Twisted mass QCD for weak matrix elements

Carlos Pena

Lattice 2006, Tucson

Thanks to:

P. Dimopoulos
M. Guagnelli
J. Heitger
F. Palombi
M. Papinutto
S. Sint
A. Vladikas
H. Wittig

ALPHA
Collaboration

Motivation

- LQCD impact on SM testing via Flavour Physics requires increased control of systematic uncertainties.
  - Light light quarks.
  - Control of symmetries / Renormalisation.
  - Cutoff dependences.
  - Conceptual issues.

- Wilson fermions are able to access large volumes and explore dependence on lattice spacing with current computing capabilities.

  See talk by L. Giusti

- tmQCD offers potential advantages related to the control of chiral symmetry breaking and the renormalisation of composite operators.

- Not a review: I will mainly report on Alpha Collaboration work.

  See talk by W. Lee

Outline

- B_K
  - Motivation from UT analysis.
  - B_K with Wilson fermions.
  - tmQCD
  - The ALPHA Collaboration computation.

- (Ongoing work on) B_B
  - Strategy.
  - Status.

- Some remarks on tmQCD for $K \rightarrow \pi\pi$ decays.

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ΔS=2 transitions: ϵ_K

$$|\epsilon_K| = \frac{\mathcal{A}(K_L \to (\pi\pi)_{I=0})}{\mathcal{A}(K_S \to (\pi\pi)_{I=0})} \stackrel{\text{exp}}{=} [2.282(17) \times 10^{-3}]\, e^{i\pi/4}$$

$$|\epsilon_K| \approx C_\epsilon\, \hat{B}_K\, \mathrm{Im}\{V_{td}^* V_{ts}\}\, \{\mathrm{Re}\{V_{cd}^* V_{cs}\}[\eta_1\, S_0(x_c) - \eta_3\, S_0(x_c, x_t)] - \mathrm{Re}\{V_{td}^* V_{ts}\}\eta_2\, S_0(x_t)]\}$$

$$\boxed{\hat{B}_K = \frac{\langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle}{\frac{8}{3} F_K^2\, m_K^2}}$$

Put in NLO PT + Cabibbo angle + A + $m_{c,t}$:

$$\bar{\eta}(1.4 - \bar{\rho})\, \hat{B}_K \approx 0.40$$

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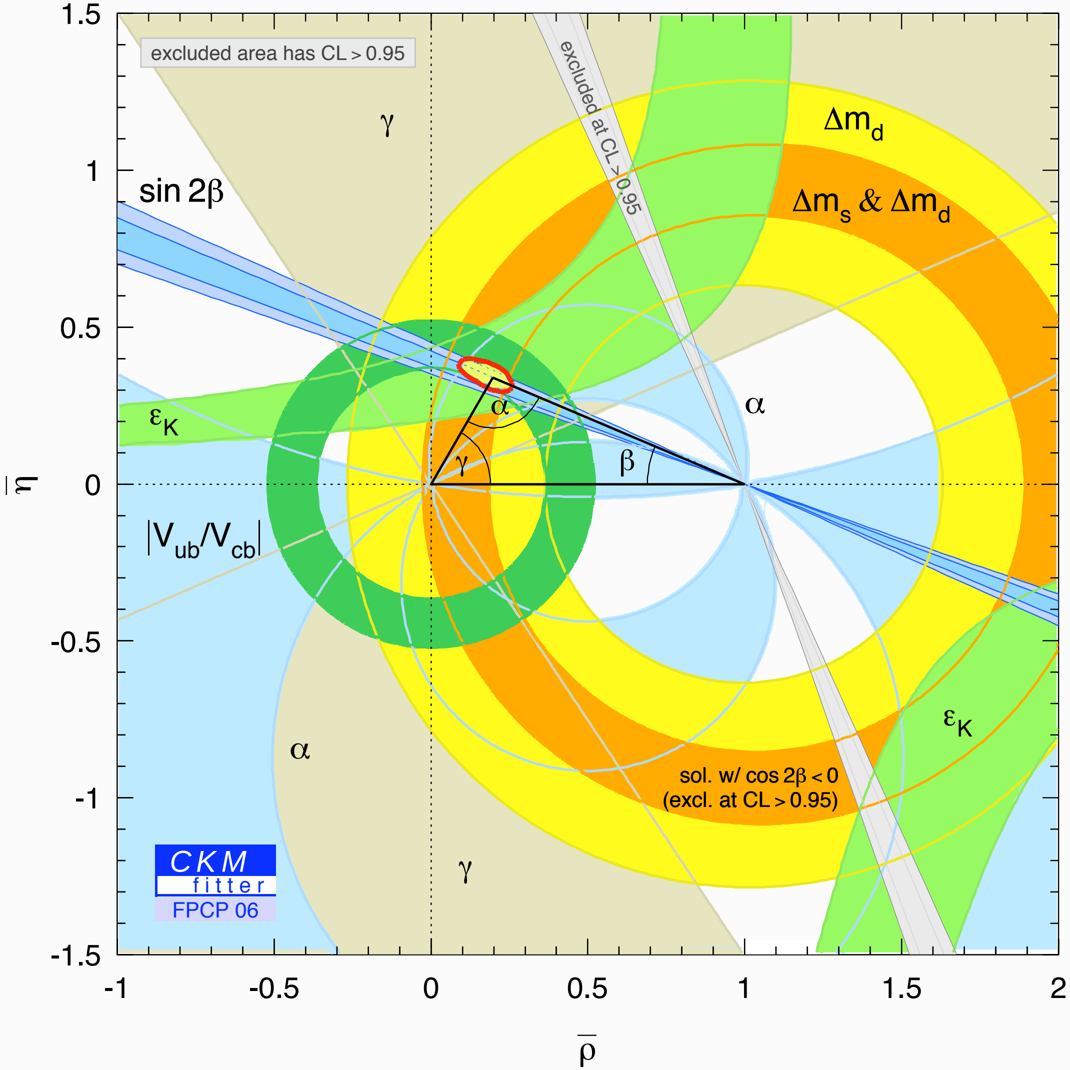

The case for a precise quenched Wilson computation of B_K

- Conceptual uncertainties minimised.
- Numerically cheap $\Rightarrow$ control cutoff dependence.
- Mature non-perturbative renormalisation techniques.
- Control/understanding of all quenched systematics essential to set up techniques and set target precision in unquenched computation.

B_K – a renormalisation classic

In the presence of explicit chiral symmetry breaking four-fermion operators of different chiralities mix under renormalisation.

Martinelli 1984; Bernard, Draper, (Hockney), Soni 1987, 1998; Gupta et al. 1993; Donini et al. 1999

$$O^{\Delta S=2} = \underbrace{[(\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu d) + (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu\gamma_5 d)]}_{O_{\mathrm{VV+AA}}} - \underbrace{[(\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu\gamma_5 d) + (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu d)]}_{O_{\mathrm{VA+AV}}}$$

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$$\bar{O}_{\mathrm{VV+AA}} = \lim_{a\to 0} Z_{\mathrm{VV+AA}}(g_0^2, a\mu)\left[O_{\mathrm{VV+AA}}(a) + \sum_{k=1}^{4}\Delta_k(g_0^2)O_k(a)\right]$$

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Vanishes if chiral symmetry is preserved (at least partially)

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$$\bar{O}_{\mathrm{VA+AV}} = \lim_{a\to 0} Z_{\mathrm{VA+AV}}(g_0^2, a\mu)\, O_{\mathrm{VA+AV}}(a)$$

Protected from mixing by discrete symmetries

Getting rid of mixing

- Straightforward option: preserve chiral symmetry — possibly exactly.

- Wilson 1: axial Ward identity (3-point function with O_{VV+AA} → 4-point function with O_{VA+AV}).

  Becirevic et al. 2000

- Wilson 2: tmQCD (3-point function with O_{VA+AV}).

  ALPHA, Frezzotti, Grassi, Sint & Weisz, 2001

  ALPHA, Dimopoulos et al. 2004

  ALPHA, Dimopoulos et al. 2006

- tmQCD bonus: push safely towards low quark masses in quenched simulations.

Twistəd mass QCD

Break flavour symmetry in non-trivial direction in flavour space → preserve different subgroup.
No free lunch: break P,T, vector symmetries.

- Originally (re)proposed to avoid exceptional configurations in quenched computations.

  Frezzotti, Grassi, Sint, Weisz 2001

- Control of chiral symmetry breaking allows for simpler renormalisation properties of d=6 operators → "mimic" exact chiral symmetry.

  Frezzotti, Grassi, Sint, Weisz 2001

  CP, Sint, Vladikas 2004

  Frezzotti, Rossi, 2004

- Interest outburst after Frezzotti and Rossi's argument for automatic O(a) improvement.

  Frezzotti, Rossi 2004

Twistəd mass QCD

Basic setup: two mass-degenerate light flavours.

$$S_{\mathrm{F}}^{\mathrm{ph}} = a^4 \sum_{x,y} \bar{\psi}(x) \Big\{ \tfrac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*) + M_q + e^{-i\alpha\gamma_5\tau_3} \left(-\frac{ar}{2}\nabla_\mu^*\nabla_\mu + M_{\mathrm{cr}} \right) \Big\}(x,y)\psi(y)$$

$$\psi \to e^{-i\alpha\gamma_5\tau_3/2}\psi, \quad \bar{\psi} \to \bar{\psi}e^{-i\alpha\gamma_5\tau_3/2}$$

$$m_q = M_q \cos\alpha, \quad \mu_q = M_q \sin\alpha$$

$$S_{\mathrm{F}}^{\mathrm{tm}} = a^4 \sum_{x,y} \bar{\psi}(x) \Big\{ \tfrac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*) + m_q + i\mu_q\gamma_5\tau_3 + \left(-\frac{ar}{2}\nabla_\mu^*\nabla_\mu + M_{\mathrm{cr}} \right) \Big\}(x,y)\psi(y)$$

Twistəd mass QCD

Basic setup: two mass-degenerate light flavours.

Tune standard mass parameter + twist angle α.

$$S_{\rm F}^{\rm ph} = a^4 \sum_{x,y} \bar{\psi}(x) \Big\{ \tfrac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*) + M_q$$

$$+ e^{-i\alpha\gamma_5\tau_3} \left(-\frac{ar}{2} \nabla_\mu^* \nabla_\mu + M_{\rm cr} \right) \Big\}(x,y)\psi(y)$$

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$$S_{\rm F}^{\rm tm} = a^4 \sum_{x,y} \bar{\psi}(x) \Big\{ \tfrac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*) + m_q + i\mu_q\gamma_5\tau_3$$

$$+ \left(-\frac{ar}{2} \nabla_\mu^* \nabla_\mu + M_{\rm cr} \right) \Big\}(x,y)\psi(y)$$

(Renormalised) composite operators are rotated.

Tuning of masses

Untwisted quark: $m_R = Z_m \left[m_q (1 + b_m a m_q) \right]$

Twisted quark: $m_R = Z_m \left[m_q (1 + b_m a m_q) + \tilde{b}_m a \mu_q^2 \right]$

$\mu_R = Z_\mu \left[\mu_q (1 + b_\mu a m_q) \right]$

$$m_q = \frac{1}{2} \left(\frac{1}{\kappa} - \frac{1}{\kappa_c} \right)$$

Precise knowledge of κ_c, renormalisation constants, improvement coefficients required.

ALPHA Collaboration quenched computation of B_K

Guagnelli, Heitger, CP, Sint, Vladikas JHEP 03 (2006) 088

Palombi, CP, Sint JHEP 03 (2006) 089

Dimopoulos, Heitger, Palombi, CP, Sint, Vladikas NPB 749 (2006) 69

- tmQCD → no operator mixing, no exceptional configurations.
- SF non-perturbative renormalisation.
- Various physical volumes: check control of finite volume effects.
- Two different regularisations: check control of the continuum limit.
- N.B.: action is O(a) improved, but four-fermion operator is *not* ⇒ continuum limit approached linearly in a.
- Computations performed on the APEMille installation @ DESY-Zeuthen.

tmQCD regularisations for B_K

<u>$\pi/2$ strategy</u>:

$$S = \sum_{x,y} \{ \bar{\psi}_\ell(x) \left[D_{\rm w,sw} + m_\ell + i\mu_\ell \gamma_5 \tau_3 \right](x,y) \psi_\ell(y) \; + \; \bar{s}(x) \left[D_{\rm w,sw} + m_s \right](x,y) s(y) \}$$

$m_\ell,\ \mu_\ell$ tuned to have $m_{\ell,\rm R} = 0$

<u>$\pi/4$ strategy</u> (specially devised for quenched case):

$$S = \sum_{x,y} \{ \bar{\psi}(x) \left[D_{\rm w,sw} + m_{\hat{0}} + i\mu_{\hat{q}} \gamma_5 \tau_3 \right](x,y) \psi(y) \}$$

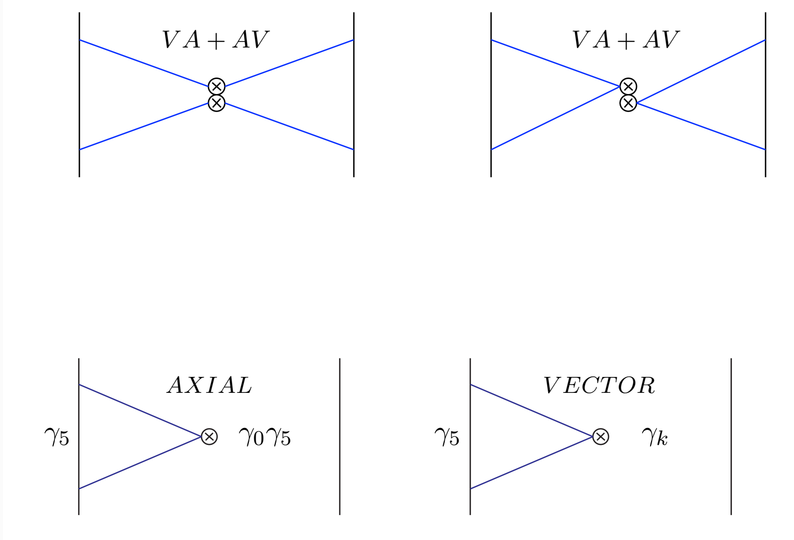
$m_{\hat{0}},\ \mu_q$ tuned to have $m_{\rm R} = \mu_{\rm R}$

in both cases: $O_{\rm VV+AA} \xrightarrow{\rm twist} O_{\rm VA+AV}$

NB: we never have *only* fully twisted quarks → Frezzotti-Rossi O(a) improvement argument does not apply.

Bare matrix element

Correlation functions computed with Dirichlet b.c. (SF framework)



B_K extracted from the ratio

$$R(x_0) = \frac{3}{16}\frac{-i f_{\mathrm{VA+AV}}(x_0)}{[Z_{\mathrm{A}}(f_{\mathrm{A}}(x_0) + c_{\mathrm{A}}\, a\,\partial_0 f_{\mathrm{P}}(x_0)) - iZ_{\mathrm{V}} f_{\mathrm{V}}(x_0)][Z_{\mathrm{A}}(f'_{\mathrm{A}}(x_0) + c_{\mathrm{A}}\, a\,\partial_0 f'_{\mathrm{P}}(x_0)) - iZ_{\mathrm{V}} f'_{\mathrm{V}}(x_0)]}$$

Systematics related to improvement coefficients / current normalisations → compare ALPHA vs LANL.

Quenched simulations

- $\pi/2$:

  $4 \times \beta,\ a \sim 0.06 - 0.09\,\text{fm},\ L \sim 1.4 - 1.9\,\text{fm},\ T/L \sim 2.3 - 3.0,\ m_{\text{PS}} \sim 640 - 830\,\text{MeV}$

- $\pi/4$:

  $5 \times \beta,\ a \sim 0.05 - 0.09\,\text{fm},\ L \sim 1.9 - 2.2\,\text{fm},\ T/L \sim 2.0 - 2.6,\ m_{\text{PS}} \sim 460 - 540\,\text{MeV}$

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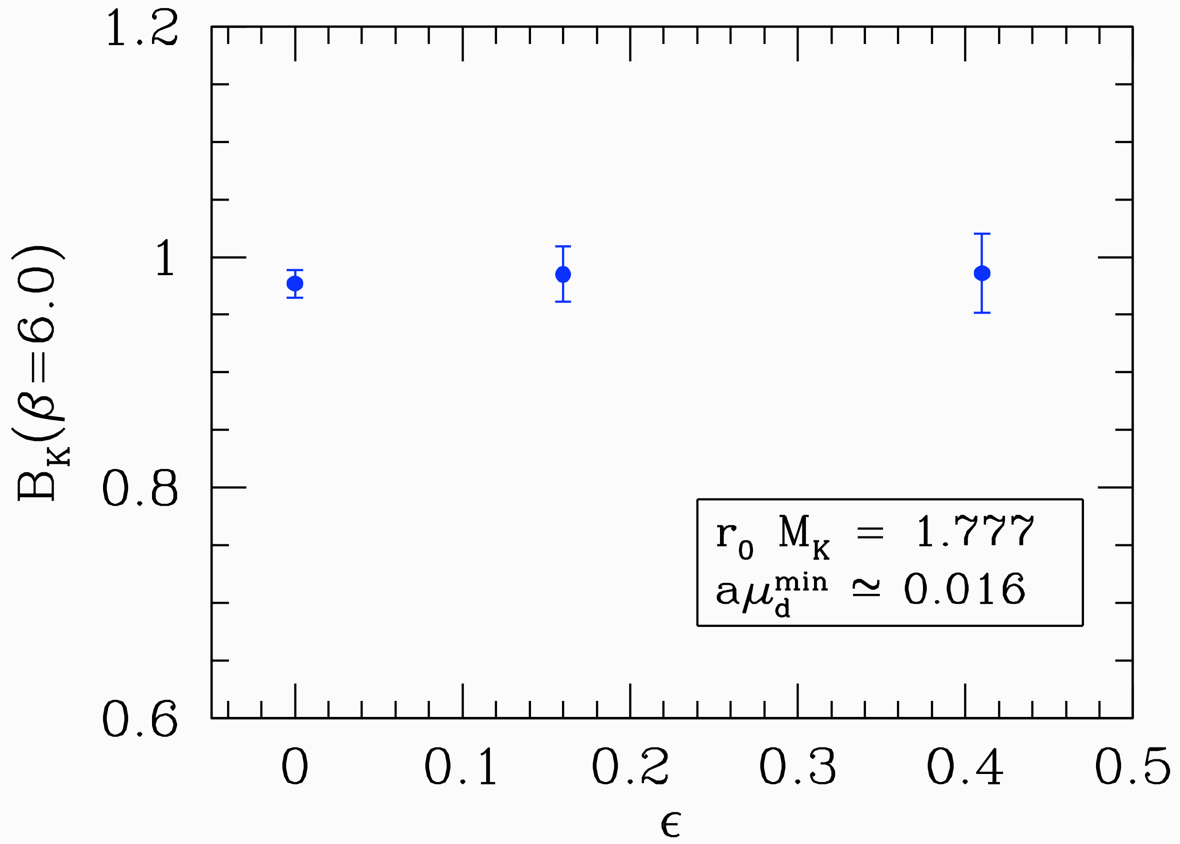
- $m_s = m_d$ limit. Physical SU(3) breaking effects checked to be small up to moderate values for the strange-down splitting.

Physical SU(3) breaking effects



$$\epsilon = \frac{M_s - M_d}{M_s + M_d}$$

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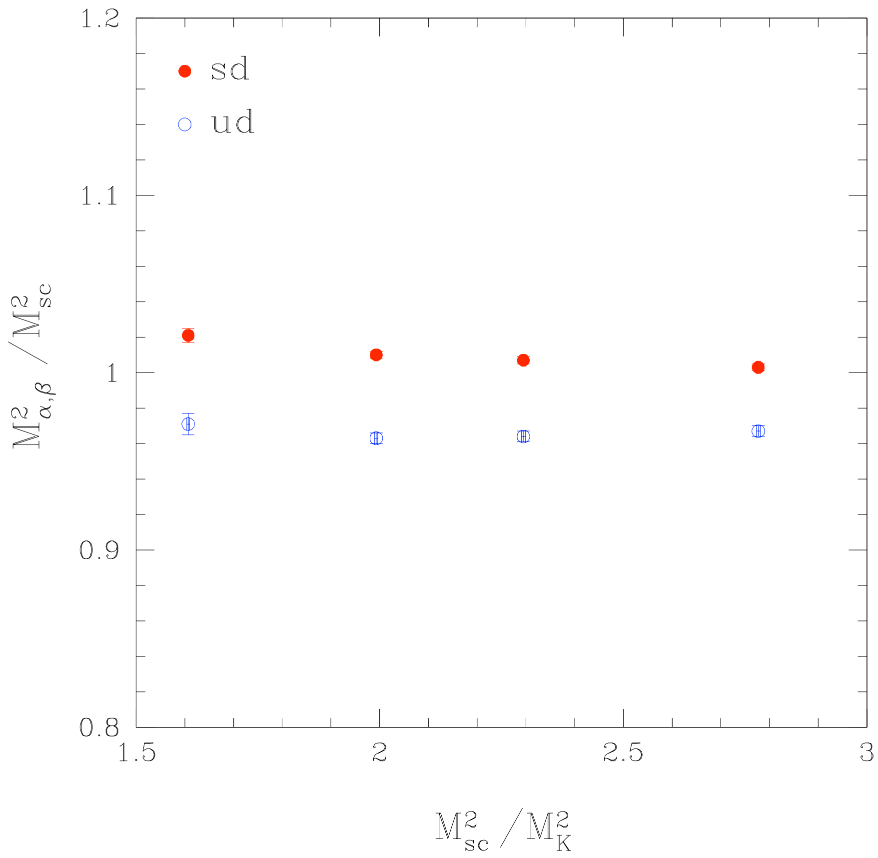
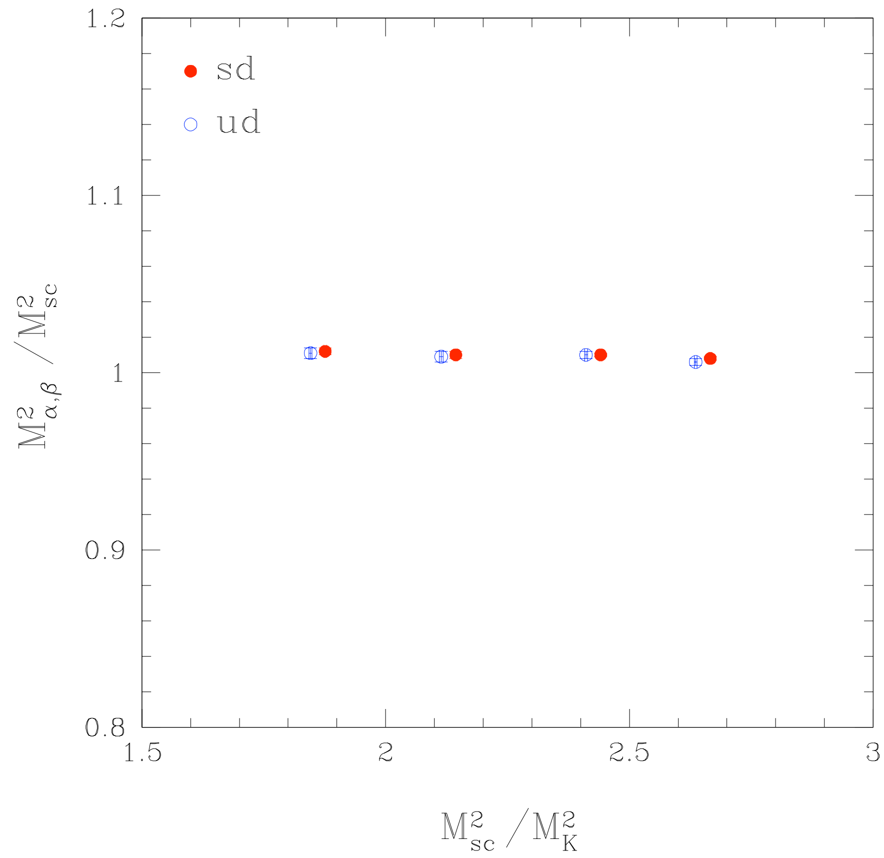
Twisted mass-induced vector symmetry breaking



$\beta = 6.0$



$\beta = 6.3$

Quenched simulations

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- Approach to continuum limit remains delicate.

Approach to continuum: non-perturbative renormalisation

ALPHA, JHEP 03 (2006) 088 & 089

- SF technique via finite size scaling: split renormalisation into
  - Renormalisation at a low, hadronic scale where contact with typical large-volume values of β is made.
  - NP running to very high scales (~100 GeV) where contact with PT is made.

$$\hat{B}_K = (\alpha_s(\mu))^{-\gamma_0/2b_0} \exp\left\{ -\int_0^{\bar{g}(\mu)} \mathrm{d}g \left[\frac{\gamma(g)}{\beta(g)} - \frac{\gamma_0}{b_0 g} \right] \right\} \left[\lim_{a\to 0} Z(g_0^2, a\mu)\, B_K(a) \right]$$

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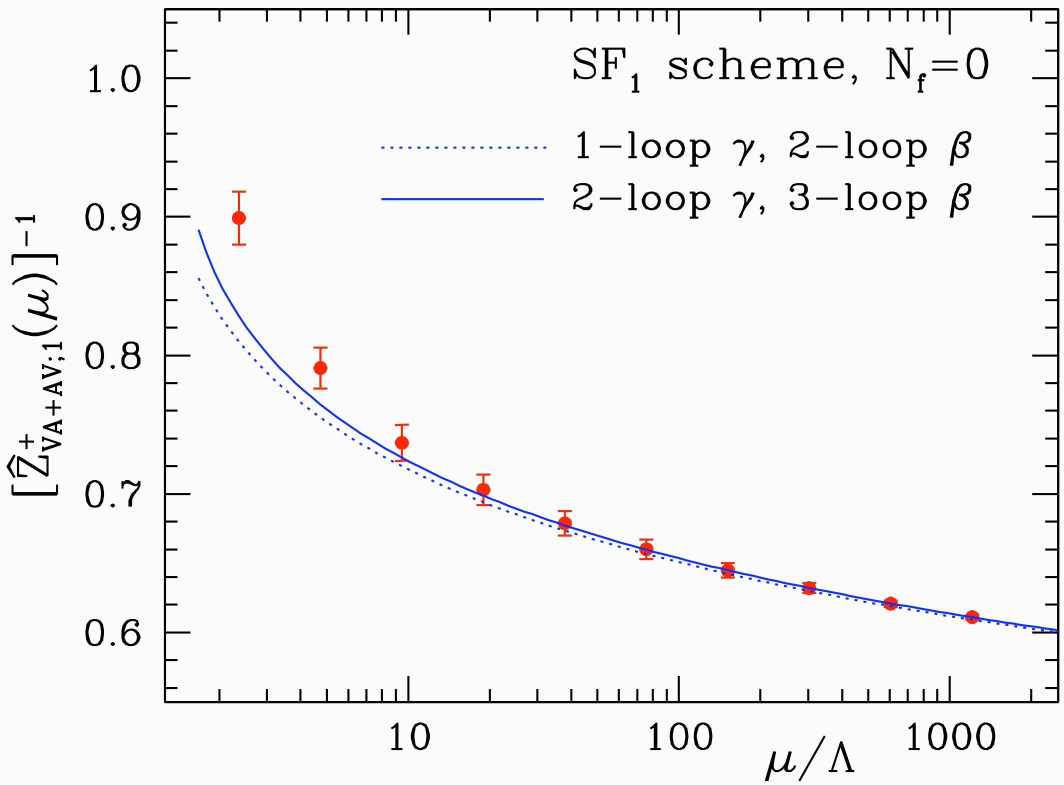
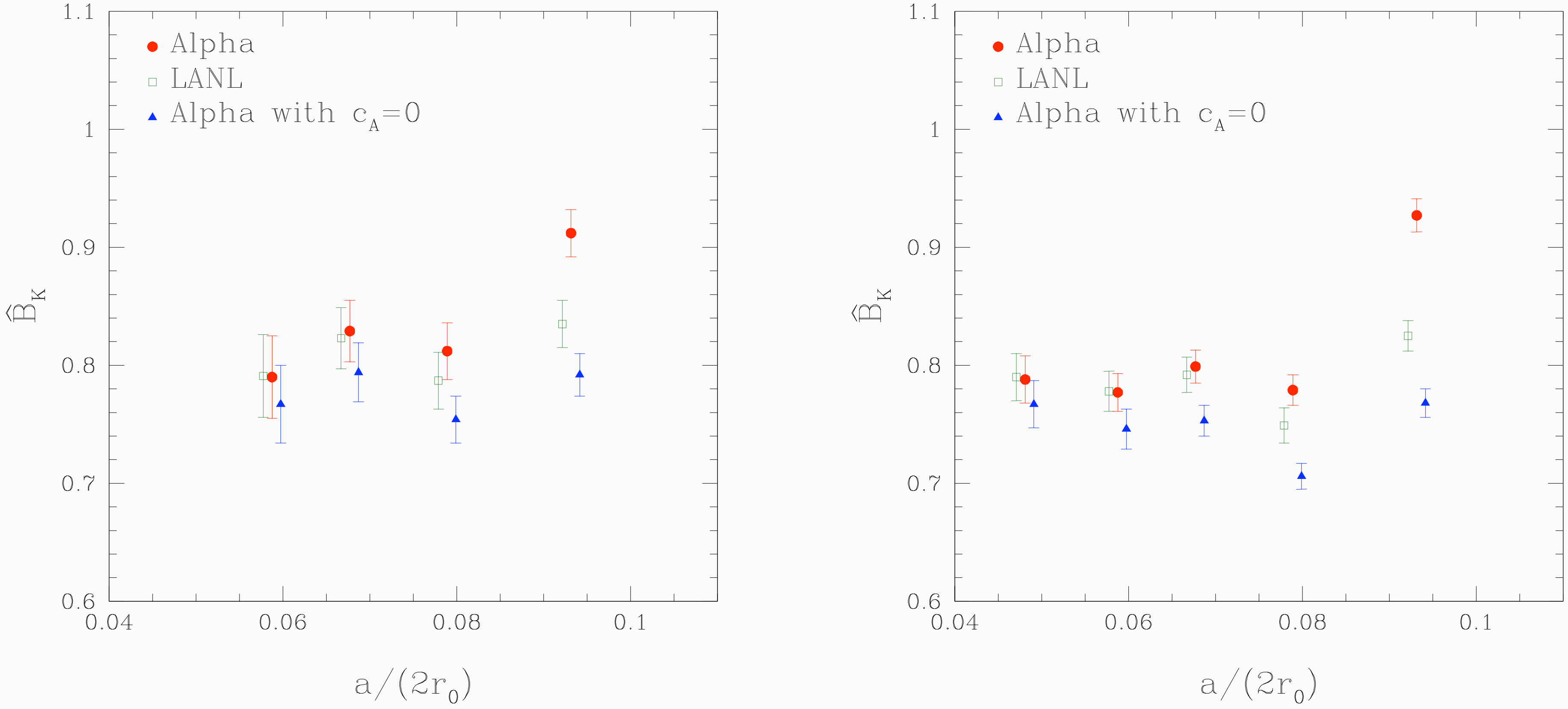


Approach to continuum: cutoff effects

Impact of O(a) ambiguities in current improvement via c_A.



α=π/2 α=π/4

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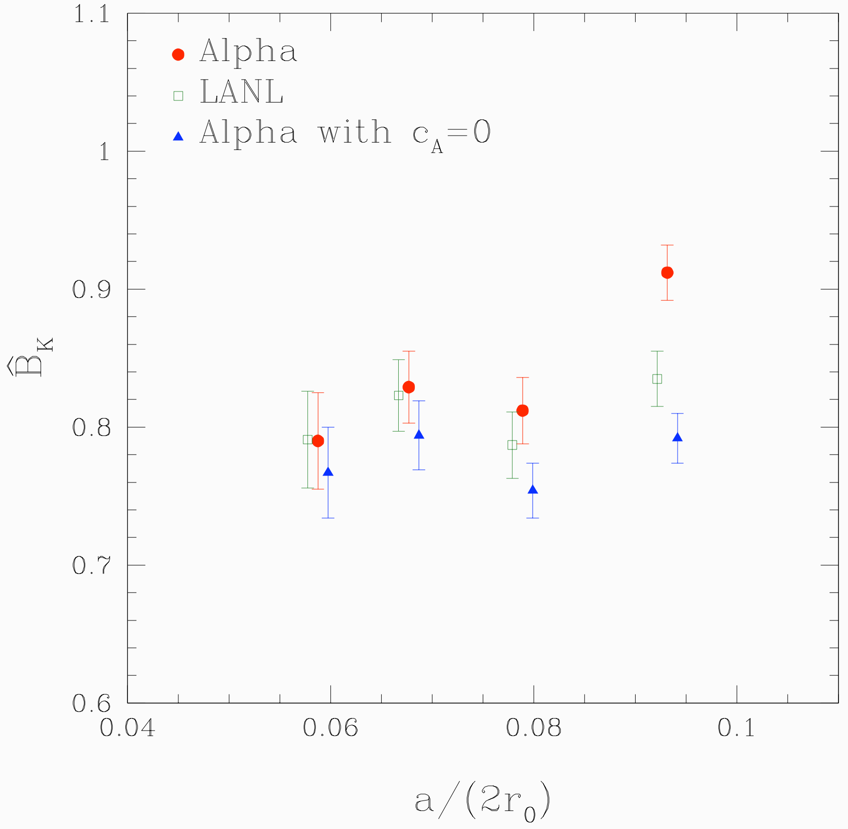

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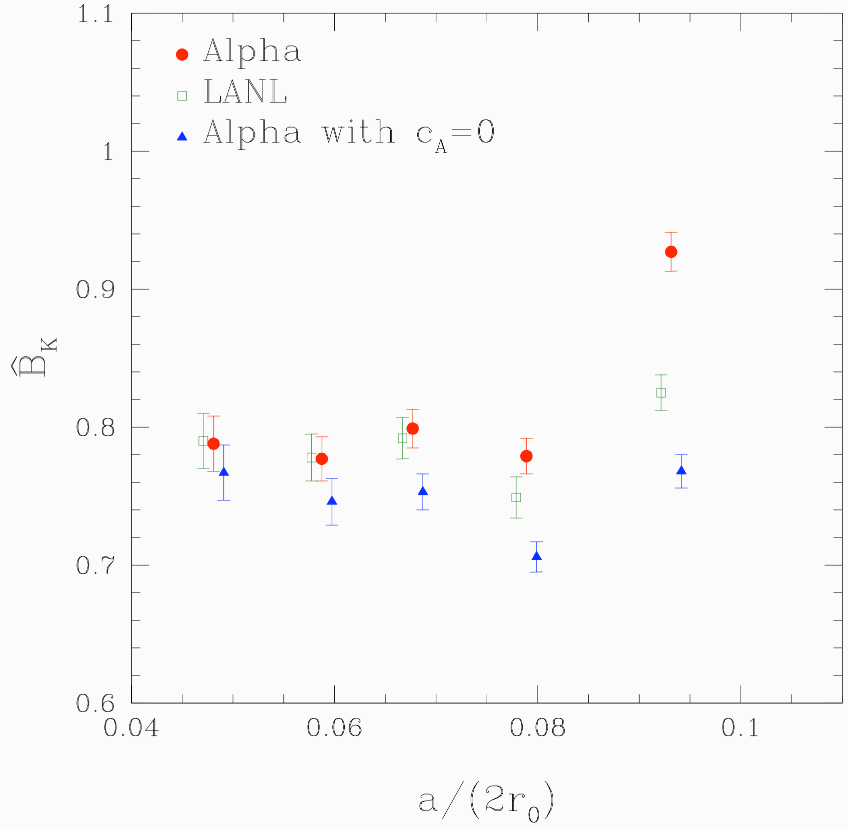

α=π/4

Problem: κ_c value at $\beta = 6.1$ found in literature turned out to be wrong.

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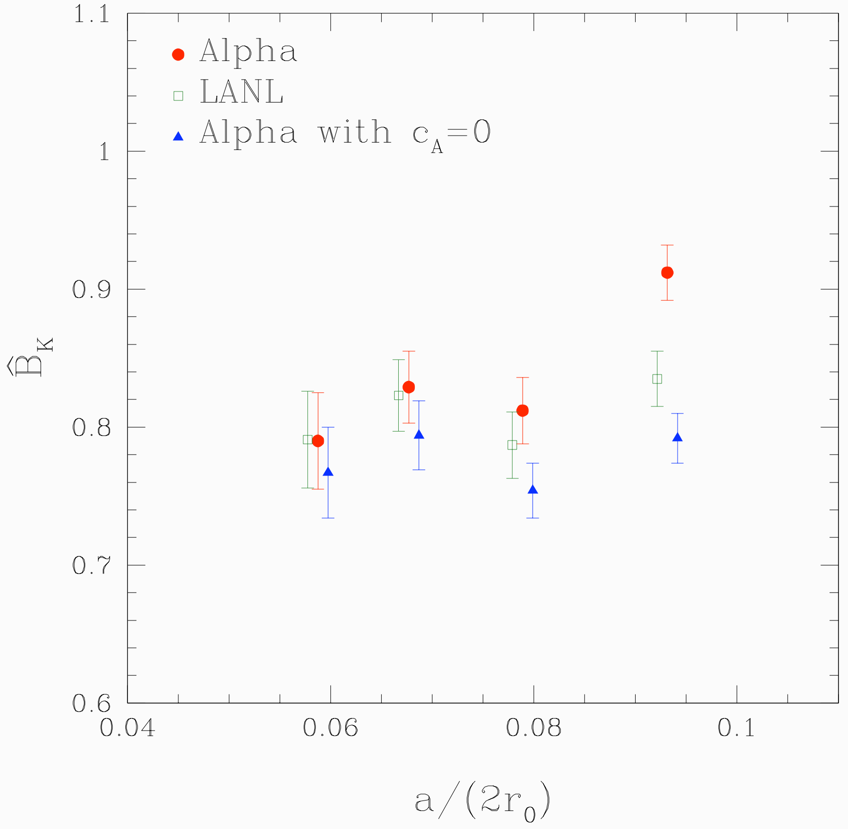

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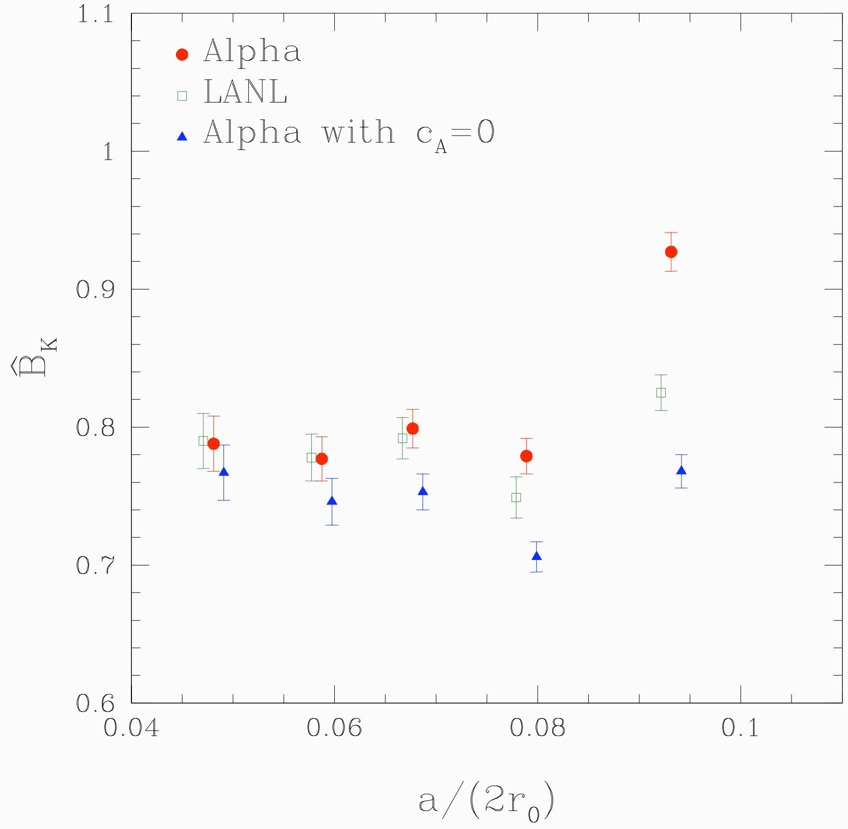


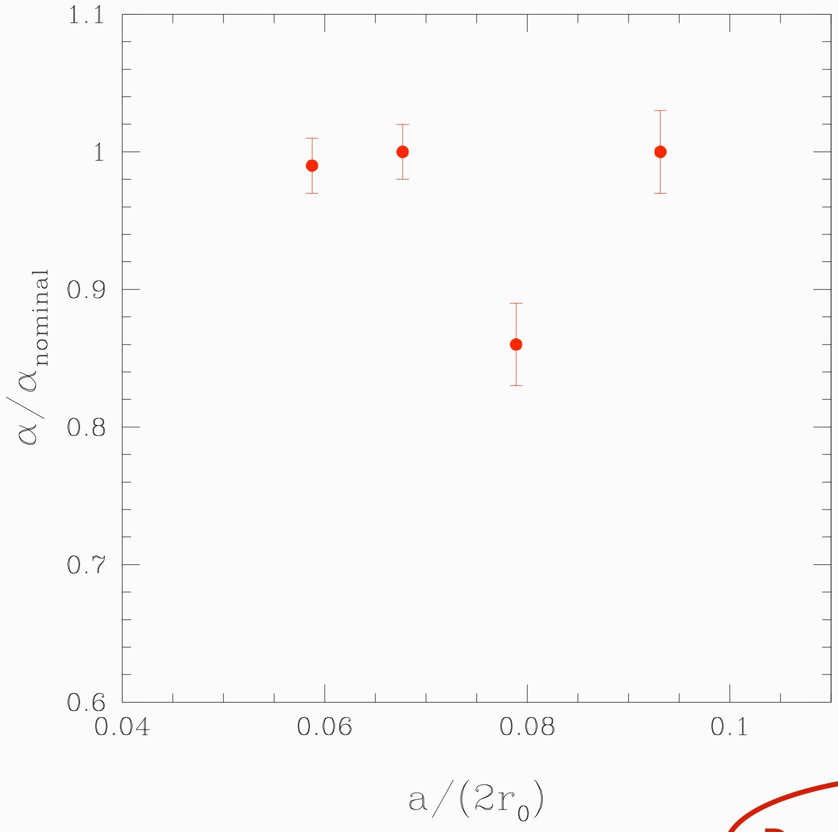
α=π/4

$\kappa_c(\beta = 6.1) = 0.135496$ Rolf, Sint 2002, interpolation from other βs

$\kappa_c(\beta = 6.1) = 0.135665(11)$ direct computation

Impact of κ_c on twist angles

Not final



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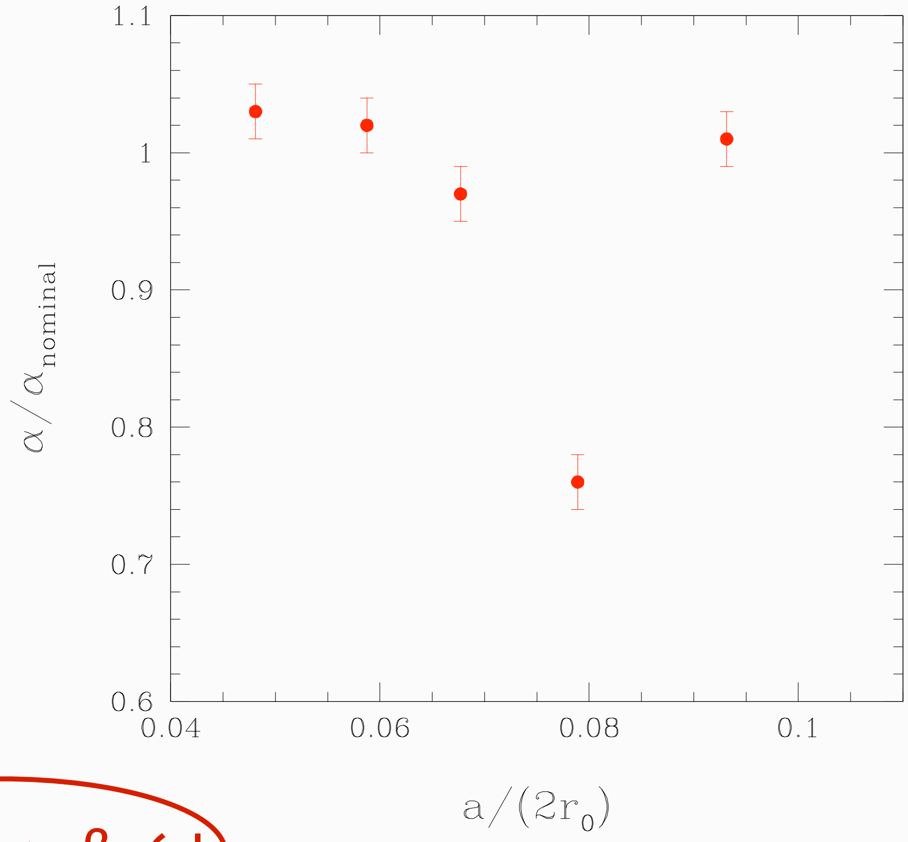

α=π/4

Recompute β=6.1

$\kappa_c(\beta = 6.1) = 0.135496$	Rolf, Sint 2002, interpolation from other βs
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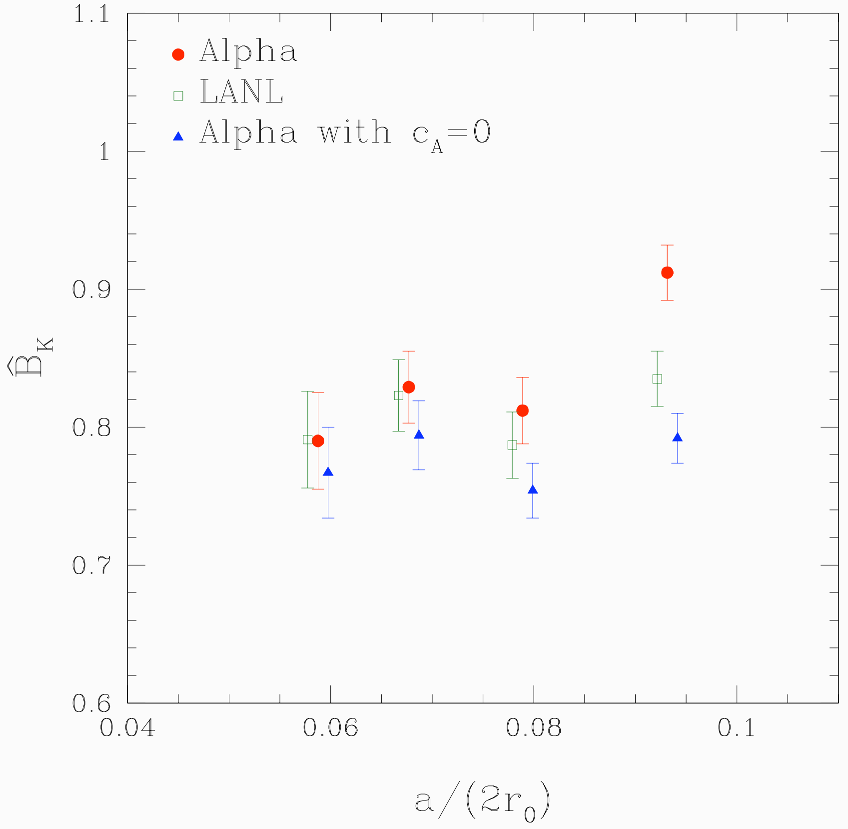

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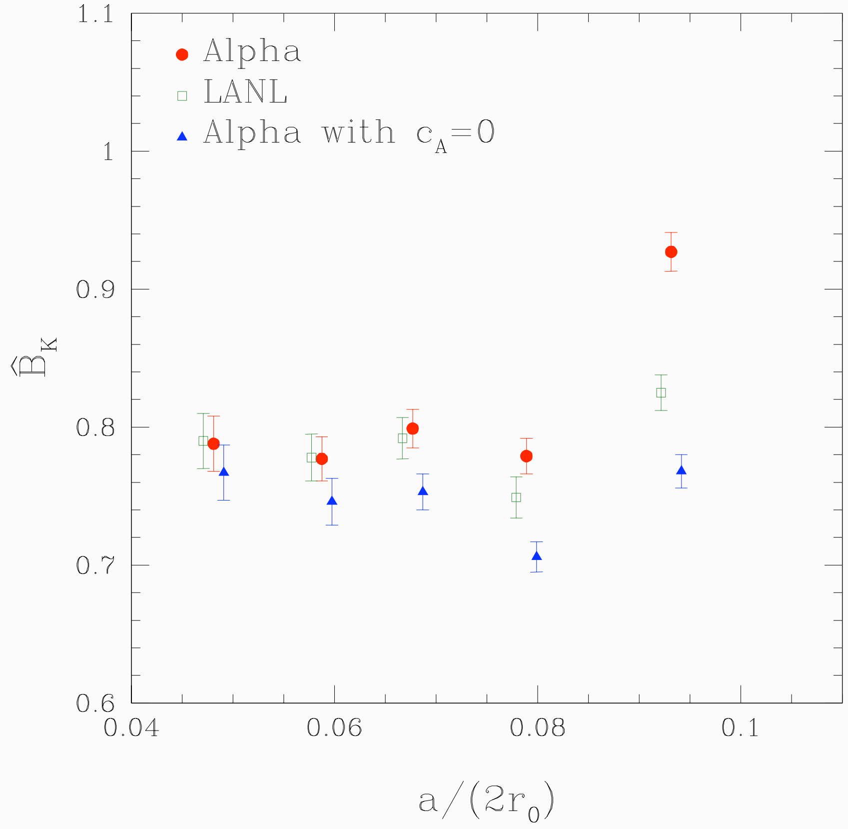

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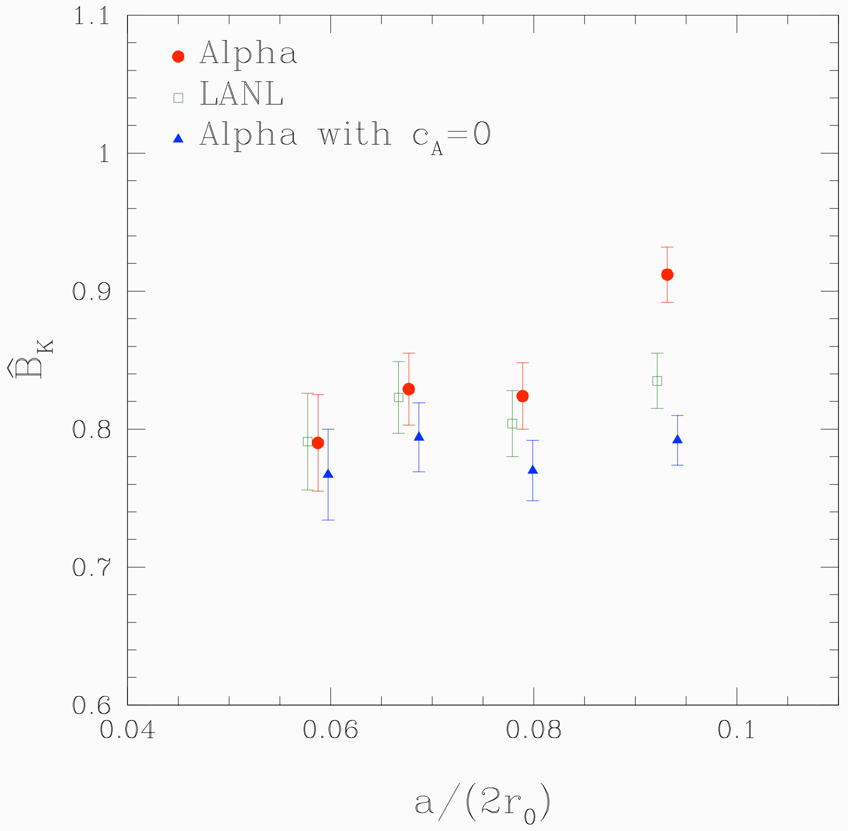

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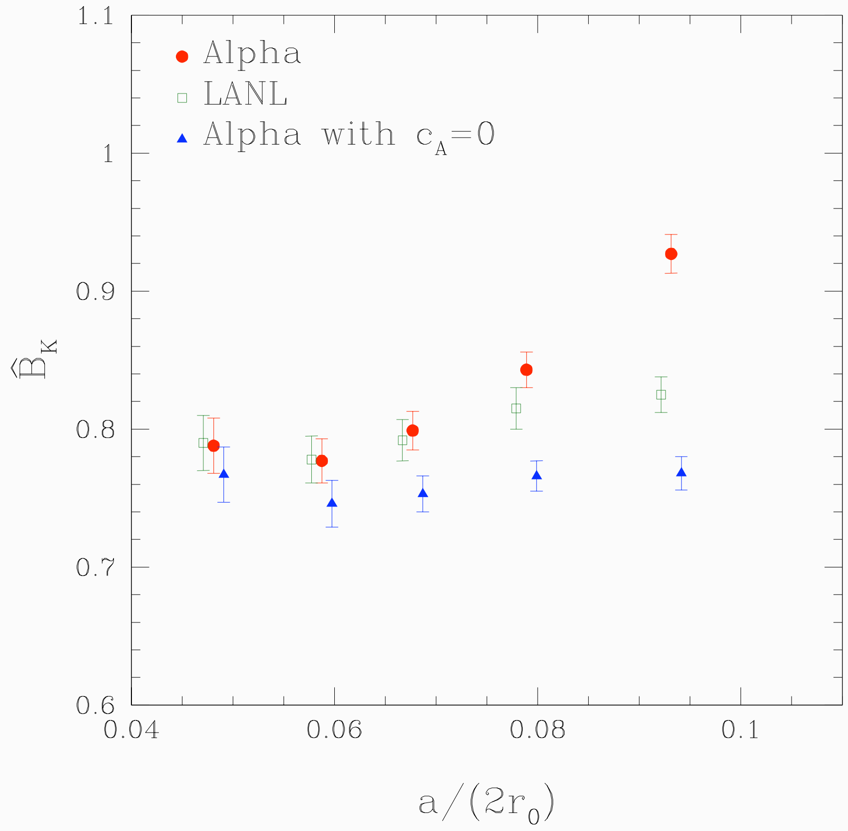

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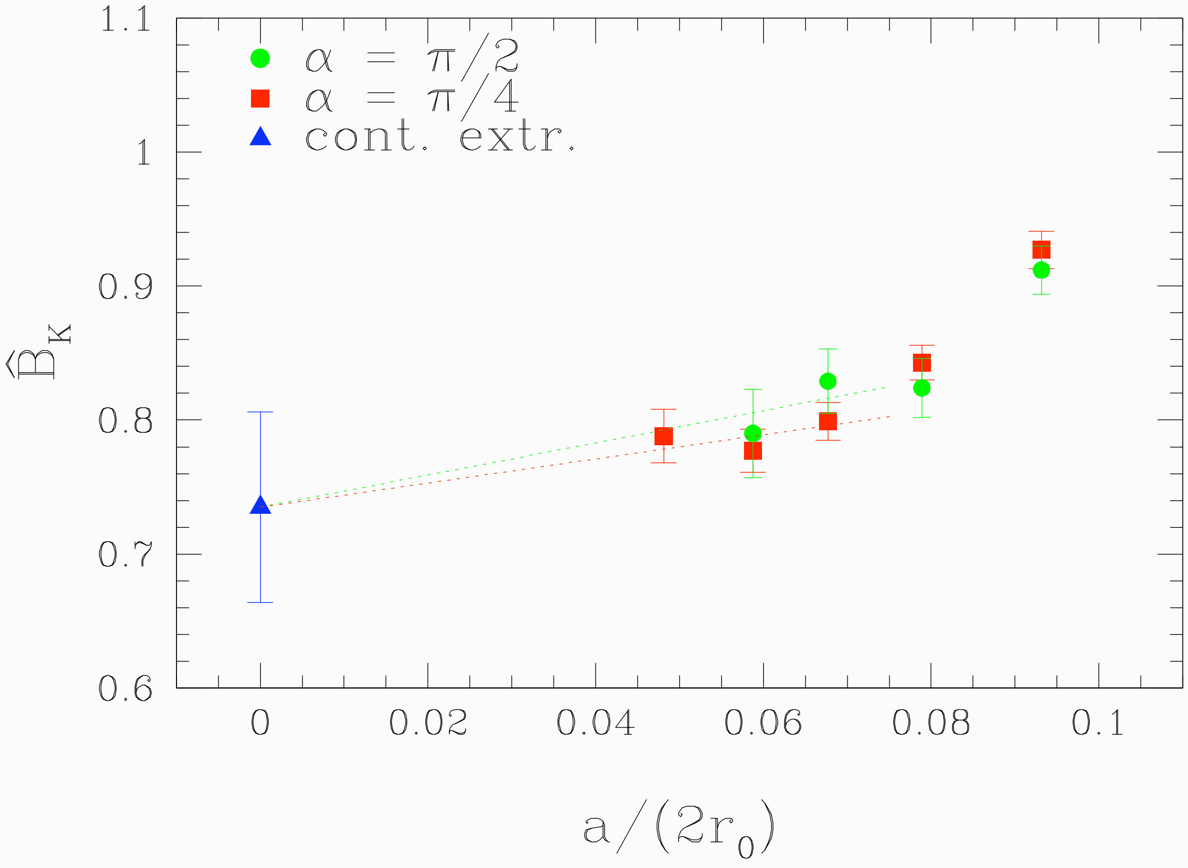
Continuum limit

- Combined linear extrapolation of the two regularisations, using ALPHA determination of current normalisations and improvement coefficients.
- Criteria:
  - Discard points on which the impact of $O(a^2)$ ambiguities from currents is well beyond the 1 sigma level.
  - Discard points for which (impossible to fit) curvature in a dependence is manifest.

Continuum limit

- Combined linear extrapolation of the two regularisations, using ALPHA determination of current normalisations and improvement coefficients.



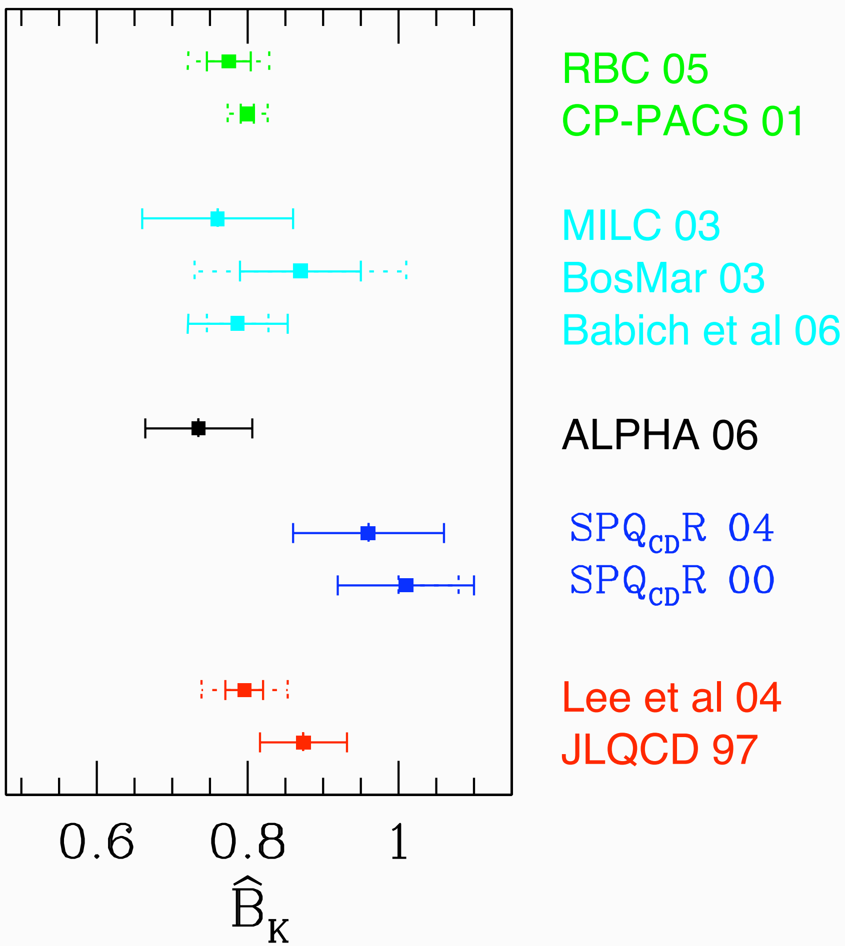
$$\hat{B}_K = 0.735(71)$$

$$\bar{B}_K^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.534(52)$$

Dimopoulos et al., in preparation

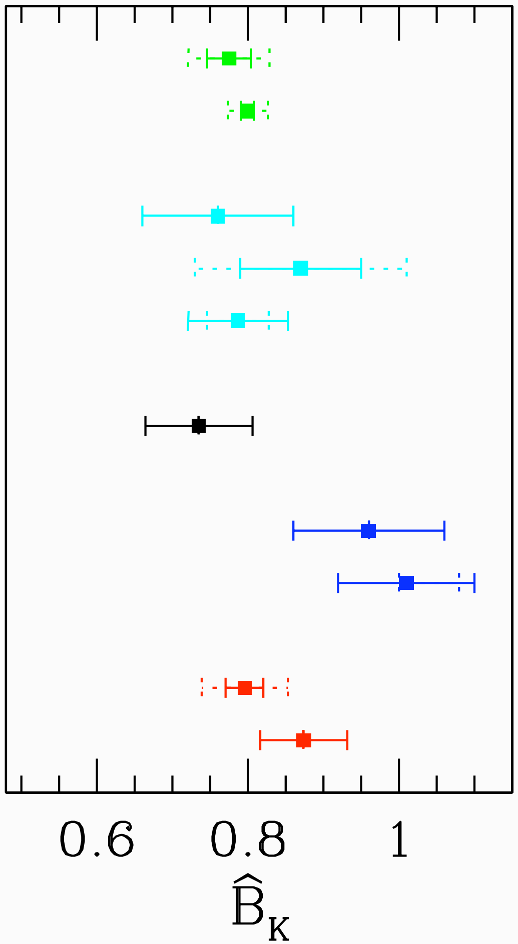
Cf. $\hat{B}_K = 0.789(46)$ quoted in NPB 749 (2006) 69.

Comparison with quenched literature



Difference with other Wilson fermion computations mainly due to method employed to extract B_K.

Comparison with quenched literature



	no mass extrap	NP renormalisation	NP RG running	test FV effects	UV cutoff dep
RBC 05	green	green	red	red	orange
CP-PACS 01	green	red	red	green	orange
MILC 03	green	red	red	red	orange
BosMar 03	green	green	red	red	red
Babich et al 06	green	green	red	red	orange
ALPHA 06	orange	green	green	green	green
$SPQ_{CD}R$ 04	red	green	red	red	green
$SPQ_{CD}R$ 00	red	green	red	red	green
Lee et al 04	green	red	red	red	red
JLQCD 97	green	red	red	green	green

Outline

- B_K
  - Motivation from UT analysis.
  - B_K with Wilson fermions.
  - tmQCD
  - The ALPHA Collaboration computation.
- (Ongoing work on) B_B
  - Strategy.
  - Status.
- Some remarks on tmQCD for $K \to \pi\pi$ decays.

ΔB=2 transitions and B mass differences

$$\Delta M_d = 0.507 \pm 0.005 \text{ ps}^{-1} \qquad \Delta M_s = 17.33^{+0.42}_{-0.41} \pm 0.07 \text{ ps}^{-1}$$

CDF measurement 2006

$$\Delta M_d = 0.50 \text{ ps}^{-1} \times \left[\frac{\sqrt{\hat{B}_{B_d}}\, F_{B_d}}{230 \text{ MeV}}\right] \left[\frac{\bar{m}_t(m_t)}{167 \text{ GeV}}\right] \left[\frac{|V_{td}|}{0.0078}\right] \left[\frac{\eta_B}{0.55}\right]$$

$$\Delta M_s = 17.2 \text{ ps}^{-1} \times \left[\frac{\sqrt{\hat{B}_{B_s}}\, F_{B_s}}{260 \text{ MeV}}\right] \left[\frac{\bar{m}_t(m_t)}{167 \text{ GeV}}\right] \left[\frac{|V_{ts}|}{0.040}\right] \left[\frac{\eta_B}{0.55}\right]$$

$$\hat{B}_{B_\ell} = \frac{\langle \bar{B}_\ell | \hat{O}^{\Delta B=2} | B_\ell \rangle}{\frac{8}{3} F^2_{B_\ell} m^2_{B_\ell}}$$

Control of systematics in B-parameters even more needed than for B_K.

ΔB=2 transitions and B mass differences

$$\Delta M_d = 0.507 \pm 0.005\ \text{ps}^{-1} \qquad \Delta M_s = 17.33^{+0.42}_{-0.41} \pm 0.07\ \text{ps}^{-1}$$

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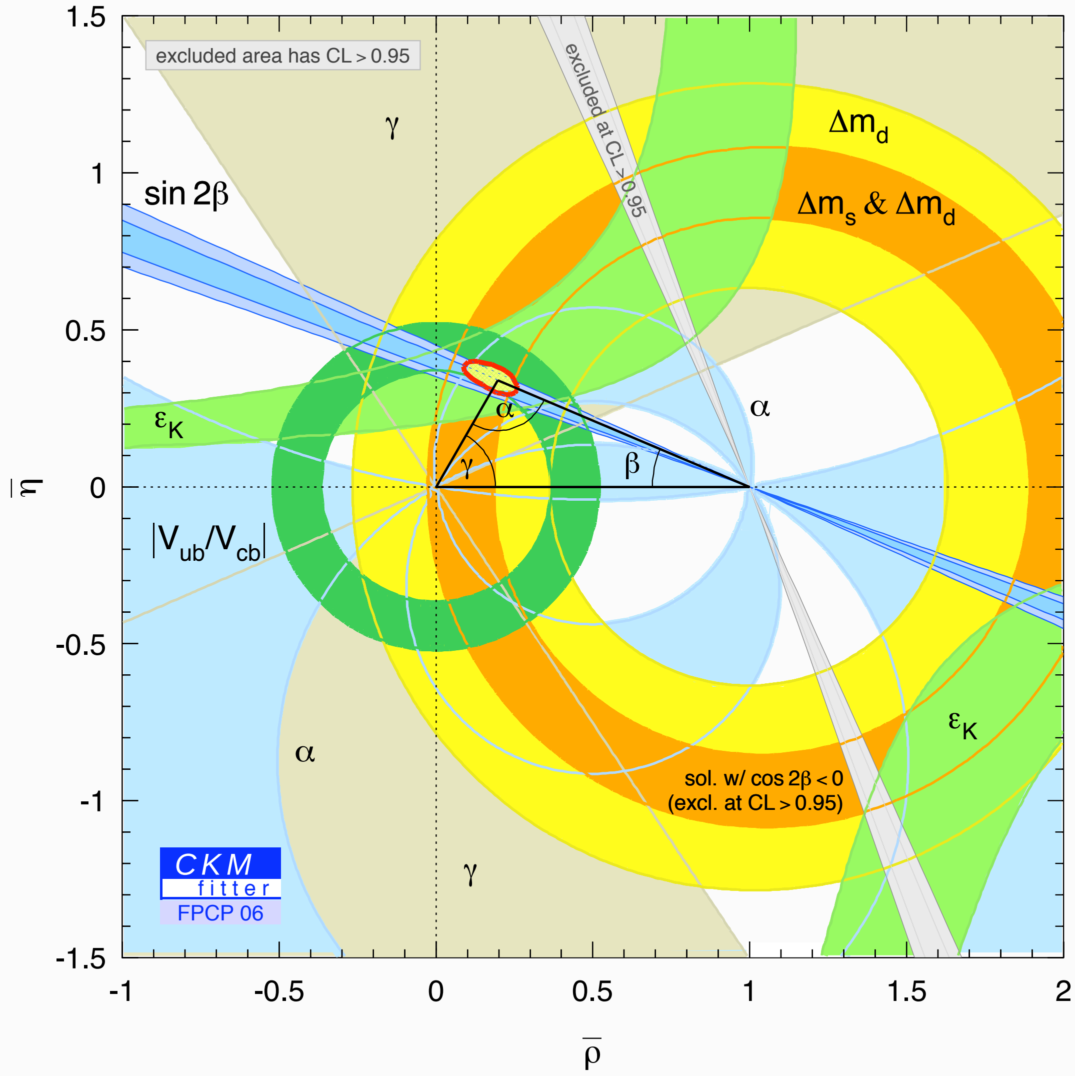


Strategy

Palombi, Papinutto, CP, Wittig JHEP in press

- Treat *b*-quark in HQET.

- Renormalisation of static-light four-fermion operators → wrong-chirality mixing absent in CP-odd sector.

- Extend B_K tmQCD strategy: static heavy quark + fully twisted light quark.
  - Multiplicatively renormalisable operators only.
  - No restrictions on light quark masses (quenched).
  - Automatic O(*a*) improvement.
  - Use static actions with good noise-to-signal ratio.

See also Della Morte 2004

ALPHA 2003-2005

Status: NP renormalisation of the full operator basis in progress, no preliminary results for matrix elements yet.

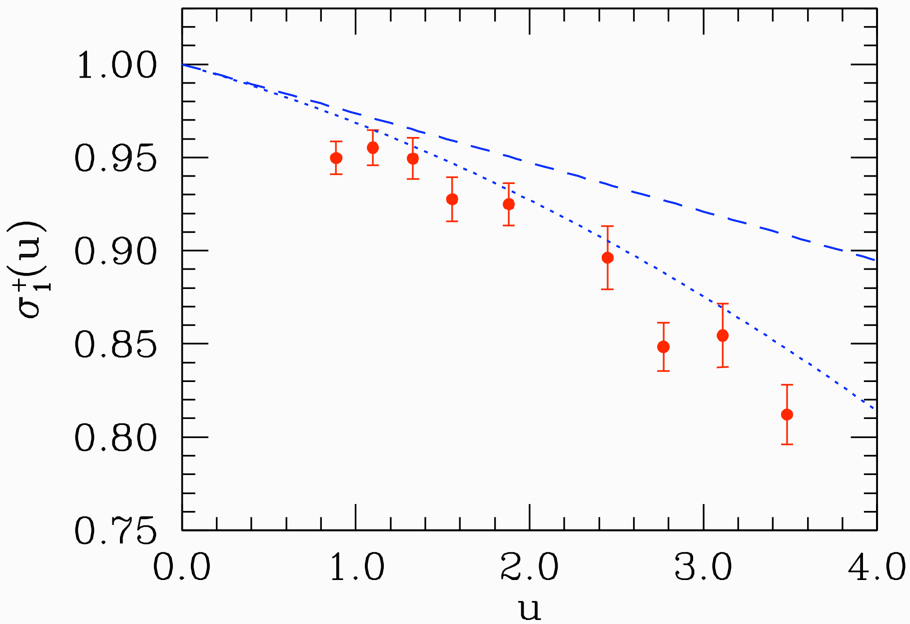
Preliminary results for RG running of operators

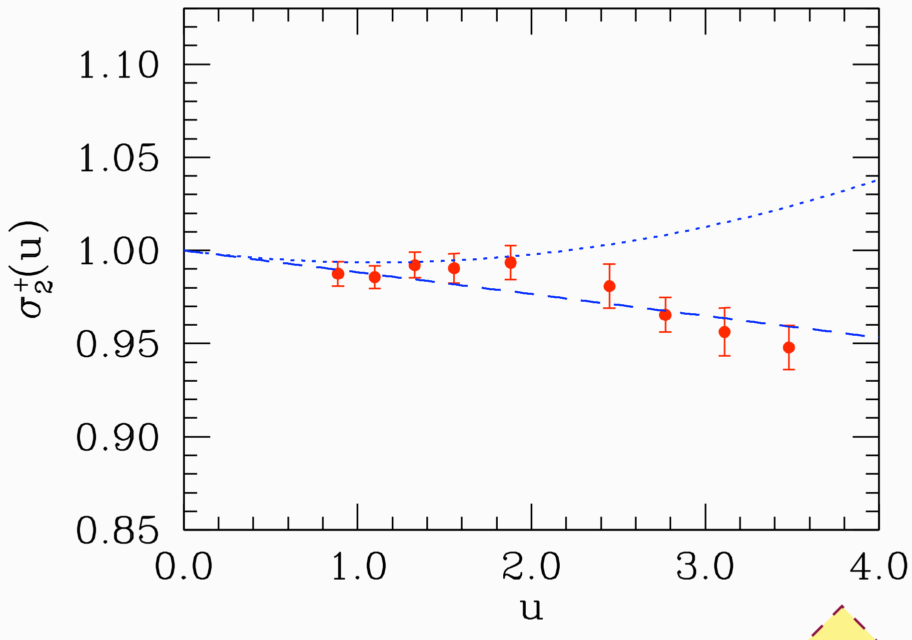
$$\bar{O}_{LL}^{\Delta B=2}(\mu) = C_{\mathrm{L}}(m_b,\mu)\bar{O}_{LL}^{\Delta B=2;\ \mathrm{HQET}}(\mu) + C_{\mathrm{S}}(m_b,\mu)\bar{O}_{LL,S}^{\Delta B=2;\ \mathrm{HQET}}(\mu) + \mathrm{O}(1/m_b)$$

$$\sigma_O(u) = \exp\left\{-\int_{\bar{g}(\mu/2)}^{\bar{g}(\mu)} \mathrm{d}g \left[\frac{\gamma_O(g)}{\beta(g)} - \frac{\gamma_O^{(0)}}{b_0 g}\right]\right\}\Bigg|_{\bar{g}^2(\mu)=u}$$

Preliminary results for RG running of operators

$$\bar{O}_{LL}^{\Delta B=2}(\mu) = C_{\rm L}(m_b,\mu)\bar{O}_{LL}^{\Delta B=2;\,\rm HQET}(\mu) + C_{\rm S}(m_b,\mu)\bar{O}_{LL,S}^{\Delta B=2;\,\rm HQET}(\mu) + {\rm O}(1/m_b)$$





$$\sigma_O(u) = \exp\left\{ -\int_{\bar{g}(\mu/2)}^{\bar{g}(\mu)} {\rm d}g \left[\frac{\gamma_O(g)}{\beta(g)} - \frac{\gamma_O^{(0)}}{b_0 g} \right] \right\}\Bigg|_{\bar{g}^2(\mu)=u}$$

Preliminary

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Wilson fermions for $K \to \pi\pi$?

- Ginsparg-Wilson fermions put at work, beautiful performance.

See talk by P. Hernández

- Wilson fermions hindered by presence of power-divergent mixing + unphysical ZMs (quenched).

- Can tmQCD help? Two different proposals:

  - Simple twist, use freedom to impose physical symmetries.

    CP, Sint, Vladikas 2004

  - Ad-hoc valence sector, exploit maximally symmetries of the tmQCD action.

    Frezzotti, Rossi 2004

Wilson fermions for $K \to \pi\pi$?

- CP, Sint, Vladikas JHEP 09 (2004) 069
  - Four-flavour theory, maximal twist on two/four flavours, enforce parity up to O(a) in correlation functions → mixing at most linearly divergent.
  - Use O(a) improved action, consistent O(a) improvement of bilinears → only finite subtractions.
- Frezzotti, Rossi JHEP 10 (2004) 070
  - Tailored valence sector on twisted sea, adjust valence twists to symmetry-kill all divergences.
  - Cancellations rely crucially on proper tuning of mass parameters.

Wilson fermions for $K \to \pi\pi$?

- Take-home message:

  - Feasibility of tmQCD approach to $K \to \pi\pi$ crucially depends on good control over tuning of mass parameters and breaking of global symmetries.

  - If approach works, it would supplement nicely the (numerically very demanding) GW results.

Conclusions and outlook

- tmQCD + state-of-the-art techniques for Wilson fermions provide benchmark quenched results for WMEs — ALPHA B_K.

- Ideas can be extended to other problems involving d=6 operators.
  - B_B in progress, result expected for next year.
  - $K \rightarrow \pi\pi$?

- Systematics currently dominated by CL extrapolation: full O(a) improvement essential in future.
  - Frezzotti-Rossi approach?
  - Can tmQCD compete with / complement exact chiral symmetry?