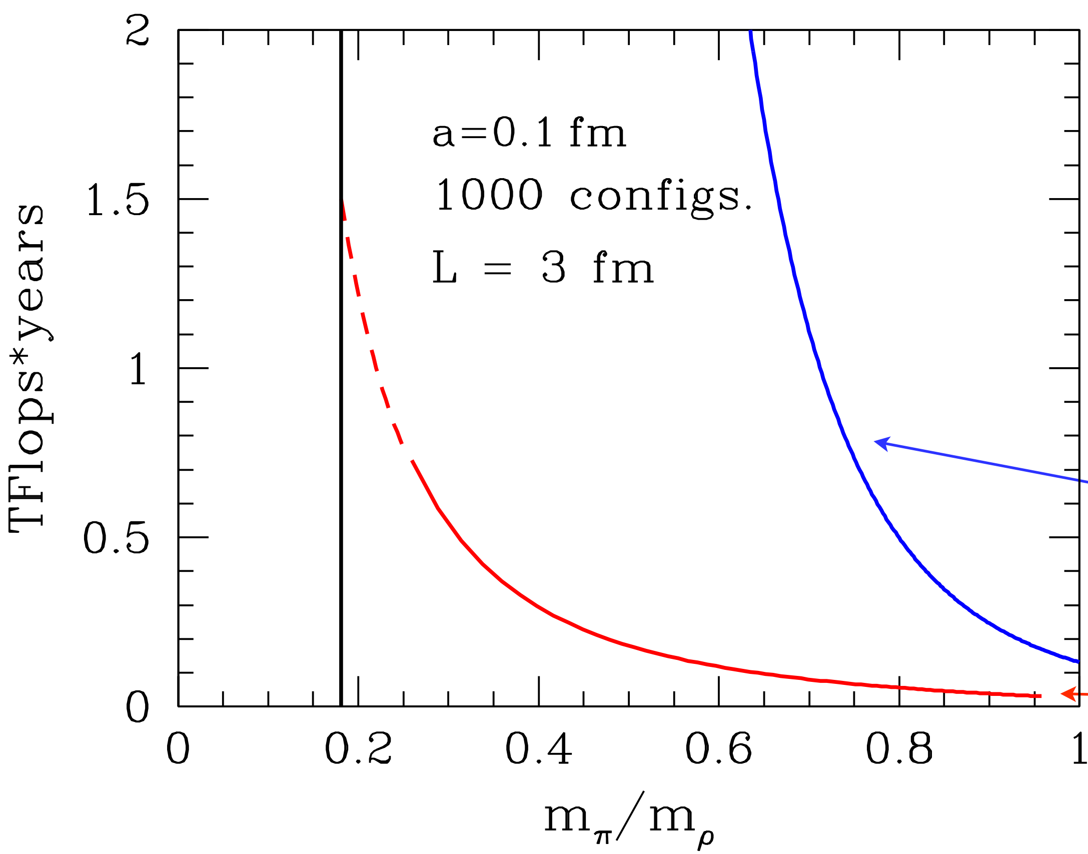
[Fukugita et al. 1977; Chivukula, Flynn, Georgi 1986]

since then, much work to refine the analysis by incorporating contributions from resonances / chiral theory effects, ...

[see review in Buras, Gérard, Bardeen EPJ C74 (2014) 5]

lattice QCD?





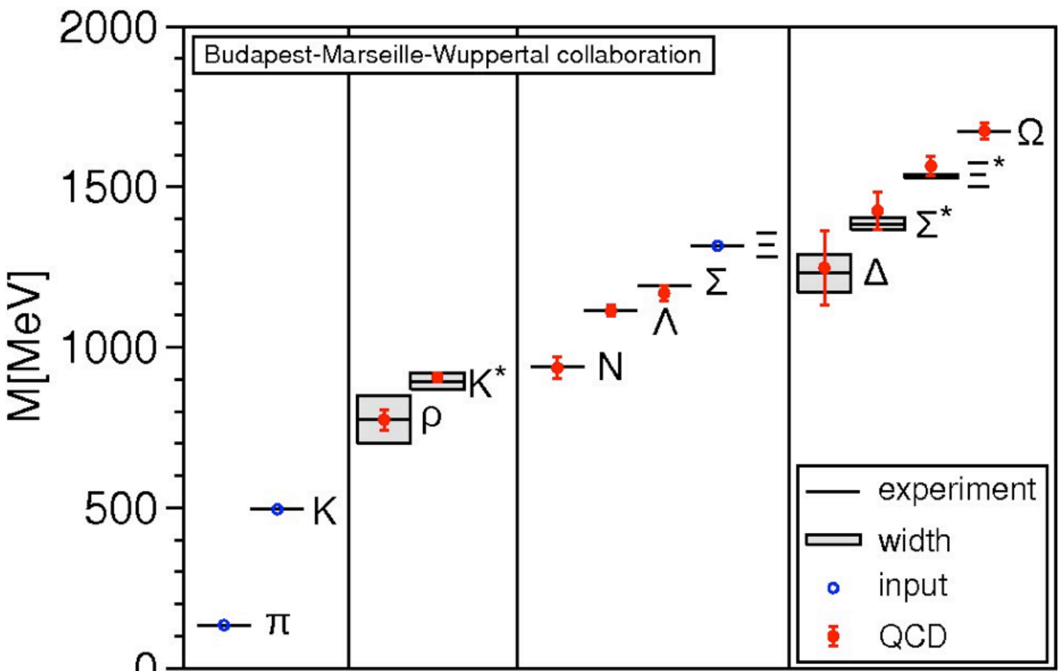
[Ukawa 2001]

$$5\left[\frac{20\ \mathrm{MeV}}{m}\right]^{3}\left[\frac{L}{3\ \mathrm{fm}}\right]^{5}\left[\frac{0.1\ \mathrm{fm}}{a}\right]^{7}$$

$$0.05\left[\frac{20\ \mathrm{MeV}}{m}\right]^{1}\left[\frac{L}{3\ \mathrm{fm}}\right]^{5}\left[\frac{0.1\ \mathrm{fm}}{a}\right]^{6}$$

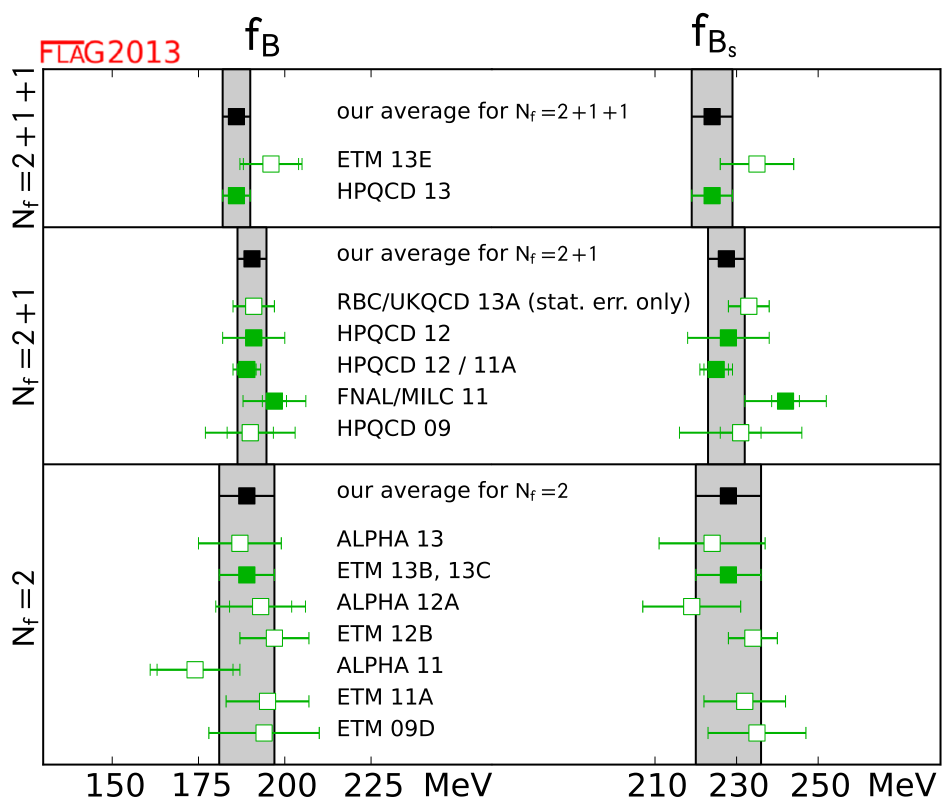
[Giusti 2006]

lattice QCD?



[BMW Collaboration 2008]

[Flavour Lattice Averaging Group 2013]



no-go theorems

Maiani-Testa: physical decay amplitudes with more than one hadron in final state cannot be extracted from Euclidean correlation functions [in infinite volume]

[Maiani, Testa 1990]

Lellouch-Lüscher: avoid by working at large finite volume to disentangle pion rescattering effects (requires volumes being reached only now)

[Lellouch, Lüscher 1998; Lin, Martinelli, Sachrajda, Testa 2001]

chiPT: use effective low-energy description of weak Hamiltonian to relate physical $K \to \pi\pi$ amplitudes to computable quantities

[Bernard et al. 1985]

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[Bernard et al. 1985]

Nielsen-Ninomiya: no ultralocal lattice regularisation preserves full chiral symmetry

[Nielsen, Ninomiya 1982]

absence of chiral symmetry leads to complicate operator mixing and severe power divergences

[Bochicchio et al. 1985; Maiani et al. 1987]

use regularisations with exact chiral symmetry (not ultralocal), or better chiral properties

[Capitani, Giusti 2001; CP, Sint, Vladikas 2004; Frezzotti, Rossi 2004]

renormalisation and chiral symmetry

active charm:

$$Q_1^{\pm} = (\bar{s}_{\rm L}\gamma_\mu u_{\rm L})(\bar{u}_{\rm L}\gamma_\mu d_{\rm L}) \pm (\bar{s}_{\rm L}\gamma_\mu d_{\rm L})(\bar{u}_{\rm L}\gamma_\mu u_{\rm L}) - [u \leftrightarrow c]$$

mixes with $(m_u^2 - m_c^2)\,\{m_d(\bar{s}_{\rm L}d_{\rm R}) + m_s(\bar{s}_{\rm R}d_{\rm L})\}$

resp. $(m_u - m_c)\bar{s}d\,,\ (m_u - m_c)(m_s - m_d)\bar{s}\gamma_5 d\,,\ Q_1^{\pm;(k)}[\Gamma\otimes\Gamma']$

charm integrated out:

Q_i always mixes with lower-dimensional operators via power divergences (no GIM factor); more severe mixing/divergences if chiral symmetry is broken

direct computations

state of the art: computation by RBC/UKQCD collaboration

[Blum et al. 2011]

- two-pion final state
- (almost) exactly chiral fermion regularisation (DW)
- effective Hamiltonian with charm integrated out

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 9.1(2.1) \text{ for } m_K = 878 \text{ MeV } m_\pi = 422 \text{ MeV}$$

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 12.0(1.7) \text{ for } m_K = 662 \text{ MeV } m_\pi = 329 \text{ MeV}$$

[plenary talk by N Garron at Lattice 2014]

improvement in the way (smaller pion masses / larger volumes)

direct computations

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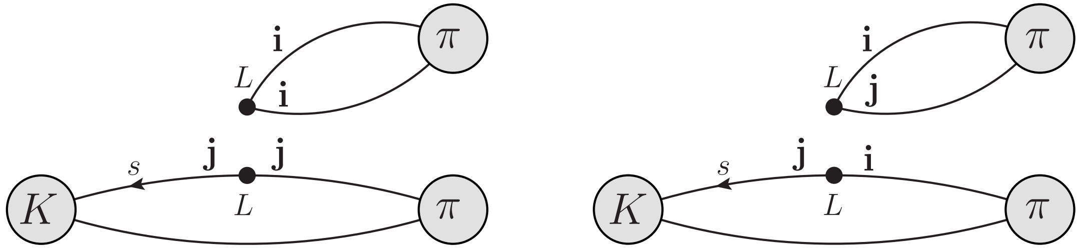
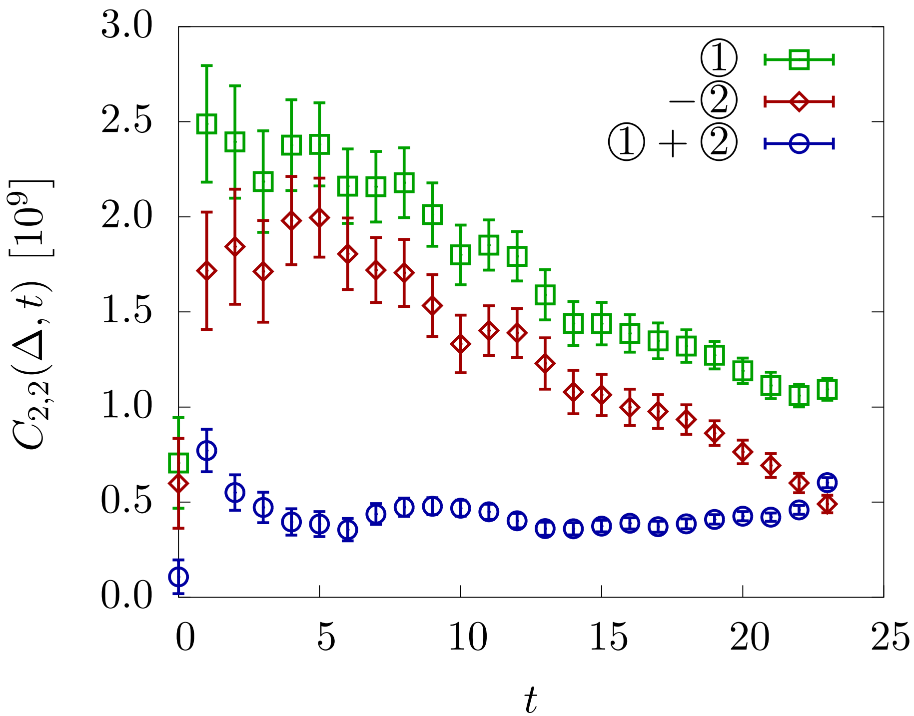
"emergent understanding"



Contraction ①   Contraction ②

- Naive factorisation approach: ② $\sim 1/3$①
- Our computation: ② $\sim -0.7$①

[Boyle et al. 2013]



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a strategy to understand the role of the charm quark

[Giusti, Hernández, Laine, Weisz, Wittig 2004]

several possible sources for $\Delta I = 1/2$ enhancement:

- physics at charm scale (penguins)
- physics at "intrinsic" QCD scale $\sim \Lambda_{\rm QCD}$
- final state interactions
- all of the above (no dominating "mechanism")

separate low-energy QCD and charm-scale physics: consider amplitudes as a function of charm mass for fixed u,d,s masses

$$m_c = m_u = m_d = m_s \quad \longrightarrow \quad m_c \gg m_u = m_d \leq m_s$$

- active charm
- use chiral fermions (good renormalisation, access to all kinematical regimes)
- give up direct computation (chiral fermions too expensive) ⇒ no control of FSI

effective low-energy description

dynamics of Goldstone bosons at LO given by chiral Lagrangian

$$\mathcal{L} = \tfrac{1}{4}F^2 \mathrm{Tr}\left[\partial_\mu U \partial_\mu U^\dagger\right] - \tfrac{1}{2}\Sigma \mathrm{Tr}\left[U M^\dagger e^{i\theta/N_\mathrm{f}} + \mathrm{h.c.}\right]$$

weak interactions accounted for by low-energy counterpart of effective Hamiltonian

light charm:

$$\mathcal{H}_\mathrm{w}^{(4)} = \frac{g_\mathrm{w}^2}{4M_W^2} V_{us}^* V_{ud} \sum_{\sigma=\pm} \{g_1^\sigma \mathcal{Q}_1^\sigma + g_2^\sigma \mathcal{Q}_2^\sigma\}$$

$$\mathcal{Q}_1^\pm = \mathcal{J}_\mu^{su}\mathcal{J}_\mu^{ud} \pm \mathcal{J}_\mu^{sd}\mathcal{J}_\mu^{uu} - [u \leftrightarrow c] \qquad \mathcal{J}_\mu = \frac{F^2}{\sqrt{2}} U \partial_\mu U^\dagger$$

heavy charm:

$$\mathcal{H}_\mathrm{w}^{(3)} = \frac{g_\mathrm{w}^2}{4M_W^2} V_{us}^* V_{ud} \{g_{27}\mathcal{Q}_{27} + g_8 \mathcal{Q}_8 + g_8' \mathcal{Q}_8'\}$$

$$\mathcal{Q}_{27} = \frac{2}{5}\mathcal{J}_\mu^{su}\mathcal{J}_\mu^{ud} + \frac{3}{5}\mathcal{J}_\mu^{sd}\mathcal{J}_\mu^{uu},$$

$$\mathcal{Q}_8 = \frac{1}{2}\sum_{q=u,d,s}\mathcal{J}_\mu^{sq}\mathcal{J}_\mu^{qd},$$

$$\mathcal{Q}_8' = m_l \Sigma F^2 \left[U e^{i\theta/N_\mathrm{f}} + U^\dagger e^{-i\theta/N_f}\right]^{sd},$$

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$$g_{27}(0) = g_1^+ \,, \qquad\qquad g_8(0) = g_1^- \; + \; \frac{1}{5}\, g_1^+$$

heavy charm:

$$\mathcal{H}_\mathrm{w}^{(3)} = \frac{g_\mathrm{w}^2}{4M_W^2} V_{us}^* V_{ud} \left\{ g_{27}\mathcal{Q}_{27} \; + \; g_8 \mathcal{Q}_8 \; + \; g_8' \mathcal{Q}_8' \right\}$$

$$|g_{27}^\mathrm{exp}(\overline{m}_c)| \sim 0.50 \,, \qquad\qquad |g_8^\mathrm{exp}(\overline{m}_c)| \sim 10.5$$

determination of low-energy constants: light charm

match suitable correlation functions in QCD and ChPT (infinite volume: $K \to \pi$ amplitudes)

QCD

$$R_i^{\pm}(x_0, y_0) = \frac{C_i^{\pm}(x_0, y_0)}{C(x_0)C(y_0)}$$

$$C_i^{\pm}(x_0, y_0) = \int \mathrm{d}^3x \int \mathrm{d}^3y \, \langle J_0^{du}(x) \, Q_i^{\pm}(0) \, J_0^{us}(y) \rangle$$

$$C(x_0) = \int \mathrm{d}^3x \, \langle J_0^{\alpha\beta}(x) \, J_0^{\beta\alpha}(0) \rangle \,,$$

SU(4) ChPT

$$\mathcal{R}_i^{\pm}(x_0, y_0) = \frac{\mathcal{C}_i^{\pm}(x_0, y_0)}{\mathcal{C}(x_0)\mathcal{C}(y_0)}$$

$$\mathcal{C}(x_0) = \int \mathrm{d}^3x \, \langle \mathcal{J}_0^{ud}(x) \, \mathcal{J}_0^{du}(0) \rangle_{\mathrm{SU}(4)} \,,$$

$$\mathcal{C}_i^{\pm}(x_0, y_0) = \int \mathrm{d}^3x \int \mathrm{d}^3y \, \langle \mathcal{J}_0^{du}(x) \, \mathcal{Q}_i^{\pm}(0) \, \mathcal{J}_0^{us}(y) \rangle_{\mathrm{SU}(4)}$$

$$\mathcal{Z}_1^{\pm} R_1^{\pm}(x_0, y_0) = g_1^{\pm} \mathcal{R}_1^{\pm}(x_0, y_0)$$

determination of low-energy constants: heavy charm

match suitable correlation functions in QCD and ChPT (infinite volume: $K \to \pi$ amplitudes)

QCD

$$R_{27} = \mathcal{Z}_1^+ R_u^+ \,,$$
$$R_8 = \mathcal{Z}_1^+ \left[R_1^+ - R_u^+ + c^+ R_2^+\right] + \mathcal{Z}_1^- \left[R_1^- + c^- R_2^-\right]$$

$$C_i^{\pm}(x_0,y_0) = \int \mathrm{d}^3x \int \mathrm{d}^3y \, \langle J_0^{du}(x)\, Q_i^{\pm}(0)\, J_0^{us}(y)\rangle$$
$$C(x_0) = \int \mathrm{d}^3x \, \langle J_0^{\alpha\beta}(x)\, J_0^{\beta\alpha}(0)\rangle \,,$$
$$C_u^{+}(x_0,y_0) = \int \mathrm{d}^3x \int \mathrm{d}^3y \, \langle J_0^{du}(x)\, Q_u^{+}(0)\, J_0^{us}(y)\rangle$$

SU(3) ChPT

$$\mathcal{R}_i^{\pm}(x_0,y_0) = \frac{\mathcal{C}_i^{\pm}(x_0,y_0)}{\mathcal{C}(x_0)\mathcal{C}(y_0)}$$

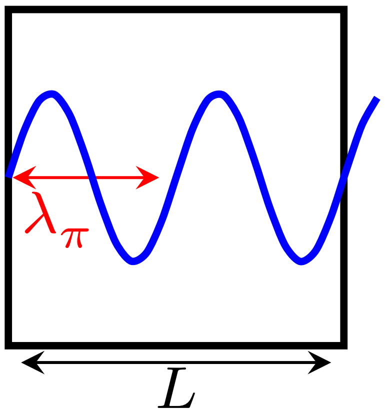
$$\mathcal{C}_{27}(x_0,y_0) = \int \mathrm{d}^3x \int \mathrm{d}^3y \, \langle \mathcal{J}_0^{du}(x)\, \mathcal{Q}_{27}(0)\, \mathcal{J}_0^{us}(y)\rangle_{\mathrm{SU(3)}}$$
$$\mathcal{C}_{8}(x_0,y_0) = \int \mathrm{d}^3x \int \mathrm{d}^3y \, \langle \mathcal{J}_0^{du}(x)\, \mathcal{Q}_{8}(0)\, \mathcal{J}_0^{us}(y)\rangle_{\mathrm{SU(3)}} \,,$$
$$\mathcal{C}_{8}'(x_0,y_0) = \int \mathrm{d}^3x \int \mathrm{d}^3y \, \langle \mathcal{J}_0^{du}(x)\, \mathcal{Q}_{8}'(0)\, \mathcal{J}_0^{us}(y)\rangle_{\mathrm{SU(3)}} \,,$$

$$R_{27}(x_0,y_0) = g_{27}\mathcal{R}_{27}(x_0,y_0) \,,$$
$$R_8(x_0,y_0) = g_8\mathcal{R}_8(x_0,y_0) + g_8'\mathcal{R}_8'(x_0,y_0)$$

kinematical regimes in ChPT

[Gasser, Leutwyler 1987; Hansen 1990; Hansen, Leutwyler 1991]

p-regime: $m_\pi L \gtrsim 1$

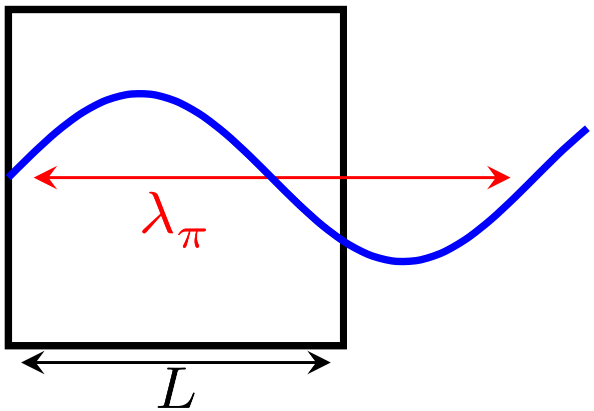


$m\Sigma V \gg 1$

standard ChPT in finite volume:

$m \sim p^2 \quad L^{-1}, T^{-1} \sim p$

ϵ-regime: $m_\pi L \lesssim 1$



$m\Sigma V \lesssim 1$

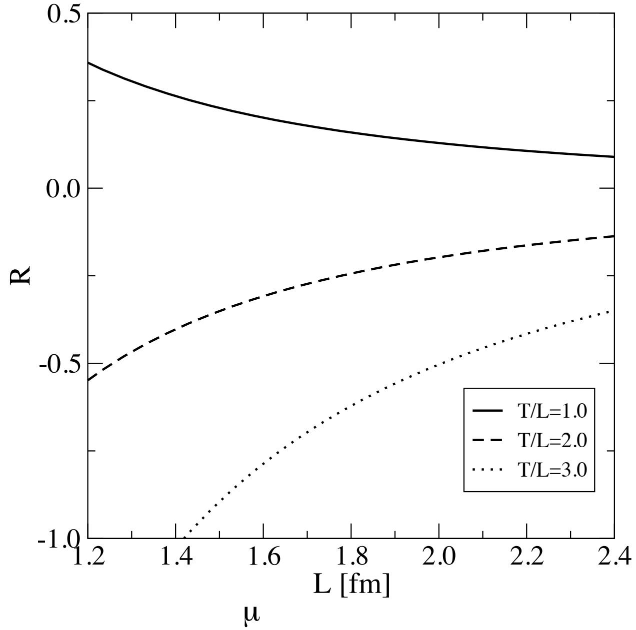
reordering of expansion:

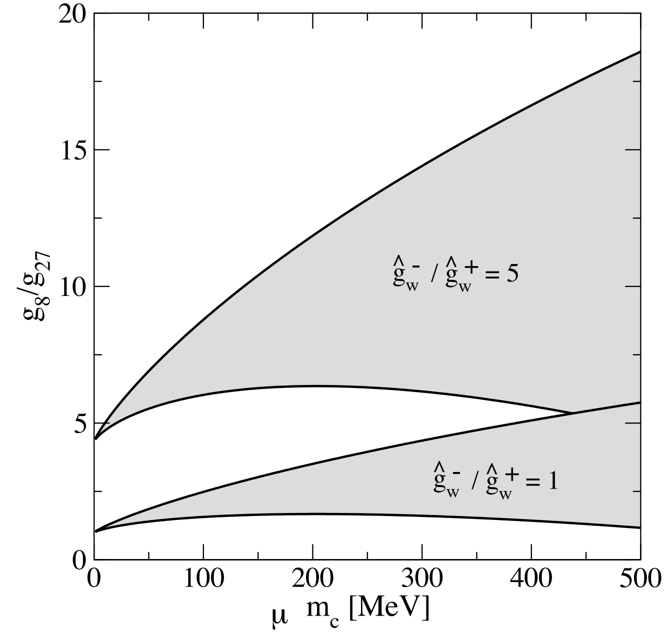
$m \sim p^4 \sim \epsilon^4 \quad L^{-1}, T^{-1} \sim \epsilon$

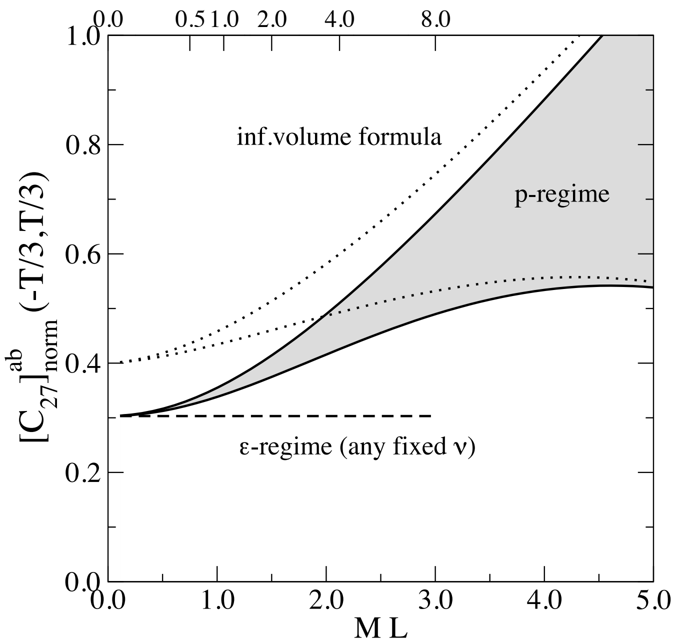
- LECs universal
- only a subset of NLO terms in chiral Lagrangian survive in the ϵ-regime
- chiral effective Hamiltonian has no NLO terms in the ϵ-regime

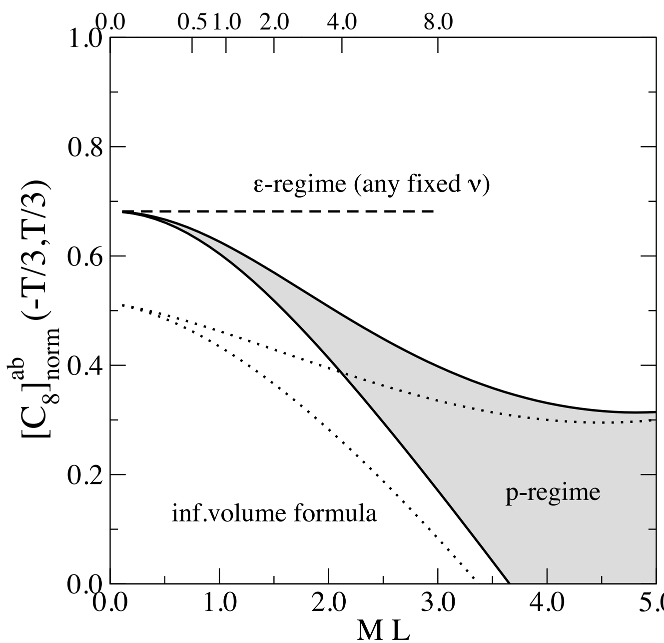
[Giusti, Hernández, Laine, Weisz, Wittig 2004; Hernández, Laine 2006]

kinematical regimes in ChPT









[Giusti, Hernández, Laine, Weisz, Wittig 2004; Hernández, Laine 2002/2004/2006]

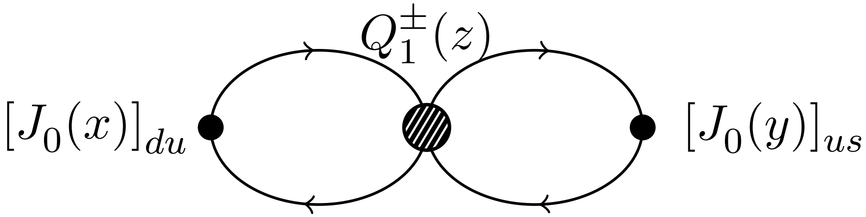
setup summary + remarks

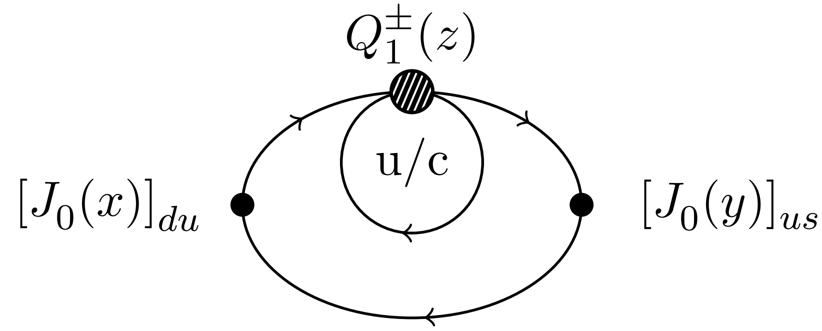
- we are after a computation of chiral effective couplings governing kaon decay
- setup allows to disentangle charm scale physics from low-energy QCD physics, not including FSI
- keeping active charm crucial for disentanglement + renormalisation
- access to different kinematical regimes crucial to control systematics
- (expensive) exactly chiral fermions mandatory (use of ChPT, renormalisation)
- only QCD computation of matrix elements of four-quark operators needed, other pieces available
  - Wilson coefficients [Ciuchini et al. 1998; Buras, Misiak, Urban 2000]
  - ChPT computations [Hernández, Laine 2002-2006]
  - non-perturbative composite operator renormalisation [Dimopoulos et al. 2006]

outline

- computing kaon decay amplitudes
  - EW effective Hamiltonian analysis
  - why is it so difficult?
  - status
- understanding the anatomy: strategy
  - disentangling scales
  - low-energy effective description and the role of chiral symmetry
  - can's and cannot's
- some results
  - long-distance effects in GIM limit
  - towards the physical charm mass scale
- conclusions and outlook

lattice computation





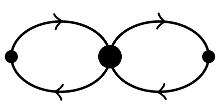
(vanishes in GIM limit)

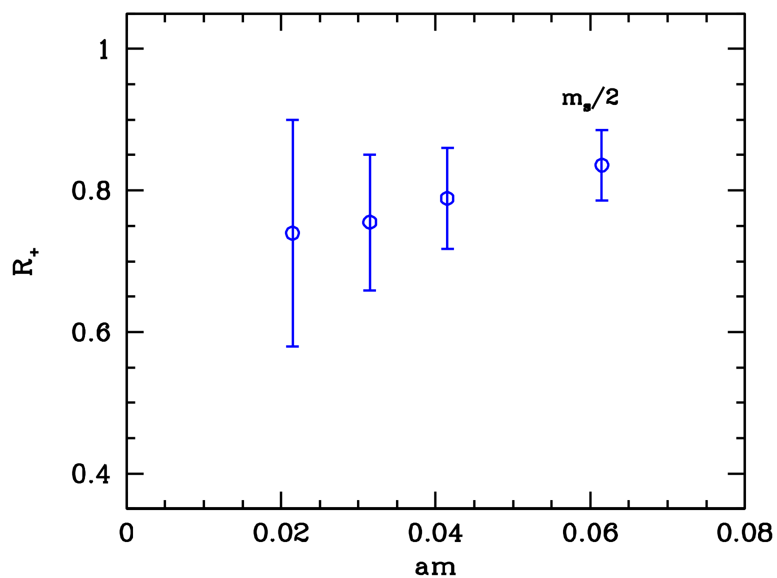
proof of concept + qualitative exploration:

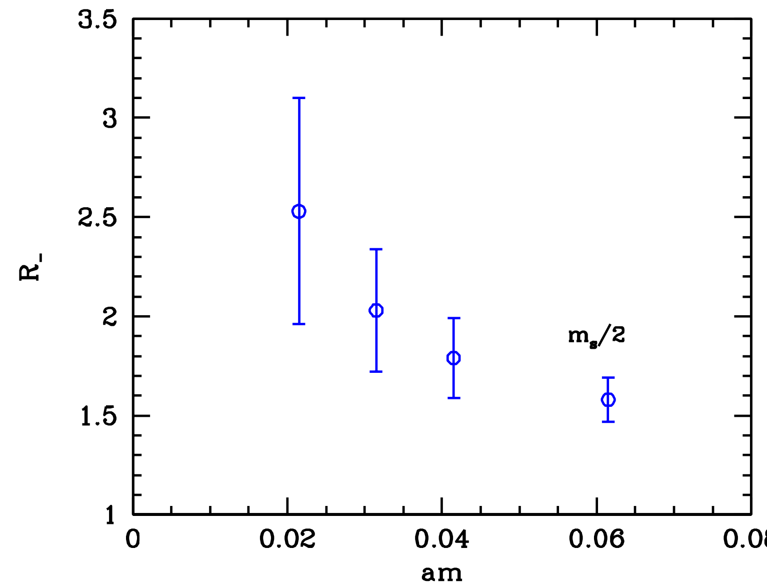
- stay quenched (dynamical fermion effects not crucial, chiral reg. expensive)
- keep all three light quarks degenerate $m_s = m_u = m_d \equiv m_l$
- access very light masses (ϵ-regime): severe variance problem
- outside GIM limit ⇒ penguin (“eye”) contractions: severe variance problem

lattice computation: variance problems

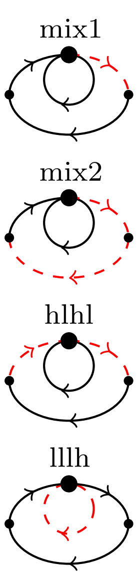
light charm: strong signal-to-noise ratio dependence on quark mass

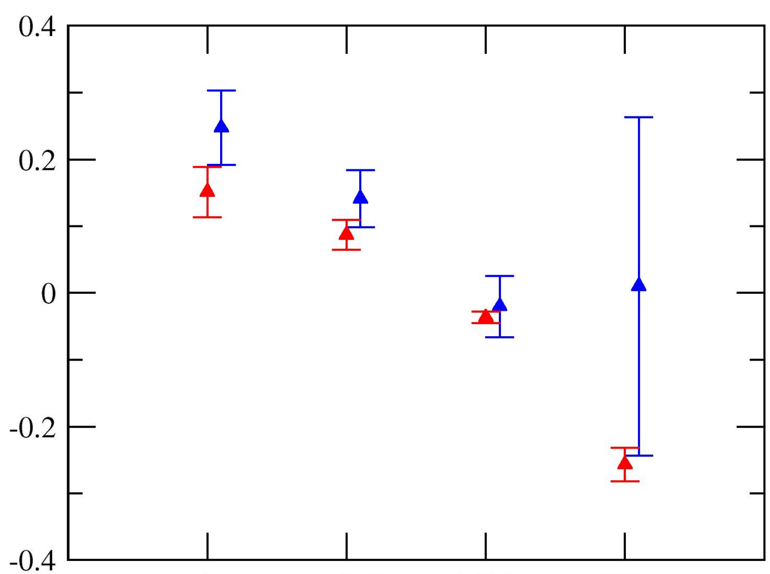


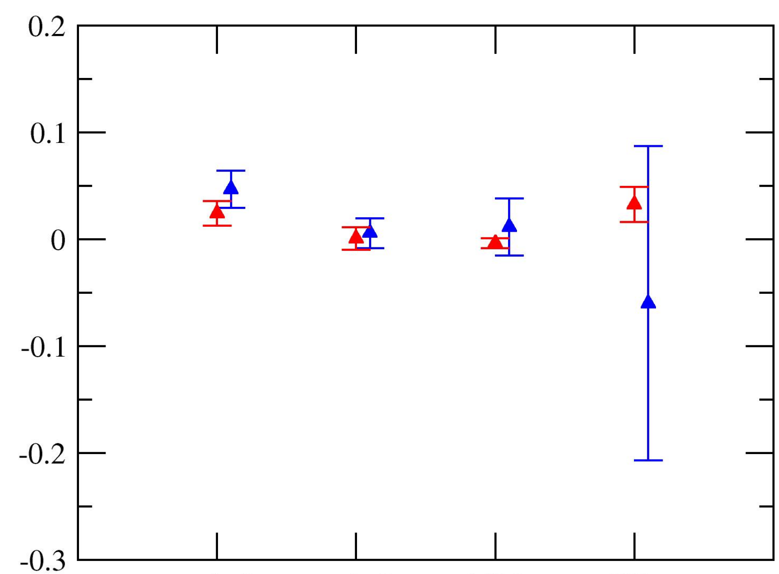




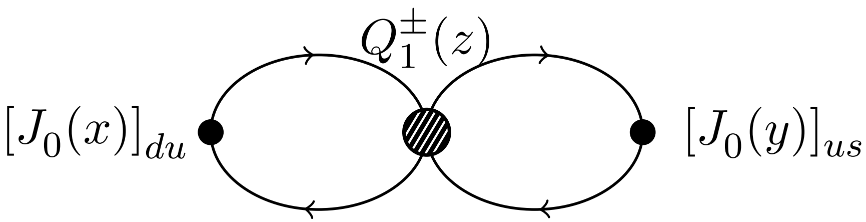
heavy charm: simple computational techniques do not yield a signal at all

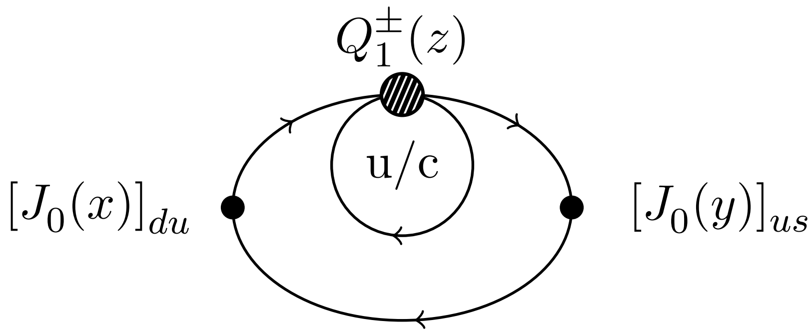




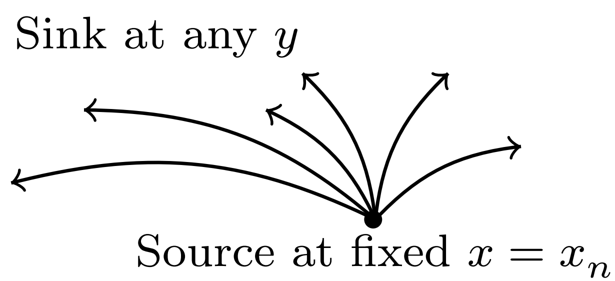


lattice computation: variance problems

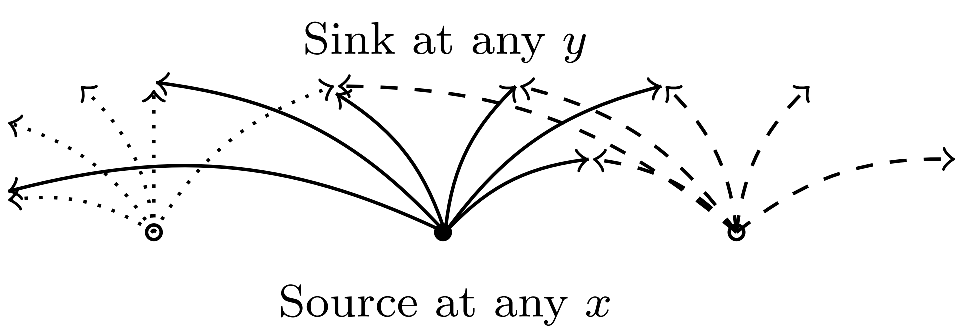




Traditional way: point-to-all

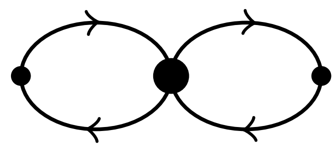


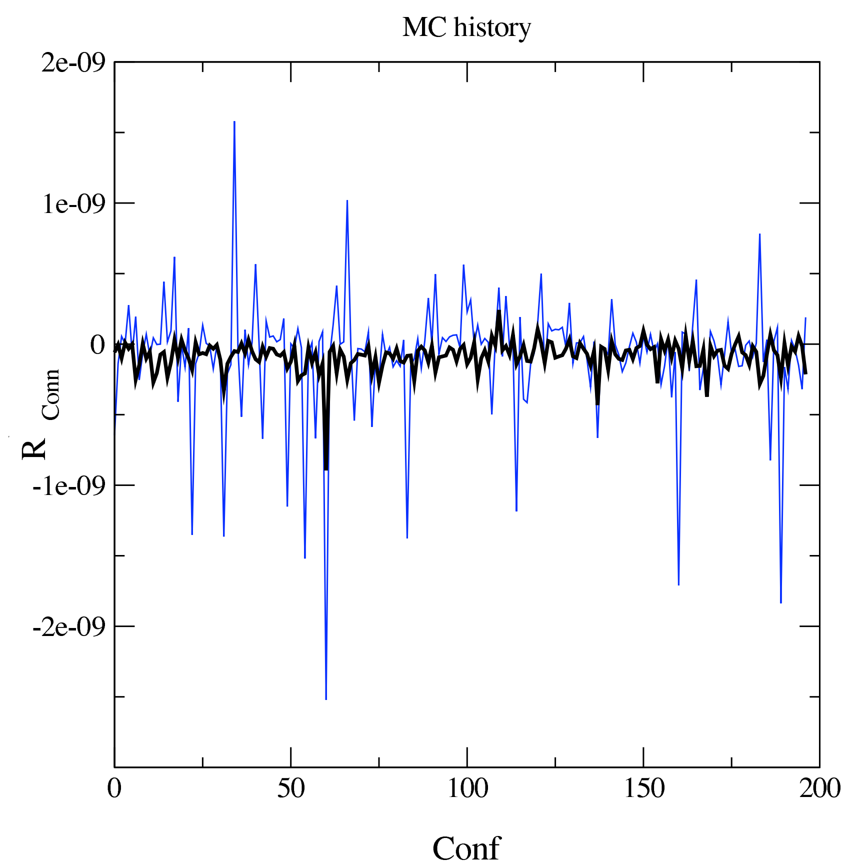
Goal: all-to-all

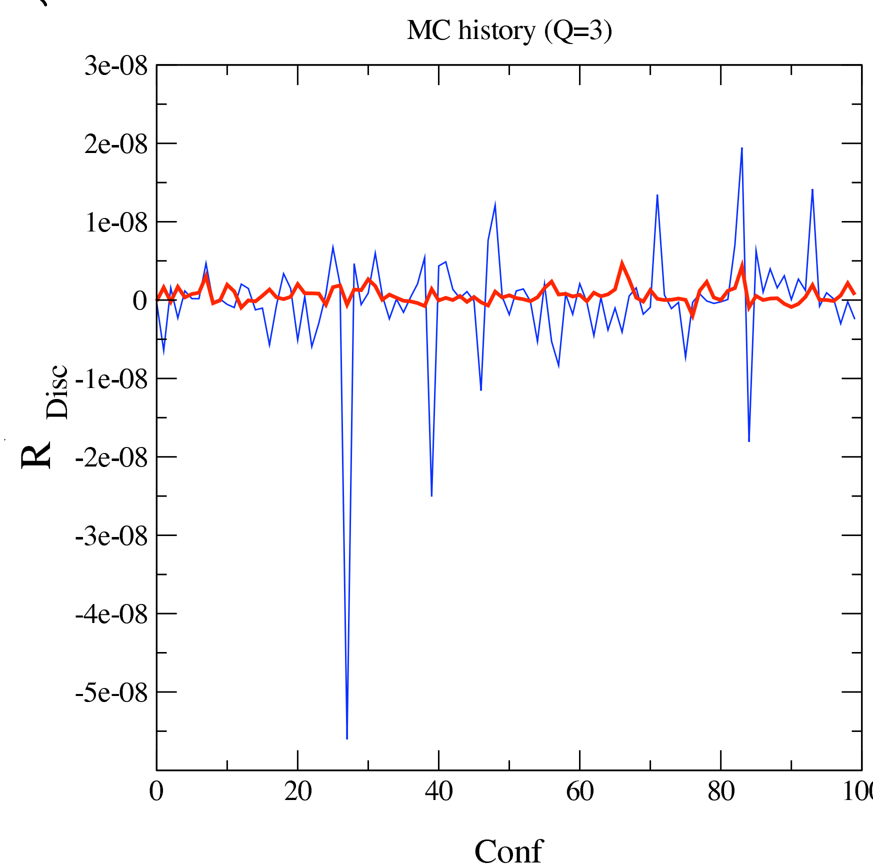


lattice computation: variance problems

solution for large variances related to very light quark masses: accurate all-to-all propagators in space of low Dirac modes (low-mode averaging)



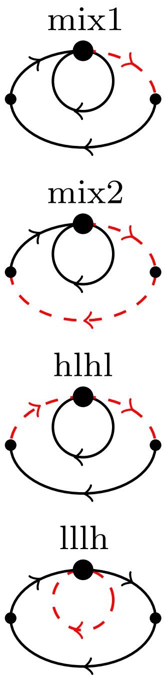


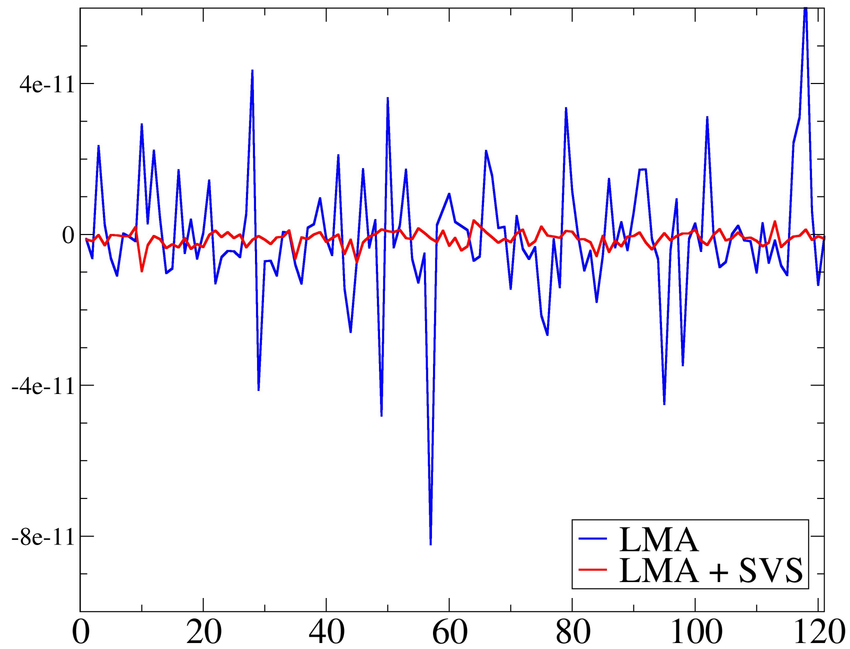


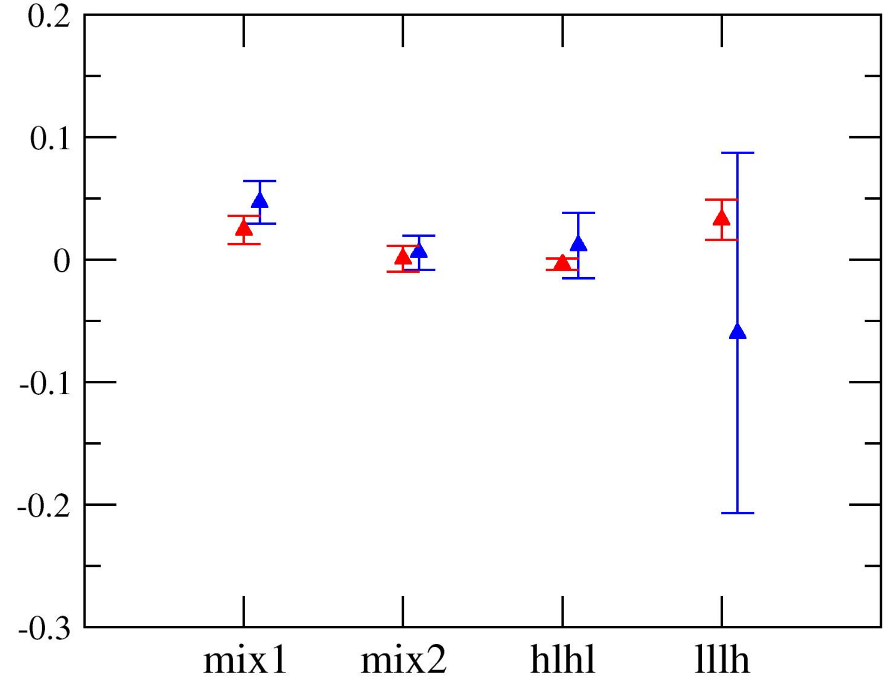
[Giusti, Hernández, Laine, Weisz, Wittig 2004]
[Giusti, Hernández, Laine, CP, Wennekers, Wittig 2005]

lattice computation: variance problems

solution for large variances related to closed quark loops: approximate all-to-all propagators involving all dirac modes (stochastic volume sources and probing)

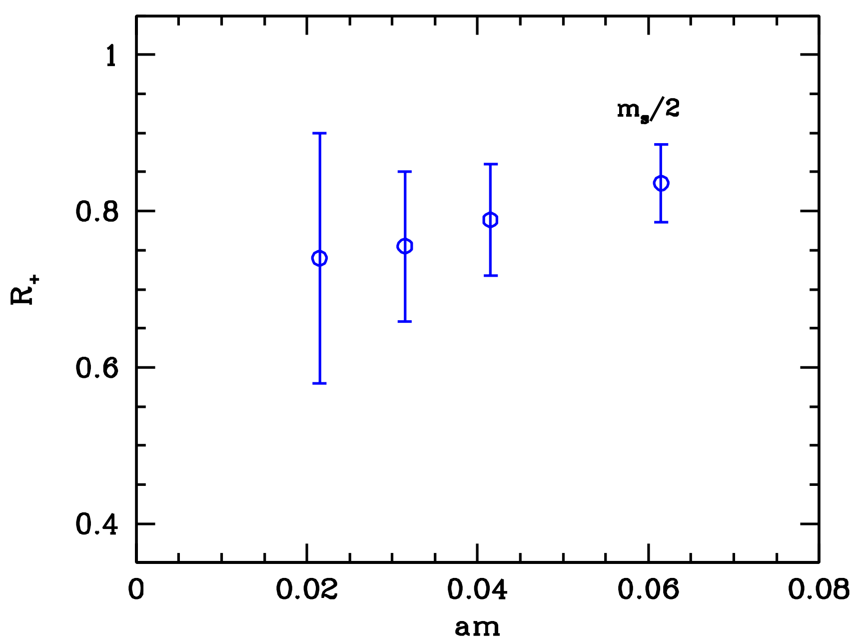


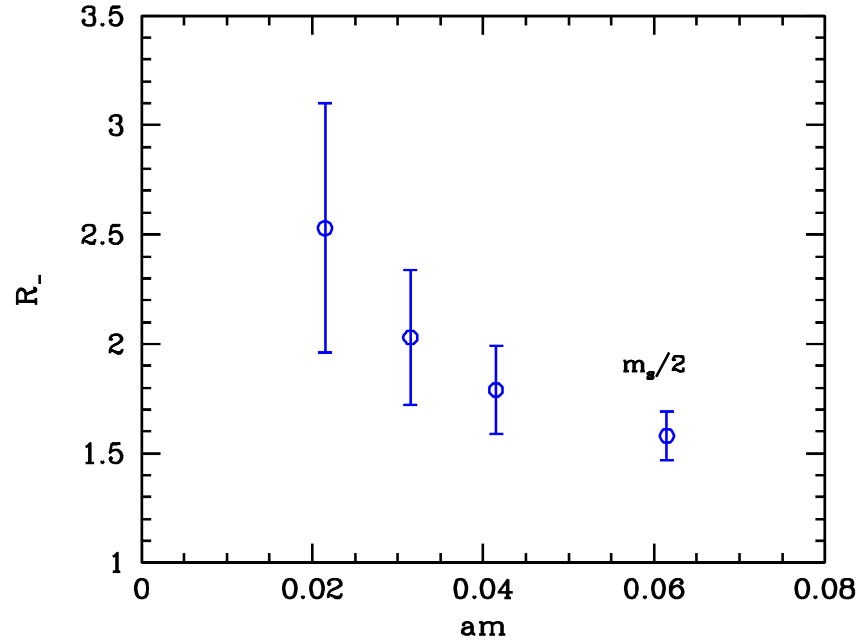




[Endress, CP 2014]

lattice computation: GIM limit





Simulation parameters:

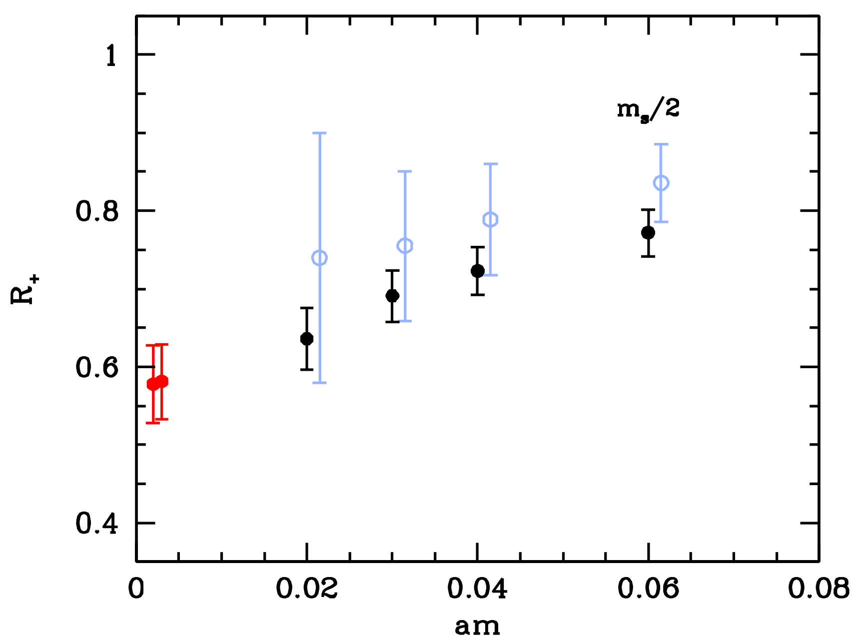
$$\beta = 5.8485 \qquad \frac{V}{a^4} = 16^3 \cdot 32 \qquad a \approx 0.125 \text{ fm} \qquad V \approx 2^3 \cdot 4 \text{ fm}^4$$

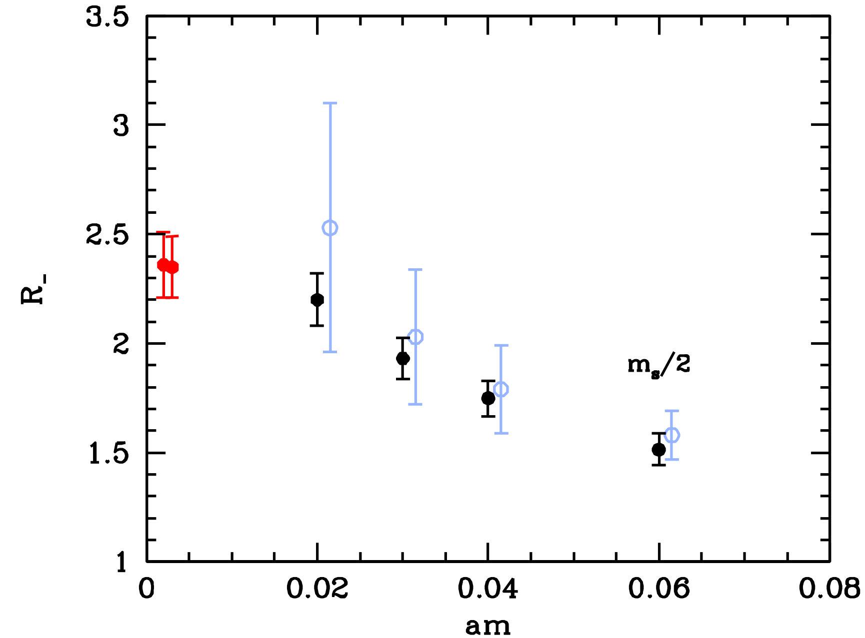
Quark masses: p-regime $m \sim m_s/2 - m_s/6$ O(200) cfgs

ε-regime $m \sim m_s/40, m_s/60$ O(800) cfgs

Quenched approximation.

lattice computation: GIM limit





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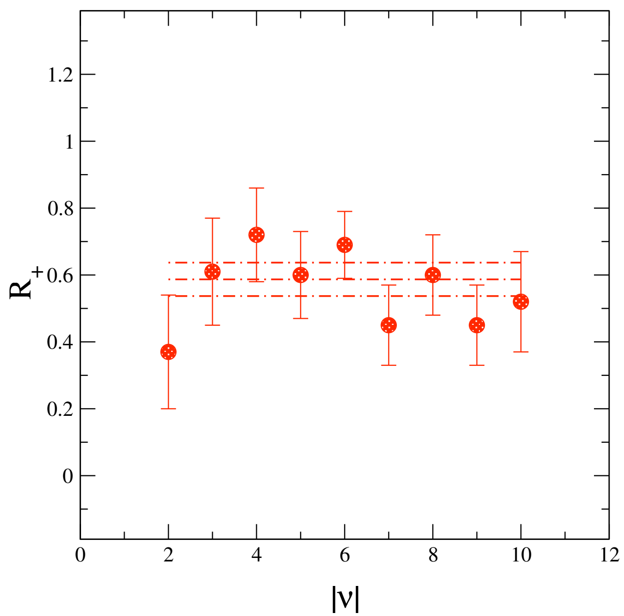
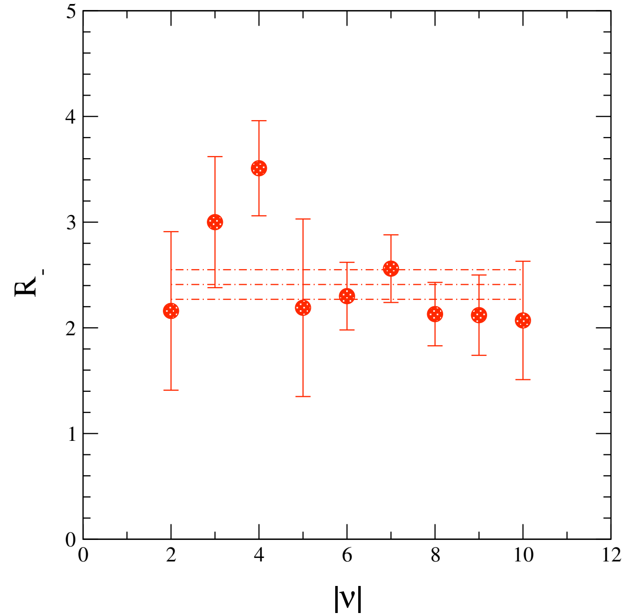
Quenched approximation.

lattice computation: GIM limit

Expected ε-regime features — independence of $R^{\pm}$ on (x_0,y_0), m and ν — are all well reproduced by the data.





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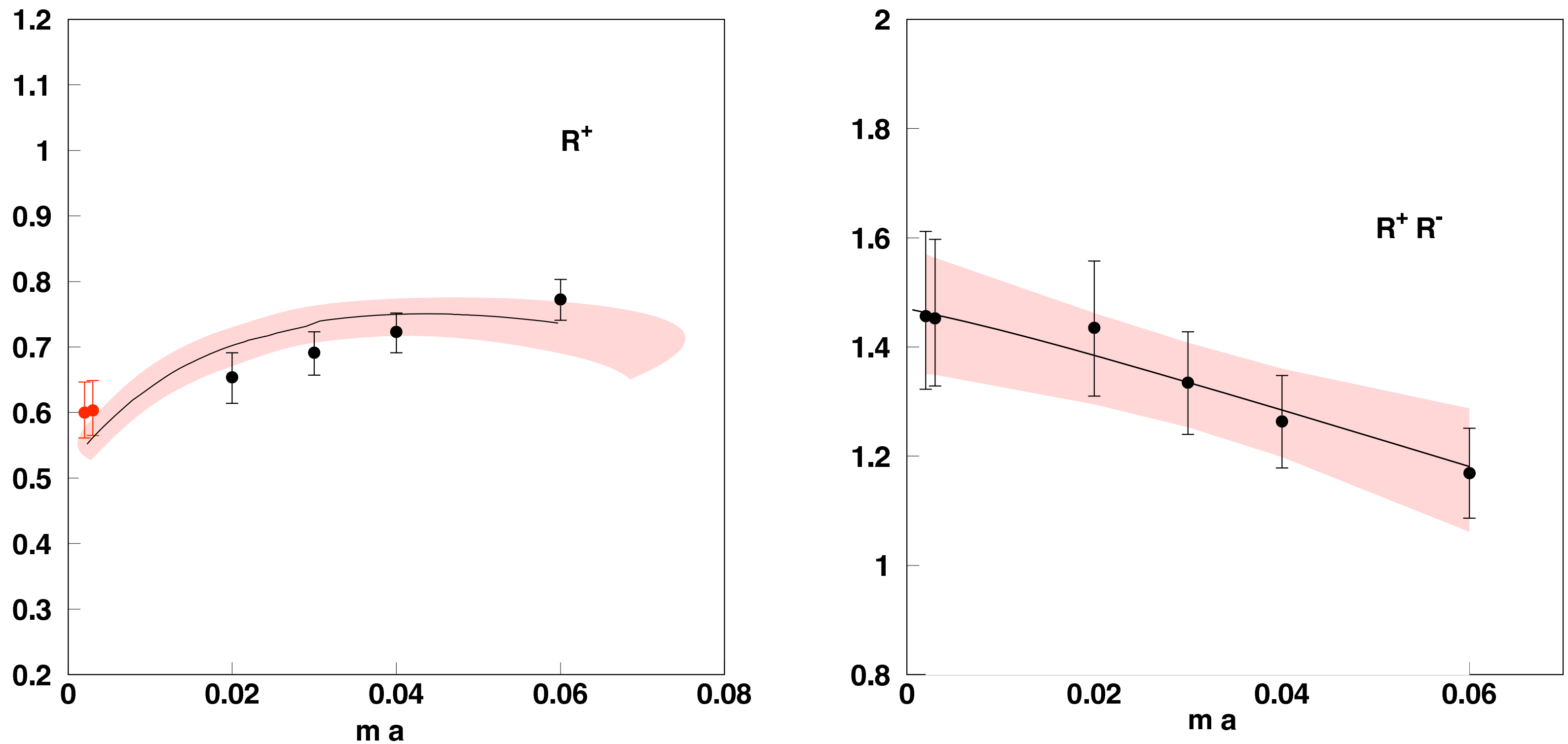
Quenched approximation.

lattice computation: GIM limit

Fits for LECs:

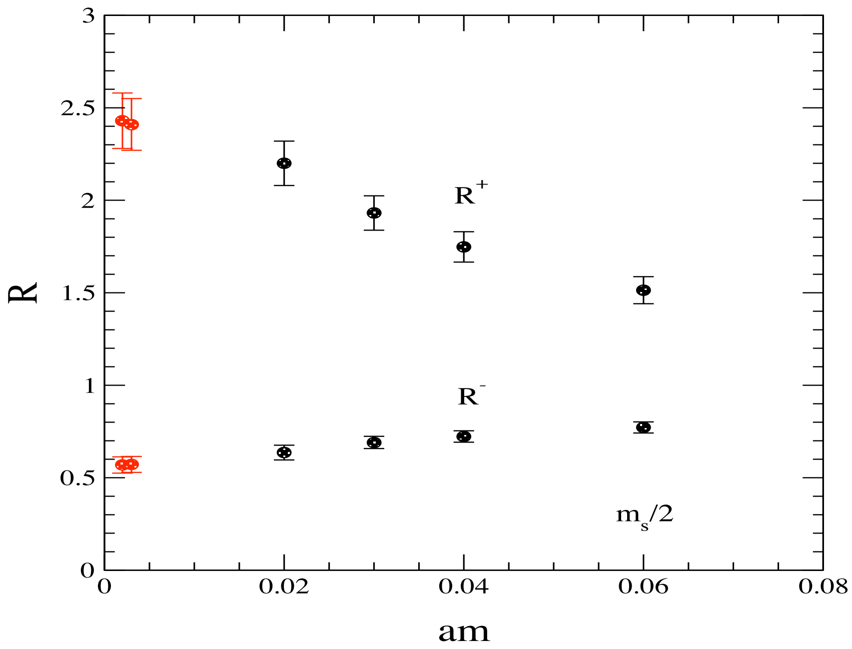
- Choose quantities with smaller mass corrections and statistical errors: $R^+,\ R^+R^-$
- Fit to NLO χPT to extract $g^\pm$ and $\Lambda^\pm$ (exploit smooth ε/p-regime transition).



Tension between ε- and p-regime may indicate non-negligible higher order corrections → systematic error included to account for this.

lattice computation: GIM limit

[Giusti, Hernández, Laine, CP, Wennekers, Wittig 2007]



	g^+	g^-
This work	0.51(3)(5)(6)	2.6(1)(3)(3)
"Exp"	~ 0.5	~ 10.4
Large N_c	1	1

- ΔI=3/2 comes in the right ballpark (n.b. charm enters only via loops — but also suggests quenching subdominant [?])
- ΔI=1/2 about a factor 4 too small to reproduce physical enhancement
- remarkable enhancement of ΔI=1/2 channel already present for light charm: pure "no-penguin" effect

lattice computation: towards a physical charm

separate low-energy QCD and charm-scale physics: consider amplitudes as a function of charm mass for fixed u,d,s masses

$$m_c = m_u = m_d = m_s \quad \longrightarrow \quad m_c \gg m_u = m_d \leq m_s$$

lattice computation: towards a physical charm

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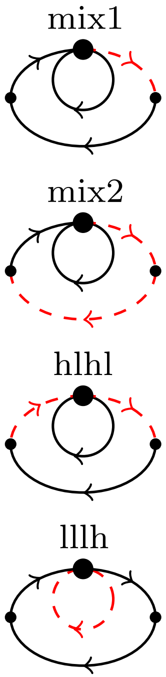

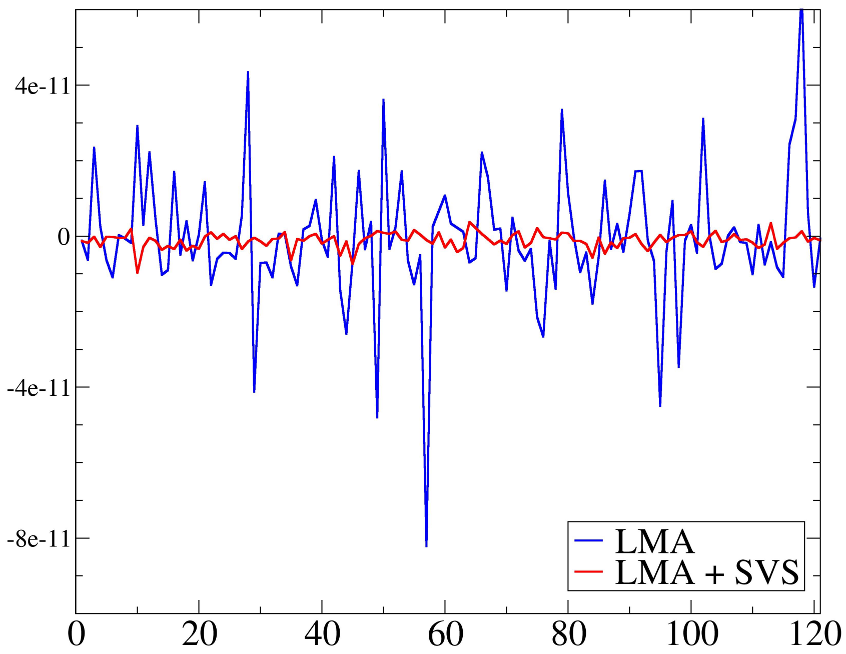

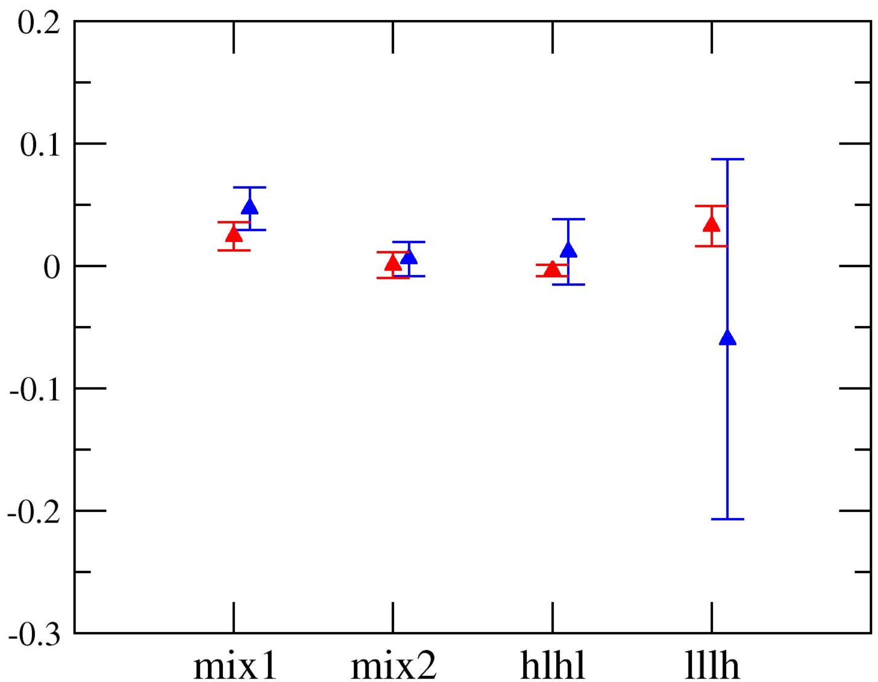

[Endress, CP 2014]

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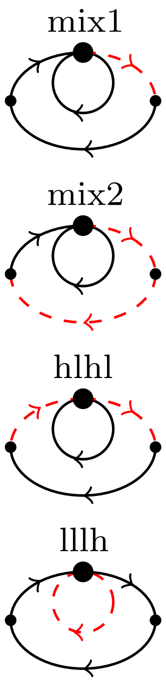
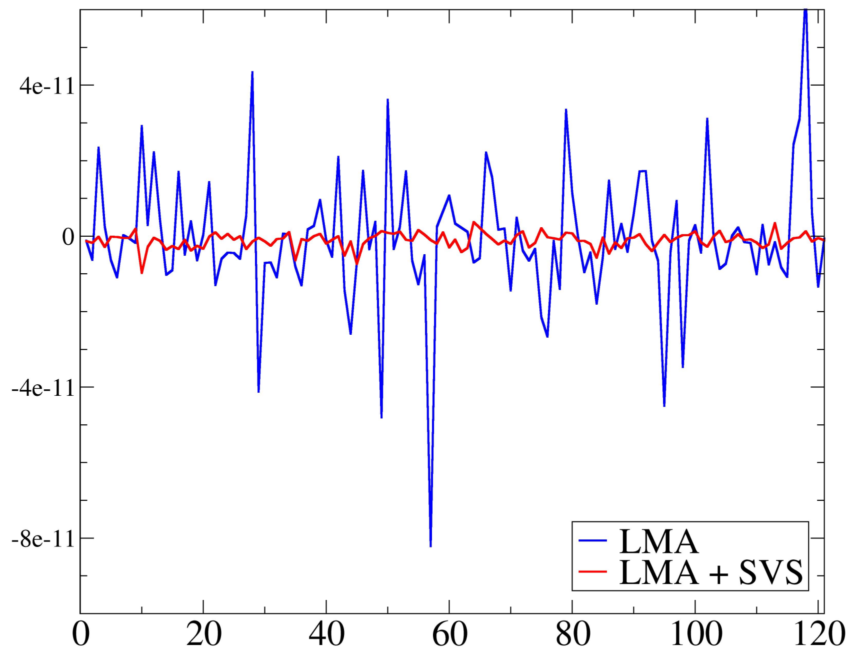
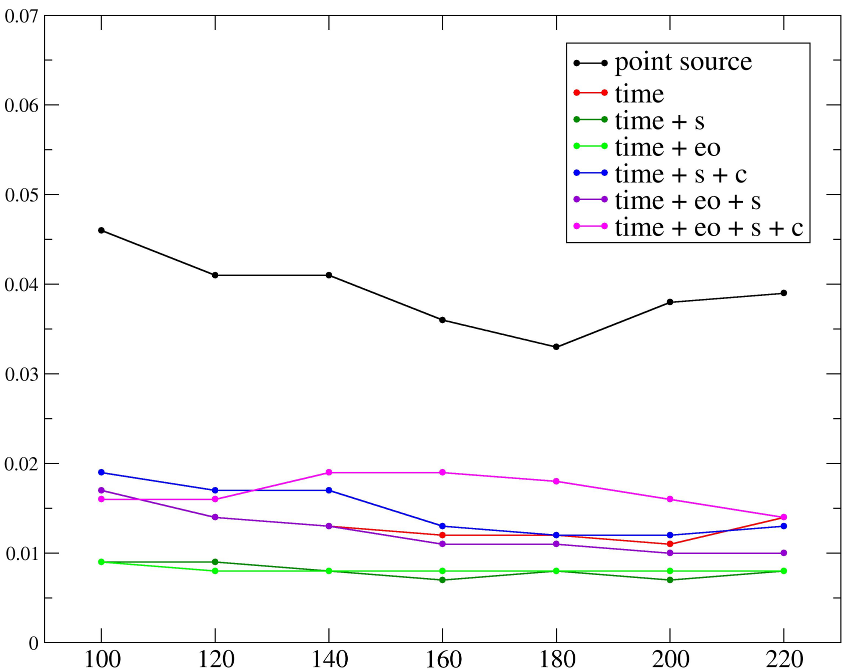
abs. error $\times \sqrt{N_{\rm cfg}}$ vs. $N_{\rm cfg}$







[Endress, CP 2014]

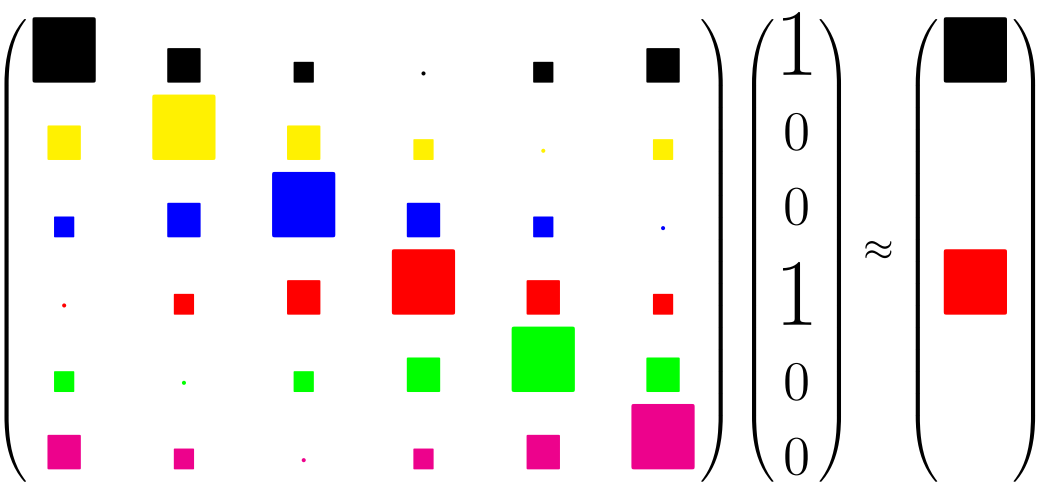
lattice computation: towards a physical charm

separate low-energy QCD and charm-scale physics: consider amplitudes as a function of charm mass for fixed u,d,s masses

$$m_c = m_u = m_d = m_s \quad \longrightarrow \quad m_c \gg m_u = m_d \leq m_s$$

probing: new technique to compute diagonal of inverse of large sparse matrices

[Tang, Saad 2012]



$$\mathcal{D}(S) \approx \mathcal{D}(SV_s V_s^T)\mathcal{D}^{-1}(V_s V_s^T)$$

$V_s := [v_1, v_2, \ldots, v_s]$, probing vectors v_i

$X_s := \{x_1, x_2, \ldots, x_s\}$: Set of solution vectors

$$\Longrightarrow \quad \mathcal{D}(S) \approx \mathcal{D}(X_s V_s^T)$$

[Stathopoulos, Laeuchli, Orginos 2013]
[Endress, CP, Sivalingam 2014]

lattice computation: towards a physical charm

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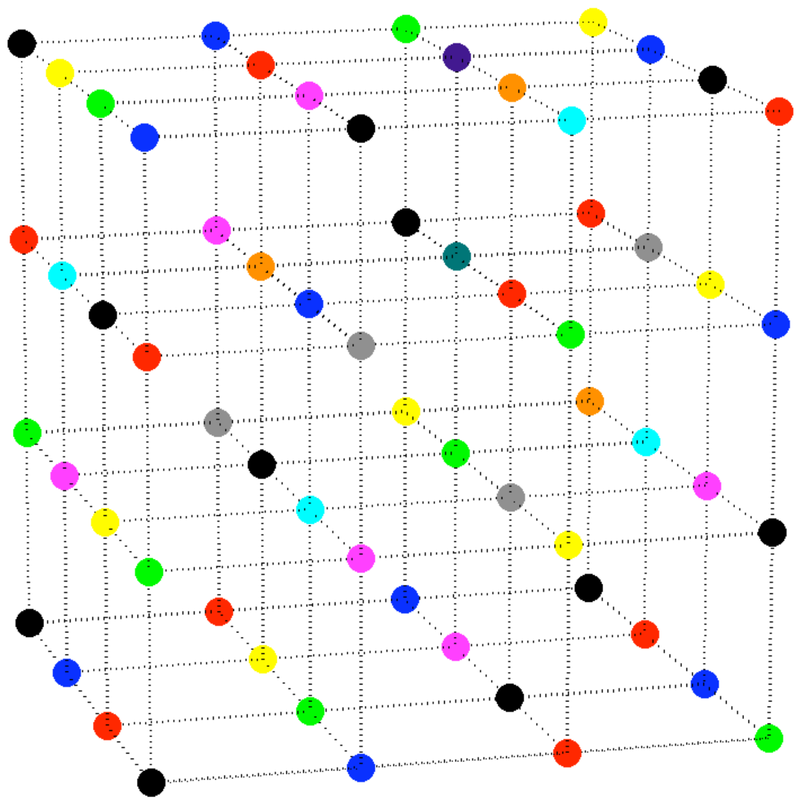


lattice computation: towards a physical charm

also need to compute contribution from the subtraction term

$$\mathcal{H}_{\rm w}^{\rm eff} = \frac{g_{\rm w}^2}{2M_W^2} V_{us}^* V_{ud} \sum_{\sigma=\pm} \{k_1^\sigma \mathcal{Q}_1^\sigma + k_2^\sigma \mathcal{Q}_2^\sigma\}$$

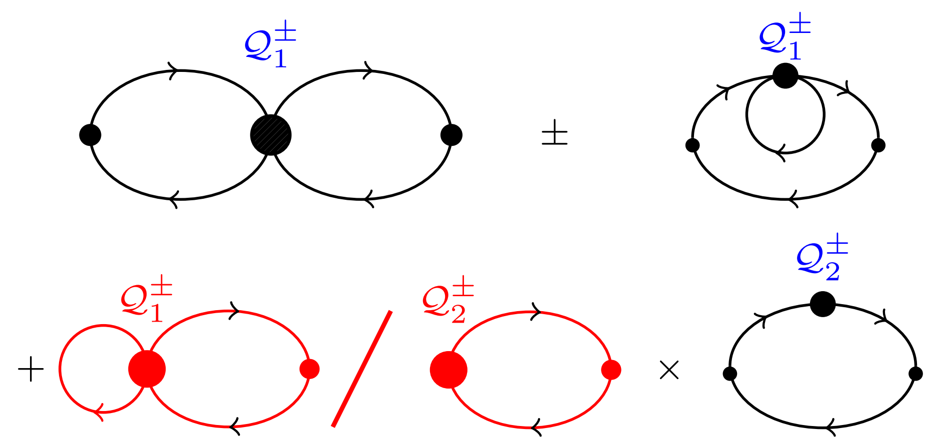
$$\mathcal{Q}_1^\pm = (\bar{s}_{\rm L}\gamma_\mu u_{\rm L})(\bar{u}_{\rm L}\gamma_\mu d_{\rm L}) \pm (\bar{s}_{\rm L}\gamma_\mu d_{\rm L})(\bar{u}_{\rm L}\gamma_\mu u_{\rm L}) - [u \leftrightarrow c]$$

$$\mathcal{Q}_2^\pm = (m_u^2 - m_c^2)\{m_d(\bar{s}_{\rm L} d_{\rm R}) + m_s(\bar{s}_{\rm R} d_{\rm L})\}$$

complicated in practice, leads to new technical issues not completely sorted out yet — work with perturbative estimate

$$\mathcal{H}_{\rm w} = \sum_{\sigma=\pm} k_1^\sigma(\mu) Z_{11}^\sigma(\mu)\{\mathcal{Q}_1^\sigma + c^\sigma \mathcal{Q}_2^\sigma\}$$

$$\left\langle \left(\mathcal{Q}_1^\pm + c^\pm \mathcal{Q}_2^\pm\right)(0)\left[J_{\rm L0}\right]_{ds}(x)\right\rangle = 0$$



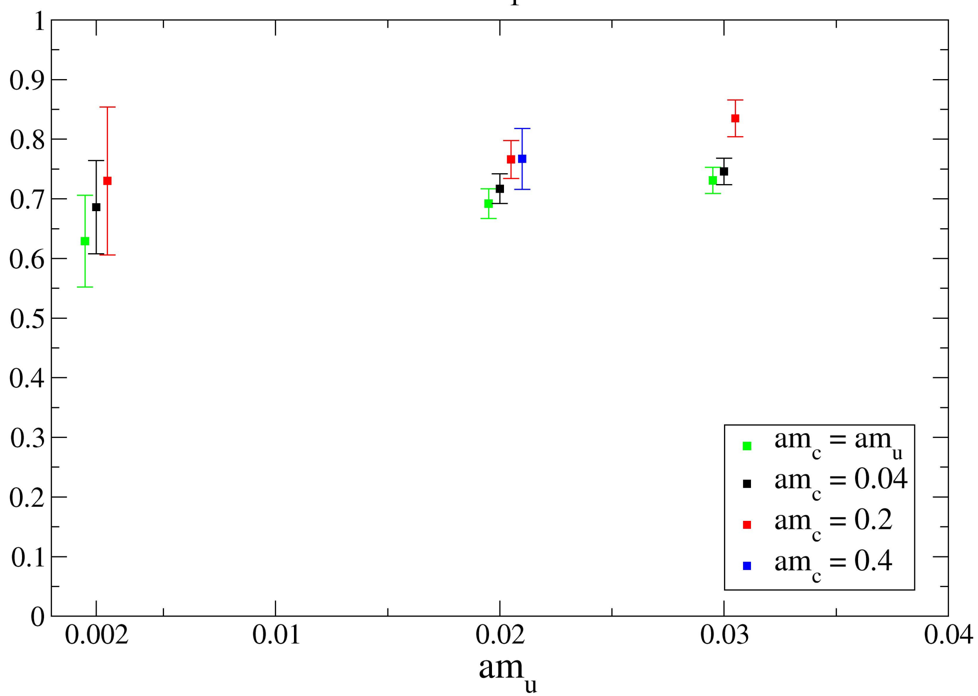
[Bernard et al. 1985]

lattice computation: towards a physical charm

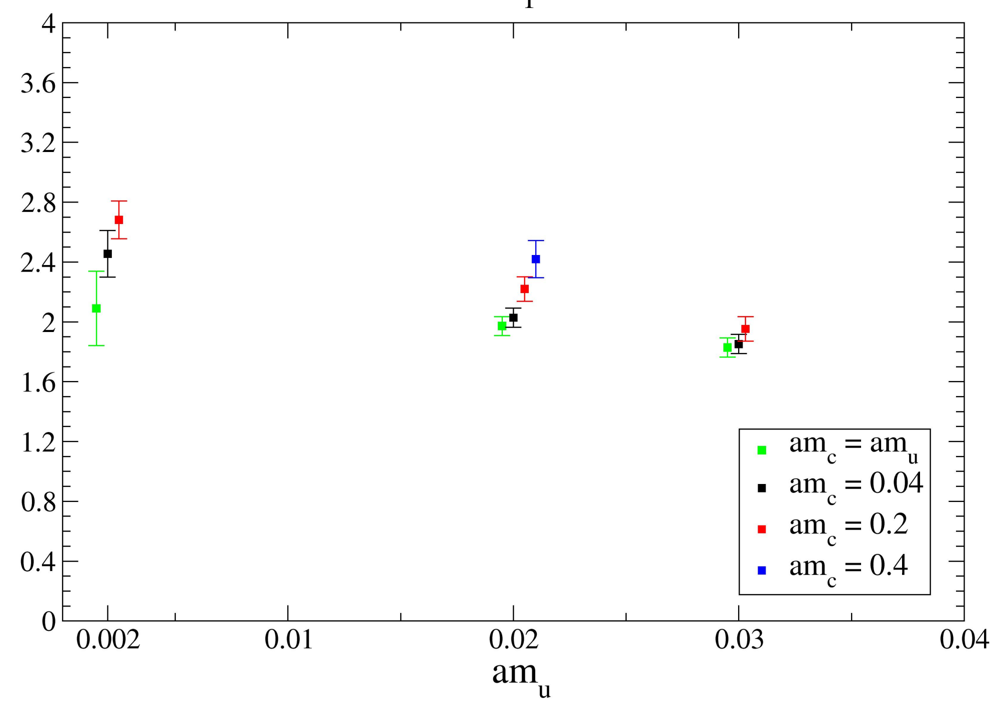
- $\beta = 5.8485$, $a \approx 0.124$ fm
- 32×16^3, $L = 2$ fm
- $N_{\text{low}} = 20$
- Dilution: time, spin, color
- Quenched approximation

	am_u	m_π[MeV]	am_c	# cfgs
ϵ-regime	0.002		$0.04, 0.2$	$\mathcal{O}(400)$
p-regime	0.02	320	$0.04, 0.2, 0, 4$ (50, 250, 500 MeV)	$\mathcal{O}(400)$
	0.03	370	$0.04, 0.2$	$\mathcal{O}(400)$

R_1^+



R_1^-

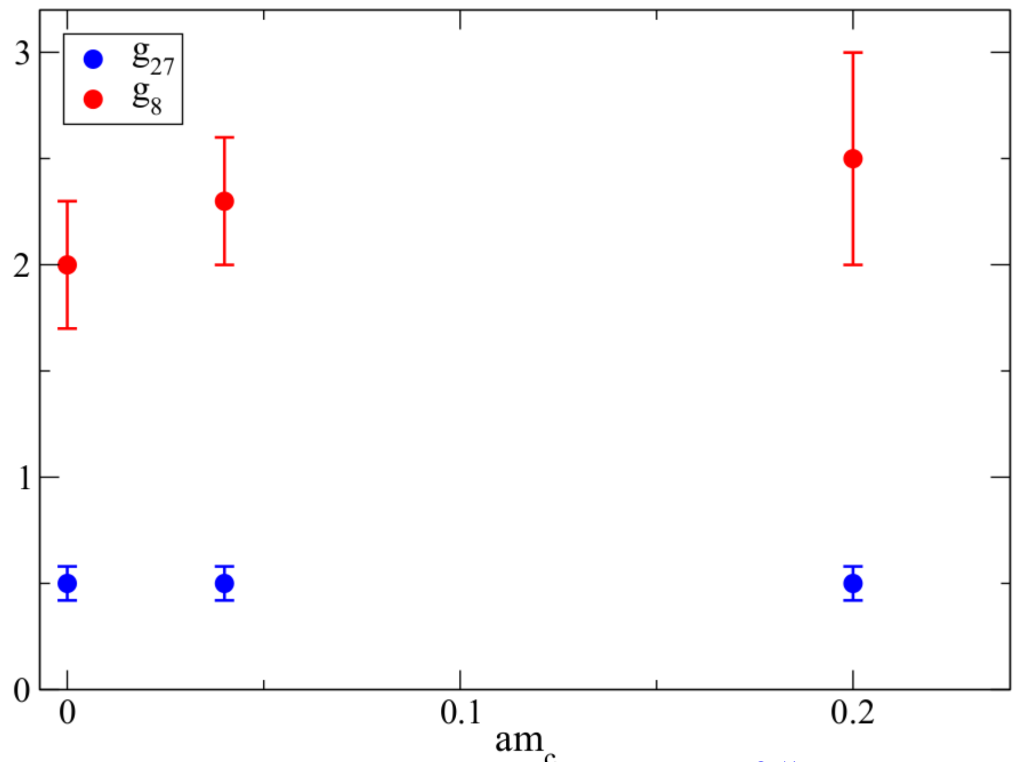


[Endress, CP 2014]

lattice computation: towards a physical charm

- $\beta = 5.8485$, $a \approx 0.124$ fm
- 32×16^3, $L = 2$ fm
- $N_{\text{low}} = 20$
- Dilution: time, spin, color
- Quenched approximation

	am_u	m_π[MeV]	am_c	# cfgs
ϵ-regime	0.002		$0.04, 0.2$	$\mathcal{O}(400)$
p-regime	0.02	320	$0.04, 0.2, 0, 4$ (50, 250, 500 MeV)	$\mathcal{O}(400)$
	0.03	370	$0.04, 0.2$	$\mathcal{O}(400)$

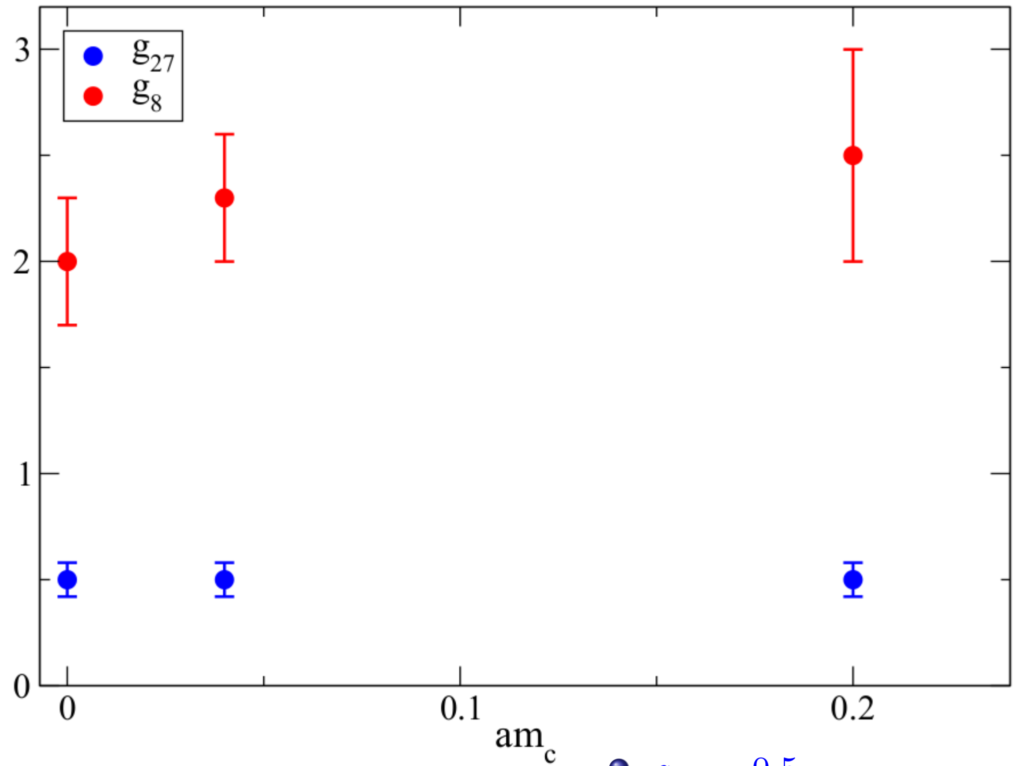


"Experimental" values:

- $g_{27} \sim 0.5$
- $g_8 \sim 10.5$

[Endress, CP 2014]

lattice computation: towards a physical charm



"Experimental" values:

- $g_{27} \sim 0.5$
- $g_8 \sim 10.5$

[Endress, CP 2014]

- did a good job at tackling eye diagrams, work missing for subtraction
- very mild extra enhancement for charm masses up to $m_c \sim \Lambda_{\rm QCD}$
- analysis of subtraction term suggests that *I*=0 amplitude grows quadratically with charm mass for $m_c \gg \Lambda_{\rm QCD}$
- better understanding of subtraction crucial

outline

- computing kaon decay amplitudes
  - EW effective Hamiltonian analysis
  - why is it so difficult?
  - status

- understanding the anatomy: strategy
  - disentangling scales
  - low-energy effective description and the role of chiral symmetry
  - can’s and cannot’s

- some results
  - long-distance effects in GIM limit
  - towards the physical charm mass scale

- conclusions and outlook

conclusions and outlook

- status of problem
  - understanding of field theory issues in late 90s crucial for progress
  - algorithmic aspects important, but not crucial
  - huge progress by RBC/UKQCD in direct computation, others in hot pursuit

- understanding of anatomy
  - our strategy works well, is easy to extend to realistic setup (unquenching, direct computation, ...)
  - significant enhancement due to pure low-energy QCD effects seen
  - role of charm / precise amount of total QCD enhancement still unclear: go to larger charm masses (smaller lattice spacings), control subtraction

- looks like the problem may be settled within a decade
  [standard conclusion for seminars in 1984, 1994, 2004]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